The Π-Theorem as a Guide to Quantity Symmetries and the Argument Against Absolutism*

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Abstract

In this paper a symmetry argument against quantity absolutism is amended. Rather than arguing against the fundamentality of intrinsic quantities on the basis of transformations of basic quantities, e.g. mass doubling, a class of symmetries defined by the Π-theorem are used. This theorem is a fundamental result of dimensional analysis and shows that all unit-invariant equations which adequately represent physical systems can be put into the form of a function of dimensionless quantities. Quantity transformations that leave those dimensionless quantities invariant are empirical and dynamical symmetries. The proposed symmetries of the original argument fail to be both dynamical and empirical symmetries and are open to counterexamples. The amendment of the original argument requires consideration of the relationships between quantity dimensions, particularly the constraint of dimensional homogeneity on our physical equations. The discussion raises a pertinent issue: what is the modal status of the constants of nature which figure in the laws? Two positions, constant necessitism and constant contingentism, are introduced and their relationships to absolutism and comparativism undergo preliminary investigation. It is argued that the absolutist can only reject the amended symmetry argument by accepting constant necessitism, which has a costly outcome: unit transformations are no longer symmetries.

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0 Introduction

There is an old question which has recently gained renewed and generalized attention; most famous is Poincaré's statement of this question regarding space:

> Suppose that in one night all the dimensions of the universe became a thousand times larger. The world will remain *similar* to itself, if we give the word *similitude* the meaning it has in the third book of Euclid. Only, what was formerly a metre long will now measure a kilometre, and what was a millimetre long will become a metre. The bed in which I went to sleep and my body itself will have grown in the same proportion. when I wake in the morning what will be my feeling in face of such an astonishing transformation? Well, I shall not notice anything at all. The most exact measures will be incapable of revealing anything of this tremendous change, since the yard measures I shall use will have varied in exactly the same proportions as the objects I shall attempt to measure. In reality the change only exists for those who argue as if space were absolute. (Poincaré, 1914, 94)

Setting aside the issue of differing metaphysical accounts of space, it is apparent that such considerations generalize to other quantity dimensions beyond the spatial ones. There is an apparent paradox: Everywhere in the laws of physics it appears that solutions depend on the absolute values of quantities. Yet, there is also an intuition behind thought experiments like Poincaré's: if all quantities of a kind are scaled by the same factor, *including those of the relevant measurement standard*, the world will be in every way empirically indistinguishable. This paper provides a general reconciliation of the absolutist form of the laws and comparativist intuitions about measurement.

A case which has recently captured the attention of some philosophers: would it make a difference if all the masses doubled overnight? The answer depends on the result of the metaphysical debate regarding quantity absolutism and quantity comparativism:

> (Absolutism) Intrinsic quantities are fundamental, qualitative properties, quantity relations supervene on them.[1]

[1]Maybe this is better put in terms of "dependence", "determination", or "grounding" (see discussion

(Comparativism) Quantity relations are at least as fundamental as intrinsic quantities and do not supervene on them.[2]

Intrinsic quantities are determinate properties of particular physical objects and not relations. We think of them as having an *essentially* monadic logical form.[3]. An object's property of being two kilograms in mass is intrinsic. Alternatively, the comparativist grounds the object's being two kilograms in mass relationally: the object stands in a relation of being twice as massive as, say, some standard kilogram in Paris. The comparativist holds that these relations are not grounded in intrinsic quantities, but are (relatively) fundamental.

Much of the debate concerns an argument against absolutism. Here it is understood as a species of symmetry argument.[4] Such arguments have a general form: some supposed fundamental feature of reality, F, varies under some symmetry transformation; so F is not a fundamental feature of reality. This argument form is applied to the supposed fundamental features of physical reality: intrinsic quantities. The comparativist argues that there is a class of symmetries that leave quantity ratios invariant while varying intrinsic quantities. In this case, the argument licenses a supervenience principle:

(Comparativist Supervenience) No change in intrinsic quantity Q of object

in: McKenzie (2014, 2020); Sider (2020)). I am not here concerned with whatever the proper relation between fundamentals and non-fundamentals is, just that there is some distinction to be drawn which at least implies a supervenience relation describable with possible world semantics.

[2]Here I focus on arguments against the fundamentality of intrinsic quantities. This corresponds to the weak absolutism and weak comparativism described in Martens (2021). Some, including Dasgupta (2016), have presented the argument against absolutism in eliminativist terms. The argument is taken to show that intrinsic quantities comprise surplus structure which ought to be eliminated from our ontology. See Ismael and van Fraassen (2003) and Dasgupta (2016) for accounts of such symmetry arguments. See Martens (2018) for an argument against mass eliminativism. See Sider (2020) and Wolff (2020, chap. 8) for accounts of the absolutist-comparativist dispute in terms of fundamentality.

[3]For an example of a relatively standard metaphysical account of intrinsicality, see Langton and Lewis (1998). See Sider (1996) for a distinction between metaphysical and syntactic criteria of intrinsicality and a discussion of the latter type.

[4]Martens (2021, 2523) usefully distinguishes three approaches to the debate regarding the empirical adequacy of comparativism which are present in the literature: (1) the symmetry approach, (2) the detectability approach, and (3) the possibility-checking (i.e. possible worlds) approach. He only discusses (1) in passing and shows that insofar as (2) is useful it is equivalent to (3). Further, I take (1) to be equivalent to (3). I understand possible worlds to provide a model theory for discussing the symmetries of physical equations. However, as will be made evident, I believe the symmetry approach in is some ways more illuminating and useful for some of the unsettled modality questions (see section 4).

O without some change in relation R between Q and Q' of some O'.[5]

The relevant symmetries are *physical* symmetries which map physical systems to identical physical systems. Well known examples of such symmetry arguments include the argument against absolute velocities due to Galilean boost symmetries and arguments against absolute space due to Leibniz shift symmetries.

Dasgupta (2013) has levied an influential symmetry argument aiming to show that intrinsic mass quantities are not fundamental properties. The argument depends on a notion of mass doubling as a transformation that doubles the mass of every massive object in some physical system and *leaves everything else unchanged*. This *ceteris paribus* condition requires mass doubling be what I call a full symmetry, meaning a dynamical and empirical symmetry. Dynamical symmetries map nomically possible systems to nomically possible systems. Empirical symmetries map systems to observationally indistinguishable systems. The argument runs so: Mass doubling is a full symmetry. Intrinsic mass quantities vary under this full symmetry. Properties that vary under full symmetries are not fundamental. Therefore, intrinsic mass quantities are not (relatively) fundamental.

Baker (2020), Martens (2018, 2021), and others have shown that this argument is unsound because its first premise is false. Counterexamples such as a two body system in which a projectile escapes a planet show that Dasgupta's *ceteris paribus* clause is untenable; either the mass doubling changes the empirical situation because the projectile fails to escape, or the projectile's trajectory breaks the laws. Mass doubling is not a full symmetry. By contrast to mass doubling—or any other basic quantity transformation—which acts on a single basic quantity dimension, I refer to any quantity transformation that leaves both the laws and the observable situation unchanged a *full* quantity symmetry. The generalization to all basic quantity transformations will be made clear by consideration of the Π-theorem.

The argument against absolutism can be rehabilitated by showing that there is a class of full quantity symmetries that shows that intrinsic quantities are not fundamental. This class of symmetries is characterized by Edgar Buckingham's (1914)

[5] The comparativist likely is committed to more than this, this is a minimal principle that excludes absolutist possibilities, e.g. a world in which all the masses are doubled and nothing else is affected. As Dasgupta puts it, the symmetries are global so there must be a change of all Qs of a type in the world. The important thing is whatever set of facts are more fundamental are not explained by the other set of facts (Dasgupta, 2013, 108-9). Dasgupta actually makes the case that this "pluralistic" grounding is less demanding than the individualistic condition I am stipulating.

Π-theorem, a foundational result of dimensional analysis. This theorem establishes a general form of physical equations which is invariant under both representational unit transformations and ontic quantity symmetries. The general structure of these quantity symmetries has implications for the nomological role of the dimensional constants. I will argue that whether these quantity symmetries are accepted as *dynamical symmetries* depends on whether or not the values of physical constants (e.g. the gravitational constant) are fixed by the laws or instead are nomically contingent. The absolutist's escape route from the amended symmetry argument requires that the values of physical constants are nomologically necessary—I argue that this is a costly move, requiring a privileged set of "natural" units.

1 The Quantity Calculus and the Argument Against Absolutism

The argument against absolutism requires the existence of ontic counterpart symmetries of quantities corresponding to the broadly accepted symmetries of the representations of quantities—unit transformations. It is necessary that these ontic quantity symmetries are both dynamical symmetries and empirical symmetries.

As terminology varies I will establish my vocabulary with a tripartite distinction:

> (Quantity) A property of a physical object that is representable by class of a number-unit pairs, generally represented by a number multiplied by a unit (e.g. my quantity of height is approximately 1.854 meters or $1.854 \times \mathrm{m}$);
>
> (Quantity Dimension) A collection of quantities which are all representable by the same set of units, i.e. commensurable (e.g. my quantity of height, yours, the length of route 66, and an Angstrom are all commensurable by various length units);
>
> (Unit) A standard value of quantity in some dimension whose assignment to the numerical representation 1 induces numerical values to all quantities in that dimension (e.g. the standard lengths defined by the meter stick,

the foot of Julius Caesar, the distance a beam of light travels in a vacuum in one second).

I have defined these terms circularly—My aim here is not an analysis but a specification of their relations so as to avoid confusion. We can see the relations as: quantities are parts (subsets) of dimensions and units provide different standards of division (partitioning) of dimensions into subparts (subsets).[6]

The representation of any quantity as a product of a number (synonymously: value, magnitude, measure) and a unit informs us that these quantities of concern exist on a ratio scale. Further we keep track of the units of some derivative quantity by not only performing algebraic operations on the numerical representations of quantities but also on their units, e.g. $\frac{5 \times meters}{2 \times seconds} = 2.5 \times \frac{meters}{seconds}$. We will see that the algebra of units obeys a necessary condition of physical equations—dimensional homogeneity. This necessary condition has to do with the dimensions of which each unit instantiates, e.g. the dimensions of force, $[N] = [dyn] = \mathrm{MLT}^{-2}$. Complex, derivative dimensions are constructed from products of powers of basic dimensions, usually M, mass, L, length, and T, duration.[7] Any quantity has a dimensionality or dimension, $[Q] = \mathrm{D}$, which can be multiplied and divided arbitrarily, e.g. $[Q_1 \times Q_2] = [Q_1] \times [Q_2] = \mathrm{D}_1 \times \mathrm{D}_2$.[8] Consider $[F] = [m] \times [a] = \mathrm{MLT}^{-2}$. However, only quantities of like dimensions can be summed or subtracted; i.e. if $k_1 Q_1 + k_2 Q_2$ is coherent, then $[Q_1] = [Q_2]$—this condition is *dimensional homogeneity*. Intuitively, it makes no sense to add a length to a force, etc. The dimensionality of a dimensionless quantity (i.e. a number) is $[1]$, which is the identity—for a quantity Q of arbitrary dimension $[Q] \times [1] = [Q]$.[9] The product of a quantity of some dimension and another of inverse dimension is dimensionless: $[Q] \times [Q]^{-1} = [1]$.

Equations are mere representations of relations between quantities, which are themselves "worldly".[10] Relations between quantities are physical systems, and equa-

[6]This is more or less the official metrological view, see JCGM (2012).

[7]Outside of mechanics, additional dimensions for electrical charge, C and for temperature, θ, are introduced. I will only deal with mechanics in this paper for simplicity.

[8]The square brackets denote the dimensionality extraction function. Basic dimensions will be denoted by un-italicized letters, like L for length (and 1 in the case of numbers). Products of powers of these basic dimensions are the values of the [X] function.

[9]Quantity dimensions form an Abelian group. The formal properties of dimensions deserves a much more thorough discussion. See Raposo (2018) for some details and a fiber bundle model. Another geometrical presentation of the structure of dimensions and quantities can be found in Tao (2012). See also Dewar (Forthcoming).

[10]My usage here is at odds with Martens (2021), for whom "quantity" refers to the representation

tions are their representative counterparts, mathematical models. A claim about adequate representation: a physical system can be adequately represented by some representation of a relationship of quantities. Quantities are either represented by variables associated with dimensions or numbers associated with units. Whether or not the units or dimensions of the quantities in some equation are literally represented, they determine the possible forms of any equation containing some set of quantities, and therefore what relations among the quantities themselves are possible: the algebra of dimensions mirrors the algebra of quantities. If so, any definable system of units on those quantity dimensions is *coherent*. This a foundational assumption of dimensional analysis and the source of its utility. As Sterrett has it:

> Thus, if it is known that the system of units is coherent, it follows that the numerical relation has the same form as the fundamental [dimensional] relation. *The form of the numerical equation can be known independently* of actually using units and numerical expressions to express the quantities and then deriving the numerical equation from the quantity equation—so long as the requirement that the system of units is coherent is met. (Sterrett, 2009, 806)

This is what generates the symmetry duality described below, which is essential to my argument, see 3.1.

The numerical representations of quantities are determined by the system of units used. We understand a unit system as a collection of maps from physical quantities to numerical representation. Each particular unit system partitions the physical quantities into equivalence classes in a way that is invariant to the particular homomorphism it adopts, e.g. mass-in-grams vs mass-in-kilograms. These unit systems are related by two kinds of isomorphisms—those that act on the quantities themselves and those that act on the unit system mappings. Distinguishing the quantities from their representatives, we can define two classes of symmetry transformations:

> (Representational Symmetries) Transformations on the assignment of numerical representatives to quantities that leaves the quantities and their ratios unchanged, e.g. unit system transformations.

and not the physical property.

> (Ontic Symmetries) Transformations on the quantities themselves which change the numerical representatives of quantities for any given unit system, leaving their ratios unchanged, e.g. Galilean boosts.[11]

Representational symmetries are transformations of mere representation. Ontic symmetries are transformations of physical systems. More concretely: a unit system transformation from CGS to SI is in part a change in representation from mass-in-kilograms to mass-in-grams. This representational symmetry is a map that can be described by an equation that holds between any *numerical representation* of a mass quantity in the two unit systems: $m_{kilograms} = 1000 \times m_{grams}$; the magnitude of the new numerical representation is increased by a factor of 1000. An ontic symmetry of the sort under consideration would be represented by the equation $m'_{grams} = 1000 \times m_{grams}$; here the transformation acts on the quantity itself, presenting a change in numerical representation that is independent of unit system. This transformation could equally well be described by the equation $m'_{kilograms} = 1000 \times m_{kilograms}$. That these ontic scale transformations of basic quantities are full symmetries, i.e. that $m = m'$, has been the fundamental premise of the argument against absolutism.

The comparativist holds that quantity ratios are more fundamental than intrinsic mass quantities, owing to their invariance under *ontic* scale transformations. The (naive) comparativist argues that scale transformations of basic quantity dimensions, like mass doubling, are full symmetries:

> (Comparativist Commitment) Basic quantity (ontic) scale transformations are full symmetries.[12]

The absolutist rejoinder shows that basic mass doubling cannot meet both criteria required of a full symmetry, so it is important to distinguish the two conditions:

> (Empirical Symmetry) An empirical symmetry is a map from one physical system to another that leaves unchanged all observable phenomena, i.e.

[11] The representational-ontic distinction corresponds to the active-passive distinction that some may be familiar with.

[12] The range of anti-absolutist views includes more than just comparativism. To accept that these scale transformations are full symmetries only requires the denial of intrinsic mass quantity *quiddities*. The denial of quiddities can be accommodated by multiple views. Most weakly it implies a sophisticated substantivalism (Wolff, 2020). More strongly there would be no quiddities if there were no intrinsic quantities at all—or at least no objective facts about them, as in a relationalist view (Dasgupta, 2020). There are also a variety of comparativisms on offer, as developed by Martens (2017, 2018, 2020).

it generates an indistinguishable system.

(Dynamical Symmetry) A dynamical symmetry is a map from one lawful physical system to another lawful physical system, i.e. a transformation that leaves the laws invariant.[13]

An explicit version of the argument against absolutism can be stated:

(1) If quantity Q is variant under some full symmetry then Q is not fundamental.

(2) Mass doubling is an empirical symmetry: If all of the mass quantities were doubled there would be no observable difference.[14]

(3) Mass doubling is a dynamical symmetry.

(4) Mass quantities are variant under a full symmetry. (1, 2, 3)

(5) Mass quantities are not fundamental. (4)

Premise (1) of this argument is a form of the more general variance-to-nonfundamentality (or unreality, or non-objectivity) principle commonly accepted by physicists and philosophers alike.[15] Premise (3) is a posit that there is a comparativist paraphrase of the laws that is genuinely Newtonian but is indifferent to mass doublings. The problem raised against this argument is the inconsistency of (2) and (3). Generally counterexamples to this argument are taken to target the empirical symmetry

[13]In this context, a criterion of a dynamical symmetry is that that the application of transformation to a system *commutes* with the lawful time evolution of the system. See Ismael and van Fraassen (2003), Roberts (2008), and Wigner (1979) for discussions of the relation between these two classes of symmetries.

[14]This argument is meant to directly parallel arguments against the existence of absolute velocity, see Dasgupta (2013, 2016). Crucially this argument depends on absolute mass, and some class of physical quantities more generally, not being observable. Roberts (2008) and Dasgupta (2016) cash this out in terms of the impossibility of constructing absolute quantity detectors. Both parties to the debate tend to accept that absolute quantities like mass are not directly observable. For criticism of the detectability interpretation see Martens (2021, 2540-44). In light of this we might drop the observable adjective and say that empirical symmetries leave all the qualitative facts unchanged (see Russell (2014)).

[15]Defenses can be found in Dasgupta (2016), Ismael and van Fraassen (2003), and Nozick (2001). This principle is also famously discussed by Born (1953). As explained above, we can understand "unreality" or "non-objectivity" as a matter of fundamentality rather than a matter of existence *tout court*. This is not to say that this principle is uncontroversial, consider those who take the debate between mereological universalists and mereological nihilists seriously, see also North (2009).

premise, (2), but my presentation below will focus on how the counterexamples can be used to more directly focus on (3). By showing that the problem with the argument against absolutism is the misclassification of a basic quantity transformation as a full symmetry, I show that the argument can avoid all proffered objections by properly identifying the class of full quantity symmetries. Onto the main counterexample.

2 Baker's Counter Example

Baker (2020) presents a counter example to comparativism, showing that mass doubling is not a full symmetry. While there is more to comparativism than the mere acceptance of a symmetry, this scale independence is the defining feature of what is under consideration:

> "When we double the values of mass, we have changed something fundamental about the world if the absolutist is right, but not if the comparativist is right. We may, therefore, say there is a sort of symmetry to the comparativist theory of quantity that the absolutist theory lacks: transformations multiplying every value of a quantity by some constant leave the comparativist's fundamental ontology invariant, but not the absolutist's fundamental ontology." (Baker, 2020, 81)

For Baker the problem cases are used to specify further commitments and consequences of comparativism and not as counterexamples that conclusively falsify the view. I will focus on the stronger characterization of the problem.[16]

Consider a two body system: a projectile traveling with velocity v away from a planet's surface. From Newton's laws we can derive an equation for the critical escape velocity such that if $v_{projectile} > v_{escape}$, the projectile will escape the orbit of the planet:

$$\text{(Escape Velocity) } v_{escape} = \sqrt{\frac{2GM}{R}},$$

where G is the gravitational constant, M is the mass of the planet and R its radius. Note that the equation for the escape velocity depends only on the mass of the planet

[16]Martens (2018, 2021) has shown this to be only one instance of a broader class of counterexamples in which two particles either collide or escape each other, depending on their absolute masses. These are all equivalent for my purposes.

and not the projectile mass. On Earth, the M and $v_{projectile}$ are such that the projectile escapes. In the mass doubled counterpart system, the planet Pandora's mass is such that the projectile does not escape. The sticking point is that the comparativist sees Earth and Pandora as *empirically equivalent*. So the comparativist cannot hold that the initial state of the two body system has a unique future as determined by the laws.

Baker presents this counterexample to comparativism as showing that comparativism introduces indeterminism into deterministic systems.[17] A different presentation will better serve our purposes: the counterexample generates an inconsistent triad. The three inconsistent propositions are:

(a) The initial states of the Earth and Pandora systems are indistinguishable;

(b) The final state of the Earth and Pandora systems are indistinguishable;

(c) The dynamics are left invariant by the transformation that maps the Earth system to the Pandora system.

The first two propositions follow from mass doubling being an empirical symmetry, the third from mass doubling being a dynamical symmetry. If mass doubling is an empirical symmetry, then it cannnot be a dynamical symmetry: the trajectory required is inconsistent with the escape velocity equation. If mass doubling is a dynamical symmetry it cannot be an empirical one: either the initial state or final state of the system must be changed for the position of the projectile to match the Earth case at the opposite temporal state while having a trajectory consistent with the escape velocity equation. Generally the issue has been characterized as one of empirical adequacy or indistinguishability, this presentation highlights the sometimes implicit assumption that empirical symmetries are a subset of the dynamical symmetries.[18]

[17] We know that Newtonian physics is not perfectly deterministic for particular systems, see Norton (2008) and Earman (1986). The system under consideration is not one of these deviant cases.

[18] Martens makes this explicit. The dynamical condition is pronounced even in the guise of the possibility checking approach "Comparativism should provide at least one metaphysically distinct (and dynamically allowed) possible world for each empirically distinct possible world allowed by absolutism. If the metaphysically distinct worlds that comparativism acknowledges fail to differentiate between those distinct empirical possibilities, then comparativism is wrong. If, on the other hand, the set of all the metaphysically distinct possible worlds acknowledged and dynamically allowed by comparativism contains all the empirically distinct possible worlds (that are dynamically allowed by absolutism), then we may opt for comparativism over absolutism based on an Occamist norm." (Martens, 2021, 2524)

Mass doubling cannot be both an empirical and a dynamical symmetry, so it is not a full symmetry.[19]

3 Rehabilitating the Argument: Lessons from Dimensional Analysis

Dimensional analysis depends on a number of basic (though not totally uncontroversial) assumptions. An account of these assumptions provides a route to a proof of the Π-theorem, which encodes all of the content of dimensional analysis.

First assumption is that all of the basic quantities that figure in equations which are representationally adequate have a ratio scale structure. This means that all quantities of a basic dimension can be related by a scalar multiplication operation of the form $f : x \mapsto \mathbb{R}x$. Mass, length, and time all share this structure and so each of these dimensions form a multiplicative group. This can be attributed to the fundamental idea that these are all extensive quantities, i.e. the quantity of a whole is an additive function of the quantity of its parts.[20] By treating these quantities as *basic* we are treating them as indefinables from which all other quantities are defined.

This is what is called a "complete" system of units (or dimensions). The derived quantities inherit some properties from the basic quantities which define them, i.e. they too are extensive quantities. The ratio scale structure of extensive quantities defines a class of unit transformations which invariant:

> (Unit Transformation) For any quantity $Q = N \times U$, there is a class of maps $Q \mapsto Q'$, $U \mapsto U'$, $N \mapsto N'$, such that $U' = xU$, $N' = x^{-1}N$, $Q' = Q$, where $x \in \mathbb{R}^+$.

An example: the representational Q-transformation $10 \times \text{kilogram} \iff 10,000 \times \text{gram}$. It involves U-transformation, $1 \times \text{kilogram} \iff 1000 \times \text{gram}$, and an N-

[19] We can also show the failure of the mass doubling transformation to meet simultaneously the conditions of being a dynamical and an empirical symmetry by considering the commutation criterion for a dynamical symmetry. Consider two scenarios, one in which the transformation is applied and the projectile escapes and another in which the projectile escapes and then the transformation is applied. By construction both the initial and the final states of these scenarios are indistinguishable, but in the early transformation scenario the projectile must violate the escape velocity equation. The mass doubling transformation does not commute with the lawful time evolution of the system and therefore is not a dynamical symmetry.

[20] I here ignore any distinction between additivity and "proper" extensivity, compare Perry (2015).

transformation, $10 \iff \frac{1}{1000} \times 10,000$. While there are an indefinite number of representations of a quantity in some dimension owing to the indefinite number of reference units, all of the representations are equivalent under the group action.

Insofar as we take quantities and equations of quantities to be describing physical phenomena and not our measurement standards we require all "objective" quantity equations to be invariant under unit transformations. Given that these unit transformations are multiplicative in nature it is clear that quantities defined by products and divisions (and iterations thereof, i.e. powers) of the basic quantities will also be so invariant. As for equations, it is also relatively clear that equations only involving terms of like dimension will be invariant under unit transformations. The equation $Z = X + Y$ remains true under transformations of the form $Z \mapsto cZ$, $X \mapsto aX$, $Y \mapsto bY$, in cases in which $a = b = c$, i.e. $Z = X + Y \iff aZ = aX + aY$. These are cases in which all three quantities share the same dimension and so the same unit transformation factor. If, say, Y was of different dimension such that for some unit transformation $a = c \neq b$, then $aZ = aX + bY$ would not remain true in any case in which $aY' \neq bY$, where $Y' = Z - X$ as defined in the original units. This violation of dimensional homogeneity does not guarantee variance under *all* unit transformations, there may be some unit transformations of some equations in which it just happens that $a = b$ even though $[X] \neq [Y]$.[21]

The present task is to show that the derived quantities in a complete systems of units have an essential form. It is established that the canonical dimensionally homogeneous equations are unit independent. We take as a constraint on the form of a derived quantity that it is a function of basic quantities: the *defining* equation is itself so invariant. That these defining equations only take the form of products of powers of the basic quantities (plus a numerical scale factor) is *Bridgman's Lemma*. The proof is presented as an analytic elaboration of "our" requirement that relative magnitudes have absolute significance—independent of numerical representation. For Bridgman (1931, 21) this naturally follows from an operationalist point of view: the

[21]Bridgman (1927) argues that the principle of dimensional homogeneity is not fundamental and rather a unit invariance principle is. He presents as a counterexample an equation which is the sum of two distinct equations of motion. The equation is not dimensionally homogeneous, but is true and unit invariant. I would argue, with Gibbings (1974, 1982), that the overdetermined equation has a surplus of independent variables. In order to make the argument in full, it is necessary to appeal to a *principle of canonical form*. This principle requires that the canonical or "true" form of an equation is the simplest one, or equivalently, the form which only introduces the terms necessary to model the system at hand.

measurement of relative magnitudes is first and foremost a comparison of bodies which could not be affected by a change in our operational standards. Consider the mechanical definition of a unit of force $F = f(m, l, t)$, where m, l, and t are units of mass, length, and time. For this equation to be unit invariant, ratios of force must remain invariant across arbitrary unit transformations of the basic units:

$$\frac{f(m,l,t)}{f(m',l',t')} = \frac{f(\alpha m, \beta l, \gamma t)}{f(\alpha m', \beta l', \gamma t')},$$

where the Greek letters are numerical scale factors representing the unit transformation and the primes distinguish the two forces. To solve for f, we rearrange the equation,

$$f(\alpha m, \beta l, \gamma t) = f(\alpha m', \beta l', \gamma t') \frac{f(m,l,t)}{f(m',l',t')}$$

and differentiate with respect to α, such that $\dot{f} = \dfrac{\partial f}{\partial \alpha}$:

$$m\dot{f}(\alpha m, \beta l, \gamma t) = m'\dot{f}(\alpha m', \beta l', \gamma t') \frac{f(m,l,t)}{f(m',l',t')} \ .$$

This holds for any unit transformation factors, so we can drop α, β, and γ and consider the simple trivial case where $\alpha = \beta = \gamma = 1$. So we get an equivalence of the product of the masses and the ratio of the forces and their derivatives relative to mass:

$$m\frac{\dot{f}(m,l,t)}{f(m,l,t)} = m'\frac{\dot{f}(m',l',t')}{f(m',l',t')}$$

As this equation is to hold for all values of the primary quantities for either object, we can hold the x' terms fixed and treat the x terms as variables, so we set the right hand side to some arbitrary constant C:

$$m\frac{\dot{f}(m,l,t)}{f(m,l,t)} = \frac{m}{f}\frac{\partial f}{\partial m} = C.$$

With some integral and logarithm trickery, the solution is then

$$f(m,l,t) = C_{l,t} m^{\phi},$$

where $C_{l,t}$ is to be determined by taking the partial derivatives of f with respect to l and t. From this partial derivation we can see that the form of $f(m,l,t)$ is $C_0 m^{\phi} l^{\psi} t^{\delta}$.

Generally derivative quantities are products of powers of basic quantities, multiplied by some dimensionless constant. *This is Bridgman's lemma.*[22]

Now we have the materials necessary for a proof of the Π-theorem. I will proceed as follows: First, an explanation of the significance and relation between the two versions of the proof to follow. This is an explanation of the core metaphysical move of this paper. Second, a proof of the Π-theorem will be given on a purely mathematical basis; the proof proceeds with purely numerical equations involving some reinterpretation of the results above. Third, a proof of the Π-theorem will be given on the understanding that equations relate quantities. It is argued that acceptance of the symmetries defined by the representational, mathematical Π-theorem entails acceptance of the symmetries defined by the ontic Π-theorem.

3.1 From the Representational to the Ontic

Here I will begin with the end and describe the main metaphysical move made by my usage of the Π-theorem. Those uninterested in any technical detail (or those who take the passive-active duality as a matter of course) can skip to 3.4 and 3.5 for the solution of the escape velocity case and a summary of the amended argument against absolutism. The theorem, in a nutshell, states that any adequate physical equation that describes a system can be put into Ur-Equation form:[23]

(Ur-Equation) $\psi(\Pi_1, \Pi_2, \ldots, \Pi_n)$,

where the Π-terms are dimensionless products of powers of the basic quantities necessary to describe the system and ψ is some arbitrary function.

I will distinguish two version of the theorem not often distinguished. Usually authors have one interpretation or another of the result,[24] all agree that there is an

[22] Alternative proofs can be found in Berberan-Santos and Pogliani (1999).

[23] This is my terminology. Sterrett (2017) calls this "The Reduced Relation Equation of 1914". Sometimes this equation is referred to as the Π-theorem itself, but I think it is more proper to consider the theorem the claim that any complete physical equation can be put in this form.

[24] For example, Bridgman (1931) takes the formalist approach indicated by the representational interpretation of the theorem, while Buckingham (1914) takes on (somewhat reluctantly) the metaphysical significance of the ontic interpretation, as does Tolman (1915), more enthusiastically. For discussions of the history of the theorem, including priority disputes see Pobedrya and Georgievskii (2006) and Sterrett (2005, 2017) and their references. Gibbings (1982, 2011) gives a typology of proofs and his own metaphysical account. See also Walter (1990) and Mitchell (2019) on the historical metaphysical dispute regarding dimensions—see Skow (2017) for a contemporary discussion.

important sense in which the result regards mathematical structure. The question regards the proper location of that structure: is this a result of the algebra of quantities or of the numbers which measure them? My argument here is that these two proofs can indeed be seen as mere differences in "interpretation" such that both readings of the transformations described are available. Further, that a commitment to the representational theorem and some minimal assumptions regarding measurement entail a commitment to the ontic theorem, i.e. commitment to a class of quantity symmetries dual to unit transformations.[25]

We first reconsider the nature of the unit transformation discussed above. Let us first specify a neutral conception of an equation between numerical and the quantitative. We take an equation to represent relations between quantities either directly or indirectly, in either case we take the representatives which figure in an equation to have the canonical form of a numerical value multiplied by a unit quantity: $Q = V \times U$. If we take the representation to be direct, then we take the associated dimension to be constitutive of Q such that the principle of dimensional homogeneity is a metaphysical instantiation of Leibniz' Law. Alternatively, we take the unit to be inessential to the representation and merely a bookkeeping device which reminds us of the conventionally decided *rules* which correspond to the principle of dimensional homogeneity. Under either interpretation of units, they are taken to represent a member of a group of homomorphic maps from quantities to numbers which represent the magnitude of the quantity, here represented by $V \in \mathbb{R}$. That the units of some dimension form a group is simply another way of saying that unit transformations are symmetries of the form $U_{trans} : V \mapsto V'$. For the represntationalist or conventionalist, this is a

[25] Sterrett (2009) has brought it to my attention that Maxwell (2002) also noted this ambiguity in the interpretation of physical equations. My understanding of these equations as *quantity equations* is in line with Sterrett's preference and the account of Lodge (1888). Accepting quantity equations means accepting the application of mathematical operations to quantities. This is already done in the representational theory of measurement (Krantz et al., 1971), where operations like addition of masses are operationalized as taking the fusion of two masses (and placing them on a scale). This avoids the awkward work around of Maxwell who gets around the supposed inapplicability of algebra to physical quantities by converting between numbers which can be so manipulated and proper quantities via the introduction and elimination of units—Bridgman (1931) takes this implicit constraint to be the total significance of dimensions. A more general account cannot be provided here, but consider this brief one: exogenous operations on quantity equations represent external actions on a physical system, like the application of work, endogenous operations either are a mere redescription of the system, or describe an internal change in the system, like the interchange of potential and kinetic energy in a pendulum. For more on the "double interpretation of physical equations" see de Courtenay (2015) and Mitchell (2019).

direct numerical transformation and the new units associated with V' merely indicate a different standard for measuring Q.

For the metaphysician there is another set of symmetries that share the form $V \mapsto V'$ with U_{trans}. Counterintuitively, this could be described as a transformation *on* units as opposed to a transformation *of* units: each unit map, $U_{map} : Q \mapsto V$, is transformed so $U_{map} \circ Q = V$ becomes $U'_{map} \circ Q' = V$. This means that a counterpart mass unit, whose identity is unchanged, e.g. gram, is increased or decreased by the same scale that every other quantity in its dimension is. For example if all masses are scaled by 1000, then the new gram standard will be equivalent to the former kilogram standard and so all numerical measurements will be unchanged: $V\,\text{grams} = V'\,\text{grams}'$, where $V' = V$ and $\text{grams}' = 1000 \times \text{grams}$. However, if we use the untransformed gram unit, then we will find all our mass measurements $V'\text{grams} = 1000 \times V\,\text{grams}$, dual to a representational unit transformation from grams to kilograms: $V\,\text{grams} = \frac{V}{1000}\text{kilograms}$.

The Π-theorem provides a bridge from the invariance of physical equations under unit transformations to the invariance of physical systems under quantity transformations. That any physical system can be represented by an equation of dimensionless quantities, is the crux of the revised argument against absolutism. All symmetries of an Ur-Equation representation of a system are dual. On the one hand we have the representational symmetries accepted by all parties—unit transformations. These change the numerical values associated with constituent dimensional quantities but they leave the dimensionless Π-terms unchanged. This requires us to understand the Π-terms as providing a semantic link between an equation and a system itself: the Π-terms represent the quantity relations of the system that have absolute significance. Therefore, there is a class of ontic symmetries which act on the constituent dimensional quantities that are themselves independent of unit transformations—these symmetries act on the system's quantities themselves. The class of ontic physical symmetries which leave the Π-terms invariant are the class of *empirical* symmetries. For this reason the Ur-Equation provides a well-tuned representation of systems—its formalism is coordinated to the physical structure of systems without excess representation. A change in the value of a Π-term necessarily represents a change in the physical system, while a change in the value of a constituent dimensional quantity may be an artifact of a purely representational change, like a change of units systems.[26]

[26] Sterett's analogy between Buckingham's theorem and Wittgenstein's *Tractatus* has greatly clarified

3.2 Proof of the Representational Π-theorem

This proof[27] proceeds on the understanding of equations as relations between numbers or representations of numbers (i.e. variables) and unit transformations as transformations of numbers, as defined above. The structure of the proof is the same in both versions. The fundamental assumption being that we are only dealing with unit invariant or dimensionally homogeneous equations. The symmetries of ratios of basic quantities will propagate to derived quantities according to Bridgman's lemma. This allows the equation to be recast enitrely in terms of dimensionless derived quantities, by way of the reduction of superfluous quantities in the expression of the equation.

A review of the requisite assumptions:

(0) *Zeroth assumption*: Any equation describing a physical system can be represented by some function of numbers which represent quantities set equal to zero:

$$f(Q_1, Q_2, \dots Q_N) = 0.$$

(1) *First assumption*: We are only concerned with "complete" equations whose algebraic form is unit-invariant. For such equations there is a class of representations:

$$f'({Q'}_1, {Q'}_2, \dots {Q'}_N) = 0,$$

where $x_i Q_i = {Q'}_i$ and the unit transformation factors $x_i \in \mathbb{R}$.

(2) *Second assumption*: If the equation describing the system is unit invariant, then the n numbers representing derivative quantities are unit-transformed by transformation factors that can be defined as products of powers of the unit-transformation factors of the numerical representations of the constituent basic quantities.

my thinking on this point. We may consider the dimensional quantities as basic objects and the dimensionless Π-terms as propositions about the relations of these objects. As it were, the world consists of facts and not things; the Π -terms are accordingly isomorphic to the physical facts while the dimensional quantities fail to represent in isolation. ψ represents higher-order propositions which are decomposable into relations, here dynamical rather than logical, between the basic propositions, Π-terms. *The equation itself serves as a model of the system.* See especially the diagrams on pages 225 and 227 of Sterrett (2005).

[27] This presentation of the proof is based on Ehrenfest-Afanassjewa (1916).

These are the fundamental assumptions of dimensional analysis; to give them up would be to forgo many important patterns of physical reasoning and would threaten the marriage of measurement and number.

The proof proceeds: We define the number of derivative quantities, r, as the difference in the total set of quantities describing the phenomena, N, and the subset of basic quantities, n: $r = N - n$. We can understand the n basic dimensions to serve as a *reduction base* for the original description of the system by N quantities. If r is non-zero, then the reduction exists, and with Bridgman's lemma, we can define the relations between the derivative and basic transformation factors with a set of r equations:

$$\left\{\begin{array}{l} x_{n+1} = x_1^{a_{n+1,1}} x_2^{a_{n+1,2}} \ldots x_n^{a_{n+1,n}} \\ \\ x_{n+2} = x_2^{a_{n+2,1}} x_2^{a_{n+2,2}} \ldots x_n^{a_{n+2,n}} \\ \qquad\qquad \vdots \\ x_{n+r} = x_1^{a_{n+r,1}} x_2^{a_{n+r,2}} \ldots x_n^{a_{n+r,n}} \end{array}\right\},$$

where the exponents $a_{i,j}$ are defined by the relation $Q_i \propto Q_j^{a_{i,j}}$, with $i = n+1, n+2, \ldots, n+r$ and $j = 1, 2, \ldots, n$.[28] These $x_{n+1}, \ldots, x_{n+r}$ factors are the numerical scale factors for unit transformations of the replacement quantity representations for the Qs: dimensionless Π-terms. The values of these factors depend on the unit transformations on the basic quantities and each how each Π-term is defined out of the basic Qs.

From this equation set, we define r derived quantities, eliminating all of the transformation factors and involving all of the relevant representations of quantities:

$$\left\{\begin{array}{l} \Pi_{n+1} = \frac{{Q'}_{n+1}}{{Q'}_1^{a_{1,1}} {Q'}_2^{a_{1,2}} \ldots {Q'}_n^{a_{1,n}}} = \frac{Q_{n+1}}{Q_1^{a_{1,1}} Q_2^{a_{1,2}} \ldots Q_n^{a_{1,n}}} \\ \\ \Pi_{n+2} = \frac{{Q'}_{n+2}}{{Q'}_1^{a_{2,1}} {Q'}_2^{a_{2,2}} \ldots {Q'}_n^{a_{2,n}}} = \frac{Q_{n+2}}{Q_1^{a_{2,1}} Q_2^{a_{2,2}} \ldots Q_n^{a_{2,n}}} \\ \qquad\qquad\qquad \vdots \\ \Pi_{n+r} = \frac{{Q'}_{n+r}}{{Q'}_1^{a_{r,1}} {Q'}_2^{a_{r,2}} \ldots {Q'}_n^{a_{r,n}}} = \frac{Q_{n+r}}{Q_1^{a_{r,1}} Q_2^{a_{r,2}} \ldots Q_n^{a_{r,n}}} \end{array}\right\}.$$

We now have r Π-terms which are independent of the transformation factors, and so we can represent the system using numbers which are unit invariant—the Πs are

[28]The exponent will be zero for all irrelevant quantities.

equivalently defined out of the Qs and the Q's—and encode the essential relations between derived and basic quantity measurements:

$$f(\Pi_1, \Pi_2, \ldots, \Pi_r) = 0.$$

This is the (representational) Π-theorem. It was derived from three assumptions: (0) that numerical representations of physical systems exist which can be described as a function of numbers set equal to zero; (1) There is a subset of such numerical equations that are unit invariant; (2) Bridgman's lemma, i.e. the numerical measures of derived quantities are products of powers of the numerical measures of basic quantities.

3.3 Proof of the Ontic Π-theorem

This proof[29] understands the of equations to directly represent quantities themselves and proceeds by considerations of ontic transformations of quantity dimensions rather than unit transformations. The relation between the two was more fully analyzed in 3.1, but the outline bares repeating. What is meant by calling this proof "ontic"? Of course, in neither the ontic or in the representational case are the *syntactic* objects which make up equation tokens taken to be quantities or numbers (rather they are variables and numerals). The ontic-representational distinction is this: either equations represent relations between quantities which are properties of physical systems or they represent relations between numbers which *measure* quantities according to some externally defined convention. Given the assumption of faithful measurement conventions, conclusions drawn under interpretations of the latter kind entail counterpart conclusions under interpretations of the former kind. This is to say: the two interpretations are in an important sense interchangeable, if numbers can measure quantities at all.

We begin again with a generalized functional form of a complete (i.e. unit-invariant) equation describing a physical system:

$$f(Q_1, Q_2, \ldots Q_N) = 0 = f(Q'_1, Q_2, \ldots, Q'_N),$$

where each Q_i is a quantity composed of a dimensionless number and a unit quantity, $Q_i = V_i U_i$, and the primed quantities are related by dimensionless transformation factors x_i. Here we abstract from the (conventional) determinancy of "value" and

[29] This presentation of the proof is based on Gibbings (1982, 2011).

the "unit" of some quantity to its magnitude, M, and dimension, D, where each unit transformed quantity counterpart is identical in these respects: $Q_i, {Q'}_i, {Q''}_i, \dots = M_i$ and $[Q_i], [{Q'}_i], [{Q''}_i] = D_i$. This abstraction serves us with a *unit-free* representation of the quantities, much like tensor calculus allows us *coordinate-free* representations of spacetime—this makes it clear that we are dealing with the ontic quantities and not their mere representations.[30]

Given Bridgman's lemma, we can define the dimensionality of each quantity as products of powers of the basic quantity dimensions $D_1, D_2, \dots, D_n$:

$$\left\{\begin{array}{c} [Q_1] = D_1^{a_{1,1}} D_2^{a_{1,2}} \dots D_n^{a_{1,n}} \\ \\ [Q_2] = D_2^{a_{2,1}} D_2^{a_{2,2}} \dots D_n^{a_{2,n}} \\ \vdots \\ [Q_N] = D_1^{a_{N,1}} D_2^{a_{N,2}} \dots D_n^{a_{N,n}} \end{array}\right\}.$$

Now we take a subset $Q_i \subset Q_N$ of quantities such that some of their exponents, $a^{i,n}$, are zero, meaning that their dimension does not require all D_n and divide through by it so as to cancel its dimension in all the other quantities. As this elimination process iterates, we will be left with dimensionless quantities. For the first dimension D_1 and each Q_j, $j \neq i$:

$$\frac{Q_j^{a_{i,1}}}{Q_i^{a_{j,1}}} \to \frac{D_1^{a_{j,1}a_{i,1}}}{D_1^{a_{i,1}a_{j,1}}} = 1.$$

The division procedure described above guarantees that the power of the dimension in the numerator and the denominator is equal, hence the dimension is eliminated in the quotient quantity. This creates the functional, complete equation:

$$f(\frac{Q_1^{a_{i,1}}}{Q_i^{a_{1,1}}}, \dots, \frac{Q_N^{a_{i,1}}}{Q_i^{a_{N,1}}}) = 0.$$

Successive cancellations up to D_n for all Q_i lead to all dimensions being eliminated and so all quantities in the function are dimensionless Π-terms of the same form as those defined in the last subsection:

[30] Note that this is merely a presentational move, the "representational" proof given above proceeds in a unit fixed representation, but defines transformations and relations which are invariant under any unit standard. There is an important sense in which these two approaches are equivalent, see Wallace (2019) and Wolff (2020, chap. 9).

$$\left\{\begin{array}{l} \Pi_{n+1} = \frac{Q'_{n+1}}{Q'^{a_{1,1}}_1 Q'^{a_{1,2}}_2 \ldots Q'^{a_{1,n}}_n} = \frac{Q_{n+1}}{Q^{a_{1,1}}_1 Q^{a_{1,2}}_2 \ldots Q^{a_{1,n}}_n} \\ \\ \Pi_{n+2} = \frac{Q'_{n+2}}{Q'^{a_{2,1}}_1 Q'^{a_{2,2}}_2 \ldots Q'^{a_{2,n}}_n} = \frac{Q_{n+2}}{Q^{a_{2,1}}_1 Q^{a_{2,2}}_2 \ldots Q^{a_{2,n}}_n} \\ \vdots \\ \Pi_{n+r} = \frac{Q'_{n+r}}{Q'^{a_{r,1}}_1 Q'^{a_{r,2}}_2 \ldots Q'^{a_{r,n}}_n} = \frac{Q_{n+r}}{Q^{a_{r,1}}_1 Q^{a_{r,2}}_2 \ldots Q^{a_{r,n}}_n} \end{array}\right\},$$

yielding a proof of the (ontic) Π-theorem:

$$f(\Pi_1, \Pi_2, \ldots, \Pi_r) = 0.$$

The divisional procedure of eliminating dimensions highlights an important aspect of the Π-theorem. It provides a definitive answer to the number of variables and the number of dimensionless groups required to describe some system. As indicated above there are $r = N - n$ dimensionless groups of variables, Π-terms, necessary to describe a system of N quantities formed by n basic dimensions. The removal of each dimension is associated with the addition of a variable to each Π-term, yielding a number of $n + 1$ variables per Π-term.[31] Such structural results are important for understanding the quantity symmetries defined by the Π-theorem.

3.4 Symmetries Defined by the Π-theorem: The Escape Velocity Case

Recall that the contextual aim of this theorem was ultimately to provide a standard for scale models in aeronautics.[32] This theorem provides a condition that must be met for one physical system to serve as a model of another, i.e. the theorem defines empirical symmetries for physical systems.[33] We can describe two systems S and S':

$$S:\ \psi(\Pi_1, \Pi_2, \ldots, \Pi_i) = 0$$
$$S':\ \psi'(\Pi'_1, \Pi'_2, \ldots, \Pi'_i) = 0$$

[31] For details and exceptions see (Gibbings, 2011, 59-61).

[32] See Sterrett (2005) for more on the historical development of the formal results of dimensional analysis.

[33] It should be noted that the notion of "physically similar systems" that the Π-theorem allows us to formalize is more fine-grained and sophisticated than the standard of empirical symmetry I am considering here. Besides dynamical similarity, there is geometrical similarity and kinematic similarity, for example. Philosophers concerned with symmetries would do well to consider physical similarity, see Sterrett (2009, 2017).

S and S' are empirically indistinguishable if and only if the relationship between the dimensionless Π-terms are unchanged under some transformation of the dimensional quantities of which they are composed, i.e. $\psi = \psi'$. These relations between the Π-terms define effective conservation principles from which equations of motion are derivable. Consider a derivation of the periodicity of a pendulum.[34] Supposing that the period of a swing of a pendulum depends only on the intensity of the gravitational field, its length, and its mass, we have the functional equation $t = f(g, l, m)$. We transform this equation into its dimensional form: $\mathrm{T} = [t] = F([g], [l], [m]) = F(\mathrm{LT}^{-2}, \mathrm{L}, \mathrm{M})$. We see that M is ineliminable and plays no role in the dimension of t so m is not to be included from any Π-term. From inspection, we find that one Π-term suffices: $\Pi = \sqrt{\frac{l^2}{g}}$. So then the period of the pendulum swing is a conserved property for a fixed length pendulum in a stable gravitational field, as shown in the equation $\Pi - t = 0$.

In our escaping projectile case $\Pi = \sqrt{\frac{kGM}{rv_{pro}^2}} + \epsilon$ and ψ is a function that subtracts one, for the Ur-Equation form $\Pi - 1 + \epsilon = 0$. So then, the ratio between the projectile's escape velocity and its actual velocity in the escape case is conserved at approximately 1: $\frac{v_{escape}}{v_{pro}} = 1 + \epsilon$. That two systems share this ψ requires that the numerical values of the Πs are equivalent between the two systems.[35]

Buckingham's argument is that changes in the basic quantity *dimensions* will leave the Π-terms unchanged in the transformation $f : S \mapsto S'$, because they are *dimensionless*. If all of the operands and the values of ψ and ψ' are identical, then the functions must be the same. This identity signifies a symmetry. In the cases we are concerned with ψ and ψ' stand in for the *dynamical laws*. This makes good on an assumption made by comparativism, that the relevant empirical symmetries of a system are a subset of its dynamical symmetries. A formerly problematic principle is justified: that measurable quantities must be invariant under dynamical symmetries is justified by the Π-theorem.[36] This licenses the inference from the existence of a

[34]Following Bridgman (1916).

[35]ϵ is simply a small constant added so we can deal with equalities rather than inequalities.

[36]This measurability-invariance-principle is the puzzle that is taken up by Roberts (2008). Roberts denies that the principle is analytic and I agree. The synthetic principles are work here are dimensional homogeneity and Bridgman's Lemma. I think it is plausible that these are equivalent or closely related to the publicity principle Roberts proposes. Note that with Roberts I take this to also provide an explanation of Earman's1989 prescription that geometrical symmetries should

class of empirical quantity symmetries to a class of *full* quantity symmetries.

We can generate a full quantity symmetry by: (i) transforming any of the n basic quantity dimensions; (ii) adjusting the r Π-terms according to the structure of Π-functions, as determined by Bridgman's Lemma; (iii) if necessary, adjusting any other quantities composing the Π-terms so that the values of the Π-terms are invariant.

Now we can return to the escape velocity case and show that a *full* mass doubling symmetry, which involves more than doubling the masses of *objects*, does not generate indeterminism or violate the laws. Consider again a situation in which the projectile escapes, $v_{projectile} = \sqrt{\frac{2GM}{r}}$.[37] In step (i) of the transformation, as mass is a basic quantity dimension, we apply an arbitrary ratio transformation: $m_i \to 2m_i$ for i massive objects. We can describe the transformed situation thus: $v_{projectile} = \sqrt{\frac{4GM}{r}}$, *but we do not stop here*. In step (ii) we change one of the derived quantities in order to preserve the relevant Π-term. The Π-term is $\Pi = \sqrt{\frac{kGM}{rv_{pro}^2}}$, or $\Pi = \frac{v_{escape}}{v_{pro}}$, and the derived quantity to be transformed is the gravitational constant G, whose dimensions $\mathrm{L}^3\mathrm{M}^{-1}\mathrm{T}^{-2}$define the compensating transformation the value as a halving, according to dimensional homogeneity.[38] Another way to understand this induced transformation of G is that G *has* (inverse) mass; its dimensionality has negative exponents in the mass dimension.

That the Π-terms are invariant under some transformation of quantity dimensions is *Buckingham's Criterion* for a full quantity symmetry:

> (Buckingham's Criterion) Only those quantity transformations which preserve the values of Π-terms that represent a physical system are full symmetries of that system.

If the absolutist is committed to the principle that physical equations are unit invariant (representational symmetries), and some fundamental principles of dimensional analysis, they are committed to the Π-theorem. This in turn commits them to ontic

not exceed dynamical ones in a "well-tuned" theory.

[37]From here on I drop the ϵ.

[38]Here's an explicit derivation modeled on Bridgman (1916): Let's define G as the product of a dimensionless number γ and its dimensions $\mathrm{L}^3\mathrm{M}^{-1}\mathrm{T}^{-2}$. If we define mass doubling as operating directly on the dimension, then $\mathrm{M}' = 2\mathrm{M}$. So then the new gravitational constant G' equals $\gamma\mathrm{L}^3\mathrm{M}'^{-1}\mathrm{T}^{-2}$, and by substitution $G' = \frac{1}{2}\gamma\mathrm{L}^3\mathrm{M}^{-1}\mathrm{T}^{-2}$. Therefore $G' = \frac{1}{2}G$.

quantity symmetries, against their position.[39]

Put somewhat more simply: a quantity transformation is a full symmetry if and only if it leaves the ratios of all quantities sharing some dimension invariant according to their exponent in that dimension.

3.5 Executive Summary of the Corrected Argument Against Absolutism

My aim is to correct a premise of the argument against absolutism. The argument fits the schema of a symmetry argument:

> (1) For any supposed fundamental property F, if F varies under a full symmetry, then F is not fundamental. (variance-to-unreality inference)
> (2) Mass doubling is a full symmetry. (naive comparativist commitment)
> (3) Intrinsic mass quantities vary under mass doubling. (definition of mass doubling)
> Therefore, intrinsic mass quantities are not fundamental. (1, 2, 3)

Premise (2) was falsified. A full symmetry is a transformation that is both a dynamical and an empirical symmetry. It was established that mass doubling cannot be both. I amend the argument by substituting (2) and (3) with (2*) and (3*):

> (2*) There are a class of *full* quantity symmetries defined by the Π-theorem, one of which, full mass doubling, doubles the masses and halves the gravitational constant. (Buckingham's Criterion)
> (3*) Intrinsic mass quantities vary under full mass doubling. (definition of full mass doubling)

Importantly my argument has been general, any quantity which is not a dimensionless ratio, is not fundamental.[40]

[39] Wolff (2020) comes to a similar conclusion, though by way of measurement theory rather than dimensional analysis. Roberts (2016) also responds to the counterexample to comparativism much the same as I do, but works on the basis of a less general principle than the Π-theorem. See also Dewar (Forthcoming)—it is not clear to me whether or not his group-theoretic sophisticated absolutism is equivalent to group theoretical presentations of the Π-theorem and the results of dimensional analysis, compare Corrsin (1951); Boyling (1979); Curtis et al. (1982); Raposo (2018).

[40] This avoids the "pushing-the-bump-under-the-carpet" objection that can be made against other comparativisms, see Martens (2020, 15).

The argument for the pivotal amended premise (2*) is the establishment of Buckingham's Criterion for a general quantity symmetry. The first part of the establishment of Buckingham's Criterion is to provide a general form for physical equations, the Ur-Equation. Its generality is justified by the assumption of dimensional homogeneity and the completeness of the physical equations in question. These are undeniable, at least for the equations we call physical laws.[41] The subsidiary proof, Bridgman's Lemma tells us the form of the quantities, Π-functions, that figure in the Ur-Equation. These are measurable quantities, dimensionless products of powers of basic quantities. With this all in place, the Π-theorem can be be proved. As the Ur-Equations which represent systems embed dynamical equations, equations of motion, quantity transformations which leave their ψ-functions invariant are by definition dynamical symmetries.[42] As only Π-functions figure in these equations, such a dynamical symmetry must leave their values invariant as well. All absolutely significant quantities are Π-terms, so any transformation that leaves the Π-terms invariant is an empirical symmetry. So we have an intersection of the dynamical and empirical quantity symmetries of a system. These are symmetries in which individual quantities may be transformed according to their ratio structure and the constraints defined by the preservation of the systems dynamics according to general law will induce transformations on other quantities such that the empirical situation, specified by Π-terms, remains invariant.

This account of the quantity symmetries takes into account *inter*-quantity relations. Quantity symmetries generally require the transformation of multiple quantities, though they may transform only a single quantity *dimension*. By ignoring inter-quantity relations, the contemporary dialectic has been built on a fallacious assumption—a primary target of one of Galileo's two new sciences:

> "Only by a miracle could nature form a horse the size of twenty horses, or a giant ten times the height of a man—unless she greatly altered the proportions of the members, especially those of the skeleton, thickening the bones far beyond their ordinary symmetry.

[41] But cf. Grozier (2020). As a matter of scientific methodology, unit invariance may not be so necessary, though it seems that it is necessary for metaphysics—for reasons already indicated. A much deeper account of the nature of dimensions and their relation to the laws and the significance of dimensionless quantities is needed. I only start to scratch the surface in 4.

[42] For example, dimensional analytic methods exploiting the results of the Π-theorem are widely used in fluid mechanics, where more formal analytically methods are intractable. Examples of such derivations of dynamical equations can be found in textbooks like Gibbings (2011).

> Similarly, to believe that in artificial machines the large and small are equally practicable and durable is a manifest error. Thus, for example, small spire, little columns, and other solid shapes can be safely extended or heightened without risk of breaking them, whereas very large ones will go to pieces at any adverse accident, or for no more cause than that of their own weight." (Galilei, 1638, 14)

4 The Nomological Role of Constants

There is one lingering issue. I cannot hope to settle it here, but I'd like to open this vista for surveying. The account of dimensional analysis above gives no special role to the constants of nature, particularly the gravitational constant; they are each merely another parameter of equations that represent physical systems. As such, they are available to be transformed by quantity symmetries. Indeed, they are only special in the sense that they are most apt to be manipulated in quantity symmetries as they describe the coupling of various logically independent basic quantity dimensions.

Let me clarify what I mean by "constants of nature". Johnson (2018)[43] distinguishes three kinds of quantities called "constants": scale factors, system-dependent parameters, and system-independent parameters. The system-independent parameters are *universal* constants and are my concern here. Scale factors are mere numerical artifacts that can be inserted or removed from equations at will by unit changes. System-dependent parameters on the other hand are true quantities that correspond to aspects of physical systems. For example, the density of a fluid ρ may be defined as the ratio of its mass and volume $\rho = m/V$. For the treatment of some particular fluid, like an idealized incompressible fluid, this quantity may indeed remain constant, but its value differs for different fluids.

Among system *independent* parameters there are three subkinds which can be distinguished, only two of whom are constants of nature in a philosophically significant sense. We distinguish:

> (Numerical Artifacts) These constants include mere numerals (like Avo-

[43]Johnson thoroughly discusses Campbell and Ellis' views on the constants and their nature. While there are many interesting and important facets of the differences between these three, it must be deferred to further work. My purpose here is just to (re)introduce these issues to the metaphysicians recently interested in the metaphysics of quantity. My discussion here follows Johnson (2018), particularly chapter 3.

gadro's constant N_A) and composite constants (like Stefan's constant σ and the fine structure constant α). I take the mere computational significance of numbers like N_A to be self-evident. Whether dimensional or dimensionless,[44] composite constants are merely *defined quantities*, much like density.

(Properties of Fundamental Particles) These constants describe properties of the fundamental elements of matter and so have universal scope. Examples: m_p, m_e, e.

(Properties of Fundamental Fields) These constants are complete general in scope and characterize the fundamental interactions. Examples: c, ϵ_0, h, G.

My concern here is solely the third class of constants, the interaction constants which describe the strength of coupling for various sorts of fields. Given that the debate between the comparativist and the absolutist has concerned the possibility of changing the basic quantities, i.e. constants describing fundamental properties the fundamental particles we can understand the question raised here as: Do transformations of the particle constants induce transformations in the interaction constants?

The problem with treating interaction constants as free parameters is that they seem to play a more significant role in all physical laws: their values seem *constitutive* of the laws. It seems that if the gravitational constant or any other constant of nature is changed, then *the laws* have changed.[45] This would mean that there is some discrepancy, though unobservable, between the two escape velocity cases, vindicating the absolutist. However, there is an ambiguity in our understanding the constants to be informative: do the constants inform us about the nature of the *actual* world, or do they inform us more broadly about the class of *nomically possible* worlds?

[44] This distinction is illusory, the Π-theorem shows how quantities of either sort can be converted into quantities of the other—All that is required is a reformulation of the defining equation, i.e. a change in what dimensions have independent *unit* quantities. For a conversion of α see Johnson (2018, 50-4). Note: Johnson doesn't think such conversions are completely conventional and depend on some empirical facts obtaining, *contra* Bridgman (1931).

[45] There is some evidence for this conception in the physics literature. In some discussions of cosmological "multiverse" models and counterfactual cosmologies it is assumed that differing constants of nature mean differing laws of nature. There is a presumption that the constants are *fundamental*. See Barrow (2004, chap. 9). Note: Barrow himself makes the case for the opposing, contingentist conception of the constants—though it seems that this goes along with a contingentism regarding the laws as well.

Alternatively: do their values determine which worlds are in fact nomically possible, or are their values fixed by the natural laws?[46] Broadly, there are two views one can have towards the gravitational constant in particular and interaction constants which appear in the laws in general:

> (Constant Contingentism) The values of the dimensional constants are independent of the laws and depend on non-nomic quantity regularities—they vary across nomically possible worlds.[47]
>
> (Constant Necessitism) The values of the dimensional constants are fundamental and necessary across nomically possible worlds. These values constrain non-nomic regularities.[48]

Contingentism naturally pairs with comparativism. For the contingentist comparativist, the "constants" are parameters in physical equations like any other; it makes perfect sense that they would transform with the basic quantity dimensions. It is only the general relation between the constants and dimensional quantities that constitutes the laws, the exact value of dimensional constants is irrelevant—in this way the laws are "structural". Some pioneers of dimensional analysis thought of the constants as properties of the environment, the gravitational constant and the permittivity constant were thought to be properties of empty space.[49] The speed of light is often understood as defining the (causal) structure of spacetime. Though one may not need to accept this sort of *ontological* grounding for physical constants: Bridgman held that the constants are *conventional* conversion factors. Conventionalist interpretations give an intuitive understanding of the contingency of the constants.

Similarly, absolutism naturally pairs with necessitism. The constants define a natural and necessary scale for magnitudes, e.g. it is nomically impossible that the gravitational force be stronger than it is. Rather than have the gravitational constant as a parameter in functions that represent gravitational systems, $f(G, x, y, z \dots)$, the

[46] I set aside issues regarding spatiotemporal variations of the constants in a single universe. See Barrow (2004) and Barrow and Webb (2005) for accessible introductions.

[47] This view has been suggested in Ehrenfest-Afanassjewa (1916, 1926) and Nordström (1914) and has recently come under criticism by Martens (2020).

[48] This is to be distinguished from Dahan's 2020 view of the constants as (defeasible) *identifiers* of universal laws—Dahan makes this point herself. Though necessitism is consistent with the idea that constants "baptize" universal laws, it is independent of it. It seems to be the case that Dahan's position entails the matching of the modality of the constants and the laws, see below.

[49] E.g. Mercadier and Vaschy, see De Clark (2017, 312-19).

gravitational constant is an essential constituent of that function which represents the dynamics of gravitational systems, $f_G(x, y, z \dots)$. One natural understanding of necessitarian absolutism is that it leads to a total determination of the facts of the world—a theory of everything would have no terms left to be determined by experiment:

> "One plausible view of the Universe, is that there is one and only one way for the constants and laws of Nature to be... The values of the constants of Nature are thus a jigsaw puzzle with only one solution and this solution is completely specified by the one true theory of Nature. If this were true then it would make no more sense to talk about other hypothetical universes in which the constants of Nature take different values than it would make sense to talk of square circles. There simply could not be other worlds."(Barrow, 2004, 178)

Assuming the world is lawful, the combination of the necessity of the laws and the absolute significance of intrinsic quantities, including the constants would entail a strict, two-way supervenience relation between the laws and the quantities they govern. As changes in the laws are, by assumption, impossible, so too are changes in the quantities.

This presentation of the these two positions is intended as a clarification of what has been at stake in debates concerning the comparativist reformulation of the laws. The laws seem intuitively to refer to absolute quantities—the escape velocity equation does not (explicitly) refer to any mass other than that of the planet.[50] Starting with Dasgupta (2013) and continuing with (Baker, 2020, 83-92) and (Sider, 2020, 145-50), many different formulations of the comparativist Newtonian laws have been proposed and criticized. To debate whether or not there is a coherent comparativist statement of the laws just is to debate the merits of constant contingentism.

Both of these "natural" pairings are superior to their mixed counterparts: necessitist comparativism and contingentist absolutism. These mixed views entail a mismatch between the metaphysics of quantities and the metaphysics of the laws in a way that generates unsynchronized changes—both violate some of our modal scruples. I will not provide knock down arguments but will show what kind of problems the mixed views face.

[50] See Martens (2020) fo a comparativist view which posits a universal reference mass, the Machian mass.

In the case of a mass doubled world, the necessitist comparativist needs to leave the laws unchanged—if the invariance of the values of the constants are a necessary condition for the invariance of the laws, then G must be unchanged. This immediately leads back to the Baker counterexamples. The necessitist comparativist will have to come up with a different solution to this problem than what is provided here.

The contingentist absolutist will have to say that in that very same case the intrinsic mass quantities and the constants have changed but the laws have not. This would seem to violate Humean supervenience: the laws become independent of the underlying facts, they are no longer generalizations of them and so they cannot then be taken to *explain* the underlying facts.[51] The facts do not counterfactually depend on the laws in the way that it seems necessary for them to be explanations. This is not an attractive position for the Humean. What of the non-Humean? They may just be ok with this, Armstrong's 1983 view allows for the laws to vary independently of the facts, e.g. if different universals are related. However there is a problem lurking that is a generalization of the worry for the Humean: If there is counterfactual slack between the facts and the laws, then it is mysterious how we could gain knowledge of them from observations of the facts.

We might describe this problem as a "mismatch problem" in which the modality of the laws and that of the constants fail to agree. The mismatch described by Loewer (2020, sec. 3)—sourced in van Fraassen's 1989 "identification problem"—is somewhat different, concerned with the matching of the distribution of natural properties with the laws (as determined by physicists), but otherwise the scenario is analogous. Here we are considering two scenarios with the same trajectories but different distributions of determinate properties (quantities). It seems as though either the laws must be different or the underlying determinables (dimensions) must have changed. Both the necessitist comparativist and the contingentist absolutists have a discrepancy between the best systematization of the "natural" (determinable) properties, the quantity dimensions, and the laws (as determined by the physicists). The laws and the optimal description of the distribution of "natural" properties describe different sets of possibilities. The sets of nomically possible worlds determined by each condition coincide for the necessitist absolutist and the contingentist comparativist, making

[51] Emery (Forthcoming) has recently made an argument that Humean laws fail to be explanatory for another reason—they cannot be *metaphysically robust* explanations of the facts. The result is the same for my purposes here, the Humean is pushed to give up one of their central theses, Humean supervenience, and so face an explanatory gap.

them more coherent than their heterogeneous counterparts.

There may be a worry that by linking the modal status of the laws and the constants so tightly I am falling into a necessitarianism regarding the laws, that there is no metaphysical or epistemic possibility that outstrips that of natural law—is it not *possible* that the same fundamental properties obey different laws?[52] I tend to find such worries and their attendant epistemology dubious.[53] For my purposes here I simply take it as an assumption here that it is required or at least preferred that the laws and the quantities which figure in them come as a "package deal" modally.[54]

In the end then, we can understand the argument against absolutism as a *reductio*, with the consequences of constant necessitism being the *absurdum*.

> "Absolutists therefore face a choice: deny comparativism even on the level of representation, or accept that there is a sense in which quantities represented on a ratio scale are invariant under some ontic transformations, contra absolutism.
>
> Denying comparativism at the level of representation amounts to insisting that not only are absolute masses undetectable and inexpressible, but there is in fact a correct numerical assignment for each magnitude. The ratio scale representation would have to be interpreted as merely reflecting our ignorance about which numerical assignment is correct." (Wolff, 2020, 149-50)[55]

Accepting constant necessitism would give sense to taking this way out of the amended symmetry argument: the laws don't only determine relations between quantities but also determine the *correct* units for describing the world, we are simply ignorant of what they are. This is not beyond the pale generally,[56] but this is a heavy cross for the

[52]See Schaffer (2005) for a typology of necessatarianist views and their discontents. I am referring to here is modal necessitarianism. Though it does seem to me that in this case modal necessitarianism and nomological necessitarianism (nomological essentialism) collapse—the necessitist absolutist could describe the modified laws and constants of the contingentist comparativist as involving *different* quantities, not just numerically but dimensionally (i.e. categorically).

[53]I mostly agree with Wilson's 2013 responses to Schaffer's objections. See also Bird (2004).

[54]I do not however here countenance the equality of fundamentality at the core of Loewer's package deal account of laws. Here I am only concerned with the modal consequences of the relative fundamentality of the dimensional constants and the relations of quantity dimensions in the laws.

[55]See ibid for a technical demonstration of this reductio: denying the existence of a non-trivial automorphism of quantities requires denying the existence of a algebraic group of unit isomorphisms (isomorphisms of the homomorphic maps).

[56]The quest for a minimal and necessary set of "natural" units determined by the constants has

philosopher to bear[57]—I leave it open that it is conceivable that physical investigation may determine such natural units.

I will not settle the issue(s) here. Let me say something more in defense of a contingentist comparativism that points the way to the work still to be done to fully flesh out what the comparativist's commitments are. The contingentist comparativist does not think that "anything goes" with respect to the real, fundamental, physical constants; there is a feature of them that is nomically necessary. What remains invariant under the comparativist symmetries is the relation between the constants and the other parameters in the laws. Their relation is encoded by their *relative* dimensionality. With $[G] = \mathrm{L}^3\mathrm{M}^{-1}\mathrm{T}^{-2}$, as required by the dimensional homogeneity of Newton's law of gravitation, $F = \dfrac{GMm}{r^2}$, it is nomically necessary that G scales inversely with M and cubicly with L. This is independent on any changes of convention regarding units or the basic quantity dimensions. It is a contingent matter of fact what values quantities have, including constants. It is matter of convention which units we use to measure them and (maybe) which dimensions we stipulate as basic. The dimensional (i.e. scaling) relation between different quantities is *necessary*.[58] A full account of comparativism must find some way for accounting for these inter-quantity relations and their boundedness by physical law.

The apparent arbitrariness in value of the constants to which the contingentist is committed, that of the *real*, fundamental constants and not merely *apparent*, eliminable ones, troubled Einstein. Part of his motivation for a unified field theory was to eliminate all real constants whose value could only be determined empirically:[59]

motivated a number of important and respectable physicists, e.g. Planck, Eddington, Einstein (see Rosenthal-Schneider (1980)).

[57] Sider (2020, 121-2) discusses this issue under the guise of fundamentality: it is "intolerably arbitrary" to suppose one relation of the class mass-in-kilograms, mass-in-grams, etc, is the fundamental mass relation between physical objects and numbers. I've avoided discussing this framework here but Sider (2020, 124) makes it clear that this "simple absolutist" cannot take on the dominant representation theory for quantities supplied by measurement theory. Further the simple or necessitist absolutist would also have to give up any nominalist aspirations regarding number, which motivated the comparativism of Field (1980). See also Eddon (2013, 82-85) on the "naive account of quantity".

[58] I cannot discuss this material fully here, but it may be worth comparing the results here and the categorization taken from Johnson (2018) to the discussion in Duff (2014). Compare also recent discussions in Grozier (2020); Riordan (2015) on the question of the fundamental constants and dimensionality.

[59] Einstein's categorization differs from the one used here. For him G is apparent while α is real. This is largely unimportant for the immediate philosophical point discussed here.

> "Of course, I cannot prove this. But I cannot imagine a unified and reasonable theory which explicitly contains a number which the whim of the Creator might just as well have chosen differently, whereby a qualitatively different lawfulness of the world would have resulted.
>
> Or one could put it like this: A theory which in its fundamental equations explicitly contains a non-basic constant would have to be somehow constructed from bits and pieces which are logically independent of each other; but I am confident that this world is not such that so ugly a construction is needed for its theoretical comprehension." (Einstein to Rosenthal-Schneider 1945, in (Rosenthal-Schneider, 1980, 37-8))

Though such a bold conjecture may prove to eliminate physical contingency altogether, likely congenial to Einstein's Spinozism.[60] Such is the price of absolutism. But this conjecture does also put the comparativist on notice: the comparativist ought to endeavor to show that the arbitrariness of the connections between logically distinct quantity dimensions is not so ugly a construction after all.

5 Conclusion

This paper presents an amendment to the symmetry argument against quantity absolutism. Rather than requiring that any universal scale transformations of basic quantities of some dimension are empirical and dynamical symmetries, the argument against absolutism depends only on those symmetries defined by the Π-theorem. These symmetries may involve scale transformations of basic quantities, but they also involve transformations of derived quantities, most notably the physical constants. The symmetries defined by the Π-theorem are transformations that leave the dimensionless quantity ratios which describe some system invariant—these are both empirical and dynamical symmetries.

The transformation of the constants in some symmetries defined by the Π-theorem raises the question of their modal status. On the one hand is constant contingentism, which states that the laws and the relations of the basic quantities determine the values of the constants—their values can vary in nomically possible worlds, supervening on variations of the relations of the basic quantities. On the other hand is constant

[60] See Paty (1986)

necessitism, which states that the values of the constants are fixed across nomically possible worlds and are fundamental—their values and dimensions fix the laws and the relations between the basic quantities. My purpose has been to introduce the debate and set some of its terms. Though I give reason to prefer constant contingentism to constant necessitism, not least of all its superior fit with quantity comparativism, the discussion here is not conclusive.

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