A new EPR-Bohm experiment with reversible measurements

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Abstract

It is usually thought that the EPR-Bohm experiment, unlike the Bell-type experiments, cannot be used to demonstrate the existence of nonlocal correlations that leads to potential incompatibility of quantum theory and special relativity. In this paper, I propose a variant of the original EPR-Bohm experiment, a new EPR-Bohm experiment with reversible measurements. It is argued that in this experiment, the correlation between the results of spacelike separated measurements depends on the temporal order of these measurements in a single-world unitary quantum theory. This violates special relativity, which requires that relativistically non-invariant relations such as the temporal order of spacelike separated events have no physical significance. Moreover, when assuming measurement results are Lorentz invariant, the temporal order dependent correlation further requires the existence of a preferred Lorentz frame. Finally, I analyze possible implications of these results for certain single-world unitary quantum theories.

1 Introduction

It has been debated whether quantum theory and special relativity are compatible. In 1964, based on the Einstein-Podolsky-Rosen (EPR) argument (Einstein et al, 1935), Bell derived an important result that was later called Bell’s theorem (Bell, 1964). It states that certain predictions of quantum mechanics cannot be accounted for by a local theory, and thus strongly suggests that quantum theory and special relativity are incompatible. On the other hand, it is usually thought that the EPR-Bohm experiment, unlike the Bell-type experiments, cannot be used to demonstrate the existence of non-local correlations that leads to potential incompatibility of quantum theory.
and special relativity. In this paper, I will propose a variant of the original EPR-Bohm experiment, a new EPR-Bohm experiment with reversible measurements. It is argued that this experiment can be used to demonstrate the incompatibility of certain quantum theories and special relativity, as well as the existence of a preferred Lorentz frame in these theories.

The rest of this paper is organized as follows. In Section 2, I propose a variant of the EPR-Bohm experiment with reversible measurements, which are permitted by a unitary quantum theory. In Section 3, I argue that in this experiment, the correlation between the results of spacelike separated measurements depends on the temporal order of these measurements in a single-world unitary quantum theory. This is inconsistent with special relativity, which requires that relativistically non-invariant relations such as the temporal order of spacelike separated events have no physical significance. In Section 4, I further argue that when assuming measurement results are Lorentz invariant, the temporal order dependent correlation further requires the existence of a preferred Lorentz frame. In Section 5, I discuss a possible issue with the proposed experiment and suggest a revised version of the experiment to fix the issue. In Section 6, I analyze possible implications of these results for certain quantum theories. Conclusions are given in the last section.

2 An EPR-Bohm experiment with reversible measurements

Let us first consider a usual EPR-Bohm experiment. There are two observers Alice and Bob who are in their separate laboratories and share an EPR pair of spin $1/2$ particles in the spin singlet state:

$$\frac{1}{\sqrt{2}}(|\uparrow\rangle_1|\downarrow\rangle_2 - |\downarrow\rangle_1|\uparrow\rangle_2).$$

(1)

Alice measures the spin of particle 1 at angle $a$, and Bob measures the spin of particle 2 at angle $b$. These two measurements can be spacelike separated. Each measurement result is $+1$ or $-1$, corresponding to spin up or spin down. Then we can calculate the probabilistic correlation function $E(a, b)$ for Alice’s and Bob’s measurement results according to the Born rule, which is $E(a, b) = -\cos(a - b)$. In particular, in the EPR anti-correlation case of $b = a$, we have $E(a, b) = -1$, which means that when Alice’s result is $+1$, Bob’s result is $-1$, and vice versa.

Now consider a variant of the above EPR-Bohm experiment in which there is an additional superobserver in Alice’s laboratory who can reverse or undo her measurement, which is permitted by a unitary quantum theory. I will consider only single-world unitary quantum theories in this paper.¹

¹A single-world unitary quantum theory can be defined as a quantum theory in which
First, suppose in the laboratory frame (in which Alice’s and Bob’s laboratories are at rest), Alice first measures the spin of particle 1 at angle $z$ and obtains her result, then Alice’s measurement is reversed by the superobserver (who restores the states of Alice and the particles to their initial states), and then Alice measures again the spin of particle 1 at angle $z$ and obtains her second result, and then Alice’s second measurement is reversed, and this process repeats a large number of times, and finally Bob measures the spin of particle 2 at the same angle $z$. Each of Alice’s measurements and the reverse operations can be formulated as follows:

$$U_A^1 U_A^1 \frac{1}{\sqrt{2}} (|\uparrow z\rangle_1 |\downarrow z\rangle_2 - |\downarrow z\rangle_1 |\uparrow z\rangle_2) |\text{ready}\rangle_A |\text{ready}\rangle_B$$

$$= U_A^1 \frac{1}{\sqrt{2}} (|\uparrow z\rangle_1 |\downarrow z\rangle_2 |\uparrow z\rangle_A - |\downarrow z\rangle_1 |\uparrow z\rangle_2 |\downarrow z\rangle_A) |\text{ready}\rangle_B$$

$$= \frac{1}{\sqrt{2}} (|\uparrow z\rangle_1 |\downarrow z\rangle_2 - |\downarrow z\rangle_1 |\uparrow z\rangle_2) |\text{ready}\rangle_A |\text{ready}\rangle_B$$  \(2\)

In this case, according to the Born rule\(^2\), the probability distribution of Alice’s results is $P(+1) = 1/2$ and $P(-1) = 1/2$. Then, after a large number of repeated-and-erased measurements, Alice will obtain two different results, spin up and spin down, with roughly equal frequency.

<table>
<thead>
<tr>
<th>Measurements</th>
<th>1</th>
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<tbody>
<tr>
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<td>+1</td>
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<td>+1</td>
<td>...</td>
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Table 1: Alice’s results when Bob finally measures

Next, suppose in the laboratory frame, Bob first measures the spin of particle 2 at angle $z$ and obtains his result, then Alice measures the spin of particle 1 at angle $z$ and obtains her result, and then the superobserver reverses Alice’s measurement, and then Alice measures again the spin of particle 1 at angle $z$ and obtains her second result, and then the superobserver reverses Alice’s second measurement, and this process repeats a large number of times. Each of Alice’s measurements and the reverse operations after Bob’s measurement can be formulated as follows:

\(^2\)There is a subtle issue here, which will be discussed later.

3
\[ U_A^{\dagger}U_A^1 \frac{1}{\sqrt{2}} (|\uparrow_z\rangle_1 |\downarrow_z\rangle_2 |\downarrow_z\rangle_B - |\downarrow_z\rangle_1 |\uparrow_z\rangle_2 |\uparrow_z\rangle_B) |\text{ready}_A \]

\[ = U_A^{\dagger} \frac{1}{\sqrt{2}} (|\uparrow_z\rangle_1 |\downarrow_z\rangle_2 |\downarrow_z\rangle_B |\uparrow_z\rangle_A - |\downarrow_z\rangle_1 |\uparrow_z\rangle_2 |\uparrow_z\rangle_B |\downarrow_z\rangle_A) \]

\[ = \frac{1}{\sqrt{2}} (|\uparrow_z\rangle_1 |\downarrow_z\rangle_2 |\downarrow_z\rangle_B - |\downarrow_z\rangle_1 |\uparrow_z\rangle_2 |\uparrow_z\rangle_B) |\text{ready}_A \] (3)

In this case, according to the Born rule, each of Alice’s results will be anti-correlated with Bob’s result, and thus the probability distribution of Alice’s results is either \( P(+1) = 0 \) and \( P(-1) = 1 \) (when Bob’s result is +1) or \( P(+1) = 1 \) and \( P(-1) = 0 \) (when Bob’s result is -1). Then, after a large number of repeated-and-erased measurements, Alice will always obtain the same result, either spin up or spin down.

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<tbody>
<tr>
<td>Results (when B = +1)</td>
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<td>-1</td>
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<tr>
<td>Results (when B = -1)</td>
<td>+1</td>
<td>+1</td>
<td>+1</td>
<td>+1</td>
<td>+1</td>
<td>+1</td>
<td>...</td>
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Table 2: Alice’s results when Bob first measures

Note that since the results of measurements are objective physical fact (relative to the measurer at least), the statistics of Alice’s measurement results exist objectively. However, since all of Alice’s measurement results are erased by the superobserver at the end of these experiments, the statistics of Alice’s results can only be calculated from a theory, and it cannot be found by experiments. This is consistent with the no-signaling theorem; otherwise Bob will be able to send a superluminal signal to Alice by measuring the spin of particle 2.

3 An incompatibility proof

The above predictions of the results of the proposed experiment are given by a single-world unitary quantum theory (SUQT in brief) which permits reversible measurements. In this section, I will argue that SUQTs are incompatible with special relativity.

According to a SUQT, in the above Gedankenexperiment, the statistics of the results of Alice’s repeated-and-erased measurements are correlated

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3It seems that the probability distribution of Alice’s results cannot be properly defined in many worlds due to world merging resulting from reversed measurements. I will not discuss the many-worlds interpretation of quantum mechanics in this paper.
with Bob’s measurement choice, and the correlation depends on the temporal order of Bob’s measurement and Alice’s measurements, which may be spacelike separated. When Bob makes a measurement after Alice’s measurements, Alice will obtain two different results, spin up and spin down, with roughly equal frequency, while when Bob makes a measurement before Alice’s measurements, Alice will always obtain the same result, either spin up or spin down.

On the other hand, according to special relativity, relativistically non-invariant relations such as the temporal order of spacelike separated events have no physical significance, and thus the correlation between the statistics of Alice’s results and Bob’s measurement choice cannot depend on the temporal order of Bob’s measurement and Alice’s measurements when these measurements are spacelike separated. This means that SUQTs are incompatible with special relativity.

4 On the existence of a preferred Lorentz frame

In this section, I will further argue that there is a preferred Lorentz frame in a SUQT based on a natural assumption that measurement results are Lorentz invariant.

Suppose in the laboratory frame, Alice and the superobserver first make their series of measurements and reverse operations and then Bob makes his measurement. Then Alice will obtain two different results, spin up and spin down, with roughly equal frequency. When Bob’s measurement is spacelike separated from Alice’s measurements, the following temporal order of events in another Lorentz frame is permitted by special relativity. In this frame, Bob first makes his measurement, and then Alice and the superobserver make their series of measurements and reverse operations. Then Alice will obtain the same result each time, either spin up or spin down. Then, when assuming that the result of a measurement (e.g. a pointer indicating +1 or −1 on a dial) is the same in all Lorentz frames, there is a contradiction.

Note again that although Alice cannot remember or report the statistics of her results, this only means an impossibility of testing certain predictions of a theory at the empirical level while what I consider here is whether the predictions of two theories are compatible. After all, the statistics of the results of Alice’s repeated-and-erased measurements in each Lorentz frame can be properly defined and also precisely predicted by a SUQT and special relativity. The result I have derived above is only that the combination

Note that when the distance between Alice’s and Bob’s laboratories is very large and the duration between Alice’s measurements and Bob’s measurement is very short, the relative velocity between this Lorentz frame and the laboratory frame may be close to zero.

This does raise an interesting issue for philosophy of science.
of these two theories will lead to a contradiction when considering their predictions for the statistics of Alice’s results in different Lorentz frames.

The existence of the above contradiction means that a SUQT can be valid only in a preferred Lorentz frame. For if there existed two Lorentz frames in which a SUQT is valid, then we could arrange the temporal order of Alice’s and Bob’s measurements so that the predictions of the theory in the two Lorentz frames contradict each other as shown above. This means that in a SUQT there must exist a preferred Lorentz frame, in which the temporal order of events is real and the predictions of the theory are always true, while in other Lorentz frames the temporal order of events is not always real and the predictions of the theory are not always true either. This result is obtained based on the assumption that measurement results are Lorentz invariant.

5 A revised version of the experiment

As noted before, there is a subtle issue for predictions when Alice’s measurements precede Bob’s measurement. In this case, in the laboratory frame, Alice first measures the spin of particle 1 at angle \(z\) and obtains her result, then Alice’s measurement is reversed by the superobserver, and then Alice measures again the spin of particle 1 at angle \(z\) and obtains her second result, and then Alice’s second measurement is reversed, and this process repeats a large number of times, and finally Bob measures the spin of particle 2 at the same angle \(z\).

Here there are in fact two possibilities. One is that Alice’s measurement results are random, and the probability distribution of Alice’s results is \(P(+1) = 1/2\) and \(P(-1) = 1/2\) according to the Born rule. This is the usual case I have discussed above. However, there is also another possibility, which is that no matter how Alice’s measurement is reversed by the superobserver during the repeated process, Alice’s measurement results are always the same. This is possible in a deterministic hidden-variable theory. In this

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\footnote{Here the invariance of the one-way speed of light or standard synchrony is assumed as usual. If one adopts the convention of nonstandard synchrony that restores the absoluteness of simultaneity (see, e.g. Gao, 2017, ch.9), then a SUQT can be valid in all Lorentz frames. But the one-way speed of light will be not isotropic in all but one Lorentz frame, and thus the non-invariance of the one-way speed of light will also single out a preferred Lorentz frame, in which the one-way speed of light is isotropic.}

\footnote{Similar results have also been obtained for Bell-type experiments (Leegwater, 2018; Lazarovici and Hubert, 2018).}

\footnote{In any deterministic hidden-variable theories including the de Broglie-Bohm theory, when the superobserver’s reverse operation is an exact time-reversal, it will restore the values of all hidden variables such as the positions of all Bohmian particles to their initial values. In this case, the results of Alice’s measurements will be all the same. However, the superobserver’s reverse operation is not necessarily an exact time-reversal, and it only needs to restore the state of Alice and the spin state of the particles 1 and 2 to their initial
case, we need a revised version of the proposed experiment to obtain the previous results.

The revised experiment is as follows. The superobserver prepares an ensemble of particles 3 in Alice’s laboratory, each of which is in the $z$-spin up state. First, suppose in the laboratory frame, the superobserver first entangles one particle 3 with particle 1 by a local interaction to form the following state:

$$
\frac{1}{\sqrt{2}}(|\uparrow_z\rangle_1 |\downarrow_z\rangle_2 |\uparrow_z\rangle_3 - |\downarrow_z\rangle_1 |\uparrow_z\rangle_2 |\downarrow_z\rangle_3),
$$

(4)

where the spin of particle 3 and the spin of particle 2 are anti-correlated in the $z$ direction. Then Alice measures the spin of particle 3 at angle $z$ and obtains her result, and then the superobserver disentangles particle 3 from particle 1 and discards it and reverse Alice’s measurement. Then the superobserver entangles another particle 3 with particle 1, and Alice measures again the spin of particle 3 at angle $z$ and obtains her second result, and then particle 3 is disentangled and discarded and Alice’s second measurement is reversed, and this process repeats a large number of times, and finally Bob measures the spin of particle 2 at the same angle $z$. Each of Alice’s measurements and the reverse operations can be formulated as follows:

$$
U_3^\dagger_A U_3^A \frac{1}{\sqrt{2}}(|\uparrow_z\rangle_1 |\downarrow_z\rangle_2 |\uparrow_z\rangle_3 - |\downarrow_z\rangle_1 |\uparrow_z\rangle_2 |\downarrow_z\rangle_3) |\text{ready}_A \rangle |\text{ready}_B \rangle
$$

$$
= U_3^\dagger_A U_3^A \frac{1}{\sqrt{2}}(|\uparrow_z\rangle_1 |\downarrow_z\rangle_2 |\uparrow_z\rangle_3 |\uparrow_z\rangle_A \rangle - |\downarrow_z\rangle_1 |\uparrow_z\rangle_2 |\downarrow_z\rangle_3 |\downarrow_z\rangle_A \rangle) |\text{ready}_B \rangle
$$

$$
= \frac{1}{\sqrt{2}}(|\uparrow_z\rangle_1 |\downarrow_z\rangle_2 |\uparrow_z\rangle_3 - |\downarrow_z\rangle_1 |\uparrow_z\rangle_2 |\downarrow_z\rangle_3) |\text{ready}_A \rangle |\text{ready}_B \rangle
$$

(5)

In this case, according to the Born rule, the probability distribution of Alice’s results is $P(+1) = 1/2$ and $P(-1) = 1/2$. Then, after a large number of repeated-and-erased measurements, Alice will obtain two different results, spin up and spin down, with roughly equal frequency.

Next, suppose in the laboratory frame, Bob first measures the spin of particle 2 at angle $z$ and obtains his result, then the superobserver entangles one particle 3 with particle 1 by the same local interaction as above to form the state:

$$
\frac{1}{\sqrt{2}}(|\uparrow_z\rangle_1 |\downarrow_z\rangle_2 |\downarrow_z\rangle_B |\uparrow_z\rangle_3 - |\downarrow_z\rangle_1 |\uparrow_z\rangle_2 |\downarrow_z\rangle_B |\uparrow_z\rangle_3).\tag{6}
$$

states. In this case, if the hidden variables are not the values of spin, then they may not be restored to their initial values in general, and thus the results of Alice’s repeated-and-erased measurements on particle 1 may be random and satisfy the Born rule.
Then Alice measures the spin of particle 3 at angle $z$ and obtains her result, and then particle 3 is disentangled and discarded and Alice’s measurement is reversed, and then the superobserver entangles another particle 3 with particle 1, and Alice measures again the spin of particle 3 at angle $z$ and obtains her second result, and this process repeats a large number of times. Each of Alice’s measurements and the reverse operations after Bob’s measurement can be formulated as follows:

$$U_A^3 U_A^3 \frac{1}{\sqrt{2}} (|\uparrow_z\rangle_1 |\downarrow_z\rangle_2 |\downarrow_z\rangle_B |\uparrow_z\rangle_3 - |\downarrow_z\rangle_1 |\uparrow_z\rangle_2 |\uparrow_z\rangle_B |\downarrow_z\rangle_3) |\text{ready}\rangle_A$$

In this case, according to the Born rule, each of Alice’s results will be anti-correlated with Bob’s result, and thus the probability distribution of Alice’s results is either $P(+1) = 0$ and $P(-1) = 1$ (when Bob’s result is +1) or $P(+1) = 1$ and $P(-1) = 0$ (when Bob’s result is -1). Then, after a large number of repeated-and-erased measurements, Alice will always obtain the same result, either spin up or spin down.

It can be seen that the difference between the original version and the revised version of the proposed experiment is that in the revised version, the values of the hidden variable will be generally different for different particles 3, whose distribution satisfies the Born rule, while in the original version, the value of the hidden variable may be the same each time for the same particle 2 as in a deterministic hidden-variable theory. Thus, the revised experiment will fix the issue of the original experiment for deterministic hidden-variable theories.

### 6 Further discussion

It is widely thought that there are single-world quantum theories which can explain the Bell inequality-violating correlations predicted by quantum mechanics and are also compatible with special relativity. Examples include relational quantum theories, such as relational quantum mechanics (Rovelli, 1996; Smerlak and Rovelli, 2017) and perspectivalism (Dieks, 2018, 2019), retrocausal theories (Price, 1996; Corry, 2015; Sen, 2019; Wharton and Argaman, 2020) and superdeterminism (‘t Hooft, 2016). This is because there are supplementary assumptions besides the locality assumption in the proof of Bell’s theorem, such as the measurement independence assumption (Myrvold et al, 2019), while these theories drop one of these supplementary assumptions.
The above proof of the incompatibility between SUQTs and special relativity does not rely on the supplementary measurement independence assumption of Bell’s theorem. According to a SUQT, in the proposed EPR-Bohm experiment with reversible measurements, the correlation between the results of two spacelike separated measurements depends on the temporal order of these measurements. On the other hand, special relativity requires that relativistically non-invariant relations such as the temporal order of spacelike separated events have no physical significance. Thus SUQTs and special relativity are incompatible. Since this inconsistency proof does not concern the complete state of the systems, but only concerns the measurement results, it does not rely on the measurement independence assumption of Bell’s theorem, which assumes the independence of the complete state of the systems and the experimental settings.

Here I discuss two examples of a SUQT. The first example is the de Broglie-Bohm theory and the modal interpretation. It has been shown that in the de Broglie-Bohm theory, the joint distributions given by the Born rule for position measurements cannot in general agree with the distributions of the actual Bohmian particle positions in all Lorentz frames (Bernel et al, 1996). Moreover, it is shown that in the modal interpretation, special relativity is violated and a preferred Lorentz frame exists at the assumed ontological level (Myrvold, 2002). By comparison, the above analysis provides a more general proof of the existence of a preferred Lorentz frame in hidden-variable theories by considering only measurement results (see also Leegwater, 2018).

The second example is relational quantum theories. These theories assume that measurement results may be different relative to different physical systems (Rovelli, 1996; Smerlak and Rovelli, 2017; Dieks, 2018, 2019). In particular, perspectivalism assumes that measurement results are frame-dependent or hyperplane-dependent (Dieks, 2018, 2019). Thus, these theories may avoid the existence of a preferred Lorentz frame in principle; the above proof of the existence of a preferred Lorentz frame relies on the assumption that measurement results are frame-independent. However, since the incompatibility result is derived only in one Lorentz frame and for only one observer, these theories are also incompatible with special relativity in the sense that the correlation between the results of spacelike separated measurements depends on the temporal order of these measurements in these theories.

Finally, it is worth noting that it is not so obvious whether a quantum theory belongs to SUQTs. For example, in retrocausal theories and superdeterministic theories, it is unclear whether a measurement can be reversed, and whether there are even wave functions in the formulations of these theories. Moreover, even though there are wave functions in some non-$\psi$-ontic quantum theories, such as consistent histories (Griffiths, 2011), $\psi$-epistemic models (Spekkens, 2007), pragmatist approaches to quantum
mechanics (Healey, 2017), and QBism (Fuchs et al, 2014), it is unclear whether the collapse of the wave function (at the epistemic level) in these theories prohibits reversible measurements. I will investigate these issues in future work.

7 Conclusions

It has been debated whether quantum theory and special relativity are compatible and whether there is a preferred Lorentz frame if they are incompatible. Bell’s theorem does not give us a definite answer due to the existence of supplementary assumptions. It seems that a single-world quantum theory may be compatible with special relativity by dropping one of these assumptions. Examples include relational quantum theories, retrocausal theories and superdeterminism.

In this paper, I re-examine the issue of whether a single-world unitary quantum theory is compatible with special relativity. I propose a new Gedankenexperiment, a variant of the EPR-Bohm experiment with a superobserver who can reverse a measurement. According to a single-world unitary quantum theory, in this experiment the correlation between the results of spacelike separated measurements depends on the temporal order of these measurements. Since special relativity requires that relativistically non-invariant relations such as the temporal order of spacelike separated events have no physical significance, this result means that a single-world unitary quantum theory is incompatible with special relativity. Moreover, I argue that when assuming measurement results are Lorentz invariant, the temporal order dependent correlation further requires the existence of a preferred Lorentz frame. Since this new analysis does not rely on the supplementary measurement independence assumption in the proof of Bell’s theorem, it provides a more general proof of the incompatibility between a single-world unitary quantum theory and special relativity. However, more work needs to be done to determine whether this incompatibility result is valid in other single-world quantum theories which can avoid the nonlocality result of Bell’s theorem.

References


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