

A Conjecture on the Origin of Superselection Rules — with a Comment on “The Global Phase Is Real”

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Abstract

It is argued that superselection rules originate not from symmetry but from the initial state of the universe. This solves the puzzle posed by Schroeren in his recent paper “The Global Phase Is Real”; since the $SO(3)$ symmetry does not imply the univalence superselection rule, his argument for the reality of the global phase does not succeed.

Suppose our universe has a zero total energy, charge and angular momentum at an initial instant. Since these quantities are conserved quantities, their values will keep constant and be still zero during the evolution of the universe. Then, if the state of the universe is a pure state, there will be no superpositions of different energies/masses, charges and angular momenta in this universe. On the other hand, if we can find or prepare such superpositions, then our universe will not have a zero total energy, charge and angular momentum. It is a big surprise indeed that our local experiments can tell us the state of the whole universe. This is possible when the state of the universe is a pure state.

This simple observation provides a possible unified explanation of superselection rules (for mass, charge and angular momentum).¹ If our universe has a zero total energy, charge and angular momentum at an initial instant, then there will be no superpositions of different masses, charges and angular momenta in the universe. This explains superselection rules and says more. For example, the univalence superselection rule says that there are no superpositions of integer and half-integer spin states, while the above analysis says that there are no superpositions of any spin states including different integer spin states and different half-integer spin states. Moreover,

¹For a helpful review of superselection rules see Earman (2008).

this analysis also says that there are no superpositions of different energies, not only different masses.

These additional predictions seem to contradict experiments; one may point out that we can find and prepare superpositions of different energies, different integer/half-integer spin states. But the contradiction may be an illusion. These superpositions may be not pure states but mixed states; it is possible that the systems being in these superpositions are entangled with other systems, and the interference can still be detected for the systems when the other systems are much bigger and the decoherence is extremely slow. On the other hand, there may also exist entangled superpositions of integer and half-integer spins or different masses and charges, but the interference may be hardly detected for the systems due to fast decoherence in these cases (see also Joos, 1996).

The above explanation of superselection rules, even if it is true, does not invalidate the popular symmetry argument for superselection rules by itself.² We still need to examine the argument. Take the univalence superselection rule as an example. It is widely recognized that the univalence superselection rule is established on the basis of rotational symmetry, namely the $SO(3)$ symmetry in the projective Hilbert space (Wick, Wightman, and Wigner, 1952; Hegerfeldt, Kraus, and Wigner, 1968; Wightman, 1995). The basic argument can be formulated as follows (Giulini, 1996). Consider a superposition of spin-1/2 and spin-1 states along a given direction $\alpha |up\rangle_{1/2} + \beta |up\rangle_1$, where α and β are arbitrary nonzero coefficients and satisfy the normalization condition $|\alpha|^2 + |\beta|^2 = 1$. Under a $SO(3)$ 2π rotation along the given direction in the projective Hilbert space, this state will become $-\alpha |up\rangle_{1/2} + \beta |up\rangle_1$, which is different from the original superposition even modulus an overall phase. Note that the rotation keeps the spin-1 states unchanged but changes the spin-1/2 state by adding an overall phase $e^{i\pi}$ or multiplying -1 . Then when assuming that the rotation group $SO(3)$ is a physical symmetry, this requires that superpositions of spin-1/2 and spin-1 states cannot exist since they do not satisfy the $SO(3)$ symmetry.

It is usually thought that the $SO(3)$ symmetry assumption is incontrovertible, and thus the symmetry argument for the univalence superselection rule is valid. In my view, however, the $SO(3)$ symmetry assumption is debatable. That $SO(3)$ is a symmetry in each of the space of spin-1/2 states, denoted by $H_{1/2}$, and the space of spin-1 states, denoted by H_1 , does not imply that it is also a symmetry in the larger space $H_{1/2} \oplus H_1$. In fact, the above analysis already shows that $SO(3)$ is not a rotation symmetry in the space $H_{1/2} \oplus H_1$; a $SO(3)$ 2π rotation does not keep all states unchanged in the projective Hilbert space. Then, why do people reach the opposite conclusion that $SO(3)$ is a symmetry and the states in a space where $SO(3)$

²There have been debates on the validity of the symmetry approach to superselection rules (see Weinberg, 1995; Joos, 1996; Giulini, 1996; Earman, 2008 and references therein).

is not a symmetry do not exist? It may be related to the fact that no such states have been observed. But I think people make a mistake here.

Whether $SO(3)$ is a physical symmetry in quantum mechanics or not can only be determined by experiments. Think about the discovery of parity violation of Lee and Yang, and Wu in 1956-57. We have found that $SO(3)$ is a symmetry in both spaces $H_{1/2}$ and H_1 . But we have no evidence that $SO(3)$ is a symmetry in the space $H_{1/2} \oplus H_1$, since we fail to find or prepare the superpositions of spin-1/2 and spin-1 states. The failure does not mean that these states must not exist. And it certainly does not support the assumption that $SO(3)$ is a symmetry in the space $H_{1/2} \oplus H_1$ either.

Note that if the symmetry argument is the only possible explanation of superselection rules, then this may support the validity of the argument. However, as I have argued above, a natural initial state of the universe provides a more direct and unified explanation of superselection rules, which does not rely on the debatable symmetry assumption. Moreover, the new explanation of superselection rules also gives additional predictions such as there are no energy superposed states, and they can be tested in experiments. The existence of other explanations of superselection rules will cast more doubt on the symmetry argument.

Finally, let me give a brief comment on Schroeren's recent paper "The Global Phase Is Real" (Schroeren, 2022). In the paper, Schroeren presents a compelling argument for the reality of the global phase based on the symmetry argument for superselection rules. His argument can be formulated in the form of the following syllogism:

A1. If spin states correspond to rays in the Hilbert space (or no two vectors that belong to the same ray correspond to distinct spin states), then there are two ways of characterizing rotational symmetry, namely $SO(3)$ and $SU(2)$, which imply all and only the same physically substantive propositions.

A2. $SO(3)$, not $SU(2)$, implies the univalence superselection rule.

A3. The univalence superselection rule is physically substantive.

C. Two vectors that belong to the same ray correspond to distinct spin states, and thus the global phase is real.

According to my above analysis, $SO(3)$ is arguably not a physical symmetry and it does not imply the univalence superselection rule, and thus A2, as well as A1, is not true. This will block Schroeren's argument for the reality of the global phase.

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