On the gnoseologic principles of Bertrand Russell*

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Summary: Exposed in 1948, within his masterpiece on the scope and limits of human knowledge, the epistemological tenets that Bertrand Russell regarded as fundamental elements in the construction of scientific knowledge, are still worthy of a detailed discussion today. Given the excellence of the author, it will not be surprising to see that Russell's gnoseologic postulates, even for the present scientific view, address some of the most controversial questions still to be solved in the theory of knowledge.

Keywords: Russell; gnoseology; knowledge; science

1. Introduction

The English philosopher and mathematician Bertrand Russell (1872 - 1970) was one of the most outstanding thinkers of the 20th century, whose work rivals in length and depth with any of the classics that preceded him. Russell never became as influential in politics as Sartre or Marcuse, because he was never dominated by an overtly partisan spirit. His analyzes were much too rational to be manipulated in the hands of any political movement, since he never neglected his unwavering commitment to the objective study of reality, natural or social. In this sense, it can be said that Russell always happened to be intelectually modern, without temptations to become post-modern.

Also in the gnoseological field his thoughts evolved starting from a philosophical education, at Trinity College, Cambridge, greatly influenced by Kant and Hegel. Through the works of the great British idealists J. M. E. McTaggart (1866 - 1925) and F. H. Bradley (1846 - 1924). Russell absorbed the doctrine of internal relationships, according to which any relationship between two objects is ultimately an intrinsic property of the related objects. Grounded on this premise, the conclusion is finally that the only genuine reality is Absolute Consciousness - whatever that may mean - and that any knowledge of particular aspects of nature is purely illusory (Russell, 1982).

Suffocated by the narrowness of this doctrine, Russell abandoned it to discover the power of the methods of modern logic when applied to philosophy (Gödel, 2006). In the wake of G. Peano (1858 - 1932) and F.L.G. Frege (1848 - 1925), he first tried to strictly build up mathematics only on logic basements, in the so-called "logicist program", and later physics on perceptions, in the "phenomenalist program”, with a splendid failure in both attempts (Griffin, 2003). The magnificence of Russell's defeats

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lies in the conclusions he was able to draw from his unfruitful efforts. That mathematics cannot be founded exclusively on logic reveals a problem - perhaps unsolvable - of philosophical importance that can be hardly exaggerated. And the exhaustion of the phenomenalist program doomed logical empiricism to a slow decline while delimiting somewhat better the scope and limits of human knowledge. This was precisely the title of the mature work that best collects Russell's contributions to the field of gnoseology, a text on which we will focus in the forthcoming paragraphs.

2. The scope and limits of human knowledge

Originally published in 1948, the book *Human Knowledge. Its scope and limits* (Russell, 1983) epitomized the ultimate considerations of the great British philosopher on epistemology. Russell's views were intellectually supported by advances in four major scientific fields: physics, physiology, psychology, and logic. On this background, the book's approach revolves around the possibility of building bridges between human perceptions and the more abstract - and therefore more comprehensive - theoretical constructions of modern science. Russell aims to achieve this by skillfully working in both directions simultaneously. On the one hand, it starts from sensory data to progress towards ever greater levels of abstraction. On the other hand, it takes the most general concepts of physics and dive into its theoretical foundations, in search of the point of connection with sensible experience that would justify such generalizations.

The main body of the text is divided into six parts, the first one containing a brief but satisfactory exposition of the state of the sciences at the time the book was written. After distinguishing between individual and social knowledge, the reader is guided along a fascinating journey through the essential ideas of astronomy, physics, biological evolution, the physiology of perceptions and, finally, psychology as a science of mind. Much of what is said there continues to retain its validity today, an extreme that reveals the author's enviable wit. Perhaps the only disappointment in this first part is due to Russell's purely empiricist interpretation of Heisenberg's inequalities, regarded as a prescription on the inherent limitations of the measurement process applied to elementary particles. This view, in line with the traditional quantum orthodoxy of Bohr and Heisenberg, has been strongly disproved over the years (Bunge, 1983), although its reputation persists even nowadays.

The second section brings to the reader eleven chapters on language as a vehicle and mediator of the expression of human knowledge. Through its pages, the enrichment of Russell's ideas in this regard can be verified, since these chapters extend and polish the theses presented eight years earlier in *An Inquiry into Meaning and Truth* (Russell, 1946*a*). To what extent the structure of our language reflects and reproduces the structure of the material world is the object of Russell's investigation in this second part. For this purpose, the English philosopher not only uses logical analysis but also epistemological and psychological considerations. A conclusion is inevitably imposed on him: truth is based on the correspondence between a belief and a fact (although there are events that no one can experience directly). Those who defend the ability of language to "create worlds" or to become completely independent of emitters and receivers should bear this in mind.

The third part, entitled "Science and perception", deals with investigating the relationship between physics -regarded as the most developed science- and experience. It does so by taking into account the contributions of both physics and physiology for the understanding of the psychological aspects of our knowledge as a construction based, at least in part, on sensory data immediately accessible to every human being. Russell rightly believes that instead of starting from the way we know to determine what we can know, the path must be traveled in reverse order. If the human sensory apparatus, alongside the brain to which it is connected, are part of the physical world, then they must be subjected to the same general physical laws that govern all matter in the universe. The fact that living organisms also obey specific laws of biological systems does not exempt them from complying with primordial physical laws on a fundamental level.

The ten chapters of the fourth block test the architecture of basic scientific concepts in relation to the sensory experience that contrasts them. If in the previous section perception was considered as a material process necessarily compatible with ordinary physical laws, now the link is sought between the formal abstractions of theoretical physics and the set of experimental observations that have to verify them. Unlike his earlier forays into the subject, as in *Our knowledge of the external world* (Russell, 1946*b*), originally published in 1914, here Russell admits that a strict adherence to empiricism does not always happens to be the best possible path towards a reasonable reconstruction of the human way of knowing.

The fifth part of the book deals with the concept of probability, in its very diverse meanings, as well as with its applications to the material universe. Russell reviews probability theory, as a branch of pure mathematics, comparing it to the version founded on finite statistical frequencies and the interpretation of probabilities as degrees of credibility. But the main intention of this set of seven chapters is not to emphasize the importance of probability theory in describing the stochastic processes of the physical world. Russell's real purpose is to delineate the extent to which it is possible to assign some kind of basis to inductive reasoning in the calculus of probabilities.

The validity of inductive reasoning has been the subject of countless discussions, and it will undoubtedly remain so for a long time, if it ever reaches a definitive solution. Well aware of this, Russell attaches a capital importance to non-demonstrative inference, one in which the truth of the premises does not guarantee the truth of the conclusions and only gives them a certain probability. This point is where the basic argumentative core of the work rests, because if such non-demonstrative inferences give us some kind of knowledge, there must be some set of principles that justify the probable certainty of its conclusions. It is precisely these postulates -or a version of them- that Russell pursues in the sixth and last part of his work. As he himself explained elsewhere:

> […]. The question to be investigated is this: given a number of A that are B, and no opposite example, under what circumstances does the probability of the generalization "all A is B" approximate to certainty, as a limit, when the number of A which are B is continually increasing? The conclusion reached is that two conditions must be met if this is to occur. The first and most important of these two conditions is that, before we know any examples of A that are B, the generalization 'all A is B' must have a finite probability based on the rest of our knowledge. […].
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> How are we to know that a certain suggested generalization has a finite probability in its favor before we have examined some of the evidence for or against it? […]. I wanted the postulates that I arrived at by analyzing examples of non-demonstrative inference to be such as to confer this a priori finite probability to certain generalizations and not to others. It will be observed that, in order for the postulates in question to fulfill their function, they need not be true; they only need to have a finite probability. […]. (Russell, 1982: 210 – 1)

Three important questions are now raised from the British philosopher's considerations: (1st) To what extent are scientific inferences "non-demonstrative" ?; (2nd) what is the meaning of the probabilities assigned, a priori or a posteriori, to an empirical generalization ?; and (3rd) what is the degree of validity of the postulates proposed by Russell to cement this theoretical edifice? The remainder of this article will be devoted to the attempt to point in which direction the answer to these three questions could be found.

3. The character of scientific inference

By clasiffying inferences into demonstrative and non-demonstrative we are perhaps imposing too restricted a pattern on scientific reasoning, which -as Russell himself points out in various places- cannot be reduced merely to the techniques of formal logic. The conclusions of a well-constructed syllogism are demonstrative inasmuch as its veracity follows logically -that is, deductively- from the truth of the premises. It is in this sense that they are also called analytical propositions and are taken necessarily granted for true.

As a good logician and mathematician, as well as a philosopher, Russell knew that natural sciences cannot be reduced to a collection of syllogisms, since scientific conclusions provide new knowledge, a capacity that symbolic logic lacks. Consequently, scientific knowledge must come from non-demonstrative inferences, the truth of which is not logically necessary since they are not analytical propositions. It is precisely at this extreme that the empiricist baggage that to some extent weighs down Russell's thought is revealed, despite the fact that he does dedicate a chapter to discussing the insufficiencies of empiricism as a philosophy of nature. Just as Kant had made the legitimacy of natural science depend on synthetic a priori judgments, Russell takes as one of the cornerstones of his gnoseology the existence of non-demonstrative inferences whose veracity can be estimated more or less probable, especially since it is not even we cannot be ven certain about the correctness of the premises.

Certainly, in everyday life most of our inferences -or almost all of them- are of the non-demonstrative kind mentioned by the British philosopher. Every day we deal with reasonings that, although logically correct, are based on unclear premises, fuzzy conjectures, intuitions and prejudices. And the fact that most of the time we get along very well pays sincere tribute to the human brain as a predictor of behaviors. But it is hard to believe that we do the like when stating scientific inferences.

If we assume as a premise the hypothesis that the Newtonian law of universal gravitation is correct, nobody would dare to say that the calculated value of the force of attraction between two objects has a high probability of being true. No doubt, scientific reasoning does not work that way. We will see in the following sections that Russell makes the typical empiricist error of assigning probabilities to legal statements, when probability only makes sense for events (real or ideal). It makes no sense to attribute probabilities to linguistic propositions or mathematical algorithms, since chance is an originally physical concept that, in tle last term, must be linked to material facts.

A more realistic approach reveals that scientists start from a more or less general theory (a hypothetical-deductive system), applied to some concrete material system represented by an idealized simplification (a "model"). The researchers also take extraordinary care in controlling the possible sources of experimental error, just because they consider that these errors require a statistical treatment that confirms the validity of the data. Quite the contrary, at no time do they assume that the theories employed, or the laws they include, make their conclusions true with a certain probability.

Scientific conceptions can undoubtedly be altered, since science only provides partial, inaccurate and provisional knowledge, and because of this the invalidated theories will have to be replaced by more powerful and comprehensive ones. But that does not mean that an experimentally disproved hypothesis - for example, the existence of the electromagnetic aether - has provided predictions with a very low probability of being true; it is simply a wrong hypothesis.

However, it is true that in certain cases scientists make inferences whose results are characterized by a certain probability, as in stochastic theories. Classical statistical mechanics or quantum physics do not usually make predictions of specific values, but rather of probability distributions. This is the case because these theories include among their postulates at least one that takes into account the random or semi-aleatory behavior of the objects they describe.

A handle that Russell never seriously renounced was the validity of the principle of causality and its link with space-time, as we will see later when considering the close relationship that he establishes between causal lines and series of space-time events. A very interesting question thus arises: could we define the space-time order of physical events from the causal connections between them? In Special Relativity, concepts like "space distance", "time interval", or "simultaneity" are characterizable in terms of causal relationships. Now, when passing to General Relativity, two manifolds with different metrics could share the same causal structure, which shows that only the causal connection is not enough to define space-time relationships (Robb, 1921, 1936).

It happens that the geometry itself does not determine the causal structure of space-time, and vice versa, the causal connections are insufficient to select a space-time geometry among several possible ones. The topological properties could be defined by causal relationships, even in a curved space-time manifold, as long as there are no closed time-curves and it is not possible to obtain them after an infinitesimal displacement of a non-closed time-curve. If so, all the topological characteristics -at least in the ordinary topology of manifolds- are determined once the series of events that constitute continuous causal paths in space-time have been specified (Sen, 2010).

The universe has traditionally been conceived as being composed of two aspects. In its space-time aspect, our cosmos is the set of everything that exists in space and time. In its causal aspect, it is the collection of all the processes in which some events determine the occurrence of others. We need space-time coordinates to establish physical laws, which are causal relationships in their highest degree of sophistication, but at the same time we need these laws to specify physically real coordinates in space and time, beyond the mere abstract construction of pure mathematics.

Everything seems to point out that metric properties of a space-time –or, at least, some of them– cannot be solely reduced to causal relationships. Quite the contrary, the notion of causality happens to contain some space-time element as a basic ingredient. We already know for sure that to specify the geometry of a certain space-time the causal structure is not enough. For that purpose we must also provide a metric structure, such as that provided by the trajectories of bodies in free fall.

The causal relationships can be considered as supplied by the arrangement of light-cones in space-time, since our physical laws claim the impossibility of exceeding the speed *c*, and this is expressed by stipulating that no world-line can leave its local light-cone. But we also need a way to link the characteristics of every local light-cone with that of its immediate neighborhood, an objective that is achieved by virtue of the set of space-time geodesics, which are local inertial frames and provide us with the affine connection necessary for this purpose (Friedman, 1991).

4. Usages and meanings of probability

The fifth part, dedicated to the interpretations of the concept of probability, is of great importance to fully understand Russell's points of view, due to both the topics he expounds and the ones omitted there. The British philosopher begins by outlining the mathematical theory of probability which he correctly considers one more branch of pure mathematics, as exact as any other, unless we wish to compare his results with the empirical data of the natural world. In such a case, like geometry’s, we will dive into the difference between abstract mathematical elucubrations and the material servitudes of experimental science.

Russell rightly distinguishes the formal use of mathematical probability (probability) and the ordinary meaning of probability as the degree of plausibility or reliability that we loosely assign to the practical assumptions of everyday life (likelihood). This second meaning gave way to the subjectivist interpretation of probability (Finetti, 1930, 1931; Ramsey, 1931), which considers probabilities as attributes of our subjective opinions on those matters about which we are not in a position to make objective statements. Thus understood, probabilities would only quantify the degree of confidence that every opinion inspires at a given moment in the person who holds it, with no other aspiration than to reach an intersubjective agreement between individuals whose reasoning is self-consistent.

Against subjectivism was raised, almost three decades later, an objectivist interpretation that judged probabilities as "propensities", that is, genuinely physical properties of material systems -on an equal footing with electric charge or mass, for example- that would reflect an inherent characteristic of the system we deal with (Popper, 1957). Developed in the second half of the 20th century and inspired by quantum paradoxes, Russell could not take this point of view into account before writing his book, although it is interesting to wonder whether he would have modified his views had he known it. On the propensities’ point of view, probabilities quantify the willingness, or tendency, of a quantum system to give rise to certain typical statistical frequencies in a specific experimental situation.

Surprisingly, Russell did not mention the axiomatization of probability carried out by the Russian Andrei Kolmogórov (1903 - 1987) three decades before the publication of the book. Perhaps that is why in the following chapters Russell moves into a certain ambiguity when he talks about statistical frequency, since sometimes he seems to use it to define the concept of probability and other times it seems to be used to justify its empirical application.

Traditionally, a certain intellectual current (Cournot, 1843; Mises, 1928, 1931) has been identifying the probability of an event with its long-term statistical frequency. In such a context, probability can only be attributed to an event if it is part of an extensive series of events of the same type, where the relative frequency of the event considered converges to a certain limit when the members of the series increase indefinitely. The limit so defined would be the probability of that event within the given series, so that, if such a limit does not exist, or the group to which it belongs has not been established, it should not make sense to speak of probabilities.

However, we should not forget that experiments only offer frequencies of results and the formulas of probability theory are not exactly satisfied by frequencies, not even in the long term, which is always a finite term. Only in a special type of random process, the Bernoulli series, it is shown that the probability of any given divergence of a frequency from the corresponding theoretical probability decreases as the sample size increases. But this second-order probability is a mathematical property of the Bernoulli series, and therefore it is not reducible itself to a frequency. Thus, the so-called “law of large numbers” determines the number n of random events that a Bernoulli series must have so that the frequency of a specific result is within an interval $p \pm \varepsilon$ around its mathematical probability p on a previously specified percentage of occasions.

On the other hand, Russell also makes the very common mistake of confusing chance with ignorance:

> […] What is the probability that a randomly chosen integer less than 10 is a prime number? There are nine integers less than ten, and five of them are prime numbers; then the probability is 5/9. What is the probability that it rained in Cambridge last year on my birthday, assuming you don't know when my birthday is? If *m* is the number of days it rained, the probability is *m*/365. […]. (Russell, 1983: 357)

The first example is completely correct, but the second is totally wrong. Probabilities can only legitimately be applied to random processes, such as when we pick a number at random in a given series. But the phrase "it had rained in Cambridge on a certain day in 1947" refers to a past event that is not at all a random one. Either it rained that day or it didn't, regardless of whether we know which specific day we are referring to. A better approach would have asked the question about the probability of hitting Russell's birthday by choosing a day of the year at random.

Not even as a forecast for the future probabilities can be assigned to rain on a specified day, unless we use statistical frequencies to estimate our expectations. Because, unlike -for example- the instant of a radioactive decay, rain is a physical phenomenon that is not at all random, even when the concurrence of numerous factors makes it difficult to limit the errors in the prediction.

At this point, it is crucial to assess as much as possible the boundaries between probability and statistics since their profiles are blurred, however closely these two disciplines are usually related. Probability is a metric concept applicable to individual random events, unlike statistics, which studies collective properties of sets on whose elements a certain probability distribution has been defined. It is easy to verify that the basic statistical parameters (mode, median, mean, standard deviations, variance, etc.) refer to group properties, not individual ones.

Russell's purpose is to use a pertinent interpretation of probability to support inductive reasoning as one of the bases of scientific knowledge. As the British philosopher did not include the objectivist interpretation of probability in his considerations, he did not propose to found inductive reasoning on statistical inference either, for we must first define a random space in which to insert the event under study. On the contrary, Russell prefers to put fordwards the five postulates that he considers indispensable and sufficient to validate the scientific method, which we will discuss below.

5. The gnoseological principles of Russell

The concept of expansive induction collects those inferences that seek to predict the properties of a possibly infinite set of elements from the data provided by a partial sample of that set. This is the kind of induction that Russell places in the foundations of the scientific method, since, although the premises do not imply the truth of the conclusion, at least they do aspire to offer good arguments to believe in it (Black, 1967). To this effect, the British philosopher relies on a very concrete interpretation of probability (Keynes, 1921) to which he dedicates the fifth chapter in part V of *Human Knowledge*. Neverthless, even these a priori probabilities have to be justified in some way, a mission entrusted to the principles of scientific inference that will be explained below:

> Every of these postulates affirms that something happens often, but not necessarily always; everyone, therefore, justifies, in a particular case, a rational expectation that does not reach certainty […]. Collectively, the postulates aim to provide the prior probabilities that are required to justify the inductions. (Russell, 1983: 491)

With the historical perspective that the passed time gives us, nowadays we know that there is no rigorous logic of inductive inference -in the same sense that there is one for deduction- and it is not very likely for it to ever exist, also for logical reasons. However, a careful examination of the postulates proposed by Russell continues to hold enormous epistemological interest in that it will reveal the scope of such a formulation and its usefulness at the present time.

5.1. The quasi-permanence postulate

The stated intention of this statement is to dispense with the philosophical notion of "substance", in which the concepts of "thing" or "person" are rooted. On this perspective, any piece of matter -living or inert- should not be considered a single individual entity persistent in time, but rather a series of events with some kind of causal connection between them. In Russell's own words: "Given an event A it occurs very frequently that, at some near time, an event very similar to A occurs in a nearby place" (Ibid., 492).

In the first place, the reasons that led Russell to reject outright the dichotomy "substance-accident" (or "substrate-property") are not evident, since at the time of writing the preceding lines he had already renounced the radical empiricism of previous years. If human knowledge must depend on some extra-empirical element, it does not matter whether it is the notion of substance or any other. Despite this, Russell's attempt is of considerable interest, as it links up with the response that might be expected after the revelation by relativistic physics of the clearly four-dimensional character of the universe..

This point of view would require considering that the identities of physical objects are given by the series of space-time events that constitute their worldline. Thus, this statement only assures us that an event is followed spatially and temporally by another one very similar, or alternatively, that the space-time events corresponding to physical entities have a certain extension in space and time (Sazanov, 1990).

It is fair to point out that, despite the time that has elapsed since Russell proposed it, this postulate beats in the guts of all the known physical laws. Even in the quantum world, without undermining the nebulous identity of its inhabitants, it is still true that at each instant their probability distribution unfolds in the vicinity of the region they occupied at the previous time (Penrose, 2006). This is the sense of the unitary evolution ruled by the Schroedinger equation (processes U) as opposed to the randomness of the result of a measure (processes R) .

5.2. Postulate of separable causal lines

This second principle establishes that from one or two members of a series of events –the aforementioned worldline– it is usually possible to obtain inferences that refer to all the other members of the series. Such series, defined internally by an intrinsic causal law, and whose mutual interactions are ruled by an extrinsic law, constitute what we call portions of matter.

Although Russell does not indicate it explicitly, it follows from his text that this postulate contains two capital statements, the first of them with a double purpose. The first idea would imply that worldlines must be characterized by an internal causal connection, otherwise any series of space-time events would form a worldline, which is patently false. This first implicit statement -whose importance cannot be overstated- serves a dual purpose, given that it explains both the persistent identity of things and people over time, and the perception that human minds have of material objects.

Indeed, this causal link connecting the events of a worldline allows them to be identified as a distinguishable entity, far from being a collection of space-time events taken at random. On the other hand, from this it follows the existence, when we perceive something, of a chain of physical events causally connected between the perceived object and the sensory organ of the perceiver, so that the object can be legitimately considered as the cause of perception.

The second claim entailed by this postulate –whose name it contributes to– admits the possibility of separating some causal lines from others, in such a way that for knowing a part of reality it is not necessary to know the entire universe as a whole. That is to say, although the natural world is a system whose components influence one another, it is not excluded that every part can be known to a certain degree with relative independence from the rest. The discovery, in the second half of the 20th century, of quantum entanglement and chaotic dynamics does not diminish the validity of this postulate.

Far away from a widespread and erroneous opinion, the phenomenon of quantum entanglement (or EPR effect) does not constitute a proper physical interaction since it does not involve energy exchange or violate the limit of the speed of light. Rather, it is the demonstration that the statistical correlations between some quantum properties do not depend on the distance that separates two micro-objects after their interaction. Surprising as it may seem, this does not prevent us from studying the evolution of entangled quantum systems without taking the rest of the universe into account (Cramer, 2016).

On the other hand, the dynamics of nonlinear systems –popularly known as "chaos theory" or "butterfly effect"– imposes a limit on the predictive capability of causal laws on certain physical systems, characterized not necessarily by a large number of components but by the particular type of their interactions. However, it must be emphasized again that non-linear systems –whether they exhibit chaotic or self-organized behavior– can be studied with the same degree of independence from the environment as ordinary linear systems. Otherwise, the very discovery of nonlinear dynamics would have been impossible (Hirsch et al, 2004).

5.3. Postulate of space-time continuity

Russell appeals to this third postulate in order to outlaw action at a distance, as has been done in physics since the advent of field theory. On this tenet we can notice that when there is a causal connection between two events separated by a finite interval, there must be a continuous process that links them. The physical tooling of the field theories along with the mathematical apparatus of the differential equations, makes the theories of action at a distance completely dispensable. Furthermore, the geometric formulation of relativistic laws makes it imperative to handle world lines, which in themselves constitute a continuous series of space-time events.

Assuming the existence of causal lines, if the second postulate made possible the extrapolation of any member of a space-time series starting from some of them, the third one opens the door to interpolations in such series. And despite the so-called

"Copenhagen interpretation", which for several decades passed as the genuine orthodoxy of quantum theory, the continuity postulate affirms that physical systems continue to exist even though no one perceives them, giving us back the right to infer their behavior between two successive observations (Bunge, 1967). We only need to have a look at the Schroedinger equation to realize that the wave-function undergoes a perfectly continuous (unitary) evolution, the *U*-process, only interrupted by the measurement intervention, the *R*-process, when the collapse of the wave functions appears as a discontinuous event.

5.4. Structural postulate

According to this principle, when a certain number of events of complex structure are arranged around a center in not very distant regions, it often happens that all of them belong to causal lines that have their origin in an event of the same structure located in that center. The probability that this is the case increases with the complexity and multiplicity of peripheral events; that is, the more complicated and numerous the events are, the more unlikely it is that they have occurred by chance or they are due to misperceptions

The structural postulate enables us, for example, to suppose that the same energy emission that we detect in stars would be perceived by any other observer located in a different orientation. And, above all, thanks to it we dare to affirm that the gravitational, electromagnetic or nuclear interactions that matter undergoes in certain places, are caused by the presence in its vicinity of other material bodies responsible for such interactions.

It is particularly striking that, having rejected the metaphysical notion of substance, Russell distinguishes between substantial structures, corresponding to a portion of matter, and other structures referring not to a material body but to a process, such as a melody. Both the light reflected by an object and the sound emitted by a musical instrument can be considered as forms of interaction with the environment that constitute our own perceptions of such objects. In both cases, light and sound waves are qualitatively very different from the objects that emit them. This makes us suspect that structural properties are what link the perceptions of our senses with the qualities of the objects that cause them. However, what is a structure, in the sense to which Russell appeals?

A structure is a finite succession of sets of objects including relations defined on such sets. Formally written, $\langle D_1, ..., D_m, R_1, ..., R_n \rangle$, with D and R representing respectively sets and relations. Two structures $\langle D, R\rangle$ y $\langle D^*, R^*\rangle$ are deemed equivalent (or "isomorphic") if there is a one-to-one correspondence, $f: D \rightarrow D^*$, such that for any pair of x and y elements belonging to D, $xRy \rightarrow f(x)R^*f(y)$. That is, when two elements x, y belonging to one of the structures are linked by the relation R, their images by f in the other structure $f(x)$, $f(y)$ are linked by the relation R^*. We then see that it is the structural equivalence between the electromagnetic signals that circulate from one telephone to another and the corresponding sound waves –for example– the condition that guarantees the possibility of using telephone as a means of communication.

5.5. Postulate of analogy

This last postulate stipulates that in the face of two classes of events *A* and *B*, with good reason to believe that *A* causes *B*, then if in a given case *A* is observed but there is no means of observing whether *B* has occurred or not, it is probable that it did occur. Also, if *B* has been observed but the presence or absence of *A* has not been observed, it is likely for A to have occurred. With such a statement it is intended to provide support for generalizations based on the repetition of a large number of events of analogous character. This repetition in which an invariable conjunction of certain events is observed leads us with irresistible force to the conjecture that there is some causal link between them, despite knowing that there is no logical necessity for this to happen.

Just because of this absence of logical necessity, Russell intends to strengthen the plausibility of the rules obtained by induction by means of a principle of uniformity that acts as the main premise of inductive reasoning. This would somehow guarantee a certain structural regularity in nature, supporting the assumption that what has happened so far in a given way will continue to happen in the same way in the future.

An interesting obstacle arises in this subjetc with regard to random sequences, in which the conjunction between several events cannot even be assured but statistically. The traditional view of probability as applied to the physical world ascribes all the epistemological value of probability theory to the fact that random phenomena, considered in their collective action on a large scale, create a non-random regularity. This thesis implies the presumption that all collective action has at least one aspect that is, in the limit, regular and not random. However, in addition to the manageable chance to which we were accustomed, there is an intractable chance caused by two main causes: the presence of irreducible fluctuations at a stable value not even at the limit, or a convergence towards regularity so slow that it is unattainable for all practical purposes.

A good example of the first circumstance is found in the so-called “atypical electrical noises”, fluctuations in the intensity of the currents that pass through electronic components that, when amplified, show that the amplitude of the signal depends on the inverse of the frequency. This kind of disturbance hides such enormous deviations that the mean does not form at all. The second form of wild chance exhibits an obvious example in the logarithmic-normal variable (the exponential of a Gaussian variable), whose limits are reached so slowly that they do not in any way reflect the collective behavior of the finite systems that we find in nature.

With all this, this final postulate is possibly the most controversial and dispensable of all, since it is reduced in fact to reformulating the belief in the validity of induction. Because that, and no other, is the name given to the thesis exposed in this fifth postulate that, ultimately, allows us to go from a collection of particular cases to a general rule inferred from them. Russell assures that this postulate intends to give solidity to our belief in the minds of others (Ibid, 497), although a cursory reading of his approach shows that it goes much further than that until it becomes a kind of question-begging in support of the inductive method.

Among the approaches to induction developed in the second half of the 20th century, it is worth highlighting that of Ray Solomonoff (1926 - 2009), based on the hypothetical possibility of simulating the entire universe using a sufficiently sophisticated computer program. Admitting it this way, any phenomenon included in that simulation would be reproducible by means of an algorithm of measurable complexity according to a certain criterion. The Solomonoff induction relies on such assumptions to establish a probability distribution that gives greater weight to less complex algorithms capable of reproducing the observations that constitute our empirical knowledge of the material world. Thus Solomonoff justifies inductive reasoning by assigning stipulated a priori probabilities by virtue of a well-defined rule.

Very useful in the intellectual framework of Artificial Intelligence and Machine Learning, Solomonoff's induction loses force beyond computer science and, of course, the definitive resolution of the induction problem cannot be fairly adjudicated to it. Not only because the possibility of simulating the entire material universe is doubtful –even theoretically– but because it implies the positivist fallacy that identifies scientific theories with a mere codification of empirical data, in this case algorithmically expressible. Quite the contrary, the great scientific advances proceed by testing original hypotheses that are not the product of any algorithm or reducible to computer programs. For this reason, the Solomonoff *a priori* probability distribution, although mathematically valid, appears as physically arbitrary as many others.

For this reason, Solomonoff's theory, as any other inductive formalism, becomes a deductive system in disguise in the very moment it chooses an *a priori* probability distribution as a starting point to make subsequent inferences. The path to arrive at this initial distribution, based on criteria of minimal algorithmic complexity, may be convincing to some authors but it is not so persuasive for others. Nor can we believe that Solomonoff's induction even reflects the real behavior of human cognition. Our reasoning does not operate as algorithmic complexity-minimizing machines; our minds are rather full of biases and inadvertent trends that can be more or less fruitful depending on the situation (as the classic studies on the matter by Amos Tversky and Daniel Kahneman showed).

It is far from obvious that it makes sense to treat the universe as a kind of computer simulation, or to consider that scientific knowledge consists of a long string of algorithms and formulas of computable variables to which an *a priori* probability can be assigned depending on their degree of complexity, provided that they are capable of yielding certain calculations. In fact it is the shortest way to fall into the computational fallacy, a much more sophisticated current version of positivism that substitutes perceptions for calculations.

Anyone who has worked at the theoretical level of science immediately realizes that hypotheses and legal statements are more –infinitely more– than mere calculation prescriptions. Theoretical postulates, regardless of their syntax, acquire full meaning when inserted into a theory, that is, they have a partially relational semantics that enables them to generate other statements that will be able to gain other contextual meanings, which is not the case for algorithms. Indeed, algorithms are interpreted by means of concepts and not the other way round, as evidenced by the fact that we have the “concept of algorithm”, but not an “algorithm of concepts”. Calculate and (scientifically) conceptualize are different logical operations that computationists confuse, since for them –as might be supposed– ultimately there are only computations.

6. Conclusions

The work *Human Knowledge* collects the most elaborate and mature thought of its author about the central problems of gnoseology. The five postulates at the end of the text are intended to synthesize our basic epistemic assumptions about the regularity of the natural world in such a way that they provide a reasonable basis for induction, which Russell places at the root of the non-demonstrative inference typical of the scientific method. The British philosopher thus affirmed his conviction that it was possible to base human knowledge on empirical foundations, at least partially, at the price of abandoning Hume's skeptical psychologism (Bishop and Trout, 2005) to embrace the logical and probabilistic elements that –on his view– endorsed the rationality and credibility of the induction. However, no one managed to define without a certain degree of arbitrariness the *a priori* probabilities that these postulates aimed to legitimize.

The concrete inference that this philosopher advocates is not induction by simple enumeration, in the style of those who sentence "all birds fly" because they have never seen an ostrich. This is what he calls “physiological inference” or “animal induction”, based on conditioned reflexes and expectations generated by the habitual repetition of a certain type of event.

> Because the world is the way it is, sometimes certain events are, in fact, evidence of certain others; and since animals are adapted to their environment, events that are in fact evidence of others tend to arouse expectations from these others. Reflecting on this process and refining it, we come to the canons of inductive inference. These canons are valid if the world has certain characteristics that we all think it has. Inferences made according to these canons are self-confirming, and it is observed that they do not contradict experience. Furthermore, they lead us to judge it probable that we have mental habits that these canons, taken altogether, will justify, since such mental habits will be biologically advantageous. (Ibid, 499)

This kind of induction puts much of our confidence down on the validity of science and a prudent common sense. In fact, that type of assumptions are at the origin of an entire research program such as evolutionary epistemology. This discipline uses the theory of evolution to understand the development of cognitive mechanisms and capabilities of higher vertebrates, particularly humans (Riedl, 1983; Bradie, 1986; Goldman, 1986; Mithen, 1996; Boy and Silk, 2000; Dieguez, 2002; Burian, 2005). Given that survival would have been impossible if our cognitive apparatus led us to error more frequently than to success, it can be deduced that realism (Bhaskar, 2008), in an epistemological and ontological sense, is endorsed by our own biological constitution.

Russell's endeavour, however, is still valid in his incitement to the search for some explanation about the operation of nature in the order of the probable, asking us what needs to be known in addition to particular facts for scientific inferences to be valid (Guttmann, 1999; Horowitz, 2006; Sosa, 2007). And either it constitutes a splendid reminder that all rational justification must not exceed its own limitations. Because of this, Russell does not try to take induction as a premise of reasoning, but as an application –not entirely specified– of mathematical probability to premises that are reached independently of induction. However, this goal is also the subject of fierce controversy, with no indication at all that it has been so far completed.

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