Putnam's Indispensability Argument

Alex LeBrun*

Abstract

Hilary Putnam (2012) boldly claimed that he never subscribed to the so-called Quine-Putnam indispensability argument for the existence of mathematical objects. All the same, he did not state exactly what his indispensability argument was. This paper presents an interpretation of Putnam's indispensability argument that is consistent with both his recent and historical writings. Through the course of his argument, Putnam requires a view of ontological commitment that is of contemporary interest. I conclude with a brief defense of this view and an application of it to a metaphysical debate.

Hilary Putnam, near the end of his life, aimed to correct the record on the eponymous Quine-Putnam indispensability argument for the existence of mathematical objects (Putnam, 2012). He considered Colyvan's version:[1]

Colyvan's version

P1. If some entity is indispensable to some of our best scientific theories, then we ought to ontologically commit to that entity.

P2. Mathematical objects are indispensable to some of our best scientific theories.

P3. So, we ought to ontologically commit to mathematical objects.

Colyvan's version of the argument aims to establish the existence of mathematical objects like sets or numbers on the grounds that such objects are indispensable to our best scientific theories. This version of the argument has influenced decades of philosophers and is often seen as its canonical formulation (Maddy, 1992; Sober, 1993; Field, 2016). Indeed,

*Draft as of June 3, 2022. The author can be reached at lebrun@ucsb.edu. Thanks especially to Thomas Barrett, Otávio Bueno, and Dan Korman. Thanks also to Steven Canet, Morgan Davies, and Katherina Gontaryuk.

[1]Colyvan's actual formulation differs in trivial ways. See Colyvan (2001, p.11). Putnam is explicitly referencing Colyvan (2008).

Putnam himself made comments that seemingly endorse this exact argument.[2]

Putnam explicitly claims that he has never endorsed Colyvan's version of the argument. As he says, Colyvan's version "is far from right" (Putnam, 2012, 182). Colyvan's conclusion is that we ought to ontologically commit to numbers. Putnam claims that this is not his conclusion. Instead, he says, "my "indispensability" argument was an argument for the objectivity of mathematics in a realist sense" (Putnam, 2012, 183). By 'the objectivity of mathematics in a realist sense' he means that there are true mathematical claims and that their truth is objective. Hereafter, I will use 'true' to exclusively mean *objective truth* of this sort. Moreover, he claims he argued that these true mathematical claims do not "have to be interpreted Platonistically", in the sense that we can accept their truth but refrain from ontologically committing to mind-independent mathematical objects. Instead of P3, Putnam's conclusion to his argument was the following:

C1. Some mathematical statements are objectively true.

One might think that C1 entails ontological commitment to mathematical objects. However, Putnam has another argument for the following conclusion (which he claims is consistent with C1):

C2. We need not be ontologically committed to mathematical objects.

I am not the first to point out that Putnam never accepted Colyvan's version. Burgess and Rosen (1997, pp. 200 - 201), Liggins (2008), Bueno (2013, 2018), Burgess (2018), Clarke-Doane (2020, p. 26), and Barrett (2020) have all noted this discrepancy between Putnam's own conclusions

[2] There is a notorious passage in Putnam (1971, p. 57):

> So far I have been developing an argument for realism along roughly the following lines: quantification over mathematical entities is indispensable for science, both formal and physical; therefore we should accept such quantification; but this commits us to accepting the existence of the mathematical entities in question.

Some have argued that we should take Putnam as presenting for consideration this indispensability argument. See, e.g, Bueno (2018, pp.204 - 205).

and P3. Bueno (2013, 2018) presents a formalization of Putnam's indispensability argument, published after Putnam repudiated Colyvan's version. Bueno's version structurally mirrors P1 and P2 but attenuates their strength to match Putnam's stated conclusions.

In this paper, I present a novel interpretation of Putnam's indispensability argument. I argue, first, that Bueno's reconstruction of the argument does not entail Putnam's C1, nor can it be non-trivially adjusted to do so. Putnam's actual indispensability argument radically departs from Colyvan's version, in such a way that we can accurately formulate it without referencing indispensability at all. (Cf. Putnam's use of scare quotes when he talks of his "indispensability" argument above.) At its core, I argue that Putnam thinks physics *presupposes* that mathematics is objectively true, in the traditional sense of presupposition. We are rationally compelled to think that mathematical claims are true, the argument goes, because our best physics theories presuppose such mathematical claims. This guarantees C1. Putnam presents another argument for how C2 can be consistent with C1. We will briefly examine Putnam's philosophy of mathematics to show how this can be done.

My interpretation of Putnam's argument is of scholarly note, but my primary aim is to illuminate contemporary metaphysical debates with this new interpretation of Putnam's argument. In particular, consider the commonplace practice of offering philosophical paraphrases. Someone may examine a theory with apparently unsavory metaphysical consequences (which include both *ontological* commitments and *structural* (or ideological) commitments) and offer a restatement or paraphrase of that theory that does not have those unsavory consequences. The standard idea is that philosophical paraphrase must fall into one of two varieties: either the paraphrase is *reconciling*, a way of restating the original in a way that is more perspicuous and shows that the original theory did not have the metaphysical consequences; or the paraphrase is *revisionary*, meant to supplant the original theory. As I will show, Putnam's argument reveals a way to drive a wedge between the two standard varieties of paraphrasing away commitment to an entity. The rough idea is that theories, prior to an "understanding", have no metaphysical consequences whatsoever; paraphrases, then, are permissible understandings of some theory. An understanding of a statement is a metaphysical explanation of how that statement could be true. I will show that this has consequences for many debates in metaphysics.

1 AGAINST THE COLYVAN-BUENO VERSION

Putnam is explicit that he does not endorse Colyvan's argument. His first comments after quoting Colyvan are as follows:

> From my point of view, Colyvan's description of my argument(s) is far from right. The fact is that in "What Is Mathematical Truth?" [(1975)] I argued that the internal success and coherence of mathematics is evidence that it is true under some interpretation, and that its indispensability for physics is evidence that it is true under a realist interpretation. . . It is true that in *Philosophy of Logic* [(1971)] I argued that at least some set theory is indispensable in physics . . . but both "What Is Mathematical Truth?" and "Mathematics without Foundations" [(1967)] were published in *Mathematics, Matter and Method* together with "Philosophy of Logic," and in both of those papers I said that set theory did not have to be interpreted Platonistically. (Putnam, 2012, p. 182, emphasis removed)

Putnam here rejects Colyvan's indispensability argument as his own. He instead claims that C1 was always his conclusion and that C2 was consistent with his indispensability argument. C1 is the claim that some mathematical statements are true. Putnam straightforwardly claims this: "[mathematics] is true under a realist interpretation". C2 is the claim that we need not be ontologically committed to mathematical objects. In this passage, Putnam says that we need not be platonists with respect to mathematics.

If Putnam rejects Colyvan's conclusion, then he must reject at least one of P1 or P2 in favor of some other premises that entail his own conclusions. Yet nowhere does he explicitly state these premises.

Bueno (2013, 2018) reconstructed Putnam as endorsing an argument that has a similar structure to Colyvan's version. Bueno's primary change is to weaken P1 and P2 to only guarantee that the resulting mathematical theories are *truth apt*. Generally, a statement or theory is "truth apt" if it is capable of truth or falsity. In the context of Bueno's argument, we take the truth aptness of a statement or theory to entail that the statement or theory is capable of truth or falsity and its truth or falsity is *objective* (Bueno, 2018, p. 205). Consider this reconstruction of Bueno's interpretation (Bueno, 2013, p.228), (Bueno, 2018, p.209):

P1′. If any theory (existentially) quantifies over some entity that is indispensable to one of our best scientific theories, then that theory is truth apt.

P2′. Some mathematical theories (existentially) quantify over entities that are indispensable to our best theories.

C1′ So, some mathematical theories are truth apt.

The idea behind Bueno's version is as follows. Just like with Colyvan's, what matters is whether quantification over mathematical objects is indispensable to our best scientific theories. With P1, indispensable quantification over objects entails that we should ontologically commit to mathematical objects. With P1′, the entailment is weaker. All that is entailed by indispensable quantification over some entity is that the theories which quantify over those indispensable entities are truth apt. Bueno does not give an argument for P1′, but we can see the idea behind it. Consider the case of electrons. Electrons are indispensable to our best scientific theories. Per P1′, this entails that any theory that quantifies over electrons should be interpreted as a theory that, either correctly or incorrectly, attempts to describe the world. Theories that quantify over electrons are thus truth apt in Bueno's sense.

P2′ requires some explanation. Strictly, all it says is that there are mathematical theories that *do* quantify over entities that are indispensable to our best scientific theories. Bueno clarifies that this includes pure mathematical theories (Bueno, 2018, p. 209). So we should interpret P2′ as including two claims. First, that mathematical objects, e.g., sets or numbers, are indispensable to our best scientific theories, and second, that pure mathematical theories quantify over mathematical objects. Again, here, we see a parallel with Colyvan's P2.

Bueno's version of Putnam's indispensability argument is primarily meant to motivate a different project, in particular the prospect of a modal mathematical picture. But as an artifact of Putnam scholarship, I think we should reject Bueno's reconstruction as accurately capturing the relationship Putnam thinks mathematics bears to our best scientific theories. There are two reasons for this.

The first is that P1′ and P2′ together do not guarantee Putnam's conclusion C1. All that Bueno's version of the argument can show is that mathematical theories are truth *apt*, that they are capable of truth and falsity, not that they are true. It is consistent with Bueno's C1′ that all mathematical theories are false. And as we saw in the quote above, Putnam is explicit that he thinks his indispensability argument shows that mathematics is true. Putnam, separately in 1975, straightforwardly said as much: "Mathematical experience says that mathematics is true under some interpretation; physical experience says that that interpretation is a realistic one" (Putnam, 1975, p. 74). It is clear that Putnam endorses C1, not C1′, and Bueno's reconstruction wrongly claims C1′ to be the conclusion of Putnam's indispensability argument as presented in Putnam (1967, 1971, 1975).

One may wish to adjust Bueno's version of the argument in light of this objection. Suppose we replace P1′ with the following:

P1″ If some theory (existentially) quantifies over some entity that is indispensable to one of our best scientific theories, then that theory is true.

P1″ together with P2′ entail C1, that some mathematical theories are true. Bueno's version of the argument can then be simply adjusted to match Putnam's stated conclusions.

The problem with P1″ is that it entails a contradiction (on the supposition that there are any entities which are indispensable to our best scientific theories). Suppose integers are indispensable to our best scientific theories. According to P1″, any theory that existentially quantifies over integers is true. Both the claim that there is a counterexample to Polya's conjecture and the claim that there is no counterexample to Polya's conjecture existentially quantify over integers.[3] Given P1″, both are true. But this is a contradiction. So we should reject P1″ on these grounds.

There is another reason we should reject Bueno's version of the argument for C1. It concerns the way Bueno interprets Putnam's argument for C2. Bueno roughly interprets Putnam as endorsing the following:[4]

[3]Polya's conjecture is the claim that over 50% of the natural numbers less than any given number have an odd number of prime factors. Both it and its negation existentially quantify over integers. Because the conjecture is a 'most' claim, both it and its negation existentially quantify over numbers.

[4]Bueno does not give numbered premises, but we can fairly easily fill in this argument. See (Bueno, 2018, p. 206): "The truth of mathematical statements does not require a platonist ontology. After all, there is a suitable interpretation of such statements in which they come out true independently of the existence of mathematical objects."

4. There are non-platonistic understandings of the truth of mathematical claims.

5. If so, then we need not be ontologically committed to mathematical objects.

C2. So, we need not be ontologically committed to mathematical objects.

I will expound upon this argument below, but the basic idea is that we are rationally free to choose between different understandings of the truth of mathematical claims. Recall that an understanding is a metaphysical explanation of a statement's truth. Putnam believes he has shown that there are non-platonistic understandings of the truth of mathematical claims. He argues that any mathematical claim can be interpreted as a claim about mathematical *possibility*, rather than being a claim about mathematical objects (Putnam, 1967). If so, then we can abstain from ontologically committing to mathematical objects. This is how Putnam argues for C2.

I agree with Bueno that 4 and 5 are how Putnam establishes his "objectivity without objects" conclusions.[5] However, this argument is entirely mysterious if all Putnam establishes is C1′ as Bueno thinks.

We do not worry about giving understandings of *some* truth apt claims. For example, consider claims that existentially quantify over ghosts, like "The ghost is in the room with me". This claim is truth apt, and we are not concerned with providing an understanding of how it could be truth apt. This is because the claim is false. Only if there are *true* claims that existentially quantify over ghosts must we provide an understanding of their truth. Similarly, suppose that we have established the truth aptness of mathematical theories. Just this alone does not demand an understanding of its truth aptness. Accordingly, there is no motivation to provide an understanding, let alone a non-platonistic understanding of the truth aptness of mathematical theories. It is only if mathematical theories are truth apt and *some are true* that we are motivated to give arguments like Putnam's for C2. So, Bueno's version of the argument for C1′ makes the argument for C2 mysterious.

In sum, I think we should reject Bueno's version of Putnam's indispensability argument. It has two problems. The first is that it does not

[5]See also Putnam (2004) for similar arguments.

guarantee Putnam's actual conclusion, which is that mathematics is *true*. And Bueno's version cannot be made to guarantee this conclusion, as we saw with P1″.[6] Second, if Bueno's version of the argument is correct, then it becomes entirely mysterious why Putnam would be motivated to argue for C2. I think what has gone wrong is this. We have been considering versions of Putnam's argument that structurally match Colyvan's presentation, with one premise that establishes the metaphysical upshots of indispensable reference to entities and another premise that establishes the indispensable reference to mathematical entities. And we have found that there is no good way to shoehorn Putnam's conclusions into this structure. We should, instead, abandon the Colyvan-Bueno structure and look closely at Putnam's comments from those original three papers (Putnam, 1967, 1971, 1975).

2 THE PRESUPPOSITION VERSION

The presupposition version of Putnam's indispensability argument establishes the truth of mathematical claims on the basis of the role mathematics plays in our best scientific theories. At its core, the idea is that our best theories *presuppose* the truth of mathematical claims, in the traditional sense of presupposition from philosophy of language.

I propose that the following argument is Putnam's indispensability argument for the truth of mathematics:

1. Our best physical theories are truth apt.
2. Our best physical theories presuppose some mathematical statements.
3. If 1 and 2, then some mathematical statements are true.

C1. So, some mathematical statements are true.

The idea is relatively straightforward. The definition of presupposition entails 3, and these three premises necessitate C1.

[6]There may be a way to infer from C1′ to C1. Namely, if one accepts certain premises about what mathematical truth amounts to. Suppose we accept the following claim: "If a mathematical theory is consistent and truth apt, then it is true." Such a premise would, together with C1′, necessitate C1. There is, as far as I can tell, no textual evidence that Putnam accepted this premise.

Before showing textual evidence for this interpretation, let us define presupposition:

Presupposition. A statement S presupposes a statement p IFF for S to be true or false, p must be true.

Roughly, one statement presupposes another statement if for the first to be *truth apt*, the second must be *true.* Strawson (1950) is the locus classicus of this sense of presupposition. Traditionally, philosophers were concerned with existential presuppositions triggered by definite noun phrases, e.g., that 'The king of France is bald' presupposes that the king of France exists. There are, though, many statements that have non-existential presuppositions. For example, the statement 'Dylan quit vaping' presupposes that Dylan once vaped. Similarly, Putnam thinks that there are statements in our best physics that presuppose the truth of mathematical claims. The relevant presupposition for Putnam is *not* the existence of mathematical objects, but the truth of mathematical claims. It is unclear whether Putnam was aware of Strawson's technical definition of presupposition.[7] Nonetheless, several comments and arguments Putnam made in the context of his indispensability argument were employing the logic of presupposition.

Consider the following, one from each of his three seminal works on mathematics:

> We will be justified in accepting classical propositional calculus or Peano number theory... because a great deal of science presupposes these statements. (Putnam, 1967, 13)

> [L]et us consider what is involved [in providing a mathematics-free version of Newton's theory of universal gravitation], and let us consider not only the law of gravitation itself, but also the obvious presuppositions of the law. The law presupposes, in the first place, the existence of forces, distances, and masses—not, perhaps, as real entities but as things that can somehow be measured by real numbers. (Putnam, 1971, p 37)

[7] There is a passage in Putnam (1971, pp. 28 - 30) that indicates some familiarity with this technical definition. However, Putnam seems to be using 'meaningless' and 'neither true nor false' interchangeably, which makes it difficult to pin any view on him.

> If one ... wants to say that the Law of Universal Gravitation makes an objective statement about bodies ... What is the statement? It is just that bodies behave in such a way that the quotient of two numbers associated with the bodies is equal to a third number associated with the bodies. But how can such a statement have any objective content at all if numbers and 'associations' (i.e. functions) are alike mere fictions? (Putnam, 1975, p. 74)

In each of these cases, Putnam seems to be pushing the claim that the relationship that physics bears to mathematics is one of *presupposition.* Though the first two mention presupposition explicitly, it is the third which is most clearly an argument from the logic of presupposition. Putnam is arguing that the Law of Universal Gravitation is neither true nor false (does not have objective content) if mathematical claims are not true, a straightforward implication of the thesis that the law presupposes mathematics.

Let us now turn to 1 and 2 in more detail. Putnam explicitly endorses 1 as his first premise. He says,

> I shall assume here a "realistic" philosophy of physics; that is, I shall assume that one of our important purposes in doing physics is to try to state "true or very nearly true" (the phrase is Newton's) laws, and not merely to build bridges or predict experiences. (Putnam, 1971, 36)[8]

It is clear that Putnam took his first premise to be the statement of scientific realism. In a similar vein, the canonical argument for scientific realism, the no miracles argument, was first formulated in the very same pape. We note that the Bueno-Colyvan formulation has no premise endorsing this kind of scientific realism.

Putnam's primary argument for 2 is rather flat-footed. He considers Newton's law of universal gravitation. Simply *reading* the law shows how it presupposes mathematical claims:

[8]See also Putnam (2012, 183, emphasis removed): "Nevertheless, there was a common premise in my argument and Quine's, even if the conclusions of those arguments were not the same. That premise was "scientific realism," by which I meant the rejection of operationalism and kindred forms of "instrumentalism." I believed (and in a sense Quine also believed) that fundamental physical theories are intended to tell the truth about physical reality, and not merely to imply true observation sentences."

> Newton's law, as everyone knows, asserts that there is a force f_{ab} exerted by any body a on any other body b. The direction of the force f_{ah} is towards a, and its magnitude F is given by
>
> $$F = \frac{gM_aM_b}{d^2} \tag{N}$$
>
> where g is a universal constant, M_a is the mass of a, M_b is the mass of b, and d is the distance which separates a and b. (Putnam, 1971, p. 36)

According to Putnam, (N) straightforwardly presupposes mathematics in the sense that it "has a mathematical structure" (*ibid.*, p. 37).

One fears that this is too easy. What is so special about the formulation of the law presented in (N)? It seems prima facie possible that there might be a reformulation of (in the sense of providing an equivalent statement of) Newton's law of universal gravitation that does *not* have a mathematical structure. If there is such a reformulation of (N) that does not have a mathematical structure, then plausibly (N) itself does not presuppose mathematics.

To this end, Putnam argues that it is impossible to provide a non-mathematical reformulation of Newton's law of universal gravitation. He argues:

2a. Some true physical theories presuppose an arbitrary amount of true facts of the form "the distance between a and b is d".

2b. The only way to formulate arbitrarily many facts of the form "the distance between a and b is d" is with mathematics.

2c. If 2a and 2b, then some of our best physics theories presuppose mathematics.

2. So, some of our best physics theories presuppose mathematics.

According to Putnam, among the presuppositions of (N) is an arbitrary amount of non-equivalent facts of the form "the distance between a and b is d". The argument for this is a little nebulous. Here's one attempt at explicating: since (N) is universal, it necessarily permits a solution given any value of d. Putnam does not think anything is special about *distance* predicates; he also thinks that physics presupposes arbitrarily many force, mass, and charge facts. Putnam here is pointing out what has been called the *indexing* role that mathematics plays (Melia, 2000, p. 473). (Cf. Baker and Colyvan (2011), Daly and Langford (2009).) The idea is that mathematics helps us index various concrete, physical facts. Putnam takes this indexing role of mathematics to be a presupposition of our formulations of the laws of physics. This is his defense of 2a.

Putnam then attempts to prove that the only way to countenance this presupposition is with mathematics. Premise 2b is the "indispensability" premise: in science, we must countenance these distance facts, and *the only way to do so* is with mathematics. He gives a short proof of this premise 2b. The basic idea is that, given some constraints, one cannot state all the presuppositions of (N)—pairwise inequivalent distance statements—unless one uses the apparatus of mathematics. Barrett (2020) examines this proof in detail and shows that it is not especially convincing.

There's some more textual evidence for this interpretation of Putnam's argument for 2. Field (2016) rejected Putnam's claim 2b. Field showed that one *could* formulate arbitrarily many facts of the form "the distance between a and b is d" without the use of numbers. He did this by quantifying over an infinite number of spacetime points and using relative distances. So, if 2b was part of Putnam's argument, then we should expect that he would concede that Field's project succeeded in responding to this argument for 2.

And indeed this is exactly what happened. Putnam says,

> Hartry Field understands very well what my arguments were, and he attempts to meet them on their own terms ... I agree that, assuming the nominalistic acceptability of [his assumptions] ..., Field has shown that much or perhaps even all of classical physics can avoid any use of set theory at all. (Putnam, 2012, pp. 190 - 191)

Per this passage, Putnam concedes that Field has shown 2b to be false. However, Putnam immediately pivots and argues that Field's strategy cannot be extended to account for the mathematical presuppositions of quantum gravity theory (*ibid.*). I take this to mean that Putnam has retreated from his original argument for 2 but ultimately stands by the form of it, modulo the indexing role mathematics plays in more contemporary physics.

Let us back up. Here I have argued for a new interpretation of Putnam's argument for the objectivity of mathematics on grounds of its "indispensability to science". The basic idea, following close investigation of his original papers and his recent remarks, is that our best theories in physics presuppose the use of mathematics: To even state our best physical theories, we must use mathematical claims. And given the reigning definition of presupposition, this entails that these mathematical claims are *true*. I have also reconstructed Putnam's argument for the claim that physics presupposes math, connecting it to his recent concession to Field.

One may worry how my reconstruction constitutes an "indispensability" argument. Nowhere does the term 'indispensable' appear in the premises. This is in part because, as Barrett (2020) has shown, Putnam uses 'indispensable' in a non-standard way. Nonetheless, we see that there is a sense in which Putnam is giving an indispensability argument, in the sense of *not being able to do without*. This is in his 2b. He effectively argues that there is a theory T such that T is among our best physical theories and if any T' is equivalent to T, then T' presupposes mathematics. Or, in other words, there is no way to formulate our best physical theories without presupposing mathematics. In this sense, Putnam has shown that mathematics is "indispensable" to science.

2.1 Objectivity without objects

Naturally, one might think that Putnam's argument for C1 entails that we must be ontologically committed to mathematical objects. After all, the thought goes, if we presuppose the *truth* of 2+2=4, it is a short logical hop to committing to the existence of 2. Putnam disagrees; he believes we can have objectivity without objects.

This line of thought provides the motivation Putnam's argument for C2. Here is Bueno's reconstruction, which I agree is Putnam's:

4. There are non-platonistic understandings of the truth of mathematical claims.

5. If so, then we need not be ontologically committed to mathematical objects.

C2. So, we need not be ontologically committed to mathematical objects.

Putnam devotes a paper (Putnam (1967)) to defending 4, and continually refers back to it as evidence for his position that we need not be committed to mathematical objects. (See, e.g., (Putnam, 1971, pp. 75 - 76), (Putnam, 1975, pp. 70 - 72), (Putnam, 2004, pp. 66 - 67), and (Putnam, 2012, pp. 182 - 183, 190).) I will attempt to reconstruct 5 on Putnam's behalf, as I too am interested in its defense. Let us first consider 4.

Putnam argued that we can provide a non-platonistic understanding of the truth of mathematical claims. (The term 'understanding' is my own.) The idea here is that we need not think that true mathematical claims are made true by a platonic realm full of mathematical objects. Rather, we can think that true mathematical claims are made true by the fact that certain mathematical structures are possibly satisfied. The idea is that, in mathematics, the existence of a given *object* is completely fungible with the possible existence of a certain *structure* (Burgess, 2018, p. 12). We can, the story goes, reformulate all true mathematical claims in a modal second-order language. For example, take the statement that Peano's axioms entail that there are infinitely many prime numbers. Putnam's modal reformulation would be something like the following:

Peano$_{modal}$ There are possible structures where Peano's axioms are satisfied.

Prime$_{modal}$ Any possible structure where Peano's axioms are satisfied is a possible structure where there are infinitely many prime numbers.

This second reformulation, Prime$_{modal}$, is meant to capture the same content as "Peano's axioms entail that there are infinitely many prime numbers". In this way, Putnam thinks mathematical claims like these can be understood as describing entailment among possible structures, rather than describing a realm of mathematical objects.

The tenability of Putnam's modal mathematics is controversial (Kreisel, 1972; Burgess and Rosen, 1997; Hellman, 1989; Bueno, 2018). Luckily, we need not worry about it here, as we are only concerned with the form of Putnam's argument. Let's proceed, then, on the assumption that Putnam has succeeded in giving a satisfactory gloss of every true mathematical claim in terms of mathematical possibility.

Crucially, Putnam does not wish to supplant ordinary mathematics with his modal mathematics. As he says,

> My purpose is not to start a new school in the foundations of mathematics (say, "modalism"). Even if in some contexts the modal logic picture is more helpful than the mathematical-objects picture, in other contexts the reverse is the case. Sometimes we have a clearer notion of what 'possible' means than of what 'set' means; in other cases the reverse is true; and in many, many cases both notions seem as clear as notions ever get in science. Looking at things from the standpoint of many different "equivalent descriptions," considering what is suggested by all the pictures, is both a healthy antidote to foundationalism and of real heuristic value in the study of first order scientific questions. (Putnam, 1967, pp. 19 - 20)

We should not think of Putnam as offering a traditional paraphrase in the sense of *replacing* the original statements. Nor does Putnam think his replacements reveal the actual commitments of the original statements; he thinks the two *understandings* of mathematics are incompatible.[9] In this way, Putnam is neither offering a revisionary nor reconciling paraphrase: he seeks neither to supplant the original theory nor to reformulate the original theory in a way that reveals its actual commitments.

What, then, is Putnam trying to accomplish by providing non-platonistic replacements of every mathematical claim? Why does it matter to one's metaphysical picture whether we have objectual statements that are fungible with modal statements? This brings us to what I take to be the most philosophically fruitful aspect of Putnam's arguments about mathematics. In what follows, I depart from worrying about Putnam scholarship, and focus instead on arguments I am inclined to endorse.

I shall sketch a picture of metaphysical commitment that is consistent with 5: One's metaphysical commitments in accepting some statement as true depend on one's understanding of the truth of those statements. There are easy cases:

(i) Electrons are negatively charged.

[9] See: "In short, if one fastens on the first picture (the "object" picture), then mathematics is wholly extensional, but presupposes a vast totality of eternal objects; while if one fastens on the second picture (the "modal" picture), then mathematics has no special objects of its own, but simply tells us what follows from what." (Putnam, 1967, p.11).

(*i*) is objectively true. A scientific realist plausibly thinks that (*i*) is true because there actually are electrons and they actually have the property of being negatively charged. This is an understanding of the truth of (*i*). Call it the 'ordinary understanding'. On the ordinary understanding of the truth of some claim, the claim is true because the things the subject terms refer to actually exist and have the properties so predicated.[10] For most claims we think are true, we accept the ordinary understanding of their truth.

But there are statements we take to be true where one need not accept the ordinary understandings of their truth. Consider:

(*ii*) The average star has 2.4 planets.[11]

Let's say I accept that (*ii*) is true and that its truth is objective, a fact about the world. The ordinary understanding of this statement gets the wrong result. The ordinary understanding says that (*ii*) is true because there actually is the average star and that it actually has the property of having 2.4 planets.[12] However, we don't think that the subject term 'the average star' refers to some average star, floating somewhere in the universe or the platonic realm with exactly 2.4 planets orbiting it. Our rejection of the ordinary understanding of the truth of (*ii*), though, does not entail that we reject the objective truth of (*ii*). Instead, we offer an alternative understanding of the truth of this statement: the ratio of planets to stars is 2.4. What explains why (*ii*) is true is that there are 2.4 times as many planets as there are stars. This alternative understanding does not commit us to the existence of some spooky *average star*. So, we can say this: (*ii*) is objectively true, and its truth is explained by the ratio of planets to stars.

Note what we have done here. There are two permissible understandings of the truth of (*ii*). The first is the ordinary understanding, that there is an average star with the property mentioned. The second is that the ratio of planets to stars is 2.4. Both are satisfactory explanations of how 'The average star has 2.4 planets' can be true, and so both are *permissible*

[10] Cf. Tarski (1936).

[11] Cf. Melia (1995).

[12] Kennedy and Stanley (2009) argue that 'average' is semantically not an ordinary adjective, and so 'the average star' doesn't actually serve to refer in the same way that 'electron' does. I'll ignore this complication here.

understandings. We reject the first, though, because it has untoward metaphysical consequences. It commits us to the existence of a spooky entity. There are three lessons from this exercise. First, there can be multiple permissible understanding of a statement's truth. Second, we are rationally permitted to choose between those permissible understandings. Third, and most radically, that a given statement is true does not by itself determine our metaphysical commitments in accepting its truth.

The existence of this alternative paraphrase strategy is controversial. Some may argue that I have not offered an alternative understanding of the truth of (*ii*). But, rather, they may argue that I have simply *rejected* (*ii*) in favor of the metaphysically palatable alternative that the ratio of planets to stars is 2.4.[13] I want to set this response aside, for I wish only to sketch a view that is consistent with Putnam's arguments for C2. We return briefly to this topic in the final section.

Putnam's modal mathematical view fits nicely in this sketch. For Putnam wishes to retain that the following is objectively true:

(*iii*) Peano arithmetic entails that there are infinitely many prime numbers.

Putnam accepts (*iii*). And as a metaphysician, Putnam must present some understanding of its truth. There is, of course, the ordinary understanding, which will entail the existence of infinitely many prime numbers. Putnam, though, has a couple reasons to be suspicious of the ordinary understanding. For one reason, there is a candidate understanding of mathematical claims that does not entail the existence of mathematical objects. Prime_{modal} presents an understanding of the objective truth of (*iii*), according to Putnam. Moreover, this alternative understanding does not entail the existence of mathematical objects. So, because Putnam is rationally free to choose among permissible understandings, he can accept that Prime_{modal} explains the truth of (*iii*), and thus that we need not be ontologically committed to mathematical objects.

This, I take it, is Putnam's argument for C2. He takes himself to have shown that there are non-platonistic understandings of the truth of mathematical claims and that this permits us to accept the truth of those claims while abstaining from committing to mathematical objects.

[13] This would amount to a "revisionary" paraphrase like Argle's replacement of 'This cheese has holes' with 'This cheese is perforated' in Lewis and Lewis (1970).

Burgess (2018) presents an objection to Putnam's modal mathematical picture that I believe can be dispelled by appreciating the sketch I just gave. Burgess says:

> As the Council of Nicæa declared that the Father and the Son are somehow the same and yet somehow different, so Putnam declares the "mathematics as set theory" and "mathematics as modal logic" pictures ... are somehow the same and somehow different. I find the Nicene Creed easier to understand than Putnam's notion of equivalent descriptions. (Burgess, 2018, pp. 16 - 17)

As he goes on to say, Burgess is confused what Putnam's larger project could even be. Putnam does not wish to *supplant* ordinary mathematics with his modal picture; nor does he think that the two are *equivalent*, for then they could not have different metaphysical consequences (and 5 would be false). The picture I just presented answers Burgess's questions. Putnam is attempting to give a metaphysical understanding of the truth of mathematical claims. What is "somehow the same and yet somehow different" is the ordinary understanding of mathematics and the modal understanding of mathematics. Both are (purportedly permissible) understandings of the same truths but have different metaphysical implications.

3 UPSHOTS

My presupposition interpretation of Putnam's indispensability argument has consequences for current metaphysical debates. In particular, it clears the way for a new way to paraphrase.

Philosophical paraphrase is ubiquitous. Whenever acceptance of some statement or theory ontologically commits one to some untoward entity, philosophers seek to paraphrase away reference to that entity. This paraphrase consists of providing an alternative statement or formulation of that theory that is equivalent on some front but fails to appeal to the problematic entity. Consider the following example:

(iv) This cheese has a hole.

Suppose someone accepts (iv) as true. *Prima facie*, accepting (iv) ontologically commits one to *holes*.

This is not the end of the story if one is averse to committing to holes. Philosophers tend to allow for one to *paraphrase away* commitment to certain entities. The idea is that we can restate the problematic claim in a way that is no longer problematic. In the literature, there are two disjoint and exhaustive varieties of paraphrase. The first is a reconciling paraphrase. The second is a revisionary paraphrase.[14]

Accepting (iv) seems to commit one to the existence of holes. A reconciling paraphraser would argue that this entailment is only apparent, that (iv) does not actually commit one to the existence of holes. For, they continue, what one *really* accepts when one accepts (iv) is the following:

(iv') This cheese is perforated.

The reconciling paraphraser offers (iv') as a paraphrase of (iv); they intend for the former to be synonymous (or something close to synonymous) with the latter. For example, "When I say that there are holes in something, I mean nothing more nor less than that it is perforated" (Lewis and Lewis, 1970). In this way, a reconciling paraphrase dispels any apparent untoward metaphysical consequences. The paraphrase is compatible with the original claim.

The revisionary takes a different tack. They will first claim, in opposition to the reconciler, that accepting (iv) genuinely does ontologically commit one to holes. The revisionary paraphraser will then offer a paraphrase such as (iv') as a *replacement* of the original claim. They will say that they deny (iv) and accept (iv') instead. The two claims, for the revisionary paraphraser, are in tension with one another. What kind of paraphrase one is giving, and the success conditions of the paraphrase, depends on their intentions.

There are known problems for both varieties of paraphrase. Reconciling paraphrases are meant to simultaneously match the originals along some semantic dimension while having different metaphysical commitments (or apparently different metaphysical commitments).[15] On the other

[14]Different philosophers use different names for these paraphrases. Metaphysicians use the compatibilist-incompatibilist phrasing (O'Leary-Hawthorne and Michael, 1996; Korman, 2009; Bagwell, 2020). Philosophers of math use call these same strategies *hermeneutic* and *revolutionary* paraphrases (Burgess and Rosen, 1997). And some call them *reconciling* and *revisionary* (Keller, 2015, 2017).

[15]Cf. Keller (2017) for a response to this problem.

hand, revisionary paraphrasers are forced to deny the truth of the original claim that they had sought to preserve. A revisionary paraphraser counterintuitively says that (*iv*) is false.

Putnam's picture presented here allows for a third way to paraphrase. According to this "third way" paraphrase, we first distinguish between a statement's being true and an understanding of that statement's truth. Recall that an understanding is a metaphysical explanation of how that statement could be true. Second, we note that true statements do not entail one particular metaphysical understanding. There may be multiple understandings of the same statement. Third, one's metaphysical commitments are given by one's *understandings* of true claims. Different understandings are articulations of possible permissible metaphysical explanations of a true claim. An understanding is neither an articulation of the actual metaphysical entailments of a claim (*pace* the reconciler) nor a replacement of the original claim (*pace* the revisionary).

Here's how the third way paraphrase works. We accept that (*iv*) is true. This alone does not compel any metaphysical commitment. We then consider what the possible permissible understandings of the truth of (*iv*) are. One is the ordinary understanding, which does ontologically commit one to holes. Another understanding is (*iv'*), which does not ontologically commit one to holes. The "paraphrases" are actually just different metaphysical understandings of the same claim, and neither has more priority than the other.

The third way paraphrase can allow some to solve particular philosophical problems. In object metaphysics, one position entails that many of the ordinary objects we take there to be actually don't exist.[16] This position, eliminativism, seems to have a big problem: many of our beliefs seem to be about the very ordinary objects that this position says don't exist. For example, it has been claimed that eliminativists must say that many ordinary beliefs, like that there is a table in the other room, are false[17]; or, more worryingly, that because our scientific beliefs reference ordinary objects, the eliminativist must reject present scientific theories.[18] The third way paraphrase says that there is no such problem. The eliminativist can say the following:

[16]See, e.g., Inwagen (1990); Merricks (2001); Rosen and Dorr (2002).
[17]Inwagen (1990).
[18]See Bagwell (2020) and LeBrun (2021).

> It is true that there is a table in the other room, and it is true that the barometer reads 30 inHg. These do not commit me to there actually *being* tables and barometers. I do not accept the ordinary understanding of the truth of these claims. Rather, I accept that what makes it true that there is a table is that there are mereological simples arranged table-wise. And what makes it true that the barometer reads 30 inHg is that there are simples arranged in a certain way. I do not have to say that many ordinary beliefs are false, nor must I be committed to some thesis that we *mean* there are simples arranged tablewise when we say that there is a table. I take a third route.

Given Putnam's argument for C2, this path opens up to the eliminativist. Of course, many difficult questions remain, about e.g. what the standards are for permissible understandings. But this is also a problem faced by any advocate of paraphrase. What is being preserved by a successful paraphrase is difficult to articulate, and the third way has no more problem than any other view of paraphrase.[19]

In all, I hoped to have shown this. Putnam's argument for the indispensability of mathematics to science is subtle, and it has been historically been wrenched from its context in Putnam's program. Putnam, from 1967 until 2012, believed that our best physical theories presuppose mathematics, and this presupposition entails that there are some objectively true mathematical claims. All the same, because of Putnam's complex picture of metaphysical commitment and truth, he affirms that Peano's axioms entail infinitely many primes but refrains from committing to prime numbers. This picture of metaphysical commitment is also of contemporary interest. Metaphysicians currently demand that a paraphrase is either reconciling or revisionary. But on Putnam's picture, a third way appears: we can offer paraphrases that are merely alternative understandings of the

[19]My third way paraphrase bears many similarities to the truthmaker view of ontological commitment. In particular, some have argued that we can accept the truth of claims that straightforwardly existentially quantify over some entity but not be ontologically committed to that thing. See, e.g., Azzouni (2004), Melia (2005), Cameron (2008), Cameron (2010), and Rettler (2016). The idea is that our ontological commitments are determined not by the existentially quantified entities, but by the *truthmakers* of the claims we accept as true. It seems to me that there are important differences between what I am proposing and truthmaker views, though these extend beyond the bounds of the present project.

truth of some proposition. We may yet have objectivity without the relevant objects.

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