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Mathematics Embodied: Merleau-Ponty on Geometry and Algebra as Fields of Motor Enaction

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Abstract

This paper aims to clarify Merleau-Ponty's contribution to an embodied-enactive account of mathematical cognition. I first identify the main points of interest in the current discussions of embodied higher cognition and explain how they relate to Merleau-Ponty and his sources, in particular Husserl's late works. Subsequently, I explain these convergences in greater detail by more specifically discussing the domains of geometry and algebra and by clarifying the role of gestalt psychology in Merleau-Ponty's account. Beyond that, I explain how, for Merleau-Ponty, mathematical cognition requires not only the presence and actual manipulation of some concrete perceptible symbols but, more strongly, how it is fundamentally linked to the structural transformation of the concrete configurations of symbolic systems to which these symbols appertain. Furthermore, I fill a gap in the literature by explaining Merleau-Ponty's claim that these structural transformations are operated through motor intentionality. This makes it possible, in turn, to contrast Merleau-Ponty's approach to ontologically idealistic and realistic views on mathematical objects. On Merleau-Ponty's account, mathematical objects are relational entities, that is, gestalts that necessarily imply situated cognizers to whom they afford a specific type of engagement in the world and on whom they depend in their eventual structural transformations. I argue that, by attributing a strongly constitutive role to phenomenal configurations and their motor transformation in mathematical thinking, Merleau-Ponty contributes to clarifying the worldly, historical, and socio-cultural aspects of mathematical truths without compromising what we perceive as their universality, certainty, and necessity.

Keywords: Merleau-Ponty; phenomenology; enactivism; higher-order cognition; motor intentionality; mathematical reasoning

1. Introduction

In classical cognitive science as well as in other philosophical traditions, mathematical reasoning has been viewed as the prototype of "higher-order" cognition that requires the possession of mental representations, along with memory, imagination, or abstract thought. On this view, the abstract nature of mathematics and the universality, certainty, and necessity that characterize its truths constitute an unsurmountable difficulty for theories grounding cognition in environmentally situated perception and concrete bodily action. However, a number of authors have challenged this view, providing suggestions on how to theorize mathematical reasoning and other types of abstract cognition from an embodied enactive perspective (see Gallagher, 2015; 2017, 204-212; 2019; Fabry, 2018; Hutto, 2019; Zahidi & Myin, 2016; Zahidi, 2021).

Taking up the approach of the early proponents of embodied cognition, many authors have returned to phenomenology and gestalt psychology to critically review and elaborate on the current embodied frameworks (see Isaac & Ward, 2019). Merleau-Ponty often plays an important role in these endeavors (see, e.g., Muller, 2021; Kee, 2021; Sheredos, 2017). Kiverstein and Rietveld (2021), for example, draw on Merleau-Ponty's philosophy of language to argue that there is no need for mental representation in abstract, absent, or counterfactual thinking, since these types of cognition are best understood in terms of skilled intentionality (178) through which we interact with an "enlanguaged environment" (176).

Following this line of inquiry, my aim in this paper is to clarify Merleau-Ponty's contribution to an embodied and enactive account of mathematical cognition. Much like current theorists, Merleau-Ponty refers to mathematical reasoning as one of the paradigmatic examples of higher-order cognition (see, e.g., 1973, 118; 2010, 51). More importantly, Merleau-Ponty provides concrete arguments on how embodied motor action contributes to mathematical reasoning. He thereby engages in discussions that are central for enactivism and faces similar challenges, such as the question of the potential "scaling up" from or "reuse" of sensorimotor experience in abstract and symbol-based cognition. I argue that by formulating a strongly constitutive explanation of the role of phenomenal configurations and their motor transformation in mathematical thinking, Merleau-Ponty contributes to clarifying the worldly, historical, and socio-cultural aspects of mathematical truths without compromising what we perceive as their universality, certainty, and necessity.

However, it is important to note that Merleau-Ponty did not formulate a unified account of mathematical cognition himself and that the position attributed to him in this paper is my reconstruction. I believe that Merleau-Ponty's potential for the philosophy of mathematics has been sufficiently described within neither the specialized scholarship nor the discussions on embodied cognition. When the topic was addressed, it elicited contradictory reactions, in particular because of Merleau-Ponty's emphasis on the role of embodiment and perception.[1] My presentation primarily revolves around two main passages from Merleau-Ponty's works that directly deal with reasoning in geometry and algebra. However, I synthesize the explicit arguments from these passages with many fragmentary analyses and comments dispersed throughout Merleau-Ponty's official texts, posthumously published lecture notes, and published or unpublished working manuscripts. Additionally, I consider Merleau-Ponty's interpretations of topics such as neuropsychological impairments or motor intentionality, which have not been previously connected to his discussion of mathematics. Therefore, apart from clarifying Merleau-Ponty's potential to enrich discussions of contemporary embodied and enactive accounts of mathematics, I aspire to deepen scholarly understanding of his philosophical contribution in this area.

To clearly articulate Merleau-Ponty's account of mathematics and connect it with current discussions in embodied and enactive cognition theories, the following section (2) briefly identifies the main points of interest in these approaches to mathematics and explains how they relate to Merleau-Ponty and his sources, in particular Husserl's late works. In the two subsequent sections, I explain these convergences in greater detail by more specifically discussing the domains of geometry (3) and algebra (4). Throughout these discussions, I focus particularly on clarifying

[1] For example, Cassou-Noguès (1998) is mostly critical, while Hass and Hass (2000) and Matherne (2018) are appreciative. See also Besmer (2007) and Baldwin (2013), whose positions are mixed.

Merleau-Ponty's original transformation of Husserl's idea of "institution" of abstract objects (*Stiftung*) and the fundamental role of gestalt psychology in Merleau-Ponty's account.

2. Enactivist themes in Husserl and Merleau-Ponty

Some contemporary authors have recognized the relevance of Husserl's phenomenology for an enactivist account of higher cognition. However, there is no agreement on the limitations of its relevance, particularly because of Husserl's inclination to transcendental idealism. My aim in this section is to identify the most important aspects of this discussion and establish the context for demonstrating how these aspects are transformed in Merleau-Ponty's account.

In his attempt to outline an enactivist approach to mathematics, Gallagher (2015; 2017, 204-212) draws on a number of contemporary authors, but also refers appreciatively to Husserl's late texts on geometry and mathematized science (1965; 1989). However, in his response to Gallagher, Hutto (2019) argues that some of Gallagher's sources seem incompatible with the central tenets of enactivism. In particular, Hutto claims that enactivists should reject mind-centered constructivism and idealism (2019, 835)[2] and emphasizes that they should instead place "the greater weight on the contributions of socio-cultural practices" (2019; 835) as suggested by Zahidi and Myin (2016). In his response, Gallagher (2019, 845-849) generally agrees on the points raised by Hutto (2019),[3] but indicates that his use of source literature is motivated by his effort to bolster the interpretation of mathematics as a "doing" or an embodied practice within a particular type of "affordance space" that includes physical and social aspects. Within this framework, Gallagher's aim is to show that embodied practices serve "as the source for mathematical practices and operations" (2019, 848).

In effect, Gallagher (2015, 343-345; 2017, 207-212) appreciates Husserl for recognizing the pragmatic roots of mathematics and considering abstract mathematical conceptions as having been derived from concrete living practices such as the art of surveying. Gallagher also perceives a convergence between recent attempts to ground abstract thinking and mathematics in embodied-environmental processes (Lakoff & Johnson, 2003; Lakoff & Núñez, 2000) and Husserl's appeal for a clarification of how mathematics and mathematized science acquire their sense "based on life and the intuitive living environing world" (Husserl, 1965, 186; cited by Gallagher, 2017, 208). Moreover, Gallagher positively values Husserl's inquiry into mathematics as a cultural accomplishment and a tradition that is passed on by social learning and training, since it prefigures contemporary research on the role of enculturation in the development of mathematical abilities (e.g., Menary, 2013; 2015; see also Fabry, 2018; Hohol, 2020, 121-142; Zahidi, 2021; Zahidi & Myin, 2016).

Gallagher's demonstration of Husserl's relevance for enactivism can be elaborated further. As already well known, Husserl presents a theory of a three-step "institution" (*Stiftung*) of ideal meaning, which consists in an original insight, intersubjective linguistic expression, and a "documentation" by "writing-down," which assures the persistence of ideality independently of an actual presence of concrete subjects and their communication (see Husserl, 1989, 164; cf. Blomberg, 2019; Lawlor, 2002). Interpreting these passages, both Derrida (1989, 189) and Merleau-Ponty

2 I note that Hutto does not explicitly connect idealism with Husserl. However, among the authors discussed by Gallagher, Husserl seems to be one of those who may be labelled an idealist.

3 In connection to this discussion, see also Hohol (2020, 89-108), who provides an analogical criticism of the idea of neural simulation of sensory experiences based on his review of literature that includes the works discussed by Gallagher and Hutto. However, Hohol adopts a moderately embodied stance and does not entirely reject the notion of representation.

(2002, 25-26) have independently stressed that the physical existence of signs in a written form is therefore *necessary* for the constitution of mathematical objects in their persisting existence. Hence, in my view, enactivists can draw on Husserl at least to the extent to which he describes how mathematics involves pragmatic, material, genetic, intersubjective, and cultural factors.

However, Gallagher (2017, 208-209) also hints at several aspects that limit Husserl's contribution to an embodied account of mathematics. In particular, mathematical truths have an ideal objectivity for Husserl and are ultimately anchored in intra-subjective insights. In effect, Husserl's engagement with mathematics is oriented toward anamnesis, recollection, and recovery of what was once given as self-evident (see Baldwin, 2013, 308-309, 323-324; Blomberg, 2019, 85; Hass & Hass, 2000, 184-185; Watson, 2016, 48). His inquiry focuses on unearthing, below the materially and socio-culturally transferred tradition, the transcendental conditions of the possibility of geometry (Gallagher, 2017, 209; cf. Baldwin, 2013, 309; Hass, 2008, 165-166). From a current perspective, mathematical cognition seems to be constituted through co-speech gesturing (Goldin-Meadow et al., 2001), the actual physical movement of body parts, and the manipulation of external material objects and perceptible symbols (Gallagher, 2019, 847; see also Fabry, 2018; Malafouris, 2013; Menary, 2013; 2015; Overman, 2016; 2018; Zahidi, 2021). For Husserl, however, much as for the traditional cognitive science, perceptible symbols acquire their meaning from meaningful thoughts, and their actual manipulation does not by itself add anything to the signification they convey (cf. Gallagher, 2017, 205, 211).[4] Thus, while enactivists argue that different material forms have a productive role in mathematical cognition in that they "impose order and structure" (Zahidi, 2021, 542), for Husserl, the writing-down "changes the mode of being of the original [geometric] sense-structure" (1989, 164), but does not determine this structure itself. Geometry ultimately remains an a priori science for him (180). As Baldwin (2013, 314-15) points out, since the role of "material propositions" is, for Husserl, to recollect the original self-standing evidence, their value in mathematics is primarily just "instrumental" for him.

Hence, it becomes clear that Husserl's account of mathematics shares a common trait with Hutto's in that they are both variants of ontological objectivism. Hutto (2019, 835) criticizes Gallagher (2017, 6) for endorsing the idea that we enact our worlds and that the world is therefore not pre-given. Hutto instead appeals to mathematical realism, since in his view, all other alternatives undermine the objectivity of mathematical truth (2019, 835). More specifically, he recommends that we "embrace both mathematical realism and conceptual constructivism at the same time" (835). For him, the role of mathematical techniques, tools, concepts, and symbolic practices is to "get a grip" on "the subject matter of mathematics [that] might be objective and mind-independent" (835). The role of mathematical formulas and practices is instrumental for Hutto, much like for Husserl. In contrast, Gallagher seems to allow for a stronger sense of enaction. In any case, the adoption of ontological objectivism and epistemological instrumentalism with regard to mathematics is far from self-evident. In alignment with the emphasis on socio-cultural practices, the universality and general validity of mathematical truths has also been interpreted as the intersubjective repeatability of mathematical demonstrations (see, e.g., Hohol, 2020, 134-138). Importantly, Merleau-Ponty develops Husserl's (1989) ideas on mathematics in a similar direction (see, e.g., Merleau-Ponty 2002, 6-8; cf. Lawlor, 2002, 217-18; Robert, 2000, 361-62).

Considering these discussions, it is not difficult to see how Merleau-Ponty not only prefigured many of the ideas on higher cognition reclaimed by enactivism, but also offered

4 Husserl's thoughts on this matter developed over time. While in his late discussions on the role of language and writing, Husserl (1989) comes close to the idea that expression plays a constitutive role with regard to ideal meaning, he explicitly rejects this in his earlier works (e.g., 1982, 296).

important clues on how to elaborate them. For example, Merleau-Ponty's interpretation of speaking speech as "operative" intentionality or a "doing" that subtends abstract thought[5] converges with the enactivist emphasis on practices in all domains of cognition. Correspondingly, Merleau-Ponty's early gestural theory of speech and, more convincingly, his interpretation of speaking speech as a coherent divergence from the established structures of symbolic systems allows for an integration of gestural sense-making into language (see Cuffari, 2012; Kiverstein & Rietveld, 2021). Further, although Merleau-Ponty did not specifically emphasize materiality, his analyses of the perceptible structure of symbols and our bodily interaction with them is clearly compatible with theories of material engagement and enculturation. Merleau-Ponty argues that thoughts do not preexist, but are rather continuously brought into existence through an environmentally, linguistically, and socially situated "expression."[6]

Correspondingly, Merleau-Ponty considers symbols, diagrams, and other mathematical notations to be culturally constructed "apparatuses of knowledge" that belong to a system of signification that "is not timeless" and always involves "historicity" (2010, 54, 58). For Merleau-Ponty, the impression of a timeless signification, which is paradigmatically evident in mathematical objects, does not stem from them being objective realities, but from the "retroactive effect of the true" that is produced by spatio-temporally situated expressive acts (1988, 29).[7] Merleau-Ponty thereby points to the fact that what will have been invented in mathematics will be seen as "operative" before this invention and thus constitute a sort of "retrospective illusion" (2010, 52, 56).[8] With respect to his views on the fundamental historicity of mathematical knowledge, Merleau-Ponty is of course significantly influenced by Husserl.[9] Yet, for Merleau-Ponty, the historically conditioned expression does not threaten an original intra-subjective evidence, but is productive with regard to signification and is therefore constitutively necessary for mathematical cognition (see Baldwin, 2013, 321; Hass, 2008, 148-55). In contrast to Husserl's backward-oriented, regressive inquiry into the transcendental origin of geometrical tradition, Merleau-Ponty's theory of abstract cognition is future-oriented, explorative, and rooted in a "productive epistemology" (Hass 2008, 168; cf. 151).[10] As Hass and Hass (2000, 183) have argued, Merleau-Ponty's philosophy thereby shows the ability to celebrate rather than suppress the plurality of formal systems in mathematics, which is difficult to achieve from an ontologically objectivist stance.

5 Speech "is a *praxis*," Merleau-Ponty claims, and the mathematical ideality appears "at the edge of speech" (2002, 56, 46). Regarding speech as operative intentionality implicated in abstract thought, including mathematics, see also Merleau-Ponty (1968b, 155, 188). For analyzes of Merleau-Ponty's notion of speaking speech in a broader context, see Baldwin (2007); Kee (2018); Kiverstein and Rietveld (2021).

6 While Merleau-Ponty emphasizes the role of perceptual and bodily expression from early on, he properly grasped the social and linguistic aspects only after 1947, when he adopted elements of structural linguistics. See, for example, Merleau-Ponty's description of the process through which we express our thoughts within a dialogue (1973, 133-46). For interpretations of Merleau-Ponty's theory of expression, see in particular Fóti (2013), Hass (2008, 146-92), Kee (2018), and Landes (2013).

7 Merleau-Ponty is alluding here to Bergson's (1946, 7-31) idea of the "retrograde movement of the true."

8 For a more elaborate discussion of this point, see section 4.3.

9 Regarding this point, see Merleau-Ponty's discussion of the idea of *Stiftung* (1970, 161-67; 2002, 16-65; 2010, 50-61). For interpretations of Merleau-Ponty's relationship to Husserl (1989), see Baldwin (2013); Besmer (2007); Hass and Hass (2000), Hass (2008, 146-69); Lawlor (2002); Robert (2000); Vallier (2005); Watson (2016).

10 In contrast to this view, Besmer's (2007) reading of Merleau-Ponty, for example, emphasizes his continuity with Husserl and is correlatively origin-nostalgic, past-oriented, and teleologically objectivistic (see, e.g., 135-136). However, I believe that since Merleau-Ponty ultimately defined our relation to reality as "interrogative" and thus open-ended, he would not accept a teleologically objectivistic view (see, e.g., 1996, 375).

Beyond an elaboration of these convergences, my goal in the following two sections is to clarify what Merleau-Ponty has to add to an enactivist account of higher cognition from his distinctive perspective. Above all, I explain how, for Merleau-Ponty, mathematical cognition not only requires the presence and actual manipulation of some concrete perceptible symbols but, more strongly, how it is fundamentally linked to a *structural transformation of concrete configurations* of symbolic systems to which these symbols appertain. Moreover, I fill a gap in literature by explaining Merleau-Ponty's claim that these structural transformations are *fundamentally linked to motor intentionality*. This makes it possible, in turn, to contrast Merleau-Ponty's approach to both idealism and realism, and to outline a *relational account of mathematical cognition*. Following this idea, I aim to show how Merleau-Ponty contributes to our understanding of mathematical truths as sensorimotorically, socio-culturally, and linguistically structured gestalts that acquire and maintain their organization in relation to a community of human subjects and that reciprocally afford these subjects certain type of engagement in the world.

3. Geometry embodied

Merleau-Ponty argues that there are "close links" between our motricity (*motricité*) and our practical relationship with space on the one hand and "all symbolic functions" such as our geometrical knowledge on the other (Merleau-Ponty, 1970/1968a, 8/17–18).[11] A demonstration of this idea can be found in the well-known chapter "The Cogito" from the *Phenomenology of Perception* (2012). My goal in this part of the paper is to explain the fundamental claim Merleau-Ponty makes in "The Cogito," according to which the "subject of geometry is a motor subject" (2012, 406). I aim to clarify this idea by connecting it with Merleau-Ponty's subsequent analyses from *The Prose of the World* (1973/1969) and his lectures from the Collège de France (2002; 2010; 2020a; 2020b).

3.1. The geometrical object is a structure

Before directly examining Merleau-Ponty's reasons for supporting the idea that geometry is fundamentally linked to our embodiment and motricity, it is necessary to clarify how he understands the *object* to which the geometer relates. After first explaining why Merleau-Ponty rejects two symmetrical approaches, which, in his view, define the geometrical object inaccurately and miss its spatio-temporal situatedness or its ideality, I build on this symmetrical critique to formulate a positive answer on the role of the body in geometry according to Merleau-Ponty.

Merleau-Ponty argues that the cognitive operations yielding specifically geometric evidence concern neither definitions, concepts, ideas, nor essences of geometric objects (2012, 403-407; 1973, 123/173). He qualifies all these entities as products of formalization, which he understands as a spatio-temporally situated process through which we "construct increasingly general expressions of the same fact" (1973, 106/150).[12] This view has two important implications. First, the types of

[11] The available English translations of Merleau-Ponty's *Résumés de cours* (1968a) and *La prose du monde* (1969) contain many inaccuracies. I have modified the passages cited from these texts as noted and I invite the reader to compare the French originals by following the pagination indicated after the slash in the citations.

[12] Cassou-Noguès (1998, 398) and Baldwin (2013, 305, 325) criticize Merleau-Ponty for not acknowledging formalization as a specifically mathematical procedure and as an evidence-providing instrument. Yet neither Cassou-Noguès nor Baldwin discusses Merleau-Ponty's own interpretation of formalization and his explicit criticism of this procedure. Moreover, Cassou-Noguès' and Baldwin's interpretations seem to lack an accurate

description used for geometrical objects need to be understood as founded on more original, less formalized types of experience of these objects. Since symbolic systems such as mathematics are "never more than relatively formal," Merleau-Ponty argues, the validity of the more formalized modes of description is necessarily retrospective, derivative, and abstract (2007, 288; cf. 1964a, 102; 2012, 405; 2002, 66). Consequently, Merleau-Ponty refuses to accept the idea that highly formalized mathematical descriptions contain "in advance" or "implicitly" all the evidence that will in fact become demonstrated throughout the history of geometry (2012, 407; cf. Hass & Hass, 2000, 182, 184-85).

Merleau-Ponty contends that the more formalized symbols always draw their meaning from "qualitatively defined," spatio-temporally situated configurations of geometric problems (2007, 288). In other words, mathematical truth is not *behind* and *beyond* mathematical symbols, as a reality behind and beyond its phenomenon. Such truth is always embedded in a specific natural and cultural situation and circumscribed by mathematical signs or symbols that constitute "a certain field of thought" for the person who thinks (1973, 105/148; cf. 2002, 18-19; 2010, 58-61). Far from delivering intuitively accessible apodictic evidence and a priori truths, geometric thought belongs to a certain tradition of cultural construction, which has a horizon necessarily involving the past and the future.[13] It involves acquired results of previous collective human endeavors—no longer perceived as *products of* creative efforts—and a promise of future results (2002, 28-31).

On the other hand, geometric knowledge does not involve a mere "drawing" either. A geometric symbol is irreducible to "an assemblage of lines fortuitously born beneath my hand," and is not simply a result of "the actual movement of my hand and my pen upon the paper" (2012, 403, 405). An ideal signification is not included *in* the material signs of a symbolic system (2010, 58), nor does it correspond to a mere "factual presence" of its signs (1969, 149).[14] Merleau-Ponty thus unreservedly acknowledges a fundamental difference between a perceptual and a mathematical signification, that is, between a drawing and a geometric figure.[15] Elaborating on Wertheimer's (1938) and Gurwitsch's (2009, 56-57) reflections, Merleau-Ponty points out that a geometric object is significantly more independent from its phenomenal context than a perceptual object: when one introduces new lines into a drawing (see 2012, 403; 1973, 119-120/167-68), or when the context of a perceived object is significantly altered (see 1973, 104/147-48; 2010, 54), the perceptual signification may become completely altered as well. In contrast to that, the phenomenal transformations related to a geometric object do not transform it into a *different* object (2012, 404).

account of the presumed counterpart of formal thought, namely, "intuitive" thought (a term Merleau-Ponty uses only very rarely, e.g., 2012, 405). Baldwin (2013, 326) claims that Merleau-Ponty rejects formal thought in favor of "informal" thought, but this is unconvincing. Unlike Husserl, Merleau-Ponty does not proceed with the idea of an intuitive insight, since he considers all experience dependent on habitual bodily schematizations and, more strongly, on socio-cultural acquisitions, which always formalize our experience at least to some degree. In contrast to Cassou-Noguès and Baldwin, Hass and Hass (2000, 182) explicitly analyze Merleau-Ponty's account of geometry in relation to formalism in mathematics and find Merleau-Ponty's position persuasive. In their view, a transition from premises to conclusions is never a purely formal but rather an "expressive" or structurally productive operation (cf. Watson, 2007, 536-37; Gallagher, 2017, 208). Moreover, as Hohol (2020, 135) explains, the contemporary formal approach to proving practice in geometry was completely unknown to the inventors of Euclidian geometry.

[13] Similarly, Hohol (2020, chapter 4) recently argued that Euclidian geometry is a cognitive artefact that cannot be explained universalistically.

[14] The English translation (1973, 105) of this passage is incorrect.

[15] In my opinion, Cassou-Noguès fails to recognize this point (see, e.g., 1998, 382).

Rather, insofar as these events have any relevance to the geometrical object at all, they *become integrated* into the signification of that object (see 2010, 54; 1973, 104-105/148).[16]

In that respect, Merleau-Ponty (2012, 404) explicitly acknowledges Gurwitsch's argument (2009, 58-61) against Wertheimer (1938) and gestalt psychology in general, according to which a mathematical entity such as a triangle cannot be interpreted as a perceptual gestalt, because its significance is not directly dependent on the perceptual context. However, Merleau-Ponty also refuses Gurwitsch's (2009, 54-61) Husserlian account, according to which an ideal mathematical entity is the correlate of a "categorial thought" or attitude and consequently detachable from the spatio-temporally concrete, historically established, and forever-open phenomenal field. Merleau-Ponty (2012, 126) argues that the presumed symbolic or representational function that enables a "categorial" attitude "rests upon a certain ground" and the error of intellectualism is "to make it depend upon itself, to separate it from the materials in which is realized." In contrast, Merleau-Ponty (2020b, 121) argues that the categorial attitude results from the labor accomplished through active symbol-based structuration (*Gestaltung*) of the relationship between a subject and the world (see also 2020b, 114-35; 1970, 22-23).

In short, the discovery of a mathematical truth is a particular case of a "cognitive process [that is] trans-phenomenal without being open to essences" (2010, 58). In other words, the ideal objects of geometry are not perceptible as such but can be accessed only from within the perceptual world; they are on the horizon of perceptual entities rather than constituting a separate domain. Perceptually accessible mathematical signs are, thus, "neither primary nor secondary" with regard to mathematical signification (1973, 111/158). They are two inseparable aspects of one experience that have a very particular structural relationship that must be described. However, while a symbolic signification such as a geometric principle can never become fully detached from the spatio-temporal field and the perceptible signs situated within it, it does have distinctive characteristics that make it possible to substitute the signs expressing it indefinitely, in a particular spatio-temporal situation, through using different signs (see 1973, 110-111/157). Having once grasped the signification of a triangle, one can obviously refer to it by diverse means such as material drawings, imaginary figures, or linguistic statements. Yet this does not mean that the ideal signification exists or is accessible without them. On the contrary, the ideal meaning subsists precisely by specifically organizing our embodied-perceptual and socio-cultural experiences and the figures and signs that correspond to them: it is a standard for a certain type of meaningful experience or a relationship with the world that presents itself as if it "will never wear out" (2010, 53; cf. 54) and will be "forever taken up" (2012, 414). It henceforth imposes itself as a quasi-universal means of understanding the world, which cannot be overcome by a simple cancellation: the discipline can only think differently by inventing a more comprehensive and thus universal means of understanding the world and relating to it (cf. 1964b, 154).[17]

[16] It is important to note that in the passages interpreted here, Merleau-Ponty speaks of drawing in a very narrow sense. However, in his other writings, he considers drawing and painting as expressive operations in their own right and acknowledges their capacity to bring forth a specific type of generality. Therefore, the difference between the generality of an expressive drawing and a geometric figure should be understood as of a degree and not of a kind.

[17] For example, the development of a non-Euclidian geometry enables a transition from a still naïve Euclidian expression of space to a *less naïve* alternative, not to an absolute truth (cf. Merleau-Ponty 2012, 414; 1973, 100/141, 103/146, 127-28/178-79). Euclidian space is by no means a priori for Merleau-Ponty, and even if it might be considered privileged in relation to other expressions of space, it is a historical invention and its privileged position is not absolute (1968b, 213; cf. Hass, 2008, 166). As Hass and Hass (2000, 180-81) note, "mathematical truths are historically and geographically located" and the demonstrative value of mathematical proofs comes from the fact that the mathematical object perseveres throughout the structural transformations involved in the proof (cf.

In contrast to a concrete perceptual object, therefore, a mathematical object has the power to inaugurate an order of signification in which phenomenal changes either are irrelevant or manifest the same object more comprehensively. An ideal signification is never fully detached from its phenomenal context, but the specific nature of the signs presenting it raises the threshold, as it were, above which the phenomenal context affects their meaning.[18] Additionally, however, the peculiar subsidiarity of the signs with respect to a mathematical meaning, that is, the possibility for us to largely substitute one set of signs for another while still relating to the same object, also makes ideal signification vulnerable to a time-bound obfuscation. As Husserl clarified, the possibility of ideality inherently contains the possibility of forgetfulness (cf., e.g., Merleau-Ponty, 1970, 120/167; 2020b, 134). The procedure of formalization, for example, weakens the demonstrative power of the original configuration. As an instrument capable of producing trans-phenomenal significations, idealization *ipso facto* threatens us with the possibility of impoverished thought and of repetition without understanding.

3.2. The geometrical object is a modality of one's relationship to the world

Once we have identified the status assigned by Merleau-Ponty to ideal entities such as geometrical objects, we are better situated to clarify in what sense the body plays, in his view, a fundamental role in our relationship to them. For this purpose, it is instructive to analyze more closely Merleau-Ponty's interpretation of the demonstration of the sum of a triangle's angles being equal to two right angles (2012, 403-408; based on Wertheimer, 1938, 279-80).[19]

Merleau-Ponty (2012) argues that one can only grasp a properly geometrical signification by relating to a "configuration" of space circumscribed by a triangle situated in the oriented space of one's visual field or in the field of one's visual imagination (405).[20] Moreover, Merleau-Ponty (2012) asserts that the system of "spatial positions" circumscribed by the triangle is also "a field of possible movements" for a motor subject (406).

As we have seen, a geometric figure is given neither as a positive fact registered by the senses nor as a transcendent essence intuited by reason. Rather, Merleau-Ponty calls it a *concrete essence*, that is, a *gestalt* situated in one's perceptual field or a certain *structure* of that field (2012, 406, 404). Unlike an idealized essence, a concrete essence expresses both the generality and the particularity of a given phenomenon (cf. 127). Consequently, a geometrical figure appertains to one's *relation to* the world rather than being a part of the world, a material or ideal object. As Merleau-Ponty explains, a triangle is related to my perceptual and motor fields as "the formula of an attitude, a certain modality of my hold on the world," and as a "motor formula" (406). Taking up

Merleau-Ponty 2012, 405). In accordance with this view, Hohol (2020, chapter 4.4.) argues that the main source of generality and necessity of Euclid's proofs is the intersubjective repeatability of reasonings "scaffolded on the consistent use of [cognitive] artifacts" such as geometric diagrams and formulas (2020, 137).

18 One may understand this phenomenon by analogy with threshold phenomena in perception (cf., e.g., Merleau-Ponty 2012, 9).

19 Merleau-Ponty later briefly interpreted an analogous geometric problem, namely, the calculation of the area of a parallelogram (2010, 55; based on Wertheimer, 2020, 14-78).

20 In his interpretation of geometric demonstration, Merleau-Ponty does not address the differences between the actual visual field and the imaginary visual field. Such an explanation is, however, necessary if Merleau-Ponty's argument is to be made entirely plausible. Generally, Merleau-Ponty interprets the imaginary field as founded on some elements of the perceptual world and refuses to conceive of the imaginary world as a purely mental domain (cf., e.g., 2010, 46-50; 1970, 48, 68-69). From this point of view, a transition to the field of imagination does not affect the fundamental aspects of Merleau-Ponty's argument concerning geometry. For an analogical contemporary attempt to ground imagination in embodied action, see Rucińska and Gallagher (2021).

what Merleau-Ponty writes about a memory, one could say that a triangle circumscribes "a certain unique position of the index of being-in-the-world" (1970, 51/72). A triangle makes our situation appear as "a particular case in a family of situations" (1973, 107/151) and thus specifically modifies our perspective on the world.[21]

More precisely, a triangle as a perceptual *figure* has a systematic relationship to our "body schema," which plays the role of providing a dynamic perceptual *ground*.[22] The body understood in this manner is "a power of various regions of the world" (2012, 108), that is, a system of sensorimotor capacities that allows us to accommodate a certain range of perceptual figures. For example, as one's body sensorimotorically situates itself in space, it establishes the "anchorage points" of one's visual field and thereby the spatial level necessary to maintain one's perceptual orientation in space (see 2012, 254-65; 2020a, 33-42). As a perceptible phenomenon, a geometric figure is correlative to a certain engagement of our body in the world in the same manner as regular perceptual figures are. For this reason, the figure of a triangle is not "congealed and dead" and remains open to transfigurations depending on how we motorically explore the "untraced yet possible directions" it initially circumscribes (2012, 406).

Since the mode of givenness of a geometric figure is involved with the perceptual aspect of experience, Merleau-Ponty contends that specific geometric properties are never accessible to us on the basis of a simple logical analysis of a geometric object as a mental representation. Rather, one grasps particular aspects of a geometric object such as a triangle by *transforming* the phenomenally concrete configuration of a triangle. That is, the geometer intervenes in the figure of the triangle "as the pole toward which [their] movements are directed" (2012, 405); they "explore" the spatial configuration the triangle opens for them, they situate themselves "at one point and from there tend toward another point" (406); they extend a side of the triangle, then draw a line through the vertex that is parallel with the opposite side, and so on (404). It is, Merleau-Ponty argues, through *reorganizing* a concrete phenomenal configuration (*Neugestaltung*), for example by constructing auxiliary lines, that the geometer comes to see that the sum of the angles of a triangle is equal to two right angles.[23] The evidence one experiences in this way is of a perceptual type even though it does not exclusively concern concrete objects.

In short, Merleau-Ponty argues that one is brought to grasp a certain type of distinctively geometric (and not just perceptual) evidence by organizing *a specific perceptual configuration* of a triangle. To grasp a geometric truth, it is not sufficient to rely on *just any* among all the possible notations and representations of a triangle: an original access to geometric truth is correlative to a *specific* phenomenal configuration (cf. 2010, 56). Once a particular geometric property is made accessible, the particular phenomenal configuration that made it evident is substitutable with different configurations, yet nothing changes in relation to the fundamental role of the original configuration in granting us access to the geometric signification in the first place. Since a triangle, for example, appertains to my relationship to the world rather than being merely an ideal or real object, its signification is, to a certain degree, dependent on how I relate to it and how I situate it within specific and possibly evolving phenomenal configurations. Thus, despite the fact that an ideal signification is not simply contained in the signs of a given symbolic system insofar as they are perceptible, the geometrical meaning remains open to the dynamic organization of the perceptual field and, above a certain threshold, is affected by the field's transformations.

21 Building on Merleau-Ponty, Irwin (2017) has developed an analogical interpretation of abstract words.

22 For a detailed explanation of how the body schema provides the ground for perceptual figures according to Merleau-Ponty, see Halák (2021a, 35-38).

23 Cf. Wertheimer's detailed discussions of analogic geometrical examples (e.g., 2020, 18).

3.3. The subject of geometry is a motor subject

Given the clarifications presented in the previous section, it is possible to identify more precisely two main reasons why the body is necessary for thinking in geometry. First, the event of grasping a geometric signification is not "without a place in the world" but involves a "movement" that proceeds "from a certain here toward a certain there" (2012, 407). Geometric figures and their relationships always remain spatio-temporally situated in the experiential field opened in relation to a sensorimotor body, despite transcending the perceptual level of signification of that field and relying on culturally developed systems of signification. Second, the body is an agent capable of reorganizing the experiential field of geometry by changing the relationships pertinent to specifically geometric, not just perceptual, configurations. As Merleau-Ponty (2012) puts it, the acquisition of a certain type of geometric evidence is an act that requires the geometer to "trac[e] out the spatio-temporal distance by crossing it" (407), at least virtually, "with [their] body" (406). Our inherited cultural schemes of geometry neither contain these transformations in advance nor do they inherently require us to deduce the transformations from them. It is in this sense that the body is required for the transformations to happen. In short, one's cognitive relationship to a geometric entity such as a triangle is embodied because it is originally dependent on a concrete perceptual configuration, which is itself, at least in part, dynamically correlated to our sensorimotor activity, and provides a perceptual norm for its development.

Merleau-Ponty thereby rejects the intellectualist idea that "it is a matter of indifference how among the diverse manners the triangle can be drawn" (Gurwitsch, 2009, 60). It is only below a certain threshold and not absolutely, Merleau-Ponty argues, that mathematical objects are independent of how they appear and are given in perspective. Merleau-Ponty holds that we can genuinely think in mathematics, and that there is "life" in mathematics as a discipline, only through changing the way in which traditionally inherited mathematical structures are concretely organized in our actual field of experience. Geometric understanding results from a specific exploration of a phenomenal field correlative to a given geometric space, and the acquisitions resulting from this exploration, therefore, always remain open for further exploration.

Moreover, if mathematical acquisitions are fundamentally linked to embodiment in this way, our access to them is also endangered by conditions that reduce the complexity of our bodily relationship to the world (cf. Kiverstein & Rietveld, 2021, 182-84). In people affected with neurological bodily pathologies, for example, the reorganization of geometric structures required for grasping a geometric truth may become impossible. This occurred in the case of Gelb and Goldstein's brain-injured patient Schneider, whose condition was also interpreted by Merleau-Ponty.

Apart from a range of sensorimotor difficulties, Schneider suffered from higher-cognitive impairments that limited his capacity to use language productively and to carry out geometrical and arithmetical operations. However, Merleau-Ponty refuses to link his impairment to either purely physiological or intellectual processes and argues that it is situated *at their junction* (2012, 132; cf. Halák 2021b). Schneider did not simply lose a presumed general capacity for thinking or adopting a "categorial attitude," as Gelb and Goldstein claimed. His difficulties became apparent when he was required to undertake what Goldstein has called an "abstract movement," which is "not directed towards any actual situation" and is not carried out against the background of the "given world" but on a "constructed" background (Merleau-Ponty, 2012, 105, 113). Thus, Schneider intellectually understood what a triangle or a square is, and the relationship between these significations did not escape him (Merleau-Ponty, 2012, 133), but he could not access any geometrical properties beyond those evident from the geometric structures as they were factually presented to him. Such insight requires performing an "abstract movement," that is, transforming the phenomenal structure of the geometric object. For example, Schneider understood that "triangles fit inside squares, *but not if the triangles [had] to be rotated*" (Hass, 2008, 82; emphasis added; cf. Merleau-Ponty, 2012, 133). Schneider's corporeal deficiency expressed itself as a lowered capacity for the structural transformation of geometric acquisitions available to him.

Schneider's intellectual cognition was affected because his phenomenal field had lost the "plasticity" that makes it possible for a healthy individual to accommodate types of organization that transcend the level of complexity of the perceived world as it is given at any one moment (Merleau-Ponty, 2012, 113). For Schneider, as for everyone else, a particular positioning of a geometric object in the phenomenal field affords the comprehension of some properties and constrains access to others; yet Schneider was unable actively to explore a given configuration and articulate a geometric object in a different way. As a pathological inversion, Schneider's case thus confirms Merleau-Ponty's contention that the subject of geometry is a motor subject: a decreased capacity to articulate objects through motor intentionality, caused by a brain injury, correlates with Schneider's decreased capacity to understand geometric relationships.

3.4. Motor intentionality is not a sufficient condition for geometry

Merleau-Ponty (2012) consequently holds that "insofar as [the body] moves itself, that is, insofar as it is inseparable from a perspective [*une vue du monde*] and is this very perspective brought into existence," it is "the condition of possibility" for the production of geometric evidence (408).

In defending what seems a strong foundational claim, Merleau-Ponty is well aware that ideal entities such as triangles are cultural acquisitions linked to a certain human history and do not directly result from an operative motor intentionality (see Merleau-Ponty, 2012, 413-14; regarding enculturation, cf. Zahidi & Myin, 2016; Fabry, 2018). In "The Cogito" chapter, however, he does not take the step to clarify that geometric figures consequently cannot be fully explained in terms of sensorimotor activity (even if we understand this activity as historically conditioned by social or habitual factors). When I "motorically explore" a triangle situated within Euclidian space, an abstraction and idealization is already in place and operating. Merleau-Ponty, however, does not explain the transition from the oriented practical space of sensorimotor exploration to the idealized space. That is, he speaks of the body and perception as necessary conditions for mathematical thinking without directly clarifying *what else* is necessary for it. In a later period, Merleau-Ponty criticizes the "The Cogito" chapter precisely because it remained disconnected from his interpretation of language and the processes of expression and idealization. Retrospectively, he finds that this passage merely shows how language is not impossible but it cannot make comprehensible how language is possible (Merleau-Ponty, 1968b, 176). An analogical argument is required concerning his treatment of geometry.

The one-sidedness of Merleau-Ponty's explanation might be perceived as a drawback (see Baldwin, 2013, 318-20; Besmer, 2007, 140-41; Hass, 2008, 169; Saint-Aubert, 2011, 31). However, we cannot conclude from it that the body is simply a *sufficient* condition of geometry for Merleau-Ponty (Cassou-Noguès, 1998; Besmer, 2007; Baldwin, 2013, 318), but rather that the body is "presupposed" by it as a necessary condition (Hass & Hass, 2000, 179). Merleau-Ponty's explanation from "The Cogito" chapter is thus partial rather than inadequate (cf. Baldwin, 2013, 321-22). That is because, as is often the case, Merleau-Ponty concentrates on showing that an opposing position is not acceptable rather than providing a full-fledged account of his own position.

In Merleau-Ponty's (2012) view, motricity possesses an "elementary power of sense-giving," but "in what follows [*dans la suite*], thought and the perception of space *are liberated from motricity and from being toward space* [*l'être à l'espace*]" (143; transl. modified; emphasis added; cf. 141-43). Integrated within the human cultural world, motor intentionality "hides behind the objective world that it *contributes to* constituting" (523 n99; emphasis added). Correspondingly, space can be present as an object of rational analysis without being present to the body and its motor exploration. Merleau-Ponty points out that in cases of apraxia, for example, patients can relate to objects in space, and to space itself, as abstract representations and linguistic referents without being capable of grasping them motorically through their bodies (cf. 140, 142; 2020a, 109-117). Conversely, we ordinarily distinguish a circular figure from other figures through its "circular physiognomy," but the perception of this "style" should not be confused with the act of grasping a set of geometrical "properties" (2012, 287; cf. 6, 11; 2003, 153).[24] The diameters of two circular tree trunks are not *perceived* as equal or unequal in the geometrical sense: before one learns Euclidian geometry, the diameters of perceived tree trunks are "neither equal nor unequal" (2012, 287).[25] Outside geometry, trees do not have "diameters," and the equality or inequality of their diameters "as such does not exist absolutely" (2010, 56, 52; cf. 1973, 122/171). That is, tree trunks "*had* the properties of the circle before the circle was known" (2010, 52; emphasis original), and the properties are therefore mind-independent. Yet, this finding makes sense only retrospectively and requires that the tradition of geometry be already established. Thus, on the one hand, Merleau-Ponty identifies phenomenological reasons for refusing an absolute existence of mathematical objects by linking them to our perceptual life. On the other hand, however, Merleau-Ponty is well aware that we neither produce a geometrical signification nor relate to it through simply moving our perceiving body: there are fundamental differences and a certain degree of independence between a perceptual correlate of our sensorimotor body and a geometric object.

It is therefore impossible to claim that in Merleau-Ponty's view, motor intentionality is sufficient for producing geometric knowledge. A sensorimotor action does not make us pass from a perceptual circularity to a geometric circle *by itself*. Something else is necessary. In his writings after *Phenomenology of Perception*, Merleau-Ponty explicitly supports the idea that enculturation is necessary for the development of numerical abilities. Although he recognizes that there is room to speak of an animal culture (2003, 198), he notes that the development of knowledge, language, and algorithm "is foreign to animality" since it is tied to human historical and cultural development (2010, 52). Anyone who learns geometry knows that it is irreducible to an order of empirical "events" (2010, 51): it "is not natural like a rock or a mountain," but is "engendered by human activity" (2002, 28). He or she "who understands geometry is ... not a mind without a situation in the natural world and in culture, ... he is the heir, in the best of cases the founder, of *a certain language*" (1969, 148-49; emphasis added).[26] It is, therefore, necessary to clarify the relationship between motor intentionality and the "language" of geometry from Merleau-Ponty's point of view.

24 Hohol (2020, 46), for example, does not seem to distinguish between perceptual figures and geometric properties. In contrast, see Zahidi and Myin's (2016) critique of this widespread cognitive bias.

25 Alternatively, it is possible that painters such as Cézanne were capable of liberating their perception from the influence of previously learned geometry. Cf. Merleau-Ponty's analysis of "perspectival distortions" in Cézanne's paintings (1964c, 13-15) or his thoughts on topological space (1968b, 212-13).

26 The English translation (1970, 105) of this passage is incorrect.

3.5. The symbolic system of geometry specifies the field of motor intentionality

In effect, Merleau-Ponty's analysis of geometric demonstration from "The Cogito" chapter is entirely based on the "language" of geometry. Explanation occurs within the framework of Euclidian space and relies on significations expressed by notions such as "triangle," "plane," "straight line," "secant," "parallels," "right angle," and "equality." Yet, as Cassou-Noguès (1998, 385) points out, when I draw "a parallel" in the geometrical sense, I do not simply see drawn lines, but rather "straight lines" in the mathematical sense. That is, I understand that the lines I see should be dealt with as lines situated on a two-dimensional plane, that do not deviate in their direction, that continue infinitely, and so on. The infinity of a line, for example, is constructed in relation to a perceived line, but it is not given in the line as it is grasped sensorimotorically. The triangle still appertains to the oriented space that one perceives and explores sensorimotorically, but it also appertains to a socio-culturally constructed Euclidian space that is differently structured and offers a range of affordances that cannot be taken up with one's body alone (cf. Kiverstein & Rietveld, 2021, 180-81).

In the post-war period, Merleau-Ponty outlines an original interpretation of how the perceptual and symbolic dimensions intertwine, thereby attenuating the one-sidedness of his account from the *Phenomenology of Perception*. This development is facilitated by Merleau-Ponty's adoption of a structuralist interpretation of language as a system of "differences without terms" and the way in which he links it to gestalt psychological ideas about perceptual differentiation. On the one hand, Merleau-Ponty continues to emphasize that "the elementary notions of point, surface, and contour have meaning in the last analysis only for a subject affected by locality and situated himself in the space whose spectacle he develops" (1973, 8/16-17; cf. 2012, 143). Thus, a geometer fundamentally relies on their body schema as a "diacritical system" that situates them in the world and thus allows for differentiating here from there, left from right, figures from grounds (2020a, 132). On the other hand, however, the relatively global perceptual values articulated body-schematically are *more specifically organized* through socio-culturally developed systems that use symbols as secondary vehicles of discrimination and differentiation.

Consequently, the symbolic system of geometry is neither an external addition to the system of one's bodily relationship to the spatial environment nor a mere reactivation or simulation of this relationship in a different context. Taking up the structuralist idea that language signs are "arbitrary" or "conventional" (see 2020b, 84, 133-34; cf. 70), Merleau-Ponty holds that symbolic systems establish a threshold that breaks the continuity between the perceptual and symbolic domains (regarding this discontinuity, see Hohol, 2020, 113-15). Similarly, drawing on neuropsychological research, Merleau-Ponty concludes that there is a relative independence between apraxias and agnosias, and consequently between the sensorimotor "praxis" and the "gnosis" that includes symbol-based cognition (see 2020a, 105-107). However, Merleau-Ponty is careful not to separate the two types of cognition into two completely independent layers (cf. 2020b, 64, 86-88).[27] Instead, he argues that symbol-based "articulated thought" is a finer differentiation of the relatively "polymorphous" structures articulated in the sensorimotorical domain (1959, 179 verso). The use of symbolic systems thus "resolves the ambiguities" of the sensorimotor domain while simultaneously opening a field where other ambiguities arise and thus suggest new questions and problems (2020b, 123). In Merleau-Ponty's view, a symbolic system such as language or mathematics is therefore a "reiteration at a higher power of [the] process of articulation" that we find in perception, an

[27] Cf. Kiverstein and Rietveld's (2021, 178-80) critique of Gibson on this point.

"underpinning" or reworking of the intentional infrastructure opened and maintained by the body (*reprise en sous œuvre*; 123).

Thus, on the most general level, the relationship between the perceptual and symbolic dimensions is of reciprocal "foundation," in which a cognitively "higher" level remains dependent on a "lower" level even though it is not reducible to it (see, e.g., 2012, 128-129; cf. Matherne, 2018; Robert, 2000). However, Merleau-Ponty's dynamically structural account ultimately calls for a revision of these hierarchical metaphors. The relationship between the two systems is asymmetrical, because the more global system of organization is presupposed by the more finely organized one,[28] and geometry thus presupposes perception. However, each of the two systems also modifies the range and type of affordances available in the other. We perceive according to the cultural artefacts of geometry, such as when circular physiognomies are unreflectively seen as circles in the geometrical sense. However, symbol-based cognition remains open to phenomenal structuration operated through concrete sensorimotor praxis, which is why a deterioration of this praxis due to fatigue or neural pathology impairs one's cognitive efficacy in geometry, while the reorganization of a concrete geometric figure enables novel geometric insight.

Merleau-Ponty's arguments from the *Phenomenology of Perception* can therefore be specified with the help of his writings from the post-war period. Geometric symbols never entirely detach themselves from the perceptual dimension, but the subject relating to the geometric figure is not simply a motor subject. Rather, it is a motor subject *who has incorporated a socio-culturally developed diacritical system that allows to more finely structure perceptual figures.* The body's sensorimotor exploration is a necessary condition of geometry, but geometry also requires that the perceptual figures articulated by the body become additionally structured according to what Merleau-Ponty calls "a nonnatural system of equivalence and of discrimination" (2003, 222; transl. modified). In this sense, the system of geometry is a "superstructure" of the body schema.[29]

4. Algebra embodied

Geometry might seem to be particularly well suited to Merleau-Ponty's type of argumentation since it is concerned with spatial relationships that are also accessible through the body. However, Merleau-Ponty argues that motor intentionality, and consequently embodiment, plays an analogic role in all symbolic systems (e.g., 1970, 9/18). While the case of language would require discussions that go well beyond the scope of a single article, Merleau-Ponty's argument can be briefly outlined for algebra, which he analyzes in *The Prose of the World*.[30] Approximately six years after writing "The Cogito" chapter, Merleau-Ponty contends that the configuration of algebraic signs plays a fundamental role in our access to algebraic insights and that this configuration is a perceptual phenomenon. In this part of the paper, I consider Merleau-Ponty's claims from *The Prose of the World* that shed light on the role of the body in the transformation of the configurations of algebraic signs and in higher cognition generally.

28 Merleau-Ponty would therefore satisfy the requirement for a relative independence of the two systems as presented by Hohol (2020), based on his review of recent empirical experiments.

29 Merleau-Ponty (2020a, 123) makes a similar point regarding the system of speech.

30 See Merleau-Ponty (1973, 105-106/149-50; 125-26/176-77; cf. 2010, 55-56).

4.1. Algebraic signification is founded on "perceptual" experience

Hass and Hass (2000, 180) note that, for Merleau-Ponty, algebra "presupposes that there are corporeal vectors of *temporality*" just as geometry presupposes that there is *spatiality* in our sensorimotor bodily fields (emphasis added; with reference to Merleau-Ponty, 1973, 100–13/142–60). Hass (2008, 152) explains more specifically that the vectors of temporality should be understood as "'next,' 'succession,' and 'progression'." Unfortunately, the authors do not provide any further explanation concerning this point. On the one hand, they are right in that for Merleau-Ponty, algebraic significations are temporally acquired and thus, linked to situated acts of cognition. However, in the case of geometry, Hass and Hass do not link corporeality to temporality and choose to refer to the spatiality of our sensorimotor fields instead, although the temporal relationships involved in the acquisition of geometric signification are clearly those same relationships that subtend the development of algebra (cf. 2012, 408-13; 1973, 104-105/148). Therefore, it remains unclear what makes the temporal aspects of algebra precisely "corporeal."

In fact, Merleau-Ponty relates algebra to embodiment in another way. He points out that an arithmetical or algebraic signification "actually means nothing and has no truth at all" unless we refer it to a perceptual configuration (1973, 106/150-51). It is necessarily related to "our field of presence, to the actual existence of a perceived object [*un perçu*]," "some situation or some structure" (107/151); that is, to the "factual presence of [mathematical] signs" or some "concrete figures" (105/149). The relationship between algebra and embodiment can, therefore, be clarified through an analysis of the dimension of mathematical sign Merleau-Ponty calls "perceptual." In what sense the "perceptual" aspect of an arithmetical or algebraic configuration relates to our bodily intentionality needs explaining.

Before addressing this question, it is necessary to emphasize that Merleau-Ponty's claims concerning the perceptual character of algebra cannot be understood in a reductionist way. As he had acknowledged already in the *Phenomenology of Perception*, geometric thought "transcends perceptual consciousness" (2012, 407). Merleau-Ponty respects the fact that our representational awareness of Euclidian space is, to a certain degree, independent of our praxic sensorimotor relationship with space. In *The Prose of the World* and later texts, Merleau-Ponty eventually sets out his position explicitly: he is neither "reducing mathematical evidence to perceptual evidence" nor denying "the originality of the order of knowledge vis-à-vis the perceptual order," that is, "the sensible" world (1973, 123/173, 126/177). Mathematical or any other knowledge "is not perception" for him (129/181). There is, therefore, a fundamental difference between bodily perception and mathematical knowledge for Merleau-Ponty. It is from this perspective that we must approach his claim that arithmetical and algebraic significations are related to a specific "perceptual" infrastructure (see 106/150-51).

4.2. Algebraic insight is correlative to a particular phenomenal configuration and its transformations

Merleau-Ponty's brief interpretation of algebraic discovery clearly constitutes a development and radicalization of his argument concerning reasoning in geometry. For Merleau-Ponty, a given arithmetical and, eventually, algebraic insight is fundamentally linked to a certain phenomenal configuration and its transformations, much like a geometrical insight. In the case of algebra, he again builds on an account given by Wertheimer, more specifically in the chapter "The famous story of young Gauss" from *The Productive Thinking* (2020, 108-142).

Following Wertheimer, Merleau-Ponty observes that when one notices, for example, that "the progression from 1 to 5 is exactly symmetrical with the regression from 10 to 5," one performs a "transformation" of the series of numbers 1-10 (Merleau-Ponty, 1973, 125/176). Such an insight can be expressed by rearranging the linear numerical progression (1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 9 + 10) into a mathematically equivalent but structurally different formula ([1 + 10] + [2 + 9] + [3 + 8] + [4 + 7] + [5 + 6]). In the latter formula, the ten members of the series form five pairs of the same value (10 + 1), while the first members of the pairs increase proportionally to how the second decrease. After the transformation, the identical arithmetical situation is shown in a new light. For this reason, Merleau-Ponty claims that the structural change produced by the transformation is "equivalent in the arithmetical object to a geometric construction" (1973, 125-26/176). The key implications of this approach for our understanding of thinking in algebra are next discussed.

On the one hand, the operation by which we determine the sum of a series of n numbers can always be accomplished through progressive steps. Similarly, a linear series of n numbers can be arranged into a series half its length comprising pairs of the same value. Now, Gauss' creative algebraic accomplishment consists of his having contracted the successive steps of the latter reorganization of the numerical series into a single algebraic formula $(n \div 2) \times (n + 1)$. Through producing this formula, Gauss demonstrates that the relationship discovered in a particular arithmetical or algebraic situation can be attributed to any continuous series of numbers without the need to carry out the progressive steps for each individual series. He thereby establishes this type of algebraic evidence and augments the field of mathematical truths. Merleau-Ponty emphasizes that anyone who applies algebra after Gauss is assured of having discovered the essence and truth of any series of numbers because, through this formula, one "*sees* the pairs of constant value derived from the series of numbers that he will count, instead of performing the sum" (1973, 106/150; emphasis original). That is, the algebraic signification Gauss discovered is made available as a general truth precisely because of the particular arrangement of the formula $(n \div 2) \times (n + 1)$.

Thanks to Gauss' formula, a certain mathematical aspect of the series of numbers no longer has a form that requires a successive performance of operations on the sum of a series of numbers: that particular mathematical aspect is available, contracted into a more concise, even if structurally more complex, formula, which saves the effort of performing certain mathematical operations. Additionally, while the formula removes the necessity of carrying out successive operations, it still allows for the perception that they are possible. Based on the formula, the reorganizing operation *does not need* to be carried out anymore, but it also always *can* be. One can obtain the desired result by just applying the formula instead of performing the sum, but one can also verify why the formula is correct by linking the formula to the original transformative operation, that is, by following the way in which it structurally reorganizes the linear series into pairs of the same value.[31]

In Husserlian terms, the exploratory operations related to the numerical series are "sedimented"[32] in the formula and can be "reactivated" based on it. Following Husserl, Merleau-Ponty explains that the principal effect of every spatio-temporally situated act of ideation is to "make its literal repetition superfluous, launch culture towards a future, make itself forgotten" (Merleau-Ponty, 1970, 116/162). The heirs of the post-Gaussian algebraic tradition thus no longer need to make the operations contracted in his formula explicit to make them "operate in us" (1973, 107/152). Precisely for this reason, the Gaussian formula, and the insight into the numerical series it

31 In contrast to that, Schneider cannot dispense with the operation itself. See below, section 4.3.

32 Husserl (1989) considers "sedimentation" the process through which an originally experienced ideal meaning becomes stabilized in language or other symbolic systems and thus, communicable across time and space (cf. Blomberg 2019).

contracts, constitute new grounds for further mathematical thinking in which the formula can itself figure as one particular step in a group of more complex mathematical transformations (cf., e.g., 2012, 135-36).

Thus, on the one hand, Merleau-Ponty's interpretation of Wertheimer's example is Husserlian in emphasizing the importance of writing, or more precisely of producing a specific configuration of phenomenally available signs, in the production of ideality. From this point of view, ideality is a cognitive artefact, a culturally constructed affordance (see Fabry, 2018; Hohol, 2020, 121-42) that contracts exploratory operations in the world and assures the possibility of their trans-individual reactivation. On the other hand, however, Merleau-Ponty diverges from Husserl in seeing the role of the formula as not primarily instrumental. It does not just facilitate the reactivation of the original piece of self-evidence produced independently of it (cf. Baldwin, 2013, 314; Robert, 2000, 362-63). Correlatively, for Merleau-Ponty, the perception involved in mathematical reasoning is not Husserl's pure fulfillment of a production realized within the self-evidence of the consciousness (see Husserl, 1989, 163). Commenting on Gurwitsch, Merleau-Ponty rejects the Husserlian approach to language signs, according to which the role of these signs is merely "to transmit a signification of which they are not a part" (Merleau-Ponty, 2001, 182). For Merleau-Ponty, on the contrary, "the very signifiers of language function as perceptual *hylê*" (182). Consequently, the mathematical insight does not pre-exist the formula; it is *with* the structural reorganization expressed by and contracted in the formula that the mathematical evidence is produced.

The importance of the mathematical formula is thus twofold. First, a formula contracts certain structural transformations of a series of numbers and thus articulates a mathematical aspect of the series that would otherwise be unavailable. That is, without Gauss' formula, for example, one would need to count the sum of the series through undertaking progressive steps, because the Gaussian relationship is not evident in the series given as a linear progression. With the formula, the progressive execution of the steps becomes unnecessary. Second, an original formula such as Gauss' has priority over all other correct algebraic formulas that can be derived from it and that have the same mathematical validity, insofar as the derived formulas do not retain the original's "demonstrative light" (Merleau-Ponty, 2010, 56; cf. 1973, 106/150). That is, the original formula makes it possible to perform the exploratory steps that are contracted within it again, whereas the derived formulas do not make such a reactivation directly accessible.

Merleau-Ponty thereby presents a strong argument for how sensorimotor intentionality contributes to mathematical cognition. He would of course agree with embodied cognition theories that perception is necessarily involved in the phenomenalization of the mathematical symbols themselves and that this factor can variously constrain and enable mathematical cognition (see, e.g., Fabry, 2018, 801). However, Merleau-Ponty's own position is more radical. From his perspective, we are entitled to claim that mathematical cognition is the correlate of a sensorimotor exploration insofar as the *particular configuration* of a set of mathematical signs contributes to their mathematical signification. In contrast to that, when we restrain our mathematical operations to merely applying or formalizing available formulas, the specific configuration of mathematical signs does not play any role in the mathematical signification and perception is not strongly involved. However, the available formulas themselves result from structural transformations of some phenomenal configurations and are thus dependent on them. Mathematical structures contract exploratory operations, thus rendering unnecessary their full realization, yet they remain open to further such operations. In other words, Merleau-Ponty argues that in humans, arithmetical and algebraic significations are never completely separated from what has been called "perceived magnitudes" and the "subitizing" praxis that corresponds to them, even though these significations are subject to cultural construction and sedimentation.[33]

4.3. The subject of algebra is an embodied subject who performs acts of structuration

Merleau-Ponty (1973) emphasizes that the structural transformations of a linear numerical series that a mathematician performs through organizing it into pairs of equal value "are not a part" of that series as it is initially given (126/176). The particular relationship between a linear progressing series and the complex of pairs of numbers emerges "only when [they] address a certain question to the structure of the series of numbers" (126/176; original emphasis removed). The initial structure is for them "an open and incomplete situation," which "poses a question" to them and presents itself as a field in which something is to be known by adopting a certain perspective, that is, by reorganizing or restructuring the way the field is situated in relation to them (126/176). In short, the algebraic signification expressed by Gauss' formula is a particular gestalt articulated within the field of algebraic expressions *by means of an act* that reorganizes the initially given general structure of the series of numbers.

Merleau-Ponty consequently argues that the structure that results from the act of reorganization of a given field of thought, such as the numerical series, does not pre-exist this operation. Here, Merleau-Ponty diverges not only from Husserl, but also from all proponents of mathematical realism. According to Matherne (2018), for example, the role of abstract mathematical knowledge in Merleau-Ponty's philosophy is to make perceptual structures "determinate and explicit" (792) and thus transform them into "something more manageable, intersubjectively available, and systematic" (793). Although these functional changes certainly take place, it is important to note that in Merleau-Ponty's view, the reorganizing operation is not an "analysis" of the initial structure; it is instead significantly "creative" or productive (1973, 126/176; cf. Hass, 2008, 151-55, 168; Irwin, 2017, 147-48). That is, the transformation does not result merely in a new mental representation or intersubjective form of already (implicitly) available contents. Thanks to the novel configuration of the question, we do not just think differently of a reality that subsists independently of our varying grasp. Since mathematical insights are necessarily bound to the production of new phenomenal structures, Merleau-Ponty holds that mathematical truth is "structural truth" no less than perception; it is "connected to perspective, to centering, to structuring" (2010, 52; referring to Wertheimer, 2020). Moreover, since it is impossible to perform "an absolute decentering," there always remains some degree of "perceptual naiveté" even in the mathematical domain, which prevents us from resolving all mathematical problems and passing to a purely intelligible world (Merleau-Ponty, 2010, 56, 52). Consequently, Merleau-Ponty argues that the "retrograde movement"[34] of ideal meaning is not merely a transformation of our knowledge of the ideality involved, *but a transformation of this ideality itself*. In other words, mathematical reasoning is never purely formal, nor is it instrumental. Abstract mathematical objects always remain related to the spatio-temporal field and to observer-dependent changes in that field. Consequently, mathematical objects are relational rather than ontologically real entities.

In *The Prose of the World*, Merleau-Ponty unfortunately does not elaborate further on how we sensorimotorically "explore" and phenomenally transform arithmetical and algebraic structures. However, here again, the pathological disintegration of motor intentionality in Schneider's case

[33] See section 3.4. for an analogical argument regarding perceptual "physiognomies" and geometrical properties. For discussion on this relationship from the perspective of embodied cognition, see Zahidi (2021), Zahidi and Myin (2016), and Fabry (2018, 796-98).

[34] See above, note 7.

provides an instructive contrast to Merleau-Ponty's positive argument. While productive thinking in mathematics characteristically involves performing structural reorganizations of previous mathematical acquisitions, Schneider is distinctive in that his capacity for such reorganizations is significantly deficient due to his bodily injury.

Schneider's limited capacity for structural reorganizations of his phenomenal environment forces him to rely on various points of support for the exploration of mathematical relationships. He was capable of adding, subtracting, multiplying, and dividing, but only "with regard to objects placed in front of him" (Merleau-Ponty, 2012, 135). The more abstract, purely arithmetical problems he solved "without any intuition of numbers" and only with the help of "manual operations" such as finger counting and more generally the "manipulation of signs" or other "fulcra" (150; 1963, 67). He understood that seven is more than four only because the former came "after" the latter in the series of numbers as he recited it from memory. Moreover, Schneider did not understand that "doubling half" of a number is this very same number, even though he could carry out the corresponding arithmetical operation that led him to a correct result (2012, 135). In short, much as in the sensorimotor domain, Schneider produced required mathematical outcomes by inflexibly following available formulas and scripts for action.

From a genetic perspective, Schneider had therefore regressed to an earlier stage of mathematical development (cf. Fabry, 2018, 794-96; Zahidi & Myin, 2016, 59-62). More importantly, the structure of Schneider's difficulties evidences that arithmetical and algebraic cognition requires sensorimotor transfiguration of phenomenal structures. The above examples show that Schneider was capable of numeration, but not of *contracting the processes of structuration* of a series of numbers into the type of evidence expressed by formulas such as ($7 > 4$) or ($2 \times [n \div 2] = n$). That is, instead of gaining mathematical insight into his situation by appropriately reorganizing the mathematical structures, which he socio-culturally inherited before the injury, he had to *factually carry out* the operations that produce the cognitive acquisition contracted in the corresponding formula. Schneider had to compensate for his incapacity concerning structuration at a higher order by always performing each arithmetical operation, starting from terms that were fixed for him and comparatively less complex than what was required by the task. Because he had problems contracting his exploratory activity into an abbreviated form generally, Schneider had to always replace a "simultaneous" grasp of an arithmetical structure of a certain level of complexity by a "successive" performing of structuration (cf. Merleau-Ponty, 1963, 65). Consequently, Schneider's mathematical insight was limited to the level of complexity that corresponds to the structure within which his successive progression occurred.

As Merleau-Ponty points out, it would be absurd to think that Schneider's higher-cognitive difficulties arose because "the shrapnel collided with symbolic consciousness" in him: rather, his capacity for higher cognition was "affected through vision" and, more generally, the motor intentionality of which his vision is a correlate (2012, 127). A mathematical signification is situated in a spatio-temporally located field and requires an "operator of the field" (1959, 172 verso) to structure it from within. Embodied cognition theorists rightly point out that anatomical and mechanic configuration of the human body, such as the degrees of freedom in joints and muscles, specifically constrain and enable mathematical cognition (e.g., Fabry, 2018, 799-800). However, Merleau-Ponty shows that it is necessary to argue, more radically, that the body is implicated in mathematical cognition not only as a physical entity but also as a relational phenomenon, a body-schematic power to articulate cognitive artefacts beyond the degree of complexity with which they are initially given.

5. Conclusion

My aim in this paper was to clarify how Merleau-Ponty's gestalt-inspired phenomenological account of mathematical cognition prefigures fundamental arguments of the present-day embodied enactive theories and makes it possible to further elaborate on them. While Husserl's position in the *Origin of geometry* remained ambiguous because it retained elements of a mind-centered idealism, Merleau-Ponty's phenomenology fully endorses the constitutive role of socio-cultural tradition in the production of mathematical insights. However, Merleau-Ponty offers more than just an emphasis on pragmatic and socio-cultural origins of higher cognition.

Enactivism can benefit from Merleau-Ponty's detailed analysis of how concrete perceptible elements of the environment found units of meaning that are relatively independent from their experiential context. Merleau-Ponty neither enquires into the history of mathematics, nor is he directly concerned with phylogenetic, ontogenetic, and neural questions. However, unlike the contemporary theorists of embodied cognition, he closely analyzes the process of geometric and algebraic demonstration with respect to its phenomenal structure.

Building on Merleau-Ponty's analysis, it is possible to argue that mathematical objects are neither representations of the mind nor ontologically objective entities. Rather, they are gestalts or concrete phenomenal configurations that necessarily imply situated cognizers to whom they afford a special type of engagement in the world and on whom they depend in their eventual structural transformations. Therefore, Merleau-Ponty's account might be seen as an elaboration of the enactivist tradition initiated by Varela, Thompson, and Rosch (2016) insofar as it supports the idea that mathematical objects are *relational* phenomena that pertain to how we collectively enact the sense of the world.

However, Merleau-Ponty also insightfully demonstrates that mathematical gestalts are structurally irreducible to perceptual gestalts because they have a higher degree of independence from their phenomenal context. On this account, mathematical necessity and generality do not derive from the mathematical objects being omnitemporal, but are rather due to the fact that the particular arrangement of the mathematical gestalts raises the threshold above which their unity undergoes a reorganization and consequently a change of meaning. The threshold is heightened because, in the process of sedimentation, the contraction of certain exploratory operations in a mathematical formula makes their identical repetition unnecessary. In this respect, Merleau-Ponty contributes to clarifying why mathematical objects are not simple correlates of individual bodily or sensorimotor enaction.

Building on this, I have argued alongside Merleau-Ponty that the role of mathematical constructions is not merely instrumental and involves enaction in a stronger and more productive sense. The production of mathematical insight is correlative to a mathematician's act of creating a phenomenally concrete transition from one gestalt to another by performing perceptual structural changes to an initially given configuration, for example by introducing auxiliary lines into a triangle. The generality of mathematical truths is then bound to the elaboration of the phenomenal structural transformation since a particular piece of mathematical evidence is accessible to all geometers who are capable of reiterating the construction involved in the transformation.

Furthermore, I have argued that by linking mathematical truths to spatio-temporally concrete phenomenal configurations, Merleau-Ponty neither reduces mathematical cognition to perceptual cognition, nor does he defend a unidirectional foundational explanation of the former through the latter. In this way, Merleau-Ponty elaborates an enactive model of higher cognition that is more viable than that offered by Husserl, who ultimately anchors mathematical insights in the self-evidence of the consciousness. The dimension Merleau-Ponty calls perceptual corresponds to structurally concrete factors of mathematical reasoning that are not pre- or extra-cultural, but rather correspond to those aspects of symbol-based gestalts that more complexly organize the relatively general, socio-culturally sedimented structures of human symbolic systems. Mathematical gestalts are not directly correlative to sensorimotor intentionality for Merleau-Ponty, but are results of past creative efforts that remain open to further elaboration through structural reorganization.

By integrating these elements, Merleau-Ponty elaborates a framework for interpreting the cognizer's relation to their environment as dynamically organized according to different levels of complexity. In particular, enactivists can take up Merleau-Ponty's interpretation of our situated mobility as an explorative function through which we modify the organizational complexity of our relationship to the world even at the level of higher cognition. On this account, bodily motricity is involved even in the abstract domain of symbol-based mathematical cognition, as a means of transforming mathematical gestalts and changing our relationship to the cultural symbolic environment consequently.

By interpreting symbolic systems as structures articulated within a more globally organized domain of structuring operations opened and continuously maintained by the sensorimotor system of exploration, Merleau-Ponty also answers the question of how to understand the relationship between concrete and abstract types of cognition, which is crucial for embodied and enactive theories. In the abstract domain, motor articulation of phenomenal structures is neither entirely excluded nor reverted to simulating presumably original perceptual experiences; rather, it is "sublimated" into "symbolic gesticulation" (Merleau-Ponty, 1973, 18-19). Hence, Merleau-Ponty substantiates the idea that mathematical cognition more finely differentiates what remains relatively polymorphous in the sensorimotor interaction with the world, yet this task is still accomplished by using the capacity for motor differentiation of phenomenal figures.

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