Viewing quantum charge from the classical vantage point

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Abstract

This paper argues for the value of interpreting classical gauge theories as part of the larger project of interpreting quantum field theories. It looks specifically at the benefit of studying a classical version of chromodynamics in order to better understand certain features of color charge in quantum chromodynamics. It discusses the ways in which the usual presentation of the conservation of color charge and the confinement of color charge serve to obscure the Lie-algebra-valued character of the conserved Noether charge, and it explores how we can remove these obscuring factors by studying color charge in classical chromodynamics. This key example of color charge illustrates the methodological benefit of the classical vantage point, giving us the ability to pinpoint the uniquely quantum character of certain features of charge properties.

1 Introduction

Contemporary particle physics is built up from three primary quantum field theories describing the strong, weak, and electromagnetic interactions of elementary particles. These quantum field theories are painstakingly developed, with varying degrees of mathematical rigor, by applying quantization procedures to classical versions of these theories. It is the resulting *quantum* versions that are the successful theories, and it is these theories, therefore, that should be the proper subject of philosophical investigation into what our current best physics has to say about the nature of matter at the smallest levels accessible to empirical study (given current technological limitations).

It may seem, therefore, that the classical versions of these theories are philosophically insignificant, if they are construed as mere formal precursors to the real theory born out of the process of quantization. I argue against this view, defending the methodological value of interpreting classical theories and coming to better understand their quantum counterparts from that vantage point. I give an illustration of how key features of the quantum theory are already present, prior to quantization, in the classical version. The license to interpret quantum charge from the classical vantage point comes from the shared gauge symmetry structure across the relevant classical and quantum theories. Furthermore, however, there is a benefit from making use of this license to interpret from the classical vantage point. I show how the formulation of a classical version of chromodynamics in terms of fiber bundles gives a particular vantage point for understanding the $\mathfrak{su}(3)$ character of color charge in QCD—a feature of color charge that is obscured in QCD itself due to confinement.

I take it as an uncontroversial point of agreement that any philosophical account of what contemporary physics has to say about the smallest constituents of matter that does not seriously contend with the quantum aspects of our current theories is, at best incomplete, and at worst fundamentally misguided. Any yet, quantum field theory (QFT) is a particularly tangled area of theoretical physics, and untangling many of its conceptual puzzles is made more tractable through a piecemeal of approach. Classical gauge theories can be thought of as part of the 'classical bones' beneath the 'quantum flesh' of current best physics. By separating the 'bones' from the 'flesh' we can more easily untangle those distinctively quantum features of a given quantum field theory from those features that arise from the shared geometric backbone of both classical and quantum field theory.

2 Color charge in quantum chromodynamics

Charge properties in field theories (both quantum and classical) are of philosophical interest for several reasons. Foremost, charge properties are major conceptual linchpins in their respective theories. The electric, color, and weak charges of the Standard Model (SM) govern the fundamental interaction processes of each theory, categorizing those particles that are eligible for participation in each interaction, and bearing a close conceptual connection to the coupling constants for each theory. In addition, as conserved quantities via Noether's theorem, these charge properties have a distinguished level of physical significance. And from the standpoint of metaphysics, these properties are especially relevant for questions concerning the interpretation of fundamental properties in our current best physics of the smallest empirically accessible length scales. In this section, I focus on two important features of color charge in QCD. First, the conserved Noether current for color charge in QCD is Lie-algebra-valued, taking values in $\mathfrak{su}(3)$. Second, QCD color charge is confined. In the remainder of this section, I argue that the feature of confinement (and related notions) in QCD actually obscures the

Lie-algebra-valued character of color charge. In section 3 I will argue that the $\mathfrak{su}(3)$character of color charge is made manifest in a classical version of chromodynamics.

2.1 Confinement, color neutrality, infrared slavery, and asymptotic freedom

It is well-known that the result of applying Noether's theorem to gauge symmetries results in a conserved current that takes values in the Lie algebra associated to the symmetry group.[1] In the case of electrodynamics, this reduces to the usual sense of conserved electric charge given in real numbers. This works because the relevant symmetry group is $U(1)$ whose Lie algebra is simply $\mathbb{R}$. Thus, we may not immediately recognize that electric charge is in fact a Lie-algebra-valued quantity. In chromodynamics the conserved Noether current takes values in the Lie algebra $\mathfrak{su}(3)$, since $\mathfrak{su}(3)$is the Lie algebra of the symmetry group $SU(3)$.

For the interpretation of charge in QCD, however, this $\mathfrak{su}(3)$feature of color charge is sometimes obscured by the further recognition that color charge is confined. As is well-known, quarks, anti-quark, and gluons are not observable in isolation, but rather nature only allows for them to appear in bound states with each other in such a way that the *net* color charge of the observed state is always zero, or 'white.' In discussions of the conservation of color charge, most textbook discussions proceed at the level of specific quark and anti-quark color states, (r, b, g; $\bar{r}$, $\bar{b}$, $\bar{g}$). Often in conjunction with Feynman diagrams, we speak of the conservation of color in terms of, separately, the conservation of each of these kinds of color at each vertex in a

[1]See Banados and Reyes (2016) for a recent review of Noether-type theorems. See also Kosmann-Schwarzbach (2011) for a comprehensive treatment, and see Olver (1993) for a standard mathematical presentation of the theorem.

diagram (see figure). Since $(r, b, g; \bar{r}, \bar{b}, \bar{g})$ are not elements of $\mathfrak{su}(3)$, this gives a very different way of thinking of the conservation of color charge than the sense of conservation we get from Noether's theorem.

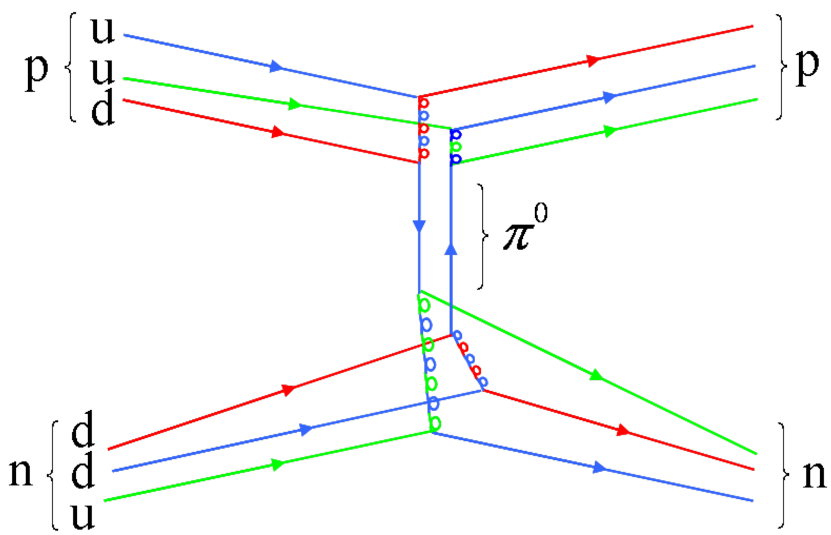

Figure 1: Color states r, b, and g accounted for at each vertex within a Feynman diagram, here given for the process of nucleon scattering via pion exchange. Straight lines with arrows depict quark states and curly lines with double coloring depict gluon states.

It is valuable to disambiguate several ideas related to confinement. First, there is the as yet unproven result of *quark confinement* as a consequence of the dynamics of QCD. Second, there is the theoretical stipulation of *color neutrality*. For example, in the construction of proton states, p, we take the tensor product of the $SU(3)$ carrier spaces for each of the three valance quarks within the proton:

$$p \in \mathbb{C}^3 \otimes \mathbb{C}^3 \otimes \mathbb{C}^3 \cong \mathbb{C}^1 \oplus \mathbb{C}^8 \oplus \mathbb{C}^8 \oplus \mathbb{C}^{10}. \tag{1}$$

On the right hand side, we have the four different *mathematical* possibilities for which representation of $SU(3)$ the proton could be in: $\mathbb{C}^1$, two copies of $\mathbb{C}^8$, or $\mathbb{C}^{10}$. The $\mathbb{C}^1$ option is the only *physical* possibility, according to the stipulation of color neutrality: all observable states must transform according to the trivial representation (whose carrier space is $\mathbb{C}^1$) of the $SU(3)$ color symmetry, i.e., be overall color-charge-neutral.

In addition, there are two well-known, key dynamical consequences of running of the strong coupling constant α_s in QCD. First, there is widespread consensus that applying perturbation theory to QCD proves that the theory has *ultraviolet asymptotic freedom*, that is, for energy levels in the ultraviolet regime (shorter length-scales, higher energies) the effective QCD coupling goes to zero as distance goes to zero.[2] Second, QCD appears to have, as demonstrated numerically in lattice QCD, *infrared slavery*: in the infrared regime (larger length-scales, lower energies) the coupling increases. These numerical results are, we might say, forcefully suggestive of quark confinement, while falling short of an airtight proof. It is expected that, if a proof of infrared slavery is found, it would further show that color neutrality is a dynamical consequence of the theory.

2.2 Obscuring $\mathfrak{su}(3)$

The target of this open question, more specifically, is the demonstration of confinement for a certain class of non-Abelian gauge theories. The theory of the weak interaction, for instance, is non-Abelian and quantum, and yet weak charge is manifestly not confined. Recent developments in mathematical quantization procedures[3] seem to be one promising line of

[2]Though see Seiler (2003) for a dissenting view.

[3]For an overview of such quantization procedures Ali and Engliš (2005), and see Hall (2013) chapters 22 and 23 for more on geometric quantization.

attack. Moreover, on that line of attack, it will be beneficial to have at one's disposal the vantage point of classical non-Abelian gauge theories. From that vantage point, one of the valuable things we can see about color charge, in stark relief, is that it's quark and anti-quark matter fields have, in a sense to be shown below, a Lie-algebra-valued quantity of charge. In classical chromodynamics, this quantity is not confined; there is no trouble with having free quark, anti-quark, or gluon fields. As a contrast class for QCD, classical chromodynamics provides clarity as to what precisely must change as a result of an appropriate quantization procedure in order to explain quark confinement.

One of the ways we speak of color neutrality ($r + b + g = white$) in QCD can obscure the Lie algebra structure of color charge for the matter fields. Since color charge is confined, its conservation amounts to always having overall 'white' states, usually accounted for by equal amounts of r, b, and g, or else through the compensation of each of these colors with their corresponding anti-colors. This suggests that what is conserved is red, and separately blue, and separately green. There may indeed be a way of showing that the conservation of the Noether charge entails these these conservation laws—but that has yet to be shown, and attempts in that direction must contend with the gauge dependence of those specific (r, b, g; $\bar{r}$, $\bar{b}$, $\bar{g}$) quantities. The converse (that the conservation of (r, b, g; $\bar{r}$, $\bar{b}$, $\bar{g}$) should imply the conservation of the Noether current) is far from apparent. By focusing on the (r, b, g; $\bar{r}$, $\bar{b}$, $\bar{g}$) properties, this way of looking at color charge conservation obscures the role of $\mathfrak{su}(3)$in this conservation law.

I argue in the next section that we can, instead, make sense of the conservation of the $\mathfrak{su}(3)$ Noether charge in QCD by investigating charge in a classical version of quantum chromodynamics. In that classical theory, the formulation of the Wong force law (the non-Abelian generalization of the Lorentz force law) leads to the ascription of an $\mathfrak{su}(3)$-valued quantity of charge for matter fields, as well as for the gauge field. This gives us a vantage point from which to see clearly the $\mathfrak{su}(3)$ character of charge for the matter fields themselves, and how they therefore contribute to the overall conserved $\mathfrak{su}(3)$ Noether charge. Moreover, by choosing to investigate charge in this classical version of chromodynamics, we have set aside the physically important but conceptually confounding issues of color neutrality, quark confinement, asymptotic freedom, and infrared slavery. By looking at this classical version of the theory with unconfined, non-neutral matter fields, we can more clearly see the $\mathfrak{su}(3)$ character of *quantum* color charge.

3 Color charge in quantum chromodynamics

In this section, I show how the formulation of a classical version of chromodynamics in terms of fiber bundles gives a particular vantage point for understanding the $\mathfrak{su}(3)$ character of color charge in QCD. This classical version of chromodynamics is not the final, correct theory, and it does not enjoy the empirical success that QCD has.[4] Nevertheless, the geometric bones of QCD are more clearly studied in classical chromodynamics than in QCD itself. And these geometric bones give a clear sense in which the color-charged particles have $\mathfrak{su}(3)$-valued charge. This serves to illustrate the interpretive advantage of the classical vantage point. By isolating and investigating the 'classical bones' supporting the 'quantum flesh' of our successful field theories, we can separate various conceptual structures and relationships in *quantum* chromodynamics. In particular, we see here that two key features of color charge picked out in section 2, namely confinement and the role of the $\mathfrak{su}(3)$, come from two different

[4] For that matter, QCD is not the final correct theory either. But it is the predictions of QCD (together with all the calculational details of renormalization, perturbation theory, lattice QCD, etc.,), rather than any of classical chromodynamics, that currently enjoy empirical success.

places in QCD. Confinement must be found in the 'quantum flesh,' whereas the Lie-algebra-valued nature of charge is manifest in the 'classical bones.'

3.1 Classical chromodynamics and the Wong force law

We may formulate the basic features of a classical version of chromodynamics as follows.[5] We will adopt the abstract index notion developed in Wald (1984) with the further notational conventions of Weatherall (2016). In particular, vectors and tensors tangent to M have lower-case Latin indices; vectors and tensors tangent to the total space P have lower-case Greek indices; and upper-case Fraktur indices are used for vectors with a Lie algebra structure. In addition, indices i, j, etc. will be used to label vectors in the carrier space V of a representation of $SU(3)$ used to construct an associated bundle. Using these conventions, fix a relativistic spacetime (M, g_{ab}) and an $SU(3)$ principal bundle $P \xrightarrow{\wp} M$ over M. In addition, fix a principal connection $\omega^{\mathfrak{A}}{}_{\alpha}$ on P and an inner product $k_{\mathfrak{A}\mathfrak{B}}$ on the Lie algebra $\mathfrak{su}(3)$ associated to $SU(3)$.

The curvature of the principal connection is interpreted as the chromodynamic field strength. This curvature is

$$\Omega^{\mathfrak{A}}{}_{\alpha\beta} = d_{\alpha}\omega^{\mathfrak{A}}{}_{\beta} + \frac{1}{2}[\omega^{\mathfrak{A}}{}_{\alpha}, \omega^{\mathfrak{A}}{}_{\beta}], \tag{2}$$

where d_{α} is the exterior derivative on P, and the bracket $[\cdot, \cdot]$ is the Lie bracket on $\mathfrak{su}(3)$.

The quark and anti-quark matter fields correspond to sections of different associated bundles $P \times_G V \xrightarrow{\wp_\rho} M$, where V is the carrier space for an irreducible representation ρ of $SU(3)$. Thus, a matter field $\Psi : M \to P \times_G V$ maps points $x \in M$ to equivalence classes $[p, v^i]$ for some $p \in P$ and $v^i \in V$. Here $[p, v^i] \cong [p', v^j]$ just in case there exists a $g \in SU(3)$

[5] See Weatherall (2016) for more technical details about formulating classical field theories using the machinery of fiber bundles.

such that $(pg, \rho(g^{-1})v^i) = (p', v^j)$. These vectors v^k describe state vectors in the internal charge space for an elementary particle of this kind of matter field (either a quark, anti-quark, or gluon).

The sections $\Psi : M \to P \times_G V$ are in one-to-one correspondence with V-valued, G-equivariant maps ψ^i on P. Given a section defined at a point x in M, $\Psi(x) = [p, v]$, we define $\psi^i : P \to V$ by $\psi^i(p) \mapsto \lambda_p^{-1}(\Psi(\wp(p)))$, where the map $\lambda_p : V \to \pi_\rho^{-1}(\wp(p))$ is defined by $\lambda_p(v) = [p, v]$. In different contexts, it is sometimes more convenient to use ψ^i on P rather than Ψ on M. In particular, it is technically simpler to define the action of the covariant derivative induced by the connection $\omega^{\mathfrak{A}}{}_\alpha$ on ψ^i than on Ψ. For a field $\psi^i : P \to V$, its covariant derivative is the ordinary exterior derivative d_α on P following the horizontal projection. That is, for any vector ξ^α on P, the covariant derivative induced by the connection is

$$\overset{\omega}{D}_\alpha \psi^i \xi^\alpha = d_\alpha \psi^i \xi^{\alpha|}, \tag{3}$$

where $\xi^{\alpha|} = \omega^{\mathfrak{A}}{}_\alpha \xi^\alpha$.

The generalization of the Lorentz force law to non-Abelian gauge theory is known as the Wong force law, first given by Wong (1970). It can be mathematically derived as follows, although the status of this derivation as a physical argument is unclear.[6] Using the inner product $k_{\mathfrak{A}\mathfrak{B}}$, the metric g_{ab} on M, and our principal connection $\omega^{\mathfrak{A}}{}_\alpha$, we can construct a metric

[6]In particular, it is unclear what physical significance we should attribute to geodesics in the total space. The mathematical derivation rehearsed here was first given by Kerner (1968). It is also given in Bleecker (2013) chapter 10 section 1. Alternative derivations can be found in Storchak (2014), Sternberg (1977), and Weinstein (1978). Wong (1970) himself arrives at the expression by extracting a classical limit of the equations of motion for the quantum fields.

on the total space known as the 'bundle metric.' First, we take g_{ab} on the base space M, and we pull this back along the projection to get a symmetric rank 2 tensor $(\wp^* g)_{\alpha\beta}$ on the total space P. That is,

$$(\wp^* g)_{\alpha\beta}\sigma^\alpha\eta^\beta = g_{ab}(\wp_*\sigma^\alpha)(\wp_*\eta^\beta) \tag{4}$$

for all vectors $\sigma^\alpha, \eta^\alpha$ at a point p in P. There is another symmetric rank 2 tensor on P, denoted $k_{\alpha\beta}$, defined in terms of the connection $\omega^{\mathfrak{A}}{}_\alpha$. It is given by

$$k_{\alpha\beta}\sigma^\alpha\eta^\beta = k_{\mathfrak{A}\mathfrak{B}}\omega^{\mathfrak{A}}{}_\alpha\sigma^\alpha\omega^{\mathfrak{B}}{}_\beta\eta^\beta. \tag{5}$$

For any two vectors σ^α and η^β at a point p in P, $k_{\alpha\beta}$ gives the inner product of their vertical projections. Adding these two tensors gives us our bundle metric $h_{\alpha\beta}$:

$$h_{\alpha\beta} = (\wp^* g)_{\alpha\beta} + k_{\alpha\beta}. \tag{6}$$

We can use the bundle metric to classify curves through the total space. Let $\gamma : [0,1] \to P$ be a geodesic relative to $h_{\alpha\beta}$ with tangent field $\xi^\alpha(t)$. It follows that $\omega^{\mathfrak{A}}{}_\alpha\xi^\alpha(t) = Q^{\mathfrak{A}} \in \mathfrak{su}(3)$ is independent of t (see Bleecker (2013), Theorem 10.1.5). Let $\tilde{\gamma} = \wp \circ \gamma$ be the projection of γ down to the base space with tangent field ξ^a. It follows that, relative to a choice of section σ, the acceleration of this curve $\tilde{\gamma}$ on spacetime obeys the Wong force law:

$$\xi^n \nabla_n \xi^b = k_{\mathfrak{A}\mathfrak{B}} g^{cb} Q^{\mathfrak{A}} \Omega^{\mathfrak{B}}{}_{ac}\xi^a = Q^{\mathfrak{A}}\Omega_{\mathfrak{A}a}{}^b\xi^a\ , \tag{7}$$

where $\Omega^{\mathfrak{A}}{}_{ab} = \sigma^*(\Omega^{\mathfrak{A}}{}_{\alpha\beta})$ is the field strength (see Bleecker (2013), Theorem 10.1.6). We interpret $\tilde{\gamma}$ as the world-line for a particle of mass m and charge $q^{\mathfrak{A}} = Q^{\mathfrak{A}}/m$. If $G = U(1)$, this reduces to the familiar Lorentz force law for electromagnetism. In that case we interpret $q^{\mathfrak{A}} \in \mathbb{R}$ as the amount of electric charge carried by the particle whose world line is $\tilde{\gamma}$. Thus, in classical chromodynamics, $q^{\mathfrak{A}}$ is the non-Abelian charge property analog of electric charge.

3.2 Interpretation

How may we interpret $q^{\mathfrak{A}}$ in this formulation of classical chromodynamics? We usually predicate color charge of a particle in the sense of a basis vector in a representation of $SU(3)$, that is, a specific color state (r, b, g; $\bar{r}$, $\bar{b}$, $\bar{g}$). Using the electromagnetic interpretation as a guide, one might then expect that $q^{\mathfrak{A}}$ gives the color state of the particle under the influence of the Wong force. And since gluons are the only type of particle which have Lie-algebra-valued color states, one might conclude that $\tilde{\gamma}$ is the world line of a gluon.

However, such reasoning is mistaken. The color-charged matter fields, representing either quarks or gluons, subject to this Wong force law couple to the gauge field via the charge-current density. This charge-current density is itself a Lie-algebra-valued 1-form $J^{\mathfrak{A}}{}_{\alpha}$ on the total space P. The charge $q^{\mathfrak{A}}$ is Lie-algebra-valued because the charge current density is Lie-algebra-valued. It is only in the case of electromagnetism that the charge $q^{\mathfrak{A}}$ reduces to a real number which can be interpreted as the amount of charge carried by an electrically charged particle.

The definition of $J^{\mathfrak{A}}_{a}$ relies upon the inner product $k_{\mathfrak{A}\mathfrak{B}}$ on $\mathfrak{su}(3)$, as well as an inner product h_{ij} on the carrier space V of the representation of G used to describe the matter field. Fix a basis $\{e^{\mathfrak{A}}\}$ of the Lie algebra $\mathfrak{su}(3)$. Then, following Bleecker (2013) 5.1.2., the current $J^{\mathfrak{A}}{}_{a}$ is given by

$$J^{\mathfrak{A}}{}_{\alpha} = k^{\mathfrak{A}\mathfrak{B}} e_{\mathfrak{B}} h_{ij} \tilde{\psi}^{j} \overset{\omega}{D}_{\alpha} \psi^{i} \ , \tag{8}$$

where $\tilde{\psi}^j = \rho_*(e_{\mathfrak{A}}) \triangleright \psi^j$. That is, $\tilde{\psi}^j$ is the result of transforming ψ^j under the representation ρ_* of $\mathfrak{su}(3)$ on V induced by the representation ρ of G on V.[7] The definition of the current in eq. (8) gives us mathematical reason to acknowledge that the charge values $q^{\mathfrak{A}}$ for charged matter fields that contribute to $J^{\mathfrak{A}}{}_{\alpha}$ are Lie-algebra-valued.

This general expression for the current associated with a charged matter field ψ^i in a non-Abelian gauge theory reduces to the more familiar current of electromagnetism as follows. Suppose now that $G = U(1)$. Further, choose $i = \sqrt{-1}$ as a basis for $U(1)$'s associated Lie algebra $\mathfrak{g} = \mathbb{R}$, and choose $k_{\mathfrak{A}\mathfrak{B}}$ such that $k_{\mathfrak{A}\mathfrak{B}} e^{\mathfrak{A}} e^{\mathfrak{B}} = 1$. Then the current $J^{\mathfrak{A}}{}_{\alpha}$ becomes,

$$J^{\mathfrak{A}}{}_{\alpha} = i h_{ij} (i\psi)^j \overset{\omega}{D}_{\alpha} \psi^i \ . \tag{9}$$

Now fix a choice of local section $\sigma : U \to P$. The vector potential $A_a = \sigma^*(\omega^{\mathfrak{A}}{}_{\alpha})$ is the pullback of the connection along the choice of section σ. Similarly, the complex scalar field on M is $\Psi = \sigma^*(\psi^i)$. With this notation, a local representation of the current on M is

$$J_a = i((i\Psi)^* \nabla_a \Psi + i\Psi(\nabla_a \Psi)^*) \tag{10}$$

$$= \Psi^* \nabla_a \Psi - \Psi(\nabla_a \Psi)^* \ , \tag{11}$$

where $\nabla_a = \partial_a - iA_a$ is the covariant derivative on M, and the star * indicates complex conjugation.

To return now to the main theme of the value of interpreting classical field theories, the foregoing discussion shows how color charge has a distinctly Lie-algebra-valued character due to the structure of the field theory that is captured in the geometry, and not from the quantum

[7] See Hamilton (2017) 2.1.12.

character of QCD. Electric charge in QED is also Lie-algebra-valued, but since the Lie algebra of $U(1)$ is $\mathbb{R}$ we do not immediately recognize the role of the Lie algebra for electric charge. The field strength (eq. 2), given by the curvature of the connection, is Lie-algebra-valued from the start, and this fits with what we expect of the gauge field for chromodynamics in relation to gluons, which transform according to the adjoint representation. Moreover, the charge $q^{\mathfrak{A}} = Q^{\mathfrak{A}}/m$ in (eq. 7) is Lie-algebra-valued, and it gives the charge of the particle whose trajectory obeys the Wong force law. Thus, charge for quarks and anti-quarks is not only a matter of states within the fundamental representations of $SU(3)$ (wherein we find (r, b, g; $\bar{r}$, $\bar{b}$, $\bar{g}$)), but also of Lie-algebra-valued contributions to the charge current density ${J^{\mathfrak{A}}}_{a}$.

Since there is, then, this sense in which charged particles have $\mathfrak{su}(3)$charge, this gives us another vantage point from which to consider the conversation of the current ${J^{\mathfrak{A}}}_{\alpha}$ (see Bleecker (2013) 5.1.5). Rather than thinking of conservation of color charge in terms of (r, b, g; $\bar{r}$, $\bar{b}$, $\bar{g}$) values in Feynman diagrams, we can think of it in terms of $\mathfrak{su}(3)$contributions to ${J^{\mathfrak{A}}}_{\alpha}$. While there is merit to the Feynman diagram viewpoint of charge conservation, the benefit of this classical vantage point is that the Lie-algebra-valued character of ${J^{\mathfrak{A}}}_{\alpha}$ is manifest.

The point is not that we can only see these features of charge from the classical vantage point, or that they are not available from the quantum field theory vantage point. Indeed, these features are present in the quantum version of the theory, and so these features of charge are expected to be preserved under any acceptable quantization procedure. They are, then, part of the quantum version of the theory precisely because they are first of all present with the quantum theory's 'classical bones.' And by extracting just the 'bones' apart from the quantum 'flesh', we are better poised to see the role the bones play in the final, finished quantum body of knowledge.

4 Conclusion

In this paper I have argued for the benefit to be gained from viewing quantum field theoretic properties from the vantage point of the classical versions of these field theories. The benefit is not absolute: the ultimate goal is a thoroughgoing understanding of the relevant properties from the standpoint of quantum field theories (and later on, presumably, of their successors). But, when we return to renew our efforts at interpreting quantum field theories as applied in particle physics, we will benefit from having first spent some time looking at these fundamental properties from the classical vantage point. I have illustrated the benefit of this approach by showing how the Lie-algebra-value character of charge properties is made manifest in classical field theories. In the case of chromodynamics, this feature of charge is obscured in the quantum version of the theory due to confinement and color neutrality. By turning attention to the unconfined matter fields in classical chromodynamics, we uncover the $\mathfrak{su}(3)$character of color charge.

Thus, from the classical vantage point, we can more easily see key features of charge properties that are preserved under the process of quantization, while also becoming more keenly aware of those features of charge and its associated interpretive issues that are essentially quantum. Classical field theories are, therefore, not merely useful fictions; we should not discard them as irrelevant for philosophical investigation for being ‘the wrong theory’. While they do not tell the full story of what current best physics has to say about the subatomic realm, classical field theories are a core part of the skeleton of the SM. Quantization (and many other theoretical and calculational procedures—e.g. renormalization) fill out the rest of the scientific achievement that is the SM. There is much we can learn about the entire body of the SM from investigating the bones themselves (classical field theories) and comparing them against the full story given by quantum field theory.

References

Ali, S. T. and M. Engliš (2005). Quantization methods: a guide for physicists and analysts. *Reviews in Mathematical Physics 17*(04), 391–490.

Banados, M. and I. Reyes (2016). A short review on noether's theorems, gauge symmetries and boundary terms. *International Journal of Modern Physics D 25*(10), 1630021.

Bleecker, D. (2013). *Gauge Theory and Variational Principles*. Dover Books on Mathematics. Dover Publications.

Hall, B. C. (2013). *Quantum theory for mathematicians*, Volume 267. Springer.

Hamilton, M. J. (2017). *Mathematical Gauge Theory: With Applications to the Standard Model of Particle Physics*. Springer.

Kerner, R. (1968). Generalization of the Kaluza-Klein Theory for an Arbitrary Nonabelian Gauge Group. *Ann. Inst. H. Poincare Phys. Theor. 9*, 143–152.

Kosmann-Schwarzbach, Y. (2011). The noether theorems. In *The Noether Theorems*, pp. 55–64. Springer.

Olver, P. J. (1993). *Applications of Lie groups to differential equations*, Volume 107. Springer Science & Business Media.

Seiler, E. (2003, 12). The case against asymptotic freedom. In *Seminar on Applications of RG Methods in Mathematical Sciences*.

Sternberg, S. (1977). Minimal coupling and the symplectic mechanics of a classical particle in the presence of a yang-Mills field. *Proceedings of the National Academy of Sciences 74*(12), 5253–5254.

Storchak, S. N. (2014, Apr). Wong's equations in Yang-Mills theory. *Central European Journal of Physics 12*(4), 233–244.

Wald, R. M. (1984). *General relativity*. Chicago, IL: Chicago Univ. Press.

Weatherall, J. O. (2016). Fiber bundles, Yang–Mills theory, and general relativity. *Synthese 193*(8), 2389–2425.

Weinstein, A. (1978). A universal phase space for particles in yang-Mills fields. *Letters in Mathematical Physics 2*(5), 417–420.

Wong, S. (1970). Field and particle equations for the classical Yang-Mills field and particles with isotopic spin. *Il Nuovo Cimento A 65*(4), 689–694.