Beyond Bell?

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Abstract

In a recent archive post ([1]) Shan Gao has argued that quantum theory is incompatible with relativity. He calls this a new proof beyond Bell's theorem, arguing elsewhere ([2]) that it closes the superdeterminism loophole in Bell's theorem. Such strong claims must be backed up by irrefutable arguments. My aim in this post to the workshop "Beyond Bell's theorem" is to refute Gao's "proof" and to show how quantum theory is compatible with relativity theory and so why Gao's "proof" does not take us beyond Bell's theorem.

1 Introduction

In a recent archive post [1] Shan Gao has argued that quantum theory is incompatible with relativity. He calls this a new proof beyond Bell's theorem, arguing elsewhere ([2]) that it closes the superdeterminism loophole in Bell's theorem. Such strong claims must be backed up by irrefutable arguments. My aim in this post to the workshop "Beyond Bell's theorem" is to refute Gao's "proof" and to show how quantum theory is compatible with relativity theory and so why Gao's "proof" does not take us beyond Bell's theorem.

Gao seeks to derive a contradiction between quantum theory and relativity by applying them both together in the scenario of a *Gedankenexperiment.* Since his argument proceeds by *reductio ad absurdum* it is irrelevant to its soundness that this *Gedankenexperiment* is totally impracticable. Alice and Bob each perform a measurement of spin-component on a (different) particle in a pair they have prepared in the spin singlet state

$$\frac{1}{\sqrt{2}}\left(|\uparrow\rangle_1 |\downarrow\rangle_2\right) - |\downarrow\rangle_1 |\uparrow\rangle_2 \tag{1}$$

In §2 of [1] he uses this familiar scenario to show why theories of the quantum world that postulate a physical process of quantum state collapse that occurs

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simultaneously in different regions of space in a preferred Lorentz frame are incompatible with fundamental Lorentz invariance. This is a familiar result, and Gao does not maintain that it takes us beyond Bell's theorem.

2 The main argument

Gao goes on in §3 of [1] to consider views of quantum theory that postulate no such physical process of quantum state collapse on measurement, maintaining on the contrary that the interaction between a system and a measuring device may be modeled as the unitary evolution of their joint quantum state. In this section he describes a less familar scenario in which Alice and Bob are joined by a superobserver who is able to reset Alice's entire laboratory to its initial pre-measurement quantum state by a local interaction that may be modeled by a unitary evolution (namely the adjoint of the unitary transformation used to model the effects on her lab of Alice's measurement on particle 1).

Gao argues that this permits the existence of a correlation between Alice's and Bob's measurement results stronger than the correlations of Bell inequalities, and that this reveals the incompatibility between unitary quantum theories and special relativity. Gao splits the class of unitary quantum theories into single-world theories and many-worlds theories before treating each subclass separately. Here I shall consider only single-world unitary quantum theories, leaving to the reader the extension of my critique of Gao's argument to many-world theories.

Suppose Alice and Bob are each in their own laboratories, totally physically isolated from the rest of the world and from each other with two exceptions: they share a pair of particles in state (1), and there is a superobserver able to apply a very carefully tailored local "reset" interaction just to Alice's laboratory. Alice and Bob are going to measure the same component (say z-spin) on the particles of a pair assigned state (1), Alice on particle 1, Bob on particle 2.

In a first scenario, suppose Alice measures (the z-spin of) 1 before (in the lab frame) Bob measures 2, and afterwards (again in the lab frame) the superobserver resets Alice's lab to its original state, thereby also resetting the state (1) of the particle pair. Before Bob measures 2, this sequence of Alice-measurement followed by superobserver reset is repeated many times. For every repetition, the Born rule yields probability 1/2 for each of the two possible outcomes of each of Alice's z-spin measurements on 1. The probability distribution over all Alice's repeated outcomes assigns a relatively high probability to a sequence with an equal number of +1 and -1 outcomes, but a very low probability to a sequence of all +1 outcomes, and to a sequence of all -1 outcomes (where a z-spin positive outcome is labeled +1, and a z-spin negative outcome is labeled -1). Neither Alice nor any other agent can *know* the actual sequence of Alice's outcomes in this scenario because the superobserver's intervention effectively erases all records of the outcome of each of Alice's measurements.

In a second scenario, suppose instead that Bob first measures (the z-spin of) 2 before (in the lab frame) Alice ever measures 1. The perfect anticorrelations predicted by the Born rule now assign probability 1 to the sequence of outcomes of Alice's repeated z-spin measurements in which every one of her outcomes is the opposite of Bob's single outcome, and probability 0 to every other possible sequence of outcomes of Alice's z-spin measurements.

No matter what the actual sequence of outcomes of Alice's z-spin measurements, the probability assigned to it by application of the Born rule differs in the two scenarios. If all Alice's measurements occurred at spacelike separation from Bob's measurement, then the only difference in these scenarios is that Alice's measurements preceded Bob's single measurement in the lab frame in the first scenario, but succeeded Bob's single measurement in the second scenario. But relativity implies that this difference can have no physical relevance because it takes the two scenarios to be physically equivalent—in special relativity they are related by a Lorentz boost symmetry. Gao concludes that quantum theory is incompatible with relativity.

He further agues that Bob's measurement instantaneously influences the outcome of Alice's measurement because after Bob's measurement the probability of any of Alice's subsequent measurements differs from what it would have been had Bob not made his measurement.

3 Refutation of the main argument

In this section I show why a correct application of quantum theory shows why Gao's main argument, restated in the previous section, fails to establish its intended conclusion. I refer the reader to Figure 1.

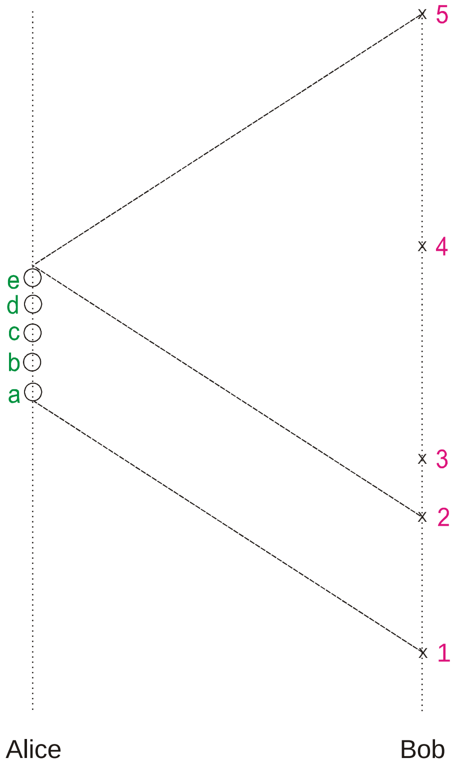

Figure 1: Space-time diagram for Gao's main argument

The vertical lines in Figure 1 represent worldlines of (the centers of) Alice's and Bob's labs, and the circles labeled a - e represent the spacetime locations of five of Alice's repeated spin-component measurements on particle 1. Light-lines emanating from some points on Alice's lab's worldline and terminating at points marked with x's on Bob's lab's worldline are represented by diagonal lines in the figure.

Gao's first scenario is represented in the figure if Bob's measurement on particle 2 occurs in the vicinity of the point labeled 4. As the figure indicates, this event succeeds Alice's measurements a - e in the lab frame, even though it is spacelike separated from all of these. The second scenario is represented if Bob's measurement on particle 2 occurs in the vicinity of the point labeled 3. This event precedes Alice's measurements a - e in the lab frame, even though it is also spacelike separated from all of these. To refute Gao's main argument and ensure the compatibility of quantum theory with relativity in this *Gedankenexperiment*, it suffices to show that correct applications of quantum theory's Born rule yield the same probability distribution for Alice's sequence of outcomes in measurements a - e in both scenarios.

In the first scenario Gao argued that the Born rule yields a probability distribution that assigns a relatively high probability to a sequence with an equal number of +1 and -1 outcomes, but a very low probability to a sequence of all

+1 outcomes, and to a sequence of all -1 outcomes. This is correct, though not because the event marked 4 occurs later than Alice's repeated measurements in the lab frame, but because that event is spacelike separated from those measurements. Since a measurement by Bob at point 4 is spacelike separated from all of Alice's measurements a - e, its outcome is not accessible to Alice before she makes any of those measurements. Consequently she is not in a position to update her assignment of a quantum state to particle 1 and should continue to apply the Born rule to that same (reduced, mixed) state to calculate the probability distribution for her sequence of measurement outcomes. (It is important to note in passing that any such updating would not commit Alice (or the one-world unitarian) to a physical process of quantum state collapse).

In the second scenario Gao argued that the Born rules yields probability 1 for the sequence of outcomes of Alice's repeated z-spin measurements in which every outcome is the opposite of Bob's single outcome of a measurement at point 3, and probability 0 to every other possible sequence of outcomes of Alice's z-spin measurements. This is incorrect. A measurement by Bob at point 3 is also spacelike separated from Alice's measurements a - e, even though it occurs earlier than them in the lab frame. The outcome of a measurement by Bob at point 3 is just as inaccessible to Alice before she makes any of those measurements as would be the outcome of a measurement by Bob at point 4. So, just as in the first scenario, Alice is not in a position to update her assignment of a quantum state to particle 1 and should continue to apply the Born rule to that same (reduced, mixed) state to calculate the probability distribution for her sequence of measurement outcomes. Correctly applied, quantum theory's Born rule predicts the same probability distribution for Alice's outcomes in both scenarios, so no contradiction ensues.

What if Bob's measurement occurred at some point between those labeled 1 and 2 in the figure? In that scenario, Bob's outcome would become accessible to Alice part way through the sequence of her repeated measurements a - e when her past light cone includes Bob's outcome—right after c, for example. Only at that point, Alice should update the quantum state she assigns to particle 1 to a pure state for which (correct) application of the Born rule yields probability 1 for the outcome of her next measurement (d in this case) to be opposite to Bob's outcome, as also for the outcomes of her subsequent measurements (e.g. e). In this third scenario Alice should continue to assign probability 1/2 to her next outcome (such as a or b) for a while, even though (in the lab frame) Bob has already made his measurement!

Bob's measurement does not instantaneously influence the outcome of Alice's measurement. It does not even alter "the" probability of that outcome. The probability of Alice's outcome conditional on a particular outcome of Bob's measurement is unchanged by Bob's actual measurement, whatever its actual outcome. The unconditional probability of Alice's outcome does change as Alice's lab's world line passes into the future light cone of Bob's measurement outcome, but this change is compatible with relativity and does not constitute a causal influence of Bob's measurement or its outcome on Alice's measurement or its outcome (see my ([3])). The argument of ([1]) does not take us beyond

Bell's theorem, and ([2]) does not close a loophole in Bell's theorem.

References

[1] Gao, Shan: "Quantum theory is incompatible with relativity: A new proof beyond Bell's theorem and a test of unitary quantum theory", http://philsci-archive.pitt.edu/16155/ (June 20, 2019).

[2] Gao, Shan: "Closing the superdeterminism loophole in Bell's theorem", http://philsci-archive.pitt.edu/16203/ (July 11, 2019).

[3] Healey, Richard: "Locality, probability and causality". In: Bell, M., Shan, G. (eds.) *Quantum Nonlocalityand Reality*, pp. 172–94. Cambridge University Press, Cambridge (2016).