Trace-Free Gravitational Theory (aka Unimodular Gravity) for Philosophers

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Trace-free gravitational theory (also known as unimodular gravity) has been touted as a means to resolving the cosmological constant problem, widely regarded as one of the most important unsolved problems of fundamental physics. The claim that the resolution takes the form of decoupling vacuum energy from the dynamics of the spacetime metric is critically examined and found wanting. The weaker claim that the theory ameliorates the fine tuning version of the cosmological constant problem is more plausible but still disputable. Unimodular gravity is observationally equivalent to standard general relativity theory at the classical level, but this observational equivalence may be broken at the quantum level, at least if some branches of the canonical quantization road to quantum gravity are followed. Unimodular gravity thus offers a quick access to foundations issues in classical and quantum gravity, as well as insights into the considerations that motivate contemporary gravitational research. As such, it deserves more attention than it has received from the philosophy of physics community. The purpose of this paper is to lay out the issues surrounding unimodular gravity in a manner that will be an invitation to more philosophical scrutiny.

1 Introduction

Trace-free gravitational theory (TFT) (aka unimodular gravity (UG)) is a variant of standard general relativity theory (GRT). Although the two theories are observationally equivalent at the classical level, the equivalence may be broken at the quantum level. Understanding how this can be the case gives a glimpse into quantum theories of gravity. It has been claimed that TFT offers a resolution to the "cosmological constant problem," a problem which Steven Weinberg (1989) deems to constitute a "virtual crisis" in physics.[1] The strongest form of this claim asserts that in TFT the dynamics of the spacetime metric is decoupled from vacuum energy. We will see, however, that there are reasons for doubting this claim, and even the weaker and causally neutral claim that TFT offers a new perspective on the cosmological constant problem has been disputed. Such disputes should be a tip off to philosophers of science that something interesting is afoot. TFT also raises anew issues about the requirement of general covariance. The crudest form of an action principle for TFT employs the unimodular coordinate condition, which clearly breaks general covariance. Full covariance can be restored by the introduction of additional geometric objects; but this procedure commits TFT to a spacetime structure that is richer than that of standard GRT. This in turn is connected with the issue of whether TFT offers a means of resolving "the problem of time" in quantum gravity, another matter that remains in dispute. Additionally, the contrast between standard GRT and TFT provides a challenge to extant philosophical accounts of the structure of scientific theories and of the equivalence/inequivalence of theories. Clearly there is much about TFT that should be of interest to philosophers of science, but surprisingly little attention has been devoted to it in the philosophy of physics literature.

The purpose of this paper is to lay out the issues surrounding TFT in a way that will be an invitation to more scrutiny from philosophers of physics. Section 2 provides summary of the formalism of TFT and some of the highlights of its history. Section 3 discusses the relationship between standard GRT and TFT. This relationship is marked by the different action principles that generate the field equations of the two theories, something not taken into account in extant philosophical accounts of the structure of scientific theories. Some of the different action principles that have been proposed for TFT are reviewed in Section 4. The generally covariant versions of these principles all utilize spacetime structure involving additions to the manifold-plus-metric structure used by standard GRT, pointing to another difference between the two theories. Section 5 introduces the cosmological constant problem as it arises in GRT and illustrates the problem in the context of

[1] The motivation for Weinberg's characterization of the cosmological constant problem as a virtual crisis may be partly explained by his conviction that "Physics thrives on crisis" (Weinberg 1989, p. 1). In the intervening years other researchers—with a few notable exceptions to be mentioned below—continue to use phrases like "crisis," "intriguing mystery," "one of the outstanding problems of fundamental physics," ... when describing the cosmological constant problem.

the Friedman-Lemaître-Robertson-Walker (FLRW) cosmologies. The problem arises because of the gross mismatch between estimates of an effective cosmological constant due to the vacuum energy of quantum fields and the relatively small value of the total cosmological constant required to explain the observed accelerated expansion of the universe. The oft-quoted estimates of vacuum energy are due some skepticism, but for present purposes the skepticism is set aside since even with smaller estimates there would be a cosmological constant problem, albeit of a less dramatic form. Section 6 takes up the question of whether, and in what sense, TFT can provide a resolution to the cosmological constant problem. The claim that TFT solves the problem because vacuum energy has no gravitational effect in TFT is rejected. However, TFT together with a dose of meta-physics may help to resolve the fine tuning version of the cosmological constant problem, but even this weaker claim is disputable. In Section 7 the focus is on the quantization TFT in loop quantum gravity and, more specifically, on the consequences for the cosmological constant in loop quantum cosmology, a symmetry reduced form of loop quantum gravity. The observational equivalence between standard GRT and TFT at the classical level appears to be broken at the quantum level. Conclusions are contained in Section 8.

2 Two theories of gravitation

2.1 Einstein's general relativity theory and trace-free gravity

The original gravitational field equations of general relativity theory used by Einstein in 1915-1916 read

$$G_{ab} := R_{ab} - \frac{1}{2} R g_{ab} = \kappa T_{ab} \tag{1}$$

where R_{ab} is the Ricci tensor, $R := R^a_a$ is the Ricci curvature scalar, g_{ab} is the spacetime metric, T_{ab} is the stress-energy tensor for matter and radiation fields, and $\kappa := \dfrac{8\pi G}{c^4}$ (Einstein 1916). Units are chosen so that $c = 1$, and the signature convention for the metric is $(+++-)$. The predictions on which the three "classical tests" are based—the advance of the perihelion of Mercury, the bending of star light, and the gravitational red shift—are derived from these equations. But in 1916 Einstein was aware that the desiderata he set for gravitational field equations would be satisfied even if what would later be called a cosmological term of the form *const* x g_{ab} is added to the Einstein tensor G_{ab} on the lhs of the original field equations (see the footnote in Einstein 1916, p. 144 of the English version).[2] And in the following year he proposed such a modification (Einstein 1917), leading to the introduction of the cosmological constant Λ:

$$R_{ab} - \frac{1}{2} R g_{ab} + \Lambda g_{ab} = \kappa T_{ab}. \tag{2}$$

Einstein's motivation for the introduction of Λ was the mistaken belief that the large scale structure of the universe is unchanging, coupled with the desire for field equations that would allow for a static cosmological model. In retrospect the Einstein static universe can be treated as a FLRW model with $k = +1$ (positive spatial curvature).[3] To produce a static universe the value of a positive $\Lambda > 0$ has to be adjusted to give an effective repulsive force that just balances the attractive force of matter. While the phrase "fine tuning"— which will crop up in the discussion below — was not used at the time, there were rumblings about the instability of the Einstein static universe. But what killed this solution as a candidate for describing the actual universe were Hubble's red-shift observations indicating that the universe is expanding. After learning in 1929 of these observations Einstein not only abandoned his static cosmological model but, as reported by George Gamow (1958, pp. 66-67), referred to his introduction of the cosmological constant as his "biggest blunder". Whether or not Einstein actually used these exact words is neither here nor there; the important point is that after 1929 he adamantly refused have anything further to do with lambda (see Earman 2001). Nevertheless, over the subsequent years the cosmological constant has made numerous appearances on the stage of cosmological theorizing, and with the discovery of the accelerating expansion of the universe in 1998 (see Riess et al. 1998) it, or some surrogate, is apparently here to stay.

It is worth noting that there is a sense in which a cosmological constant

[2] Later writers would prove a uniqueness result. For instance, Lovelock (1972) showed that in four-dimensional spacetime a linear combination of G_{ab} and g_{ab} is the most general two index tensor that is divergence free and is constructed from the metric tensor and its first two derivatives, with the upshot that field equations with cosmological constant term give "the most general modification which does not grossly alter the the basic properties of Einstein's equation [(1)]" (Wald 1984, p. 99).

[3] These models are described below in Section 5.3.

is striving to emerge on its own from the original field equations (1). Taking the trace of both sides of eq. (1) yields (for four-dimensional spacetime)

$$R = -\kappa T, \quad T := Tr(T_{ab}) \tag{3}$$

Multiplying each side of (3) by $\frac{1}{4}g_{ab}$ and adding the result to (1) yields the trace-free field equations

$$\begin{aligned} \widetilde{R}_{ab} &= \kappa \widetilde{T}_{ab} \\ \widetilde{R}_{ab} &:= R_{ab} - \frac{1}{4}Rg_{ab},\ \widetilde{T}_{ab} := (T_{ab} - \frac{1}{4}Tg_{ab}). \end{aligned} \tag{4}$$

The trace-free field equations are weaker than the originals: the conjunction of eq. (1) and the Bianchi identity

$$\nabla^a G_{ab} = 0 \tag{5}$$

entails the conservation law

$$\nabla^a T_{ab} = 0 \tag{6}$$

whereas the combination of eqs. (4) and (5) does not entail (6). To make up for this deficit, postulate the conservation law separately and take TFT to consist of the conjunction of (4) and (6).[4] Then note that TFT implies

$$\nabla^a (R + \kappa T) = 0 \tag{7}$$

with the upshot that

$$R + \kappa T = const := 4\widehat{\Lambda}. \tag{8}$$

with $\widehat{\Lambda}$ being a constant of integration. Using (8) to eliminate T from the trace-free field equations (4) gives

[4] "Trace-free theory" is a bit of a misnomer since it uses (6) rather than the conservation equation for the trace free stress-energy tensor

$$\nabla^a \widetilde{T}_{ab} = 0. \tag{6*}$$

$$R_{ab} - \frac{1}{2}Rg_{ab} + \widehat{\Lambda}g_{ab} = \kappa T_{ab}. \tag{9}$$

which is of the same form as Einstein's 1917 field equations (2).

A cosmological constant $\widehat{\Lambda}$ has made an appearance, although this constant has a different significance than the one Einstein introduced in 1917 with the modified field equations (2). The status of $\widehat{\Lambda}$ vis-à-vis Einstein's original Λ will be discussed below. Juggling the various equivalences of the equations derived above, the TFT can equally well be taken to be the combination of the trace free equations (4) and the conservation law (6); or the combination of (4) and either of the trace relations (7) or (8); or the equation (9).

Before closing this section it is worth noting that TFT or unimodular gravity (as it came to be known for reasons to be discussed below) is not some recent invention but, in fact, was already considered by Einstein in 1919. Einstein's interest in TFT was peaked by a possible connection between gravitation and a theory of matter that would provide an alternative to the account then on offer by Gustav Mie (see Einstein 1919).[5] But after this avenue proved to be a dead end, Einstein abandoned TFT, much as he had abandoned the cosmological constant term, never to return to it in later years. In both cases Einstein's attitude seems to have been shaped by a strange hubristic notion of original sin: 'I introduced X for what turned out to be erroneous considerations; therefore X is tainted and cannot have any further scientific utility.' This inference has proved to be faulty in the case of the cosmological constant, and it may also be faulty in the case of TFT.

In recent decades interest in TFT has been re-awakened by the possibility that it may help with "the problem of time in quantum gravity" and with "the cosmological constant problem." The former problem will receive passing attention below, but the latter problem together with surrounding issues will receive detailed attention. However, before turning to these matters the relation between standard GRT and TFT needs more careful examination.

[5] Casual reading of Einstein (1919) might suggest that Einstein introduced not eq. (4) but

$$\widetilde{R}_{ab} = \kappa T_{ab} \tag{4*}$$

(see equation (1a) on p. 193 of the English version of Einstein 1919). But at this point in his article he was considering matter fields for which T_{ab} is trace free.

3 The relationship between the two theories

Extant philosophical accounts of theory structure and theory equivalence are not very helpful in trying to get a grip on the relation between standard GRT and TFT. Appeal to the statement view of theories is not initially promising.[6] The Einstein field equations (2) have the same form as the TFT fields equations in the form (9). But as physical laws do they make the same assertion when the intended interpretations of their respective lambdas, Λ and $\widehat{\Lambda}$, are taken into account? And what exactly are these interpretations?

Nor is the so-called models view of theories more helpful.[7] Thinking of solutions to the field equations of the two theories as providing "models" of the theories, any pair g_{ab}, T_{ab} that satisfies the Einstein 1917 field equations (2) for any given value of $\Lambda \in (-\infty, +\infty)$ satisfies the equations of TFT in any of its guises; in particular, g_{ab}, T_{ab} will satisfy (4)&(6) or (4)&(7) in which $\widehat{\Lambda}$ does not appear, or (9) in which $\widehat{\Lambda}$ does appear if the value of $\widehat{\Lambda}$ is chosen equal to the given value of Λ. Conversely, any pair g_{ab}, T_{ab} that satisfies (4)&(6) or (4)&(7) will satisfy (2) for some value of $\Lambda \in (-\infty, +\infty)$, or that satisfies (9) for a given value of $\widehat{\Lambda}$ will satisfy (2) for a value of Λ equal to that of the given value of $\widehat{\Lambda}$. Does this mean that GRT and TFT have the same class of models and, thus, on the models view of theories GRT and TFT count as different versions of the same theory?

Alternatively, it could be claimed that the models of GRT are triples Λ, g_{ab}, T_{ab} satisfying (2) while the models of TFT are triples $\widehat{\Lambda}, g_{ab}, T_{ab}$ satisfying (9), but that while there is a natural one-one correspondence between the members of these two sets of models they are different sets because Λ and $\widehat{\Lambda}$ are different physical quantities. Or it might be claimed that, properly construed, the models of the two theories are not comparable because the models of GRT are triples Λ, g_{ab}, T_{ab} satisfying the Einstein 1917 field equations (2) whereas the models of TFT are pairs g_{ab}, T_{ab} satisfying (4) and (6). A reason that might be advanced for not including $\widehat{\Lambda}$ in the models of TFT is that it is not a quantity ontologically distinct from g_{ab} and T_{ab} but

[6] Philosophers tend to speak of the "syntactic view" rather than the "statement view." This is largely a red herring since only the severest instrumentalist thinks that a scientific theory consists of uninterpreted syntactic strings.

[7] The more common name for what I am calling the "models view" is the "semantic view". For an overview of the statement/syntactic and models/semantic views, see Winther (2016).

rather a defined quantity whose definition is given by eq. (8).[8] The upshot is that rather than looking to the models view for insight into the relationship between these two theories, we need to first understand more about the relationship in order to be able to properly apply the models view.

Much about the relationship between the two theories turns on the status of Λ vs. $\widehat{\Lambda}$. Both are spacetime constants, i.e. $\nabla_a \Lambda = \partial_a \Lambda = 0$ and $\nabla_a \widehat{\Lambda} = \partial_a \widehat{\Lambda} = 0$, taking values lying in $(-\infty, +\infty)$. But, it is said, Einstein's Λ is a universal constant, or a constant of nature, on a par with the gravitational constant G and the velocity of light c, whereas the $\widehat{\Lambda}$ of TFT is a mere constant of integration. This saying tells us more about one way of presenting TFT—via eqs. (4) and (6)—than it does about the nature of this constant. Another way to present the theory is via eq. (9) where it is said that $\widehat{\Lambda}$ functions as an "unspecified cosmological constant" (Kuchař 1991) or an "unspecified value of a variable" (Bufalo et al. 2015). The challenge is to translate these sayings into something more substantive.

One way to meet the challenge is to subject the two theories to the demand that their field equations be derivable from an action principle. The 1917 Einstein field equations (2) can be derived from an action principle of the form

$$\delta(S^{EH}_{grav} + S_\Lambda + S_{matter}) = 0 \tag{10}$$

where S^{EH}_{grav} and S_Λ are respectively the Einstein-Hilbert gravitational action for the original field equations (1) and the action for Λ with

$$S^{EH}_{grav} \ : \ = \int R\sqrt{-g}d^4x, \ \ g := \det(g_{ab}) \tag{11a}$$

$$S_\Lambda \ : \ = -\int \frac{\Lambda}{\kappa}\sqrt{-g}d^4x, \tag{11b}$$

and S_{matter} is the action for matter fields. Variation of this Einstein-Hilbert-Λ action $S^{EH}_{grav} + S_\Lambda$ with respect to the metric produces the field equations (2) when $T_{ab} := \dfrac{2}{\sqrt{-g}}\dfrac{\delta S_{matter}}{\delta g^{ab}}$ and when the variation of S_Λ does not include a variation of Λ but is given by

$$\delta S_\Lambda = -\frac{\Lambda}{2\kappa}\int \sqrt{-g}g^{ab}\delta g_{ab}d^4x = \frac{\Lambda}{2\kappa}\int \sqrt{-g}g_{ab}\delta g^{ab}d^4x. \tag{12}$$

[8]Some reasons to resist this idea will be found in Section 4.1.

So the picture is: we choose the values of the universal constants that characterize a possible world—in this instance the values of G, c, and Λ—and hold fixed their values while varying g_{ab} to get the field equations that specify the physically possible behaviors for that G-c-Λ world. This procedure is not optional for Λ, for if stationarity of the Einstein-Hilbert-Λ action under variations in Λ is required then the disastrous consequence is that $\int \sqrt{-g}d^4x = 0 \Longrightarrow \sqrt{-g} = 0$: the volume of any region of spacetime is zero. Requiring stationarity under variations of G also implies a disaster.

Einstein (1916) noted that some coordinate expressions in GRT are simplified by imposing the unimodular coordinate condition

$$\sqrt{-g} = 1. \tag{U}$$

But at the same time he was at pains to note that such a simplification does not compromise the general covariance of the theory.[9] The TFT field equations (4) can be derived from the Einstein-Hilbert-Λ action by limiting variations in the metric to those that preserve the unimodular coordinate condition (U) and, thus, are subject to the condition $\frac{\delta}{\delta g_{ab}}\sqrt{-g} = 0 \Longrightarrow g^{ab}\delta g_{ab} = 0$. This is why TFT is sometimes referred to as unimodular gravity, and in what follows I will refer to TFT/UG. But terminology aside, such a limitation on variations of the metric certainly does compromise general covariance.

As will be discussed below in Section 4 there are several candidates for generally covariant action principles for TFT/UG; and in contrast to conventional GRT they all vary $\widehat{\Lambda}$ in producing the field equations (9) of TFT/UG with a spacetime constant of undetermined value. However, it is here that another difference between standard GRT and TFT/UG emerges, for the generally covariant action principles for the latter all seem to require addi-

[9] "[I]t would be erroneous to believe that this step indicates a partial abandonment of the general postulate of relativity [the general covariance of the laws]. We do not ask 'What are the laws of nature which are covariant in face of substitutions for which the determinant is unity?' but our question is 'What are generally covariant laws of nature?' It is not until we have formulated these that we simplify their expression by a particular choice of system of reference." (Einstein 1916, p. 130 of the English translation). An apt analogy would be the choice of, say, the Lorentz gauge to simplify the expressions in Maxwellian electromagnetic theory. Such a choice in no way comprises the gauge invariance of the theory.

tional spacetime structure over and above the manifold and metric g_{ab} that characterize the spacetimes of GRT. In sum, subjecting standard GRT and TFT/UG to the demand that their field equations be derivable from an action principle reveals factors that indicate that the two theories should regarded as different theories, and in the case of TFT/UG there are potentially many different theories that use different spacetime structure in addition to the manifold-plus-metric structure of GRT spacetimes.

While there is a difference between standard GRT and TFT/UG that devolves from the different roles that Λ and $\widehat{\Lambda}$ play in the two theories, at the level of classical gravitation it would seem that standard GRT and TFT/UG are indistinguishable by any test since any classical gravitation effect that can be accommodated by some solution of the field equations of one can be accommodated by a solution of the field equations of the other (see Ellis et al. 2011 and Ellis 2014). But despite the empirical equivalence of the two theories there are reasons for being interested in the choice between them. First, it has been claimed TFT/UG is to be preferred to standard GRT because it helps to resolve the cosmological constant problem (see Ellis 2014). Some authors dispute this claim because they see an equivalence between TFT/UG and standard GRT as being very strong. For instance, Padilla and Saltas (2015) assert that there is no sense in which TFT/UG can say anything more or less than GRT about anything to do with classical gravity and, thus, they conclude that "there is no sense in which it [TFT/UG] can 'bring a new perspective' to the cosmological constant problem." This issue will receive more attention below in Section 5. The second consideration (discussed below in Section 7) is that, since GRT and TFT/UG employ different action principles, they lead via the canonical quantization route to different quantum theories of gravity and, thus, the equivalence in terms of observational testability on the classical level may be broken on the quantum level.

There is another wrinkle to theory testing which results from the distinction between three senses of the cosmological constant: Einstein's Λ—which is sometimes called the bare cosmological constant—an effective cosmological constant; and the total cosmological constant. To explain the latter two senses suppose that the stress-energy tensor T_{ab} can be split into two parts $T_{ab} = T^{(1)}_{ab} + T^{(2)}_{ab}$, where $T^{(2)}_{ab} = Cg_{ab}$ with $C = const.$ Then $\lambda := \kappa C$ acts as an effective cosmological constant, for moving the $T^{(2)}_{ab}$ part of the stress-energy tensor from the rhs of (2) to the lhs results in

$$R_{ab} - \frac{1}{2}Rg_{ab} + \Lambda' g_{ab} = \kappa T^{(1)}_{ab} \tag{2'}$$

with lower-case lambda absorbed into upper-case lambda to give the total $\Lambda' := \Lambda - \lambda$. As will be discussed in detail in Section 5, what can be determined from cosmological observations is the value of the total cosmological constant Λ'. This provides an opportunity as well as a challenge for testing GRT vs. TFT/UG. If a reliable theoretical calculation fixes the value λ^* of the effective cosmological constant λ then the observational determination of the total cosmological constant Λ' fixes a value Λ^* for the bare cosmological constant Λ.[10] If Λ^* matches the value of the universal constant that characterizes this world then GRT is confirmed, and if not GRT is refuted. But without god-like powers to know independently of observation and theoretical calculations the value of the bare Λ, there is no way to carry out this test. And even with the help of god-like powers this would-be test of GRT is not a test of GRT vs. TFT/UG for the simple reason that from the point of view of TFT/UG the cosmological constant is not a constant of Nature so there is no pre-set value of $\widehat{\Lambda}$ for the actual world to be perceived even with god-like powers.

The inconclusive conclusion is that GRT and TFT/UG are similar but distinct theories that are related in a number of ways that cannot be summarized in a twenty-words-or-less cereal box slogan. Nor are the similarities and differences easily captured by extant philosophical accounts of theory structure. Further support for this inconclusive conclusion will be found in Section 4.1.

3.1 Some meta-physics

In going forward it will be helpful to set out a piece of meta-physics which is consistent with but does not follow from the foregoing. I am *not* endorsing this way of thinking but only setting it out for later use in treating the cosmological constant problem. To begin, universal constants are construed

[10] As will be seen in Section 5.2 the calculation of λ can be quite contentious when it arises from the vacuum energy of quantum fields. Some researchers dismiss the notion of a bare cosmological constant and seek to explain the observed value of the cosmological constant as entirely due to the vacuum energy of quantum fields. This leads to one version of the "cosmological constant problem" (see Section 5).

as properties of worlds whose values are set by Nature.[11] Accordingly, if standard GRT is correct, She sets the value $G_{@}$ for the gravitational constant, the value $c_{@}$ for the velocity of light, and the value $\Lambda_{@}$ for the (bare) cosmological constant for the actual world. The different physically possible ways the actual world could be are given by all of the solutions g_{ab}, T_{ab} of the Einstein equations (2) sharing the values $G_{@}$, $c_{@}$, and $\Lambda_{@}$.[12] This is of a piece with the derivations of the Einstein field equations of standard GRT from an action principle in which the action is extremized under variations of g_{ab} with the values of G, c, and Λ held fixed and, a fortiori, the field equations for the actual world result from extremizing the action under variations of g_{ab} with the values of G, c, and Λ set to those that characterize the actual world. By contrast, if TFT/UG is correct, the different physically possible ways the actual world could be are given by all of the solutions g_{ab}, T_{ab} of the TFT/UG equations (4)&(6) or (9) sharing the values $G_{@}$ and $c_{@}$ but with $\widehat{\Lambda}$ taking any value in the range $(-\infty, +\infty)$. This is of a piece with the derivations of the field equations of TFT/UG from an action principle in which the action is extremized under variations of g_{ab} with the values of G and c but not Λ held fixed and, a fortiori, the field equations for the actual world result from extremizing the action under variations of g_{ab} with the values of G and c set to those that characterize the actual world while the value of Λ is allowed to vary. Consequently, if TFT/UG is correct, there is a much richer array of physically possible ways this world could be than if Einstein's theory is correct. This expansion of possibilities might seem to be of interest only to meta-physicians, but it is a possible source of the perceived advantage of TFT/UG in coping with one aspect of the cosmological constant problem (see Section 6.2).

[11] Philosophers have offered a variety of alternative ways of construing universal constants, e.g. as conversion factors between different dimensionful quantities. How such alternative construals would affect the issues under consideration is a project for another time.

[12] If accepted, this piece of meta-physics has implications for treating "what if" or counterfactual scenarios, i.e. what if, contrary to actual fact, X had been the case. To determine how the scenario plays out involves a decision about what conditions that actually obtain are to be carried over to the what-if scenario. The meta-physics under discussion implies that the first thing to carry over to a what-if scenario about the actual world are the physical laws with the values of the universal constants that actually obtain in this world.

4 Action principles and time in TFT/UG

4.1 Action principles for TFT/UG

As noted above the TFT/UG field equations (4) can be derived from the Einstein-Hilbert-Λ action by limiting variations in the metric to those that respect the unimodular condition (U), but the price paid for this derivation is a compromise of general covariance. As is well known, broken general covariance can in many cases be restored, at least formally, by introducing additional geometric object fields. Perhaps the most direct way to do this in the present instance is to introduce a fixed scalar density ϵ_0 where $\int \epsilon_0 d^4x$ defines a proper spacetime volume element. Unimodular coordinates are now those in which

$$\sqrt{-g} = \epsilon_0. \tag{U*}$$

An action principle for TFT/UG can be written by treating $\widehat{\Lambda}$ as a Lagrangian multiplier:

$$S^U_{grav} := \int [R\sqrt{-g} - \frac{\widehat{\Lambda}}{\kappa}(\sqrt{-g} - \epsilon_0)]d^4x. \tag{13}$$

Unrestricted variation of $S^U_{grav} + S_{matter}$ with respect to the metric produces the field equations (9) of TFT/UG, and variation with respect to $\widehat{\Lambda}$ produces the unimodular condition (U*), ensuring that $\frac{\delta}{\delta g_{ab}}\sqrt{-g} = 0$. Assuming that S_{matter} is invariant under arbitrary coordinate transformations guarantees the conservation law (6), and this in conjunction with the field equations (9) ensures that $\widehat{\Lambda}$ is a spacetime constant. However, it might be complained that although the letter of general covariance is satisfied by this maneuver the spirit is violated since the use of the fixed scalar density ϵ_0 is a barely disguised way of smuggling in a covariance-breaking coordinate condition.[13]

Some of the worry here can be assuaged by a more subtle approach explored by Henneaux and Teitelboim (1989) and Kuchař (1991) using a technique referred to in the literature as parameterization of the action. The fixed ϵ_0 is replaced by a variable vector density τ^a, and the proposed gravitational action for unimodular gravity is

[13] Stated in other terms, the diffeomorphism group is not a gauge symmetry of the theory with such a fixed background object; see Smolin (2011).

$$S_{grav}^{HTK} := \int [R\sqrt{-g} - \frac{\widehat{\Lambda}}{\kappa}(\sqrt{-g} - \partial_a \tau^a)] d^4x. \tag{14}$$

Unrestricted variation of $S_{grav}^{HTK} + S_{matter}$ with respect to the metric produces the field equations (9) of TFT/UG. Varying τ^a produces $\partial_a \widehat{\Lambda} = 0$, while variation of $\widehat{\Lambda}$ gives

$$\sqrt{-g} = \partial_a \tau^a. \tag{14}$$

The relationship between the previous approach and this one is discussed in detail in Kuchař (1991). In brief, the unimodular coordinates X^a in which $\sqrt{-g} = \epsilon_0$ holds can be treated as four independent scalar fields. If the vector density τ^a is defined by

$$\tau^a := \frac{1}{4!} \epsilon_0 \delta_b^{[a} \delta_c^e \delta_d^f \delta_e^{g]} X^b \partial_e X^c \partial_f X^d \partial_g X^e \tag{15}$$

where $\partial_a X^b$ is the Jacobian of the transformation $x^a \longrightarrow X^a(x)$ between arbitrary label coordinates x^a and the X^a, then (15) reduces to $\sqrt{-g} = \epsilon_0$ when the label coordinates are unimodular. Still other action principles for TFT/UG are available (see Smolin 2009 and Bufalo et al. 2015).

As far as I am aware there is no proof of the non-existence of a satisfactory generally covariant action principle in which lambda does not appear but which nevertheless yields directly the eqs. (4) and (6); but the fact many able investigators have failed to provide such creates a presumption against its existence. The apparent need to include $\widehat{\Lambda}$ as a configuration variable in an action principle for TFT/UG coupled with the fact that a number of different $\widehat{\Lambda}$-action principles are available might lead one to conclude that TFT/UG is not one theory but many. What experimental predictions, if any, distinguish these theories or their quantized counterparts remain to be seen. In any case, the apparent need to include $\widehat{\Lambda}$ in an action principle for TFT/UG, the additional spacetime structure needed to support such an action principle, and the role played by $\widehat{\Lambda}$ in the initial value problem for TFT/UG (see Section 6.1) argue against the notion that $\widehat{\Lambda}$ is merely a auxiliary quantity defined in terms of more fundamental quantities per eq. (8). There is no name for such a constant which is not a universal constant but certainly not just another constant; but perhaps one should be coined.

4.2 A remark on time in unimodular gravity

The extra spacetime structure needed to support a $\widehat{\Lambda}$-action principle has ramifications about the nature of time in classical and quantum gravity. To illustrate, consider a spacetime that is diffeomorphic to Σ x $\mathbb{R}$ where Σ is a compact three-manifold, and let $t \in \mathbb{R}$ be a parameter that indexes a foliation of the spacetime by spacelike hypersurfaces. Pick some fiduciary hypersurface $\Sigma(t_0)$, and for $t > t_0$ define the unimodular time associated with $\Sigma(t)$ by

$$T(t) := \int_{\mathcal{R}} \partial_a \tau^a d^4x \tag{16}$$

where τ^a is the vector density from the HTK action and where the integration is over the region spacetime $\mathcal{R}$ between $\Sigma(t_0)$ and $\Sigma(t)$. From (16) it follows that $T(t)$ is just the four-volume between the fiduciary $\Sigma(t_0)$ and $\Sigma(t)$. Of course, unimodular time is not associated with a unique foliation of the spacetime by spacelike hypersurfaces but only serves to label equivalence classes of hypersufaces, where hypersurfaces are counted as equivalent if they are separated by a zero volume. Because of this feature of unimodular time Kuchař (1991) concluded that, without supplementary conditions, it cannot be used to "set the conditions for measurement" and, therefore does not resolve the "problem of time" as it arises in canonical quantum gravity. But the switch from standard GRT to TFT/UG does help with one aspect of this problem since it "unfreezes" the dynamics in the sense that in the canonical quantization of TFT/UG the Hamiltonian does not vanish (see Unruh 1989 and Unruh and Wald 1989). I will not discuss these fraught issues here. The point of relevance here is that unimodular time is useful in treating the dynamics in some approaches to quantum gravity (see Section 7.2.2 below).

5 The cosmological constant problem

5.1 How the problem arose

The cosmological constant problem has its roots in classical GRT, but in its most pointed form it arises from trying to combine GRT with considerations coming from particle physics. In fact there are a number of versions of the problem, and commentators are not always clear which version they have in mind. To avoid confusions it is necessary to proceed slowly and pay due attention to the historical context.

Already in the early 1930s Georges Lemaître, the great champion of a cosmological constant, was speaking of Λ as corresponding to the energy density of the vacuum: in the presence of a non-zero Λ, he wrote, "Everything happens as though the energy in vacuo would be different from zero" (Lemaître 1934, p. 12).[14] A few years later Arthur Stanley Eddington speaks of Λ as "fixing a zero from which energy, momentum and stress are reckoned," the "reckoned" stress-energy(-momentum) tensor being the difference between the actual "absolute" stress-energy(-momentum) tensor T_{ab} on the rhs of eq. (2) and the "absolute" stress-energy(-momentum) tensor $\frac{\Lambda}{\kappa}g_{ab}$ of a "standard zero condition" (Eddington 1939, pp. 232-233). The common idea is that the cosmological term on the lhs of Einstein's 1917 field equations (eq. (2)) can be moved to the rhs to form a total stress-energy tensor consisting of the sum of the stress-energy tensor for matter and radiation fields plus $-\frac{\Lambda}{\kappa}g_{ab}$, the latter of which can be considered the stress-energy tensor of the vacuum since it functions in its position on the rhs of eq. (2) as the stress-energy tensor in the presence of the classical vacuum ($T_{ab} \equiv 0$).

A parallel move can be made in the opposite direction with parts of the stress-energy tensor for matter and radiation contributing to an effective cosmological constant. The standard example is a perfect fluid with an exotic equation of state. The stress-energy tensor for a perfect fluid takes the form

$$T^{fluid}_{ab} = (\varrho + p)u_a u_b + p g_{ab} \tag{17}$$

where ϱ and p are respectively the mass density and the pressure, and u_a is the normed four-velocity of the fluid flow ($u_a u^a = -1$). The pressure is assumed to be related to the mass density by an equation of state $p = w(\varrho)$. In the extreme case where $w \equiv -1$ the fluid exerts a pressure $p^* = -\varrho^*$ which is negative (assuming that $\varrho^* > 0$).[15] While such exotic fluids are

[14] To be clear, Lemaître was a champion of the Λ of GRT not the $\widehat{\Lambda}$ of TFT; indeed, I know of no evidence that Lemaître ever entertained TFT.

[15] Lemaître (1934) suggested identifying Λ with just such a fluid. That $T_{ab}V^aV^b \geq 0$ for any timelike V^a is known as the weak energy condition; for a perfect fluid it requires that $\varrho \geq 0$ and $\varrho + p \geq 0$, which is satisfied for $w = -1$ as long as $\varrho \geq 0$. However, the condition $\varrho = -p$ produces a violation of the strong energy condition $T_{ab}V^aV^b \geq -\frac{1}{2}T$ which for a perfect fluid requires that $\varrho + 3p \geq 0$.

not normally encountered they are conceptually possible, and they produce a stress-energy tensor $-\varrho^* g_{ab}$ which, assuming $\varrho^* = const$ (which it must be by the conservation law if T_{ab}^{fluid} is the total stress-energy tensor) is of the form of a cosmological constant term. According to conventional Einstein gravitational theory this term has the same gravitational effect as a "bare" cosmological constant of magnitude $\kappa\varrho^*$; and the upshot is that the total cosmological constant is the sum of the bare constant Λ plus the effective cosmological constant arising from contributions from forms of matter whose stress-energy tensor takes the form λg_{ab}, $\lambda = const$.

This analysis applies to the contribution of a scalar field φ hypothesized as the driver of early universe inflation. The stress-energy tensor for such a field is

$$T_{ab}^{\varphi} = \nabla_a \varphi \nabla_b \varphi - \frac{1}{2}[\nabla_c \varphi \nabla^c \varphi + 2V(\varphi)]g_{ab}. \tag{18}$$

With $u_a := -\frac{1}{\dot{\varphi}}\nabla_a \varphi$, this stress-energy tensor can be rewritten as

$$T_{ab}^{\varphi} = \dot{\varphi}^2 u_a u_b - [\frac{1}{2}\dot{\varphi}^2 - V(\varphi)]g_{ab} \tag{19}$$

giving the associated mass density and pressure defined by

$$\varrho_{\varphi} := \frac{1}{2}\dot{\varphi}^2 + V(\varphi), \quad p_{\varphi} := \frac{1}{2}\dot{\varphi}^2 - V(\varphi). \tag{20}$$

During an era in which the evolution of φ takes it to a local minimum of the potential where it rests, giving $\dot{\varphi} = 0$, (20) implies that $\varrho_{\varphi} = -p_{\varphi} = V(\varphi_{\min})$. Thus, if $V(\varphi_{\min}) > 0$ during such an era, the potential of the inflaton field supplies a contribution to the vacuum energy.

Thus far the discussion has been purely classical. But now enter from stage left and right the particle physicists. A convenient reference point is provided by the work of Zeldovich (1968) which linked the cosmological constant problem to the theory of elementary particles. In the 1960s the trace-free form of gravitational theory seems to have been forgotten, and in conventional GRT Einstein's Λ was generally out of favor. However, there were proposals to revive Λ in order to explain why the then observed red shifts of QSOs all fell in a narrow range about $z \approx 1.95$. Although skeptical of these proposals, Zeldovich took the opportunity to reexamine the status of the cosmological constant, leading him to ponder the "close connection between the question of Λ and the theory of elementary particles" (1968, p.

383)—that is, the connection between the energy density of the vacuum and the vacuum energy of the quantum fields describing elementary particles. The absolute value of the latter does not matter for most purposes where quantum effects depend on energy *differences* or energy *shifts*. But for gravitational effects the absolute value of the energy density of the vacuum does matter, at least according to standard GRT.

This was the beginning of a minor industry that generated papers giving estimates of the vacuum energy of quantum fields, finding values that seemed ridiculously large, followed by hand wringing and attempts to find an escape route. As already mentioned, a major milestone in this literature was Steven Weinberg's (1989) review article in which he lent his immense prestige to underscoring the importance of the problem and billing it as a virtual crisis for physics. The desperation that a number of researchers felt is shown in their willingness to try to solve the problem by resorting to anthropic reasoning, the last refuge of bankrupt physics. The idea was to argue that if the value of the total cosmological constant were very large then the universe would be inhospitable to observers such as ourselves. If true, this would explain why observers such as ourselves should not be surprised to find that we inhabit a universe where the total cosmological constant is not very much larger than it actually is (since, arguably sentient beings such as us could not have evolved in such a universe), but it does not provide a physical explanation of why it has an observed small but non-zero value.

5.2 Some skepticism about the formulation of the cosmological constant problem

Alhough the acceptance of "the cosmological constant problem" as a fundamental problem crying for a solution is widespread in the physics community there are occasional expressions of skepticism about the motivation for the problem (for particularly insightful commentary see, for example, Bianchi and Rovelli 2010 and Koberinski 2017, 2021). Here I will mention a few of the reasons that fuel the skepticism.

The problem is formulated in semi-classical quantum gravity, a half-way house between classical GRT and a full quantum theory of gravity. The ingredients for the construction of this half-way house, in which matter fields but not gravitational degrees of freedom are quantized, include a fixed background spacetime g_{ab}, the quantization of matter fields on this background, and a stress-energy tensor operator $\widehat{T}_{ab}$ counterpart of the classical T_{ab}. The QFT expectation value $\langle\widehat{T}_{ab}\rangle$ is inserted on the rhs of the Einstein eq. (2) in place of T_{ab}, and backreaction effects on the metric are computed. When the background spacetime is Minkowski spacetime, the requirement that the Minkowski vacuum state $|vac\rangle$ is Poincaré invariant implies that

$$\langle vac|\widehat{T}_{ab}|vac\rangle = \; -\varrho_{vac}\eta_{ab} \tag{21}$$

where η_{ab} is the Minkowski metric, which has the form of an effective cosmological constant term. It is suggested that in the cosmological setting (21) can be replaced by

$$\langle vac?|\widehat{T}_{ab}|vac?\rangle = -\varrho_{vac}g_{ab}. \tag{22}$$

However, in the cosmological setting Poincaré invariance is lost; indeed, the cosmological metric g_{ab} may lack any non-trivial global or even local symmetry.[16] In such circumstances the usual notion of a vacuum state becomes problematic, and a convincing basis for (22) is lacking.

This worry is connected with a worry about the procedure used to produce the much-quoted huge estimate of the vacuum energy of quantum fields. The technique is to apply a suitable ultraviolet cutoff to the sum of the vacuum state energy modes of the fields, where the sum is taken as if the modes were independent degrees of freedom. The oft-quoted 120 orders of magnitude discrepancy between the effective cosmological constant due to the vacuum energy of quantum fields and the observed total cosmological constant results from setting the cutoff at the Planck scale. Lowering the cutoff to the energy scale achieved by the current generation of accelerators would produce a diminished but still large discrepancy, so it seems that there is a robustness to the cosmological constant problem. However, Hollands and Wald (2004) have noted that in a curved spacetime the requirements of locality and covariance imply that the renormalization procedure for the stress-energy tensor $\widehat{T}_{ab}$—even in the simplest case where $\widehat{T}_{ab}$ arises from a free, massless scalar field—has a holistic character that means that the low energy modes cannot be treated as if they were independent degrees of freedom. Hollands and Wald estimate that in an adiabatic vacuum state for a slowly expanding universe the expectation value of the stress-energy tensor for a free, massless scalar field should be many orders of magnitude *lower*

[16]Local in the sense of a finite neighborhood of spacetime.

than the value of the cosmological constant needed to explain the observed accelerated expansion in our universe. So if, as some particle physicists urge, the cosmological constant is to be explained solely as the effect of the vacuum energy of quantum fields (see the following Section) then there is indeed a cosmological constant problem, albeit of a quite different form than is usually assumed.

In sum, there are good grounds for taking seriously Bianchi and Rovelli's skeptical stance:

> To trust *flat-space* QFT telling us something about the origins or the nature of a term in Einstein['s] equations which implies that spacetime cannot be flat, is a delicate and possibly misleading step. To argue that a term in Einstein's equations is "problematic" because flat-space QFT predicts it, but predicts it wrong, seems a *non sequitur* to us. (2010, p. 7)

It should also be mentioned that a rather different version of semi-classical quantum gravity does not proceed by coupling gravity to the expectation value of the renormalized stress-energy tensor but rather attempts to take into account the effect of the fluctuations in the stress energy tensor due to the vacuum fluctuations of quantum fields. In this approach the sources of gravity are "stochastic fields whose properties are determined by their quantum fluctuations" (Wang et al. 2017, 103504-6; see also Wang and Unruh 2018). In the model of Cree et al. (2018) a small acceleration of cosmological expansion is predicted when a sufficiently large number of particle fields are present.

Finally, some skepticism is due for the hinge assumption that the vacuum energy of quantum fields gravitates in the sense that it serves as a source for the gravitational field. At present there are no direct experimental tests of this assumption.[17] The theoretical support offered for the assumption consists of appeals to the fact that all other known forms of energy do gravitate and the alleged fact that the "reality" of the vacuum energy of quantum fields is demonstrated by the Casimir effect which can be given a heuristic explanation from the idea that the energy density of the quantum vacuum

[17] A test involving the weighing of a Casimir cavity has been proposed (see Calloni et al. 2014), but it has been argued that such Casimir effects cannot demonstrate that *free* vacuum energy of quantum fields gravitates and contributes to the effective cosmological constant (see Cerdino and Rovelli 2015)

is lower between the plates than outside. But as noted by Jaffe (2005) the exact expression for the Casimir force between parallel conducting plates can be calculated from the relativistic forces between the charges in the plates without any reference to vacuum energy. The exact force formula depends on the fine structure constant α, and the Casimir force vanishes as $\alpha \to 0$, whereas α does not appear in the heuristic explanation of the origin of the force using vacuum energy.

5.3 Getting a better fix on the cosmological constant problem

There are two different perspectives that produce two versions of the problem, and since oscillation between the two is the cause of confusions it is important to keep the distinction between the two in mind. The first perspective accepts that there may be a non-zero value of the bare cosmological constant of standard GRT but at the same time also acknowledges that vacuum effects of quantum fields may contribute an effective cosmological constant. This perspective leads to a version of the cosmological constant problem that makes it a species of the fine tuning problems that crop up like mushrooms in cosmology, or so I will argue below. TFT/UG may offer help with this version of the problem—or not, depending on which commentator one believes.

The second perspective has the ambition to obviate the need for a bare cosmological constant by assuming that the total cosmological constant is due entirely to the vacuum energy of quantum fields. It is this perspective that prompted Bianchi and Rovelli's skeptical commentary "Why all these prejudices against a constant?" (2010). An example of this second perspective is provided by Stephen Hawking's "The cosmological constant" (1983). In 1983 the available cosmological observations placed an upper bound on (total) lambda that made it "the quantity in physics that is most accurately measured to zero" (Hawking 1983, p. 304). After briefly flirting with an anthropic explanation of the presumed zero value of lambda, Hawking proposed an explanation involving a gauge extended supergravity mechanism. After the observations of the late 1990s revealed that the value of the total cosmological is non-zero, the goal of this second perspective became more elusive; for although it is conceivable that symmetry cancellations produce a zero value for the vacuum energy contributions of quantum fields, it is hard to see how they would produce the currently observed non-zero but small value of the total cosmological constant of the order of $10^{-47}GeV^4$. While the second perspective is certainly worth pursuing, I will not do so here because, if successful, it obviates the need for TFT/UG since the version of the cosmological constant problem stemming from this perspective is to be solved not by tinkering with classical GRT but by a better understanding of quantum fields.

We can get a better grip on the version of the cosmological constant problem that emerges from the first perspective by making the problem more concrete.[18] To this end focus on the FLRW cosmological models, used in contemporary cosmology to describe the large scale features of the actual universe. The line element for the these models has the form

$$ds^2 = a^2(t)d\sigma^2 - dt^2 \tag{23}$$

where $a(t)$ is the scale factor and $d\sigma$ is the line element of the metric for a three-space of constant curvature $k = -1$, 0, or $+1$. The symmetries for this family of spacetimes force the stress-energy tensor to take the form of a perfect fluid (eq. (15)). For present purposes it will be assumed that the total T_{ab}^{fluid} is the sum of two parts, one of which captures the contribution from ordinary matter and radiation (with $w(\varrho) \geq 0$) and the other of which is of the form $\varrho_{vac}g_{ab}$, where ϱ_{vac} ($\nabla_a \varrho_{vac} = 0$) arises from either exotic classical sources (e.g. a classical relativistic fluid with equation of state $w(\varrho) \equiv -1$) or from quantum field contributions to vacuum energy.[19] To simplify the discussion I will focus on spatially flat ($k = 0$) FLRW models and will assume that the contribution to the total stress-energy from ordinary matter arises from dust matter with mass density ϱ_d ($p_d = w(\varrho_d) \equiv 0$). Nothing important

[18]What is being discussed here and below is sometimes referred to as the "old problem." Commentators go on to discuss other problems such as the coincidence problem, i.e. why is the measured value of the cosmological constant the same order of magnitude as the current value of the (non-constant) Hubble constant? This generates a problem or puzzle only if it is thought that the coincidence is, in some appropriate sense, unlikely. For some skepticism about the employment of likelihoods in cosmology see Norton (2010) and McCoy (2017).

[19]Notice that there is a hidden assumption here in the case that the total stress energy tensor consists of the sum of two parts, one of which arises from an ordinary perfect fluid with $w \geq 0$ and the other of which arises from a perfect fluid with $w = -1$ and is thus of the form $\varrho^* g_{ab}$. It does not follow from the conservation law (the vanishing of the covariant divergence of the *total* stress-energy tensor) that $\nabla_a \varrho^* = 0$ which is needed for ϱ^* to make a contribution to the effective cosmological constant. So $\nabla_a \varrho^* = 0$ has to be assumed or guaranteed by other means.

in the conclusions will be affected by these simplifications.[20]

In these simplified cosmological models the Einstein field equations (2) reduce to two ODEs

$$\frac{3\ddot{a}}{a} = -\frac{1}{2}\kappa\varrho_d + \kappa\varrho_{vac} + \Lambda \tag{24}$$

$$\frac{3\dot{a}^2}{a^2} = \kappa\varrho_d + \kappa\varrho_{vac} + \Lambda \tag{25}$$

We know that the conservation law is a consequence of (24) and (25). To see this explicitly, multiply (25) by a^2 and differentiate with respect to t. Then using (24) to eliminate $\ddot{a}$ yields the conservation law

$$\dot{\varrho}_d + 3\varrho_d\frac{\dot{a}}{a} = 0 \tag{26}$$

which integrates to

$$\varrho_d = Ka^{-3}, \;\; K = const \tag{27}$$

giving the expected dilution of the density of dust with the expansion of the universe. Combining (25) and (27) it is seen that the values of a and ϱ_d at some initial time serve to fix a unique evolution of the scale factor once the values of the constants ϱ_{vac} and Λ are specified.[21]

The fine tuning version of the cosmological constant problem can now be posed for the considered cosmological models. It turns out that a huge value for ϱ_{vac} is not essential to generating this problem.

5.4 What exactly is the fine tuning version of the cosmological constant problem?

Equation (24) tells us that, according to standard GRT, the expansion of a $k = 0$ FLRW dust-matter universe has positive acceleration just in case the total cosmological constant $\kappa\varrho_{vac} + \Lambda$ is greater than $\frac{1}{2}\kappa\varrho_d$. Whatever the value of ϱ_{vac}, the value of the bare cosmological constant Λ can be set so that the total cosmological constant $\kappa\varrho_{vac} + \Lambda$ overbalances $-\frac{1}{2}\kappa\varrho_d$ by just

[20]But note that a static FLRW model is impossible with $k = 0$.

[21]For explicit formulas for $a(t)$ in the FLRW models, see Hobson et al. (2007, Ch. 14).

the amount needed to give any desired value of acceleration. And there (supposedly) is the rub: standard GRT certainly can accommodate the actually observed acceleration, but in order to do so the value of Λ has to be "tuned" to a value so as to almost but not quite cancel out the (allegedly) huge value of the vacuum energy of quantum fields. If this is a real problem then there is also a tuning problem, though perhaps a less dramatic one, even if ϱ_{vac} due to the vacuum energy of quantum fields is many orders of magnitude less than its (allegedly) huge value.

I confess that when talk begins about fine tuning problems I tend to tune out. For what values of the bare cosmological constant and effective cosmological constant is there *not* a fine tuning problem? And how would one decide? Once you allow yourself to worked into a state of puzzlement about one fine tuning problem then other fine tuning problems begin to sprout like dandelions, and without a principled way to decide whether weed killer or fertilizer should be applied, the good flowers in your garden are in danger of being overgrown. This is not to deny that there are sometimes interesting issues in the neighborhood of perceived fine tuning problems. For instance, the exquisite tuning of Λ needed for the Einstein static universe raises the issue of the physical stability of this model; and talk of fine tuning may be a way of raising a concern about the lack of robustness of an explanation that requires a narrow range of parameter values.

Furthermore, it is undeniable that fine tuning considerations do play an important role in motivating physicists' research, especially in deciding which line of research is worthy of pursuit. Inflationary cosmology is a relevant example since it got its initial push in the 1980s from the complaint that, although the then standard hot big bang cosmology could accommodate the then available data, it has to resort to fine tuning in order to solve the horizon and flatness problems.[22] Ironically, it has proved to be controversial whether or not inflationary cosmology offers a satisfying solution to the fine tuning problems that served as its initial motivation since to get inflation going fine tuning may be required.[23] The disanalogy between fine tuning problems in standard big bang cosmology vs. inflationary cosmology and in standard GRT vs. TFT/UG is twofold. First, the flatness and horizon problems are internal problems for standard hot big bang cosmology and, thus, insofar as

[22] Alan Guth's seminal paper on inflationary cosmology was entitled "Inflationary universe: A possible solution to the horizon and flatness problems" (Guth 1981).

[23] See McCoy (2015) for an overview of the issues.

they are matters of legitimate scientific concern they constitute (alleged) demerits of this version of cosmology; but since the most pressing version of the cosmological constant problem arises from an attempted shotgun marriage of classical GRT and QFT into the fraught union of semi-classical quantum gravity (recall Section 5.2) it is unclear that the demerits attach to GRT. Second, whatever the motivation for pursuing a line of research, ultimately the judgment of success must be made in the tribunal of empirical tests.[24] Here inflationary cosmology can point to the success in explaining the spectra of the cosmic microwave background, and it may receive decisive confirmation (or decisive disconfirmation) from tests of its predictions about gravitational radiation produced in early universe inflation. At the classical level TFT/UG cannot point to any actual or potential empirical successes not enjoyed by standard GRT since the two theories are observationally equivalent at this level. However, the line of research suggested by TFT/UG may pay off in successes at the quantum level. I will take up this matter below in Section 7.

Enough quibbling. Let us go forward to see how TFT/UG might help with various versions of the cosmological constant problem.

6 Trace-free gravitation as a solution to the cosmological constant problem

6.1 How TFT/UG does not solve the problem

It has been claimed that adopting TFT/UG in place of conventional GRT resolves the cosmological constant problem because in TFT/UG "the vacuum energy has no gravitational effect" (Ellis et. al. 2011, p. 2; Ellis 2014, p. 2).[25] Similarly it is said that TFT/UG "decouples the dynamics of the metric ... from any contribution to the energy-momentum tensor, whether classical or quantum, of the form of a constant times the spacetime metric" (Smolin 2011, p. 1). These are startling claims: if TFT/UG implies that vacuum energy has no gravitational effect then how does TFT/UG explain

[24] Or at least this used to be the credo of science. To those who would dethrone empirical success in favor of beauty and elegance, I would respond as do Ellis and Silk (2014).

[25] When they say that TFT resolves the cosmological constant problem their claim is only supposed to apply to the "old problem" and not to other versions; see note 16.

the current era accelerated cosmic expansion or the (widely presumed) early universe inflation?

These claims are motivated by the fact that trace-free field equations (4), along with the conservation law (6), are invariant under the transformation

$$T_{ab} \longrightarrow T'_{ab} = T_{ab} + Cg_{ab}, \;\; C = const. \tag{T}$$

What this symmetry means in the first instance is that if g_{ab}, T_{ab} is a solution to the TFT/UG equations (4) and (6) then so is $g_{ab}, T_{ab} + Cg_{ab}$. But correspondingly, if g_{ab}, T_{ab} is a solution to the Einstein field equations (2) then so is $g_{ab}, T_{ab} + Cg_{ab}$, albeit with a different value of Λ than in the g_{ab}, T_{ab} solution. This is on a par with the situation with respect to the TFT/UG field equations in the guise (9), where if g_{ab}, T_{ab} is a solution then so is $g_{ab}, T_{ab} + Cg_{ab}$, albeit with a different value of $\widehat{\Lambda}$ than in the g_{ab}, T_{ab} solution. It might be objected that since $\widehat{\Lambda}$ is an adjustable spacetime constant it is legitimate to construe the form of eq. (9) as preserved under the transformation (T) since the term Cg_{ab} added to the rhs of (9) can be absorbed into $\widehat{\Lambda}$, whereas a parallel move in the case of GRT is not legitimate since Einstein's Λ is a universal constant. With the help of the meta-physics of Section 3.1 this difference can be brought to bear on the fine tuning version of the cosmological constant problem (see the following Section), but it does not speak to the claim that TFT/UG decouples the dynamics of the metric from vacuum energy. A solution g_{ab}, T_{ab} to the TFT/UG field equations (9) with given value of $\widehat{\Lambda}$ is matched by a solution of the Einstein equations (2) with the value of Λ equal to said value of $\widehat{\Lambda}$. If the matching solutions represent accelerated cosmic expansion and if in the GRT case the cosmic acceleration is attributed to the total vacuum energy arising from Λg_{ab} and stress-energy contributions of matter fields of the form Cg_{ab} then in the TFT/UG case why is the cosmic acceleration not attributed to the total vacuum energy arising from $\widehat{\Lambda} g_{ab}$ and stress-energy contributions of matter fields of the form Cg_{ab}?

To adjudicate claims about the role of vacuum energy in the dynamics of the metric it is helpful to consider the initial value problem. The FLRW models discussed in the preceding section provide a simple test case. It was seen that for conventional GRT the initial value problem for the $k = 0$ FLRW dust models reduces to the initial value problem for the scale factor a, and a unique solution to eqs. (24)-(25) is determined by the values at the chosen initial time t_{in} of the scale factor $a(t_{in})$ and the dust density $\varrho_d(t_{in})$ and by the values of the constants ϱ_{vac} and Λ that combine to give the total cosmological constant. Thus, in GRT the allowed $k = 0$ FLRW models are parameterized by $(a(t_{in}), \varrho_d(t_{in}), \varrho_{vac}, \Lambda)$. The dependency of the dynamics of the spacetime geometry on the vacuum energy—in the guise of ϱ_{vac} or Λ is as plain as day.

What is the message from the initial value problem in TFT/UG? Restricting, as before, attention to spatially flat $k = 0$ FLRW dust models, the trace-free field equations (4) reduce to one second order ODE

$$\frac{\ddot{a}}{a} - \frac{\dot{a}^2}{a^2} = -\frac{\kappa}{2}\varrho_d \tag{28}$$

which contains neither ϱ_{vac} nor $\widehat{\Lambda}$. The conservation equation for TFT/UG is the same as for conventional GRT, viz. eq. (27), which also contains neither ϱ_{vac} nor $\widehat{\Lambda}$, apparently confirming the claim that TFT/UG theory decouples the dynamics of the metric from vacuum energy.

However, in the initial value problem for (27) and (28) a unique solution requires not only the initial value of the scale factor a but the initial value of $\dot{a}$ as well. The value of $\dot{a}$ at a given time is not freely specifiable but is constrained by the TFT/UG field equations. TFT/UG consists not just of the trace free field equations (4), which yield (28), but also (6) or (8) which, for the special FLRW models at issue here, entail

$$\frac{\ddot{a}}{a} + \frac{\dot{a}^2}{a^2} = \frac{\kappa}{6}\varrho_d + \frac{2}{3}\kappa\varrho_{vac} + \frac{2}{3}\widehat{\Lambda}. \tag{29}$$

The vacuum energy contribution—both in the form of the vacuum energy ϱ_{vac} of quantum fields and strange forms of classical matter and also in the form of the cosmological constant $\widehat{\Lambda}$—to the dynamics has been outed. Adding (28) and (29) gives

$$\frac{3\ddot{a}}{a} = -\frac{1}{2}\kappa\varrho_d + \kappa\varrho_{vac} + \widehat{\Lambda} \tag{30}$$

while subtracting these equations gives

$$\frac{3\dot{a}^2}{a^2} = \kappa\varrho_d + \kappa\varrho_{vac} + \widehat{\Lambda} \tag{31}$$

Equations (30) and (31) are the TFT/UG analogs respectively of eqs. (24) and (25) for standard GRT. It was obvious from the beginning, of course, that these TFT/UG analogs had to emerge since eq. (9) of TFT/UG could have been used as the starting point rather than (4) and (6). And it should be no surprise that in the TFT/UG initial value problem for the $k = 0$ FLRW models the solutions are parameterized by $(a(t_{in}), \varrho_d(t_{in}), \varrho_{vac}, \widehat{\Lambda})$ in exact parallel to standard GTR.

Furthermore, insofar as explanation in physics consists of deduction from physical laws,[26] vacuum energy plays similar roles in the explanations offered by both standard GRT and TFT/UG. For example, the explanation of the currently observed accelerated expansion of the universe according to GRT flows from eq. (24): the value of $-\frac{1}{2}\kappa\varrho_d(t_{now})$ is overbalanced by the value of the total cosmological constant $\kappa\varrho_{vac} + \Lambda$ to give the observed value for $\ddot{a}(t_{now})$—intuitively, the current attraction of gravity is overcome by the repulsive push of vacuum energy. The explanation according to TFT/UG that flows from eq. (30) has exactly the same form: the value of $-\frac{1}{2}\kappa\varrho_d(t_{now})$ is overbalanced by the value of $\kappa\varrho_{vac} + \widehat{\Lambda}$. It might be objected that, unlike the Λ of standard GRT, $\widehat{\Lambda}$ cannot play an explanatory or causal role since it is merely a constant of integration or an unspecfied value of a variable. If one is tempted by this response then the present setting is a good place to test various philosophical accounts of explanation and causation. I will not be drawn into such a discussion here since it would take us too far afield. But in any case it seems that ϱ_{vac} plays an explanatory or causal role in TFT/UG no less than in GRT.

6.2 Does TFT/UG ameliorate the fine tuning problem?

TFT/UG may offer some succor for those who take rhetoric about fine tuning seriously and who see the cosmological constant problem as a fine tuning problem, at least if the meta-physics sketched in Section 3.1 is taken on board. Consider again the FLRW dust models, and suppose that the actual universe is being described using these models. According to the proposed meta-physics, the physically possible ways the actual FLRW universe can be according to GRT are described by the models with the actual values $G_{@}$ of the gravitational constant and $\Lambda_{@}$ of the cosmological constant held fixed. But among this class of models none are able to reproduce the observed behavior of the scale factor unless Nature has "tuned" the value $\Lambda_{@}$

[26]Philosophers refer to this as the deductive nomological conception of explanation. It competes for philosophical allegiance with causal and unificationist accounts of explanation. For an overview of these competing accounts, see Woodward (2014).

to compensate for the value of ϱ_{vac} provided by particle physicists. TFT/UG offers more leeway since now the physically possible ways the actual FLRW universe can be are described by those models with $G_{@}$ held fixed; the value of $\widehat{\Lambda}$ is not held fixed since, not being a universal constant, its value is not set by Nature. No fine tuning by Nature is needed to find in this expanded class of models one that describes the observed cosmic expansion.

Some commentators are unimpressed by this new perspective TFT/UG offers on the cosmological constant problem. One particularly harsh judgment has it that the move from standard GRT to TFT/UG

> does not accomplish anything, nor does it provide a better understanding of the cosmological constant. The value of the integration constant $\widehat{\Lambda}$ has to be inserted by hand in order to arrive at the correct value. (Nobbenhuis 2006, pp. 624-625)[27]

I share the worry that shifting from GRT to TFT/UG does not accomplish anything except that instead of leaving it to the hand of Nature to do the fine tuning it is now to be done by the hand of the theorist in tuning the value of $\widehat{\Lambda}$ to make the total cosmological constant match observation. There is also the worry that the "new perspective" offered by TFT/UG uses pieces of meta-physics about the nature of universal constants and physical possibility. But since the alleged fine tuning problem itself involves meta-physics it may be appropriate to fight meta-physics with more meta-physics. How low the battle over fine tuning has brought us!

7 Theory choice, action principles, and quantization

7.1 Choosing between observationally equivalent theories

Grant now for sake of discussion that TFT/UG does help to resolve—or at the very least to offer a new perspective on—some versions of the cosmological

[27] I have taken the liberty of substituting $\widehat{\Lambda}$ for Λ to conform to my notation. And I am taking the even greater liberty of assuming that Nobbenhuis is addressing the maneuver I have sketched; apologies if I am wrong. For an equally negative verdict on TFT/UG's ability to cope with the cosmological constant problem, see Padilla and Saltas (2015) and Padilla (2015).

problem. Does it follow, as George Ellis (2014) would have it, that this virtue of TFT/UG gives a reason to "decisively prefer" TFT/UG over GRT?

One could study this question in the context of the debate between scientific realists (those who read scientific theories literally and think that we should believe what our well confirmed physical theories say not only about the observable but also what they say about unobservable entities and processes) vs. instrumentalists (those who view theories merely as instruments for making predictions about the observable).[28] Instrumentalists come in many stripes, but the stripe hardest to defeat is the epistemic anti-realist who does not quarrel with reading theories literally but who argues as follows: 1) for any scientific theory positing unobservable entities there are actual or yet-to-be-formulated rival theories that posit different unobservable entities but yield the same observational predictions as said theory, and 2) there is no good reason for believing the assertions about the unobservables posited by one rather than another theory from a class of observationally equivalent theories. To the realist response that theoretical virtues such as simplicity, unity, explanatory power, etc. can provide reasons to choose among observationally equivalent theories, the anti-realist can retort that such virtues help the realist cause only if these virtues can be presumed to support truthlikeness, a presumption no less difficult to establish than scientific realism itself.

In the present instance, it seems a stretch to believe that the ability of TFT/UG to resolve the fine tuning version of the cosmological constant problem—a problem generated by combining classical GRT with considerations from QFT—supports the truthlikeness of TFT/UG as a classical theory of gravity over its observationally equivalent rival GRT. What seems less of a stretch is that the ability of TFT/UG to resolve the cosmological problem provides a reason to prefer TFT/UG over GRT as a signpost pointing the way to a quantum theory of gravity. How valuable this signpost is perceived to be depends on which approach to quantum gravity one favors. One widely accepted perspective is that a classical relativistic theory of gravity is presumed to emerge in some appropriate low energy limit from a yet-to-be-constructed high energy theory (string/M theory?).[29] Perhaps knowing that TFT/UG rather than GRT should emerge as the low energy limit will

[28]For an overview of this debate, see Psillos (2009).

[29]Koberinski and Smeenk (2022) argue that the cosmological constant problem indicates that GRT cannot be treated as an effective field theory.

furnish clues as to the shape of this sought after high energy theory. On the other hand, from the point of view of canonical approaches to quantum gravity which seek to produce a quantum theory of gravity by quantizing a classical relativistic theory of gravity, GRT and TFT/UG serve as competing starting points of the road to a quantum theory of gravity. It is this second point of view that will be explored here.[30]

Once one starts down this path a new aspect is added to the discussion; namely, at the quantum level one may not have to cast around for virtues, such as simplicity, unity, etc., to help choose among observationally equivalent theories, for quantization may break the observational equivalence of GRT and TFT/UG that holds at the classical level. This would be a healthy development if it comes to pass.

7.2 Breaking the observational equivalence of GRT and TFT/UG at the quantum level

One reason to suspect that quantization breaks the observational equivalence of classical GRT and TFT/UG is that the $\widehat{\Lambda}$ of TFT/UG becomes a dynamical variable that is subject to quantization along with the other degrees of freedom. Thus, the quantization of TFT/UG will have implications for the expectation values of $\widehat{\Lambda}$; but since the Λ of GRT acts as a coupling constant on a par with the gravitational constant G it is not subject to quantization so presumably the quantization of GRT will not have corresponding implications for the value of Λ.

7.2.1 The path integral approach

In the Feynman path integral approach to quantization the probability amplitude for the transition from one point in configuration space to another is given by a "sum over histories," i.e. by adding up the contributions of all paths from the one configuration point to another, where the contribution of a path is weighted by $\exp(\mathcal{S}/\hbar)$ with $\mathcal{S}$ an integral over the action. Different versions of the action principle for TFT/UG give rise to different path integrals, but it is currently unknown whether or not these differences translate into differences in observational predictions (see Bufalo et al. 2015). I

[30] There are many approaches to quantum gravity (see Oriti 2009), but it seems fair to say that the leading research programs are string/M theory and loop quantum gravity, the latter of which falls under the canonical quantization program.

will bypass this matter and simply assume the HTK action (eq. (14)) as a starting point for the quantization of TFT/UG.

Using this starting point, Ng and van Dam (1990) postulated that the form of the path integral is

$$Z^{NvD} = \int Z^{GRT}(\widehat{\Lambda})d\mu(\widehat{\Lambda}) \tag{32}$$

where $Z^{GRT}(\widehat{\Lambda})$ is the partition function of GRT for some value $\widehat{\Lambda}$ of the cosmological constant and $d\mu(\widehat{\Lambda})$ is a measure over the possible values of this constant. They argue that in the absence of matter fields the use of the stationary phase approximation leads to the conclusion that (32) is dominated by solutions with $\widehat{\Lambda} \approx 0$. Subsequently Smolin (2009), also starting from the HTK action for TFT/UG, gave a justification for the conjectured form (32) of the path integral. He then argued that in the presence of matter the stationary phase approximation will be dominated by values of $\widehat{\Lambda}/\kappa$ approximately equal to the average over the spacetime volume of the history of the universe of the energy density of matter. The meaning of this average is unclear, but Smolin suggests that it can be given operational significance by applying a principle of mediocrity and estimating this average by the average matter density in our neighborhood of the universe.

Good physicists are able to extract useful information from formal expressions whose meaning and mathematical justification may be unclear, and the odds are that useful information has been gleaned from gazing at path integrals for TFT/UG. But it would be desirable to have some independent confirmation.

7.2.2 Loop quantum gravity and loop quantum cosmology

Loop quantum gravity (LQG) is a species of the canonical quantization program of Dirac using Ashtekar variables to make the handling of constraints more tractable.[31] What one would like to know is whether the quantum theory of gravity that results from LQG quantization starting from the HTK action for TFT/UG is observationally distinguishable from the quantum theory of gravity that results from LQG quantization starting from the standard Einstein-Hilbert-Λ action for GRT. The question just posed is too hard, and

[31] For an accessible introduction, see Gambinii and Pullin (2011); for more advanced topics see Rovelli (2004).

I will focus instead on what happens at the level of loop quantum cosmology (LQC) which imposes a symmetry reduction before quantizing.[32] The symmetry reduction results in a much easier road to quantization since it means that there are only a finite number of degrees of freedom to quantize; but this advantage comes with a warning since the answer one gets at the level of LQC may be misleading because it is not known whether doing a full LQG quantization and then doing a symmetry reduction will give the same answer as LQC.

Chiou and Geiller (2010) studied the LQC quantization of the empty de Sitter cosmological models starting from the HTK action for TFT/UG. Using the unimodular time variable which is conjugate to $\widehat{\Lambda}$ (recall Section 4.2) to describe evolution of the quantum state, they find that the demand of semiclassicality of a solution to the quantum equation of motion places constraints on $\widehat{\Lambda}$; specifically, a solution remains concentrated on its classical counterpart only if the expectation value of $\widehat{\Lambda}$ is small in Planck units. How to compare this to the LQC quantization of the empty de Sitter cosmological models starting from the action Einstein-Hilbert-Λ action for standard GRT is unclear. In the latter case not only is there no unimodular time for describing evolution, there do not appear to be any good candidates for "clock variables," e.g. the scale factor in de Sitter models does not behave monotonically. But for present purposes it may be enough to know that the LQC quantization starting from the Einstein-Hilbert-Λ action for the empty de Sitter models is not going to have any implications for expectation value of Λ anymore than it does for the gravitational coupling constant κ.

7.2.3 Taking stock

While the differences discussed above between the quantizations of standard GRT and TFT/UG—at least as judged from formal path integral expressions or from LQC quantization based on the HTK action for TFT/UG—do not lead to any decisive observational tests, they do lead to potential confirmations (or disconfirmations) of quantum TFT/UG, e.g., finding a value for the cosmological constant $\widehat{\Lambda}$ near (respectively, far from) the expected value predicted by quantum TFT/UG would provide confirmation (respectively, disconfirmation). The trouble is that what is experimentally ascertainable is not the value of the bare cosmological constant but the value of the total

[32]For an overview of LQC see Ashtekar (2009).

cosmological constant. This leads to a second matter.

At the classical level TFT/UG was supposed to help with the fine tuning version of the cosmological constant problem by making $\widehat{\Lambda}$ an adjustable parameter whose value can be set to balance any given vacuum energy contribution from quantum fields so as to produce a given observed value of the total lambda. But it seems from the above reported results that the considered quantization of TFT/UG at least partially undercuts the advantage offered by classical TFT/UG by making improbable some values of $\widehat{\Lambda}$ in the semiclassical regime where we make our observations. It would thus seem that to solve the fine tuning versions of the cosmological constant problem at the quantum level something rather more ambitious is required; namely, when the quantization of TFT/UG incorporates the matter fields that describe elementary particles, the expected value of the total cosmological constant in the semiclassical regime should accord with cosmological observations. And meeting this more ambitious goal is not just a condition for resolving a fine tuning problem but more importantly as a condition of empirical adequacy quantized TFT/UG.

The present case should also serve as caution when hearing talk of a multiverse composed of miniverses characterized by different values of physical constants. To implement such a scenario may require modification of action principles, which in turn may produce modifications of quantizations entailing consequences not consistent with what is observed in the miniverse we inhabit.

8 Conclusion

The cosmological constant and TFT/UG have checkered and intertwined histories. Both were introduced by Einstein, only to be later abandoned by him. Despite Einstein's shunning of the cosmological constant it had its champions, especially Georges Lemaître; and over the decades it was taken off the shelf by cosmologists in attempts to fix perceived anomalies, only to be put back on the shelf, until in the late 1990s supernovae observations indicated that it—or some surrogate ("dark energy")—has a secure role to play in cosmic evolution. TFT/UG seems to have been forgotten for several decades after Einstein's brief flirtation with it in 1919. It was rediscovered or reinvented by two culturally different groups of physicists: general relativists pursuing a quantum theory of gravity via the canonical quantization route and particle theorists bedeviled by the cosmological constant problem.

I did not weigh in on claims, pro and con, that TFT/UG helps to resolve the problem of time in canonical quantum gravity. But I did weigh in on the claim that TFT/UG offers a means to resolving the cosmological constant problem. I found wanting the version of this claim that has it that TFT/UG decouples vacuum energy from the dynamics of the spacetime metric. I did, however, find some possible merit in the claim that TFT/UG offers a new perspective on the fine tuning version of the cosmological constant problem, but that perspective depends on meta-physical stances. It seems far fetched to think that such merit is a reason for preferring TFT/UG over conventional GRT in their roles as classical theories of gravity, where they are observationally equivalent; but it may be a reason to prefer TFT/UG as a starting point in the search for a quantum theory of gravity. Some versions of the canonical quantization program apparently lead to breaking at the quantum level the observational equivalence of TFT/UG and conventional GRT that holds at the classical level. If substantiated, this would constitute a healthy transmutation of meta-physics into physics, with considerations of empirical adequacy replacing slippery debates about fine tuning.

Whether or not the reader agrees with my take on these matters, I hope it has been made apparent that the study of TFT/UG provides access to a rich array to foundations and methodological issues that are deserving of much more attention from the philosophy of science community, both for their own sake and because they resonate strongly with perennially discussed topics in philosophy including the structure of scientific theories, explanation, causation, and realism.

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