

UNITARY INTERACTIONS DO NOT YIELD OUTCOMES: ATTEMPTING TO MODEL “WIGNER’S FRIEND”

R. E. KASTNER

ABSTRACT. An experiment by Proietti *et al* purporting to instantiate the ‘Wigner’s Friend’ thought experiment is discussed. It is pointed out that the stated implications of the experiment regarding the alleged irreconcilability of facts attributed to different observers warrant critical review. In particular, violation of a Clauser-Horne-Shimony inequality by the experimental data actually shows that the attribution of measurement outcomes to the “Friends” (modeled by internal photons undergoing unitary interactions) violates quantum theory. An elementary but often overlooked result regarding improper mixtures is adduced in support of this assessment. It is further noted that under an assumption of unbroken unitarity, no measurement correlation can ever yield an outcome, since all systems remain in improper mixtures, and attributing a definite but unknown outcome contradicts their composite pure state. It is pointed out that there already exists a solution to this conundrum in the form of an alternative formulation of quantum theory, which accounts for the data showing that no outcomes occurred at the interior entangled photon level and also predicts that outcomes can and do occur at the exterior “super-observer” level in this type of experiment.

1. BACKGROUND.

This paper presents a critique of a recent attempt by Proietti *et al* (2019) to experimentally model the Wigner’s Friend experiment, a famous elaboration by Wigner of the Schrödinger’s Cat experiment. Both are thought experiments that amplify the measurement problem (MP) of standard quantum theory by showing that it leads to absurdities when entanglements are presumed to continue to the macroscopic level of everyday experience. This author has argued elsewhere (Kastner, 2020b) that these thought experiments are properly understood not as experiments proposed to be performed in the lab (since what they predict, such as cats or people in superposition, is already known to be falsified by experience), but rather as *reductio ad absurdum* arguments making explicit the arbitrariness of the so-called ‘shifty split’ that is inevitable when quantum theory is assumed to have only unitary (linear) dynamics. In what follows, I will refer to this standard approach to quantum theory as ‘UOQM’ for ‘Unitary-Only Quantum Mechanics.’ UOQM consists of any approach that assumes that quantum theory has only unitary physical processes except perhaps upon ‘observation,’ where that term is ambiguous. In addition, UOQM assumes that any non-unitary ‘collapse’ or ‘reduction’ upon ‘observation’ has no quantitative

Date: April 20, 2021; University of Maryland, College Park. rkastner@umd.edu.

physical counterpart in the theory, making it an *ad hoc* postulate. This is the approach assumed in Poietti *et al.*

Before dealing with the specifics of the experiment (which is quite an impressive technical accomplishment despite its misinterpretation), it is important to review an elementary but often-overlooked fact regarding the fraught notion of “measurement” in the context of UOQM. This fact is that, as a purely logical matter, there can be no well-defined measurement outcomes under UOQM. This was originally pointed out by Feyerabend (1962) and instructively reviewed by Hughes (1989), §5.8. We now review the basic argument.

Under UOQM, a good (sharp) measurement of some observable A on a system S is modeled as an interaction between a measuring apparatus or “pointer” P, initially in a ready state $|\phi_0\rangle$, that couples it to S such that states $|\phi_i\rangle$ of P point to the different eigenstates $|a_i\rangle$ of A, as follows:

$$|\phi_0\rangle \otimes |\psi\rangle \rightarrow c_1|\phi_1\rangle|a_1\rangle + c_2|\phi_2\rangle|a_2\rangle + c_3|\phi_3\rangle|a_3\rangle + \dots c_N|\phi_N\rangle|a_N\rangle \quad (1)$$

where the c_i are the amplitudes associated with this decomposition of the system’s prepared state $|\psi\rangle$. Thus, the pointer P enters into an entanglement with S such that states of P “copy” the eigenstates of $|a_i\rangle$ pertaining to the system, with the appropriate amplitudes. However, according to standard UOQM, nothing beyond this unitary process ever physically happens—at least nothing that can be modeled in any quantitative way. Either (i) a “projection postulate” (PP) is invoked based on an undefined notion of “observer,” or (ii) the entanglement is simply assumed to continue. Under (i), one cannot define the physical circumstances constituting an “observer” or “observation”; this is the “Heisenberg Cut” or “shifty split” in which certain systems are taken as described by quantum theory while others are not, for no principled reason. Under (ii), one cannot say that any outcome actually occurred, at least not one in which the system can be taken as represented by the corresponding eigenstate, even if we don’t know which one it is.

The reason is as follows. For simplicity, let us suppose that the observable A has only two eigenstates (like the observables in the Proietti *et al* experiment), i.e., the composite state $|\Psi\rangle$ is:

$$|\Psi\rangle = c_1|\phi_1, a_1\rangle + c_2|\phi_2, a_2\rangle \quad (2)$$

where we use abbreviated notation for the direct product.

The state of the system S, as a component of the entangled pure state (2), is given by the reduced density matrix ρ_S . This is obtained by tracing the composite density matrix ρ over the pointer basis. In this case, the composite pure state density matrix ρ is:

$$\rho = |c_1|^2|\phi_1, a_1\rangle\langle\phi_1, a_1| + c_2^*c_1|\phi_1, a_1\rangle\langle\phi_2, a_2| + c_1^*c_2|\phi_2, a_2\rangle\langle\phi_1, a_1| + |c_2|^2|\phi_2, a_2\rangle\langle\phi_2, a_2| \quad (3)$$

and the reduced density matrix for the system is

$$\rho_S = |c_1|^2 |a_1\rangle\langle a_1| + |c_2|^2 |a_2\rangle\langle a_2| \quad (4)$$

This looks like a mixed state in which outcomes occur with the relevant Born probabilities. However, it is an improper mixed state, meaning that it cannot represent a situation in which P and S really possess an outcome represented by the eigenstates $|\phi_k\rangle$ and $|a_k\rangle$ while the probabilities just represent our ignorance about which outcome has occurred. For were that the case, P and S would be in the mixed state

$$\rho = |c_1|^2 |\phi_1, a_1\rangle\langle \phi_1, a_1| + |c_2|^2 |\phi_2, a_2\rangle\langle \phi_2, a_2| \quad (5)$$

That is, the off-diagonal entries, reflecting interference obtaining in the pure state (3), are missing. Since the assumption that the system and pointer are really in well-defined eigenstates corresponding to an outcome contradicts the actual composite state, that assumption is false: a unitary interaction yielding a correlation between degrees of freedom, in which one “copies” the other, does not constitute the occurrence of a well-defined outcome. With this point in mind, we now turn to the specifics of the experiment.

2. THE EXPERIMENT

The experiment uses a source S_0 of Bell-entangled photons a and b that are prepared in the state

$$|\bar{\Psi}\rangle_{ab} = \frac{1}{\sqrt{2}} [\cos \frac{\pi}{8} (|h_a, v_b\rangle + |v_a, h_b\rangle) + \sin \frac{\pi}{8} (|h_a, h_b\rangle - |v_a, v_b\rangle)] \quad (6)$$

where $|h_{a/b}\rangle$ stands for “a/b is horizontally polarized” and $|v_{a/b}\rangle$ stands for “a/b is vertically polarized.” The setup is schematically illustrated in Figure 2 of Proietti *et al* (2019). The photons a and b are sent in opposite directions, to be ultimately detected by Alice and Bob respectively. Alice and Bob have a choice of measuring either a Bell-type observable A_1 (B_1) or a “which polarization” observable A_0 (B_0) by leaving in or removing a final beamsplitter. Between the source and the Alice/Bob detectors on each side are two additional sources of entangled photon pairs, S_A and S_B . We now describe the situation on the “Alice” side; the same interactions occur on the “Bob” side. Source S_A produces an entangled pair $\{\alpha, \alpha'\}$ in the state

$$|\Psi^-\rangle = \frac{1}{\sqrt{2}} [|h\rangle_{\alpha'} |v\rangle_{\alpha} - |v\rangle_{\alpha'} |h\rangle_{\alpha}] \quad (7)$$

while S_B produces the same state for $\{\beta, \beta'\}$. Here, the photons α and β are supposed to play the role of “Alice’s Friend” and “Bob’s Friend” respectively while α' and β' are alleged to “herald” the measurement purported to take place. In follows, we will refer to α and β collectively as “Photon Friends*,” or PF*s for short, where the asterisk denotes some question as to whether they really qualify as “Friends” in the sense of the original Wigner’s Friend experiment. The putative “measurement” by the PF*s consists of the

establishment of a correlation between α and a and β and b , which will be discussed in detail below. When the auxiliary “heralding” photons α' and β' are detected by a neutral detector, this signals that the correlation has been established.

3. CRITIQUE OF INTERPRETATION OF THE EXPERIMENT

The first thing to note about the portrayal of the PF*s’ “measurement” interactions with the source photons a and b is that they overlook the fact that these are in an entangled state, and that the interaction with the PF*s just increases the entanglement by correlating α with a and β with b , respectively. This means that all the individual photons are in improper mixed states. The “Friends” are nothing more than pointers P as discussed in the previous section, and as we have already seen, correlated pointers alone do not yield actual outcomes. Yet the authors seem to suggest that they are in proper mixed states. Specifically, the Appendix contains the following statement and characterization of the interaction:

“Depending on the state of the incoming photon, the operation performed by Alice’s friend transforms the overall state as follows:

$$\begin{aligned} |h_a, \Psi^-\rangle_{\alpha',\alpha} &= \frac{1}{\sqrt{2}}[|h\rangle_a|h\rangle_{\alpha'}|v\rangle_\alpha - |h\rangle_a|v\rangle_{\alpha'}|h\rangle_\alpha] \rightarrow \frac{1}{2}|h\rangle_a|v\rangle_\alpha, \\ |v_a, \Psi^-\rangle_{\alpha',\alpha} &= \frac{1}{\sqrt{2}}[|v\rangle_a|h\rangle_{\alpha'}|v\rangle_\alpha - |v\rangle_a|v\rangle_{\alpha'}|h\rangle_\alpha] \rightarrow \frac{1}{2}|v\rangle_a|h\rangle_\alpha \end{aligned} \quad (S3)''$$

In the excerpt above, I have italicized the problematic phrase that seems to suggest that the incoming photon a or b really is in a particular (but unknown) pure state, when that is not the case. In their equations (S3), which omit the entire entangled state of a and b , these transformations are presented as yielding well-defined outcomes for the source photons a and b , when in fact they do not. The actual transformation taking place upon the interaction with α is as follows:

$$|\bar{\Psi}\rangle_{ab}|\Psi^-\rangle_{\alpha',\alpha} \rightarrow \frac{1}{2\sqrt{2}}[\cos\frac{\pi}{8}(|h_a, v_\alpha, v_b\rangle + |v_a, h_\alpha, h_b\rangle) + \sin\frac{\pi}{8}(|h_a, v_\alpha, h_b\rangle - |v_a, h_\alpha, v_b\rangle)] \quad (8)$$

This is clearly still an entangled state, so it is unambiguous that there has been no collapse or reduction as a result of the unitary interaction between α and the source state. Applying the transformation for Bob’s “Friend” β then yields the authors’ equation (S7), an entangled state whose Bell-state properties can be confirmed via Alice and Bob’s observable $A_1 = B_1$. ((S7) is just the final version of (8) including correlation of the β states with b .)

Despite the fact that there has been no collapse or reduction as a result of the interaction of the PF*s with the source photons, the authors maintain that an outcome occurred at that point. This sort of ambiguity about whether there is an outcome, how there could be an outcome, and even “for whom” there might be an outcome, is perhaps inevitable

under the general assumption that quantum theory “really” is unitary-only, according to which even human beings must be quantum subsystems of much larger composite entangled states. The authors reveal this as their background assumption in discussing the original Wigner’s Friend thought experiment, which involves a human being as the Friend. They say:

“Concurrently however, the friend does always record a definite outcome, which suggests that the original superposition was destroyed and Wigner should not observe any interference. The friend can even tell Wigner that she recorded a definite outcome (without revealing the result), yet Wigner and his friend’s respective descriptions remain unchanged.”

The above says that “the original superposition was destroyed” but also that Wigner correctly describes the situation with a superposition: clearly a self-contradictory account. This kind of inconsistency is a feature of the “shifty split” of UOQM, in which collapse can be nothing more than a wholly mysterious, non-modeled process assumed to (somehow) occur based on observation by an “external conscious observer.” (Indeed the fact that “external conscious observer” cannot be defined constitutes the *reductio* nature of the WF experiment.) But there need be no inconsistency between the descriptions: if there really *were* non-unitary collapse as a result of the Friend’s measurement, then Wigner would *not* be able to correctly model his system as remaining in a pure state. That is, had a definite outcome occurred along with collapse, the Friend would *not* be able to “tell Wigner that she recorded a definite outcome without revealing the result” and still have Wigner correctly describe his system by a pure state. Instead, Wigner’s correct and experimentally verifiable description of the Friend and her system would be a proper mixed state, regardless of whether the Friend revealed anything (however uninformative) to Wigner or not.

Thus, the idea of the non-physical, mysterious collapse allegedly attending a “measurement” interaction leads the authors to the self-contradictory position of supposing that collapse “must have occurred” at the level of α and β even though they themselves must deny collapse in order to obtain the correct final state (A7), and their own data show that no collapse occurred. The inability of UOQM to define what counts as a “measurement” or “observation” yielding an outcome – because none can under UOQM – is the ultimate source of the confusion here.

Regarding the Clauser-Horne-Shimony (CHSH) inequality that is violated by the data obtained in this experiment: CHSH (1969) originally formulated their inequality in terms of hidden variables, which are indeed forms of “observer independent facts.” But in the current context, which does not involve any hidden variables, the inequality refers to the assumption that there were determinate measurement outcomes by the “Friends,” even though those putative outcomes did not collapse the superposition obtaining in the prepared Bell state. This assumption plays the role of a hidden variable, since it is assumed to obtain even while the quantum state does not reflect that any outcome occurred (being still an entangled pure state). Since there are no other hidden variables assumed in this experiment, and the only nonstandard assumption is that the PF*s precipitated an outcome even without collapse or reduction of the quantum state, the violation of the inequality

by the experimental statistics is properly taken as refuting that assumption. This is reinforced by the fact that the application of the quantum formalism required to match the experimentally observed results does not collapse the quantum state between the putative ‘measurement’ by the PF*s and the detections by Alice and Bob.

The point of this paper is not to quibble about whether the internal photons α and β count as “observers” or not, since UOQM is utterly helpless to say anything about what an “observer” could possibly be. If an “observer” is a system that can yield an outcome, then no system, not even a human being, can give rise to a well-defined outcome under UOQM. As we reviewed in Part 1, the mere copying of a quantum number pertaining to one degree of freedom by another degree of freedom (the definition of “observer” used by the authors) does not qualify as yielding an outcome, despite the language about “extracting information.” (This is just the measurement problem facing UOQM again.) Thus, our routine experience of definite outcomes remains at odds with UOQM. Nevertheless, it is quite clear from the data correctly predicted by the final uncollapsed state (the authors’ (S7)) that there is no collapse at the level of the PF*s. The fact that they copied quantum numbers in a superposition, i.e., acted as pointers, does not establish any fact of the matter; there is no warrant to claim that they “observed an outcome” since no outcome is possible according to quantum theory itself. That is, quantum theory itself does not permit the interpretation of the improper mixed state of a subsystem of a composite pure state as a situation in which an outcome actually occurred. Therefore, it is not tenable to claim that there was an measurement outcome at the level of α and β . The “heralding” photons α' and β' merely signal that a unitary interaction has occurred at that point, not that there was any outcome. The observed violation of the relevant CHS inequality, rather than indicating that there are irreconcilable facts among “observers,” demonstrates that the assumption that there was a “fact” at the internal Friend level violates quantum theory (which was already evident from the fact that the systems are in improper mixtures) and is therefore erroneous.

Thus, this experiment, rather than demonstrate an antirealist conclusion such as “different observers irreconcilably disagree about what happened in an experiment,” confirms what Feyerabend and Hughes pointed out decades ago: quantum theory (when taken as unitary-only) does not allow one to ascribe a fact of the matter, corresponding to an eigenvalue of a subsystem observable, to a subsystem of an entangled system. A subsystem of an entangled system is therefore not an entity that can yield a well-defined “fact” corresponding to an outcome of an observable pertaining to that system. Something more is needed for an outcome (“fact”) to occur; specifically, physical collapse of the entangled (nonseparable) state to a single separable state. That has not occurred at the “Friend” level in this experiment. However, if the heralding photon were made into a pointer to the h, v states, the final state (the authors’ S7) would not be tenable even according to UOQM due to traditional decoherence arguments (i.e., putative entanglement with the degrees of freedom correlating the heralding photon to the relevant states). Of course, that account remains inadequate since there is still no justification for any definite outcome to arise under unitary-only dynamics. For further discussion of this point, see Kastner (2014, 2020a, 2020b).

The bottom line: Establishment of a correlating entanglement and verification of the correlating entanglement at the level of the “Friend” is not equivalent to “outcome occurred” or “fact was obtained” at the Friend level. The inequality expressing this erroneous assumption is thus violated by the experiment, which verifies the probabilistic predictions of quantum theory. Had reduction and a real outcome for the polarization observable occurred at the “friend” level (or even correlation of the heralding photon with the h and v states as mentioned above), there would be no ongoing entanglement at the level of Alice and Bob and that would be reflected in the statistics, which would confirm the loss of their prepared Bell state and its replacement by a mixed state.

4. SO HOW DO WE GET OUTCOMES?

It is perfectly possible (beyond UOQM) to get a definite outcome, and of course we can all testify to the existence of definite outcomes in the lab. Rather than take the latter empirical fact as license to mis-attribute definite outcomes to systems in improper mixed states when that is logically contradictory (yet a practice that is endemic under the assumption of UOQM), a more consistent approach is to take our empirical experience as strong evidence that non-unitarity is real, and that this is why we get actual measurement results. The present author has been studying a formulation of quantum theory called the Relativistic Transactional Interpretation (RTI), which is based on the direct-action theory of fields. RTI contains quantitatively well-defined physical non-unitarity that does not involve any *ad hoc* change to quantum theory, and is empirically equivalent at the level of the Born probabilities to standard unitary-only quantum theory. While it is not the purpose of this paper to advocate any particular alternative formulation of quantum theory, the interested reader may consult Kastner (2018), (2020a) and (2021) for some details.

With regard to the experiment under discussion, the conditions that would be required for collapse or reduction under RTI are discussed in the above references. These conditions do not obtain at the “friend” level of the Proietti *et al* experiment, which detects only a photon whose state is uncorrelated to the observable of interest. They can readily obtain at the “Alice and Bob” level, which is why Alice and Bob find definite results confirming their Bell state. In Wigner’s original thought experiment, without restriction to UOQM and under a direct measurement by the macroscopic Friend of the system presented to him, collapse could easily occur at that stage. (What counts as “macroscopic” and why that level generally precipitates collapse is discussed in Kastner (2018)). Then, even if the Friend “merely” signaled to Wigner that he saw a result without revealing the result, the system would still have been collapsed, and Wigner would find his system in a proper mixed state, not a pure state. Thus, we have to be careful about assuming that such a signal (“I see a result but I’m not revealing which”) preserves entanglement. In fact it does not if reduction actually occurred at that point. On the other hand, if entanglement is preserved beyond that point, then nobody really “saw a result,” since there could be no well-defined result (as Feyerabend and Hughes point out). Thus any portrayal of a “Friend” as really

seeing a result but not disclosing it and yet continuing on as a component of an entangled state is erroneous, since no definite outcome can occur under those circumstances.

5. CONCLUSION

It has been argued that the observed violation of the relevant CHS inequality, rather than supporting claims such as “observers see irreconcilable facts” demonstrates that it is a mistake to attribute an outcome or ‘fact’ of the matter concerning any specific observable to a system that is in an improper mixed state, such as the PF*s in the Proietti *et al* experiment.

The authors do not consider this interpretation of the violation of the CHS inequality, because they do not question their claim that the PF*s did find an outcome. Yet, as we reviewed above, it is unambiguously incorrect to assert that the PF*s found an outcome, since that assumption violates quantum theory itself: a degree of freedom in an entangled pure state, like the PF*, is in an improper mixture that cannot be described by a definite but unknown observable eigenstate. The observed violation of the CHS inequality expressing the incorrect assumption that an outcome occurred at the PF* level confirms this elementary fact.

Another way to see that this is an error is to consider one of the electrons in a two-electron Bell state. Yes, the electron’s state is correlated to its partner, but that is not justification for attributing any measurement outcome, i.e., value of an observable, to the entangled electron. This mistake is made by the authors when they say:

“Recalling from Eq. (S4) how the friends’ measurement results are encoded in their polarisation states...” This is no more correct than asserting that a Bell state like

$$|\Psi\rangle = \frac{1}{\sqrt{2}}[|z \uparrow\rangle|z \downarrow\rangle - |z \downarrow\rangle|z \uparrow\rangle] \quad (9)$$

expresses an ‘encoding of measurement results.’ Yes, the electron’s state is correlated to its partner, but that does not mean that any ‘information has been extracted’ nor that there can be any value of Z-spin – i.e., no outcome – attributed to the electron.

The violation of the CHS inequality, which assumes a ‘fact’ or outcome at the level of the PF*s, demonstrates that it is a violation of quantum theory to attribute fact/outcomes to the PF*s. This is because the unitary interaction they undergo with the Alice and Bob photons preserves their pure entangled state. Thus, rather than a test of local observer independence (as stated in the title), this experiment is a test of the assumption that there is a measurement outcome based only on a non-decohering unitary correlation among photons. That assumption fails the test.

6. REFERENCES

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