A simple proof that the global phase is real

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Abstract

It is a standard view in quantum mechanics that two wave functions that differ only in the global phase represent the same physical state. In this paper, I argue that this standard view is wrong and the global phase is real in psi-ontic theories such as the de Broglie Bohm theory, the many-worlds interpretation and collapse theories of quantum mechanics.

Suppose there is a superposition of two spatially separated (normalized) wave functions of a particle such as a neutron $\frac{1}{\sqrt{2}}(|\psi_1\rangle + |\psi_2\rangle)$, which appears in many quantum interference experiments. A local interaction can be introduced to add a local phase to one branch of the superposition. For example, a local magnetic field can be introduced to rotate the spin of the neuron in one branch and add a local phase to the branch. Consider two possible situations. One is that a local interaction is introduced in the region of $|\psi_1\rangle$, which adds a local phase ϕ to the branch $|\psi_1\rangle$, where $\phi \in (0, 2\pi)$, and the superposition becomes $\frac{1}{\sqrt{2}}(e^{i\phi}|\psi_1\rangle + |\psi_2\rangle)$. The other is that a local interaction is introduced in the region of $|\psi_2\rangle$, which adds a local phase ϕ to the branch $|\psi_2\rangle$, and the superposition becomes $\frac{1}{\sqrt{2}}(|\psi_1\rangle + e^{-i\phi}|\psi_2\rangle)$. We have the relation $e^{i\phi}|\psi_1\rangle + |\psi_2\rangle = e^{i\phi}(|\psi_1\rangle + e^{-i\phi}|\psi_2\rangle)$. Now if the two superpositions in these two situations, which differ by a global phase, correspond to two different physical states, then we can prove that the global phase is real.

Consider the psi-ontic view, which says that two (nomalized) wave functions which differ not only in the global phase represent different physical states or the physical states of a single system correspond to (normalized) rays in the Hilbert space. On this view, the two superpositions $\frac{1}{\sqrt{2}}(e^{i\phi}|\psi_1\rangle + |\psi_2\rangle)$ and $\frac{1}{\sqrt{2}}(|\psi_1\rangle + |\psi_2\rangle)$ correspond to different physical states, so do $\frac{1}{\sqrt{2}}(|\psi_1\rangle + e^{-i\phi}|\psi_2\rangle)$ and $\frac{1}{\sqrt{2}}(|\psi_1\rangle + |\psi_2\rangle)$. In other words, the local interaction that changes the local phase of each branch of the initial superposition also changes the underlying physical state of the particle.

The next step is to prove that the changed physical states in the above two situations are different. The Schrödinger equation ensures that the local interaction in one region does not change the wave function of the particle in other regions. On the psi-ontic view, this means that the local interaction in one region does not change the physical state of the particle in other regions. Then, the local interaction that changes the local phase of $|\psi_1\rangle$ only changes the physical state in the region of $|\psi_1\rangle$, and the local interaction that changes the local phase of $|\psi_2\rangle$ only changes the physical state in the region of $|\psi_2\rangle$. Thus the changed physical states in the above two situations are different. This proves the reality of the global phase (for the psi-ontic view).

On the other views of the wave function (which admit the existence of the physical states) such as the psi-epistemic view, the wave function does not represent the physical state of a single system but corresponds to the state of an ensemble of identically prepared systems. In this case, we can similarly argue that different global phases correspond to different states of an ensemble under a locality assumption which says that the local interaction with a system in one region does not change the physical state of the system in other regions instantaneously.

The local phase ϕ in the superposition $\frac{1}{\sqrt{2}}(e^{i\phi}|\psi_1\rangle + |\psi_2\rangle)$ is often called the relative phase. This denomination seems to suggest that the local phase ϕ is a nonlocal property of the whole superposition. An argument supporting this viewpoint is that the local phase cannot be measured locally by measuring the corresponding branch, but be measured by measuring the whole superposition. I think this viewpoint is not right. In my view, the reason why the local phase cannot be measured locally by measuring the corresponding branch is because the global phase of the wave function containing only the branch cannot be measured. If the global phase ϕ of the wave function $e^{i\phi}|\psi_1\rangle$ can be measured, then the local phase ϕ of the superposition $\frac{1}{\sqrt{2}}(e^{i\phi}|\psi_1\rangle + |\psi_2\rangle)$ can also be measured locally. Thus, the fact that the local phase cannot be measured locally does not imply that the local phase is a nonlocal property. According to wave function realism (Albert, 2013), the wave function of a single particle is a field in our three-dimensional space, and the local phase of the wave function is obviously a local property in space.[1]

To sum up, I have argued that two wave functions, which differ only in the global phase, correspond to different physical states in psi-ontic theories.

[1]For an N-body system, the wave function is defined in the $3N$-dimensional configuration space. In this case, the above proof of the reality of the global phase may also be given in the configuration space; the local phase will be still a local property in the configuration space, but it may be a nonlocal property in our three-dimensional space (see also Aharonov and Vaidman, 2000).

In other words, if the wave function (modulus the global phase) is real, then the global phase is also real.[2]

References

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[2]For a recent discussion of this topic see Schroeren (2022), Gao (2022) and Wallace (2022).