Why the quantum equilibrium hypothesis? From Bohmian mechanics to a many-worlds theory

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Abstract

The status and justification of the quantum equilibrium hypothesis (QEH) in Bohmian mechanics is "a rather delicate matter". In this paper, I present a new analysis of this hypothesis. It is argued that the QEH should be regarded not as a mere initial condition but more appropriately as one part of the law of motion. Moreover, it is more reasonable and simpler to assume that the QEH holds true consistently at all times. The resulting theory is a many-worlds theory of random discontinuous motion of particles in three-dimensional space. This new theory agrees with experiments, and it is arguably the simplest realist version of quantum mechanics.

The quantum equilibrium hypothesis (QEH) is an important but still mysterious element of Bohmian mechanics or the de Broglie-Bohm pilot-wave theory (de Broglie, 1928; Bohm, 1952). The status and justification of this hypothesis is "a rather delicate matter." (Goldstein, 2021). There are two approaches to understanding the QEH, the dynamical approach and the typicality approach, and which one is right has been debated for several decades (see Callender, 2007 and Norsen, 2018 for helpful review). In this paper, I will present a new analysis of the QEH.

In Bohmian mechanics (BM), a complete realistic description of a quantum system is provided by the configuration defined by the positions of its particles together with its wave function. In order to explain the Born rule and ensure its empirical equivalence with standard quantum mechanics, BM assumes that the configuration of a system whose wave function is ψ is random with probability distribution $|\psi|^2$ at an initial instant. This is the QEH. Since the dynamics keeps this distribution unchanged over time and position measurements also reveal the positions of actual particles in BM, the Born rule for measurement results, which are registered configurationally, can be derived.

The issue is to understand the QEH. For simplicity, consider a universe consisting of non-interacting particles which have the same wave function $\psi(x)$ at an initial instant. The QEH requires that at the instant the position of each particle is random with probability distribution $|\psi(x)|^2$. Now the crucial question is: why is the position probability distribution of each particle related to its wave function? This question is more general and deeper than the usual one of why the distribution is $|\Psi|^2$, not $|\Psi|^3$ or others.

First of all, if the QEH holds true exactly, then the equilibrium distribution at an instant cannot arise from some earlier non-equilibrium distribution by a dynamical process, which can only generate an approximate equilibrium distribution during a finite time. Next, if each particle has no properties related to its wave function at the initial instant, then its position probability distribution cannot depend on the wave fucntion at the instant. In other words, the QEH requires that each particle has a property related to its wave function at the initial instant.[1] Concretely speaking, each particle must have an indeterministic disposition or propensity to be in a position x with probability $|\psi(x)|^2$ at the initial instant in order that the QEH holds true exactly.

Here it is worth emphasizing the important role of the QEH in BM. In the words of Callender (2007), with it "the Bohmian can explain the occurrence of frequencies in accord with Born's rule and all that follows from that, e.g., why quantum mechanics works, why the uncertainty principle holds, why the 'no superluminal signaling' theorem obtains, and more", while without it or something close to it, "the Bohmian runs a serious explanatory deficit." Yet, the great significance of the QEH is hidden in the usual formulation of BM. The formulation always states the dynamics first and then the QEH, treating it as an initial condition for the dynamics.

In fact, the QEH should be regarded not as a mere initial condition but more appropriately as one part of the law of motion; it states a lawful probability relation between the two key quantities in the formulation of the law of motion: one is the particle configuration in the guiding equation, and the other is the wave function in the Schrödinger equation. Then, a better and clearer fomulation of BM will state the QEH (about the initial position probability distribution of all particles in the universe) first, then the dynamics (that keeps this distribution unchanged over time). Here the importance of the QEH can be manifested in a new aspect. The QEH is exact and unique, while there are infinitely many possible forms of the dynamics, deterministic (Deotto and Ghirardi, 1998) or stochastic (Vink, 1993), indistinguishable in experience, all of which can preserve the probability distribution.

[1]If this is not true, then the only explanation will be that "God likes quantum equilibrium and Born's law and so put it there at the beginning of times." (Bricmont, 2001)

Now we have the QEH, one part of the law of motion, for the universe at an initial instant $t = 0$. It says that the configuration of the universe is random with probability distribution $|\Psi(0)|^2$ at the instant, where $\Psi(0)$ is the wave function of the universe then. According to the dispositionalist view of laws, this means that at the initial instant all particles in the universe have an indeterministic disposition or propensity property that determines the probability density that they appear in every possible configuration, which is equal to the modulus squared or density of their wave function in the configuration. These particles will appear more likely in a configuration with larger density, and less likely in a configuration with smaller density. On this view, one may also say that at the initial instant the wave function represents the propensity property of the particles that determines the probability density that they appear in every possible configuration. Note that this ensures the empirical equivalence between BM and standard quantum mechanics at the initial instant (when measurements are done at the instant).

Then, what is the law of motion for the following instants? BM uses the guiding equation to replace the QEH to keep the quantum equilibrium distribution unchanged over time. Correspondingly, the lawful relation between the particle configuration and the wave function is also changed. On the dispositionalist view of laws, the guiding equation means that the wave function represents a deterministic dispositional property of the particles that determines their continuous motion in space at all following instants. This deterministic dispositional property is certainly different from the indeterministic dispositional property at the initial instant.

This kind of dualism in properties and the law of motion raises a lot questions. Why does the wave function have a double role and represent different properties in BM? Why does the wave function no longer play its role in QEH immediately after the initial instant? Or why does the QEH as a law of motion hold true only for a special instant and not for all instants? If the QEH can ensure the empirical equivalence between the theory and standard quantum mechanics at each instant, then why replace it with the guiding equation for the instants following the initial instant? Even if a replacement is needed, how is the QEH replaced by the guiding equation around the initial instant? How is the indeterministic dispositional property changed to the deterministic dispositional property around the initial instant? Why is there a discontinuous change in the properties of the particles at the initial instant? and so on.

A more natural and reasonable choice of Nature will be that the QEH as a law of motion holds true consistently for all instants. This is the simplest way to keep the quantum equilibrium distribution unchanged over time. It unifies the initial condition and the dynamics for the particle configuration. And it can directly ensure the empirical equivalence between the theory and standard quantum mechanics at every instant. No additional guiding equation, which is notably ill-defined at the nodes of the wave function, is needed. The resulting picture of motion will be that in Bohm's theory without trajectories (Bell, 1981) or random discontinuous motion of particles (Gao, 2017). Since the configuration of the universe is random with probability distribution $|\Psi(t)|^2$ at every instant t, the resulting motion of particles will be random and discontinuous, not deterministic and continuous. On the dispositionalist view of laws, this means that the particles have a propensity property which is described by the wave function and which determines their random discontinuous motion in space. In a sense, we may say that the motion of particles is "guided" by their wave function in a probabilistic way.

John Bell, the great supporter of BM, once argued that the continuous particle trajectories are not an essential part of the theory, and there is no need to link successive particle configurations into a continuous trajectory (Bell, 1981). In Bell's own words,

> instantaneous classical configuration x are supposed to exist, and to be distributed in the comparison class of possible worlds with probability $|\psi|^2$. But no pairing of configuration at different times, as would be effected by the existence of trajectories, is supposed. (Bell, 1981)

This theory has been called Bohm's theory without trajectories (Barrett, 1999). In the theory, the deterministic guiding equation of BM is replaced by a random dynamics:

$$P(Q(t), t) = |\Psi(Q(t), t)|^2, \tag{1}$$

which means that at every instant t the particle configuration of the universe is random, and its probability of being a given $Q(t)$ is equal to the Born probability $|\Psi(Q(t), t)|^2$. In other words, the particles do not move in a continuous and deterministic way, but move in a discontinuous and random way (Gao, 2017).

Bell thought that this theory is consistent with quantum mechanics and experiments, although he did not like it due to the unreliability of an observer's memories in the theory (Bell, 1981). As argued by Barrett (1999), this theory is plagued by an empirical incoherence problem, namely that even if the theory were correct, one could not have an empirical justification for accepting that it is correct. Recently, I also argued that this theory contradicts experiments for the preparation and verification of quantum states (Gao, 2021a).

Although Bohm's theory without trajectories as a one-world theory has serious issues, Bell's two insightful observations may turn out to be correct: one is that the continuous particle trajectories are not an essential part of BM, and the other is that keeping the instantaneous configurations but discarding the trajectory is the essential of Everett's theory (Bell, 1981). The synthesis will be a theory of many worlds with particles in jump motion (Gao, 2021c).

Here is an example to illustrate this new theory. Consider a simple z-spin measurement, in which an observer M measures the z-spin of a spin one-half system S that is in a superposition of two different z-spins. By the linear Schrödinger evolution, the state of the composite system after the measurement will be the superposition of M resulting z-spin up and S being z-spin up and M resulting z-spin down and S being z-spin down:

$$\alpha \left|up\right\rangle_S \left|up\right\rangle_M + \beta \left|down\right\rangle_S \left|down\right\rangle_M , \tag{2}$$

where α and β are nonzero and satisfy the normalization condition $|\alpha|^2 + |\beta|^2 = 1$.

According to the picture of random discontinuous motion of particles (Gao, 2017), a quantum system is composed of particles with mass and charge which undergo random discontinuous motion (RDM) in our three-dimensional space, and the wave function represents the propensities of these particles which determine their random discontinuous motion, and as a result, the state of motion of particles is also described by the wave function.[2] At each instant all particles have a definite position, while during an infinitesimal time interval around each instant they move throughout the whole space where the wave function is nonzero in a random and discontinuous way, and the probability density that they appear in every possible group of positions in space is given by the modulus squared of the wave function there. Visually speaking, the RDM of each particle will form a mass and charge cloud in space (during an infinitesimal time interval around each instant), and the RDM of many particles being in an entangled state will form many entangled mass and charge clouds in space. Note that the clouds corresponding to different branches of an entangled superposition exist not at the same time but in different sets of instants or different time subflows.

In the above experiment, there are only one system and one observer at each instant. The positions of the particles representing the measurement result of the observer are definite at each instant. Moreover, these particles randomly jump between the two result branches $\left|up\right\rangle_M$ and $\left|down\right\rangle_M$ over time, and the probability of they being in these two branches at each instant are $|\alpha|^2$ and $|\beta|^2$, respectively. Then at each instant there is an observer who obtains a definite result corresponding to one of the two result branches in the post-measurement superposition. Moreover, which result she obtains is randomly determined at the instant, and the probability of she obtaining

[2] As noted before, there is also a picture of random discontinuous motion of particles in Bohm's theory without trajectories (Bell, 1981). In that theory, however, the wave function is regarded by Bell as a real physical field in configuration space, and the particles arguably have no mass and charge as in BM (Gao, 2017).

a particular result is equal to the modulus squared of the wave function associated with the result, namely the probability of she obtaining the result z-spin up is $|\alpha|^2$ and the probability of she obtaining the result z-spin down is $|\beta|^2$. This provides a direct derivation of the Born rule.

Now the crucial observation is that the observers who obtain different results in the two result branches are different observers, and the two result branches of the post-measurement superposition represent two different worlds. Here is the arguments. First, the two result branches of the post-measurement superposition (as two groups of clouds in space) do not exist at the same time during a time interval; rather, they exist in different sets of instants or different time subflows. This means that for each result branch, the other result branch does not exist in space and time. Next, the two result branches have no interactions with each other. The system and the observer in one result branch do not interact with the system and the observer in the other result branch. Lastly, the systems and the observers in different result branches have different interactions with each other and their environment. In particular, the observers in the two result branches have different memories. Thus, it is arguable that the two result branches of the post-measurement superposition represent two parallel worlds in space and time, in each of which there is an observer who obtains a definite, random result with the Born probability.

Here a world is defined (as usual) as the total of all entities which exist in the same space and time and interact with each other. Entities in different worlds exist in different time subflows and they do not interact with each other; for entities in one world, the entities in other worlds do not exist in space and time. Note that such worlds are not Everett's (1957) relative states or Wallace's (2012) emergent macroscopic multiplicity at the level of structure. They exist at the fundamental level and originate directly from the underlying ontology of quantum mechanics. Concretely speaking, these worlds originate from the RDM of particles and the law of motion; we have the same particles, but they can form a time division multiverse by means of their random discontinuous motion. This provides a more direct solution to the two thorny problems of the many-worlds interpretation of quantum mechanics (MWI), namely the problems of ontology and probability (see Maudlin, 2014). This new theory can be regarded as a marriage of BM and MWI.

Different from Bohm's theory without trajectories, this many-worlds theory of RDM of particles agrees with experiments, and an observer's memories of measurement results are also reliable in each world in the theory. However, being a many-world theory, its predictions are different from those of single-world theories such as BM and standard quantum mechanics in some cases. As I recently argued, the observed thermodynamic arrow of time in our universe may provide strong evidence favoring such a many-world theory (Gao, 2021b).

Certainly, decoherence will help generate stable, quasi-classical worlds as usual, although they are not necessary for the existence of the worlds defined above. In addition, observers and their interactions with the environment will also select the actual preferred basis or which world they will live in (Vaidman, 2021). Observers with quite distinct brain structures may perceive different worlds (Penrose, 2004).

A final point. If the wave function indeed represents the RDM of particles in three-dimensional space as argued by Gao (2017, 2020), then the QEH will have a firm basis for being true at every instant. In this case, no additional ontologies and postulates are introduced in the above theory, and the theory is arguably the simplest realist version of quantum mechanics.

To sum up, I have argued that the quantum equilibrium hypothesis (QEH) in Bohmian mechanics should be regarded not as a mere initial condition but more appropriately as one part of the law of motion. Moreover, it is more reasonable and simpler to assume that the QEH holds true consistently at all times. The resulting theory is a many-worlds theory of random discontinuous motion of particles in three-dimensional space.

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