Two Forms of Functional Reductionism in Physics

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Abstract

Functional reductionism characterises inter-theoretic reduction as the recovery of the upper-level behaviour described by the reduced theory in terms of the lower-level reducing theory. For instance, finding a statistical mechanical realiser that plays the functional role of thermodynamic entropy allows to establish a reductive link between thermodynamics and statistical mechanics. This view constitutes a unique approach to reduction that enjoys a number of positive features, but has received limited attention in the philosophy of science.

This paper aims to clarify the meaning of functional reductionism in science, with a focus on physics, to define both its place with respect to other approaches to reduction and its connection to ontology. To do so, we develop and explore two alternative frameworks for functional reduction, called *Syntactic Functional Reductionism* and *Semantic Functional Reductionism.* They represent two different possible stances on the view that expand and improve the basic functional reductionist approach along different lines, and make clear how the approach works in practice. The former elaborates on David Lewis' account, is connected with the syntactic view of theories, is committed to a logical characterisation of functional roles, and is embedded within Nagelian reductionism. The latter adopts a semantic view of theories, spells out functional roles mainly in terms of mathematical roles within the models of theories, and is expressed in terms of the related structuralist approach to reduction. The development of these frameworks has the final goal of advancing functional reductionism, to make it a fully-fledged alternative account for reduction in science.

Contents

1 Introduction

Functionalism is all about understanding things in terms of the roles they play. According to this view, theoretical terms are defined by the roles they have in theories, and properties are cashed out in terms of their causal roles or behaviour. Functional reductionism exploits functionalism to shed light on inter-level relations: finding a lower-level realiser for an upper-level functional role gives us a way to connect the two levels.

Functional reductionism is a view with a venerable tradition in the philosophy of mind (e.g. Lewis (1972), Kim (1998, 2005), Morris (2020)), where it has been employed to relate phenomenal and mental states. If pain is that state "that tends to be caused by bodily injury, to produce the belief that something is wrong with the body [...]" (Levin (2021)) and so on, and we individuate a brain state that fills those roles, we can functionally reduce pain to that specific kind of physical state.

This account is growing in importance within the philosophy of science as well, especially in the philosophy of physics (e.g. Esfeld and Sachse (2007), Lam and Wüthrich (2018, 2020), Butterfield and Gomes (2020a, 2020b), Robertson (2020)).[1] In this context, functional reduction is primarily used to model theoretical reduction between scientific theories. For instance, let's say we can functionally define 'temperature' in terms of its role within thermodynamics, and we find out that 'mean kinetic energy' plays the role of temperature: in that case, we can functionally reduce temperature to mean kinetic energy, and this can be regarded as a step in the reduction of thermodynamics to statistical mechanics.

The aim of this paper is to clarify the meaning of functional reductionism in the context of physics, and define both its place with respect to other approaches to scientific reduction and its connection with ontology, starting from an instance of functional reduction recently advanced by Robertson (2020), concerning the reduction of thermodynamic entropy. Functional reduction provides a unique approach to reduction, different from standard views like Nagelian or structuralist reduction, and enjoys several advantages. For example, as argued by Robertson in the context of our case study, functional

[1]The value of functionalism in physics has been also recently defended e.g. by Knox (2019) and Wallace (2012, 2021). Functional reductionism has been discussed also by Albert (2015). Moreover, Lewis' and Kim's models quoted above apply to science as well.

reduction is able to deliver a reductive account of the second law of thermodynamics to statistical mechanics, which is a historically thorny issue for reductionism.[2]

The central questions we address here to develop this approach to reduction are: how is functional reductionism related to one's view about the nature of scientific theories, i.e. with respect to the syntactic-semantic distinction? How should we formalise functional roles? What is exactly the connection between functional reduction and existing approaches to scientific reduction? Is functional reductionism a local or global form of reduction? What is the connection between theoretical reduction and ontology within this approach? These issues are not present in the philosophy of mind, and an analysis of these questions in the philosophy of science is crucial but missing.[3]

Responding to these questions will lead us to develop two alternative frameworks for functional reduction, that expand the basis of the view and address differently these topics, and that we shall confront with our case study. We call them respectively *Syntactic Functional Reductionism* and *Semantic Functional Reductionism*. The first is embedded within the syntactic view of theories, is committed to a logical characterisation of functional roles, and is a functionalist form of Nagelian reduction. The second is based on the semantic view of theories, spells out functional roles mainly in terms of mathematical roles within the models of the theory, and is expressed in terms of the structuralist approach to reduction. The former framework is based on the only full-fledged functional reductionist account in the literature, introduced by Lewis (1970) and recently defended by Butterfield and Gomes (2020a), whereas the latter framework is a completely novel proposal. We particularly focus on the syntactic-semantic divide because the view about theories one combines with functional reductionism heavily influences how we formalise functional roles, which has overarching consequences for the whole account.

The results of this paper are important for several reasons. **First**, we see that functional reduction can be articulated in different ways, depending on one's views about several different topics, such as the syntactic-semantic distinction, and one's preference for Nagelian or structuralist reduction. Given the potential of functional reduction as an alternative account for reductionism in science, developing the view in more detail is a valuable achievement in its own right, as it clarifies what the theory really amounts to. **Second**, we show that functional reduction can integrate either Nagelian or structuralist reduction and provide a revised and improved version of these approaches, embedded in the functional reductionist framework, and thus the framework is largely relevant for the debate about scientific reduction. Syntactic Functional Reductionism is indeed a form of Nagelian reduction in which bridge laws are not postulated as additional assumptions, and are thus less problematic, while Semantic Functional Reductionism

[2]We go back to the pros of functional reduction at the end of section 2, after having introduced our case study, and then in sections 4 and 5 we discuss how functional reduction can be combined with Nagelian and structuralist reduction to provide improved versions of those accounts.

[3]We thus contrast with a common attitude within the literature on functional reduction, where the transition between the two has been usually considered to be very smooth (cf. Kim (2005), Fazekas (2009)), in such a way that we can easily generalise functional reductionism about mind to philosophy of science in general. We argue here that things are not so simple, given the role that scientific theories play in functional reductionism within science.

improves on the rather broad structuralist account of reduction, providing a preciser version of it and a stronger justification for the approach. **Third**, by analysing the thermodynamics case study, we show how functional reduction works in practice, and discuss both the pros and cons of each framework and the ontological bearings of this account of reduction. The Syntactic framework delivers an approach to ontology that is very clear but also very strict, whereas the Semantic one allows for a more flexible view on the ontological aspects of reduction. These aspects mirror the situation at the theoretical level: while the former approach adopts a very rigorous and logically-formulated view on the formulation of theories, the latter embraces a model-based account. The choice between the two frameworks hinges also on these features. Most importantly, the lack of flexibility characterising the Syntactic approach is problematic insofar as we want to freely choose which elements of the theory we want to be realists about, and the logical translation aspect of the view can work against the framework as well. We argue that the Semantic framework is preferable in these respects: in this sense, the paper suggests that the semantic view of theories and the structuralist stance on reduction better suit functional reductionism.

Section 2 starts by reviewing Robertson's functional reductionist proposal concerning thermodynamics, that will be the starting point of our discussion. Having presented how functional reduction works for a real example, in section 3 we start our enquiry on functional reductionism from the foundations, by defining the central terms of the debate. Then, in sections 4 and 5, we discuss the two functional reductionist frameworks.

2 A Case Study for Functional Reductionism

We review here the instance of functional reduction recently put forward by Robertson (2020). Being a state-of-the-art example of scientific functional reduction, it provides a good introduction to the approach, and a well-suited case study for our discussion about the relationship between functional reduction and ontology in sections 4.3 and 5.2.

Robertson's aim is to reduce the thermodynamic entropy S_{TD} to some statistical mechanic quantity, as a step in the reduction of thermodynamics to statistical mechanics – in particular, to reduce the second law of thermodynamics, which can be expressed in terms of the behaviour of the thermodynamic entropy.[4] To do so, her goal is to find in statistical mechanics a realiser for the role of the thermodynamic entropy. We report here just the essential details. Let's start from the top-level theory, and in particular from thermodynamic entropy S_{TD}. This is a function of the state of a thermodynamic system, like pressure, and it is roughly said to measure the 'disorder' of the system.[5] Or, using Clausius' definition, entropy can be defined as the thing that increases by Q/T whenever heat Q enters a system at temperature T.[6] We can thus represent the change of entropy dS_{TD} in a system as:

[4] The second law of thermodynamics, according to Clausius' statement, says that it is impossible for heat to spontaneously move from a lower-temperature reservoir to a higher-temperature reservoir.

[5] We consider thermally isolated systems and reversible processes.

[6] Cf. Schroeder (1999).

$$\frac{dQ}{T} = dS_{TD}, \tag{1}$$

where dQ is the change in heat (the heat absorbed) and T is the temperature. Thermodynamic entropy can then be represented by integrating (1). In this way we represent the entropy difference between two states of the system, in this case state 0 and state B:

$$\int_0^B \frac{dQ}{T} = S_{TD}(B). \tag{2}$$

This quantity is crucial for modelling thermodynamic behaviour, and thus reducing it to statistical mechanics would be an essential step to reducing thermodynamics to statistical mechanics, as we can use this to formulate the second law of thermodynamics. The two central characteristic features of S_{TD} on which reduction is focused are related to how this function works in two kinds of situations.

On the one hand, let's look at the case of arbitrary quasi-static reversible cycles in the equilibrium space Ξ. A thermodynamic equilibrium state is a state in which no macroscopic change occurs in a system, and the equilibrium space is the space of those states. A quasi-static reversible cycle is a process in which the system moves through equilibrium states, thanks to the fact that it is evolving slowly. For these processes we expect the following to occur:

$$\oint \frac{dQ}{T} = 0. \tag{3}$$

That is, if a process P is a quasi-static reversible process, we can write:

$$\Delta S_{TD} = 0. \tag{4}$$

On the other hand, it can be proven that, if a process P (say, between state A and state B) is not quasi-static, the thermodynamic entropy is a quantity that cannot decrease:

$$S_{TD}(B) - S_{TD}(A) \geq 0, \tag{5}$$

that is:

$$\Delta S_{TD} \geq 0. \tag{6}$$

Functional reduction consists in finding a statistical mechanics realiser – a statistical mechanics function or quantity – for the roles of S_{TD} which are mathematically specified by (4) and (6). Using words to express that theoretical functional role, we can say that we have to "Find a statistical mechanics realiser which, for thermally isolated systems, is increasing in non-quasi-static processes, but non-increasing in quasi-static processes, such as those represented by curves in Ξ." (Robertson (2020), 21).

Let's thus move to statistical mechanics.[7] A key concept in statistical mechanics is that of canonical ensemble, which is used to represent the possible states in which a

[7]We slightly deviate here from Robertson's presentation, for simplicity of exposition. See e.g. Tong (2012) for an introduction.

system can be. In particular, the canonical ensemble gives the probability that a system is in a specific state n:

$$p(n) = \frac{e^{-E_n/k_BT}}{\sum_m e^{-E_m/k_BT}}, \tag{7}$$

where E is the energy of each state and k_B is Boltzmann's constant. To simplify, we can introduce a new notation, and write $\beta \equiv 1/k_BT$ and $Z = \sum_n e^{-\beta E_n}$. We can thus rewrite (7) as:

$$p(n) = \frac{e^{-\beta E_n}}{Z}. \tag{8}$$

Moving to quantum statistical mechanics, we write (8) in a slightly different way. In quantum mechanics, a system can be described via a density matrix ρ. We can thus express the canonical ensemble for a given system as:

$$\rho = \frac{e^{-\beta \hat{H}}}{Z}, \tag{9}$$

where $\hat{H}$ is the Hamiltonian operator representing the energy.[8] In both classical and quantum statistical mechanics, the canonical ensemble can be used to represent thermal equilibrium. What matters for us now is that ρ is important to the introduction of a new quantity, the quantum Gibbs entropy, since the canonical ensemble is said to maximise Gibbs entropy S_G:

$$S_G = -k_B Tr \rho \ln \rho, \tag{10}$$

where *Tr* is the trace over the density matrix. Having introduced S_G, we shall now gloss over a lot of details and just report here how the reduction of thermodynamic entropy is achieved through a functional reduction of S_{TD} to S_G. Briefly put, Robertson (2020, sect. 6) shows that, for quasi-static processes in quantum statistical mechanics, we can write:

$$\Delta S_G = 0 \tag{11}$$

On the other hand, for non-quasi-static adiabatic processes, with $t_1 - t_0 \approx 0$, we can derive:[9]

$$S_G[\rho_{can}(t_1)] - S_G[\rho_{can}(t_0)] > 0. \tag{12}$$

The presentation so far provides what we asked for: we have found a statistical mechanical function – i.e. the statistical mechanical entropy S_G – that is constant in quasi-static

[8] If we make a measurement of the energy of the system then the probability of finding ourselves in an energy eigenstate $|\phi\rangle$ is $p(\phi) = \langle\phi| \rho |\phi\rangle$, and for energy eigenstates this is indeed just (8).

[9] Provided that we have adopted a new canonical ensemble tailored to the process and different from the starting one, and we have abstracted away certain details.

processes (11) and that increases in rapid non-quasi-static processes (12). Indeed, the statistical mechanical equations (11) and (12) for S_G mathematically mirror the thermodynamic equations (4) and (6) embedding S_{TD}. The result is that these equations display the functional similarities shared by the two quantities.

Finally, to strengthen the functional correlation between the two quantities, Robertson shows that, in the right parameter regime, S_{TD} and S_G evolve in a very similar way. First, take (1), and derive the following from the first law of thermodynamics $dE = TdS - pdV$, where V is the volume:

$$dS_{TD} = \frac{1}{T_{TD}}(dE_{TD} + p_{TD}dV). \tag{13}$$

On the other hand, within Gibbsian quantum statistical mechanics, given certain assumptions and approximations, we can derive Gibbs entropy as:

$$dS_G = \frac{1}{T}(d\langle E\rangle + \langle p\rangle dV), \tag{14}$$

where the brackets denote that we are taking the average value. All in all, we can conclude that the Gibbs entropy functionally reduce the thermodynamical entropy:

> The Gibbs entropy can play the right role, since it increases in non-quasi-static processes but is constant in quasi-static processes. Furthermore, S_G is connected to heat in the right way (Robertson (2020), 31).

To recap, Robertson's goal was to find a statistical mechanical reductive basis that could reduce a specific thermodynamic behaviour, which is codified by the evolution of the thermodynamic entropy S_{TD}. To do so, she exploited the functionalist idea that, in order to reduce the thermodynamic entropy to statistical mechanics we have to find a statistical mechanical quantity which – at least approximately – plays the role of S_{TD} in the upper-theory.

The use of a functionalist model of reduction is here motivated by two main reasons, that are not limited to our specific case study. First, functional reductionism tells us what we have to do if we want to establish reduction, that is we have to focus on finding something in the low-level theory which instantiates the right patterns of behaviour within the high-level theory. In this way, functional reduction provides a clear and plausible model for reduction, that we can exploit to find a statistical mechanical underpinning for thermodynamics. Indeed, as stressed by Robertson, formulating a reductionist account for the second law of thermodynamics is a notoriously difficult task,[10] and functional reduction provides the tools to do so. Second, as Robertson (2020, 5) says: "Functionalism has the upper hand here as it specifies the differences that can be tolerated: the realiser can differ in ways that do not affect its playing the functional role". In fact, we have seen above that Gibbs entropy is quite different from thermodynamic entropy, and they also do not play exactly the same theoretical roles: most importantly, one quantity is essentially statistical and is expressed in terms of expectation values,

[10]See Callender (1999) on this topic.

while the other is not. However, provided that the bottom quantity approximately plays the role we expected it to play, no other difference matters.[11]

In sections 4 and 5 we develop two alternative frameworks for functional reduction, and this case study will be crucial to discuss how the frameworks work as applied to a real example. We see that Syntactic Functional Reductionism faces the problem of shoehorning the mathematical formalism used here into a logical formulation and the problem of accounting for the approximation required for the reduction. On top of that, the way in which the framework reformulates this example of functional reduction prompts a very specific, but also too restrictive, account of the ontological implications of the reduction. On the contrary, the formulation of functional reductionism provided by the Semantic framework is model-based and mostly mathematically formulated, and thus accommodates in a more straightforward way the case study as presented here. Framed in that way, the account also allows for a more flexible account of the ontological meaning of the reduction at stake. Before moving to that, however, we need to present more generally what functional reductionism is.

3 Laying the Foundations of Functional Reductionism

We start our presentation of functional reductionism from the basics, by providing a definition of the central terms of debate: functionalism, reductionism, and functional reductionism. This is the first step in the clarification of the meaning of functional reductionism within the philosophy of science, and provides the foundations for the development of our two frameworks. To begin with, we introduce two distinctions, which are crucial for our disambiguation of the terms in the debate.

3.1 Two kinds of distinctions about scientific theories

The first is between *formal mode* as opposed to *material mode*.[12] Discussions in the formal mode concern the theory-level, whereas discussions in the material mode concern the world-level. This distinction is pivotal in spelling out functionalism and reductionism, as well as functional reductionism. Formal and material characterisations of these theses are often mixed up, but they should be really kept apart. As this paper shows, e.g., theoretical functional reductionism can be endorsed independently from ontological functional reductionism, and the former position is tied to certain issues (for instance, about the nature of theories) which do not directly regard the latter. Finally, it is important to stress that the main focus of the present discussion is functional reduction in the formal mode, i.e. theoretical functional reduction. Indeed, functional reduction in the philosophy of science is primarily concerned with the functionalisation of theoretical concepts or quantities and with reduction between theories (or parts of theories), and talk about material reduction is often secondary. One reason is the belief that our ontological

[11]For further discussions about the advantages of functional reductionism see Lam and Wüthrich (2018) and Butterfield and Gomes (2020a, 2020b).

[12]Cf. Ladyman et al. (2007).

commitments should be guided by what scientific theories tell us about the world. So, for instance, talk about material reduction between the ontology of two theories should be dependent on formal reduction between the two theories themselves.

The second distinction is between the *syntactic* and the *semantic* view of scientific theories. We will not attempt here a complete and accurate reconstruction of the debate, especially because many versions of each view are available, and the distinction between the two is often blurry.[13] Rather, we present them in a way that can clearly distinguish between two distinct approaches to scientific theories, and that can be helpful to appreciate the difference between the two brands of functional reductionism we introduce. The detailed presentations of the frameworks in sections 4 and 5 will make clear that they eventually appeal to two distinct formulations of what a scientific theory is, that we can respectively label as syntactic and semantic view. A first way to distinguish the two views is to stress the focus of the former on sentences and the focus of the latter on models. On the one hand, we can say that for the syntactic view "the structure of a scientific theory is its reconstruction in terms of sentences cast in a meta-mathematical language" (Winther (2021)). We shall see in section 4 that a prerequisite of Syntactic Functional Reductionism is the idea of formulating scientific theories as sentences formulated in the language of second-order logic. Within this approach, inter-theoretical relations such as reductive relations between theories are formulated as deductions under a given class of logical relations. On the other hand, according to the semantic view, representing scientific theories requires mathematical tools and not predicate logic. In the context of physics, which is the focus of the present essay, theories can be said to be constituted by sets of models which are mainly mathematically formulated, in the sense of 'model' which is employed by physicists.[14] For the purpose of this paper, the two main features of the semantic approach are the following, quoting Ladyman et al. (2007, 118): "(a) The appropriate tool for the representation of scientific theories is mathematics; (b) The relationships between successive theories, and theories at different scales whether spatio-temporal or energetic, are often limiting relations and similarities of mathematical structure (formally captured by structure-preserving maps or morphisms of various kinds), rather than logical relations between propositions". In saying this, they follow Suppes' famous slogan according to which "philosophy of science should use mathematics, and not meta-mathematics".[15] The Semantic Functional Reductionist framework of section 5 follows these principles, by characterising functional roles in mathematical terms and functional reductive relations as structural relations between models. The overall distinction between the syntactic and the semantic view, as we described them here, is summed up in the following table.

[13]See Winther (2021), Halvorson (2019) for an introduction. See Halvorson (2013) and Lutz (2017) for a recent defence of the syntactic approach; and Suppes (1961, 1967), Suppe (1974), Van Fraassen (1980), Ladyman et al. (2007), Wallace (2021) for a defence of the semantic approach.

[14]By characterising the semantic view in these terms we thus broadly follow the approach of Van Fraassen (1980).

[15]As quoted in van Fraassen (1980, 65).

	Objects	**Relations**
Syntactic view	Sentences	Deduction under a given class of logical relations
Semantic view	Models	Structural relations, usually mathematical

3.2 Defining functionalism and reductionism

We are now in the position to introduce functionalism and reductionism. Functionalism is the view that 'to be x is to play the role of x'. The view can either be spelled out in formal mode or in material mode. More specifically, x can either be a theoretical term/concept or a property. We can thus distinguish between:

Definition 3.1 (**Functionalism about theoretical terms**)**.** The thesis that 'to be x is to play the role of x', where x refers to something in a scientific theory. In this case functionalism says that x should be *functionally defined.*

Definition 3.2 (**Functionalism about properties**)**.** The thesis that 'to be x is to play the role of x', where x refers to something in the world.

Let's confront these two notions with the case study of section 2. With respect to functionalism about theoretical terms, the case study provides a functional definition of the concept of 'thermodynamic entropy' in terms of its role within thermodynamics. Another example is the theoretical functional definition of 'spacetime' within spacetime functionalism.[16] With respect to functionalism about properties, in the context of the case study, functionalism claims that the property of having thermodynamic entropy is the property (that can be instantiated by a physical system) of behaving so-and-so. Functionalism about mental states is usually predicated on these terms.

Reductionism, similarly to functionalism, can be framed as a view concerning the world or as a view concerning scientific theories:

Definition 3.3 (**Theoretical reductionism**)**.** The view that upper-level theories can be reduced to lower-level theories, where reduction represents here an asymmetrical relation between two theories that can be further spelled out in different ways, depending on the view on reduction one adopts. The reducing theory is taken to subsume the domain of applicability of the reduced one. The view can be formulated either in *global* terms, concerning the reduction of whole theories, or in *local* terms. In the second case, we are concerned just with the reduction of specific laws or specific models of a theory.[17]

Definition 3.4 (**Ontological reductionism**)**.** The view according to which an entity x (usually taken to belong to a higher level, or higher scale) is reduced to another entity y (usually taken to belong to a lower level or lower scale) if x can be identified, or is dependent on, or is grounded/determined by y. If some entities are reduced to other entities, there is thus a sense in which the reduced entities are not really anything over and above the reducing entities, in a way to be specified.

[16]Knox (2019), Lam and Wüthrich (2018), Wüthrich (2019).

[17]See also Rosaler (2015) on the dichotomy between global and local approaches to reduction.

It is worth spending a few more words on theoretical reductionism, given that there is no consensus on the specific meaning of the term, and for that reason we explicitly acknowledged above that the exact definition of the view hinges on which specific approach to reduction one endorses.[18]

The main account about theoretical reductionism in the literature is **Nagelian reductionism**. According to Nagel's (1962) classic model of reduction, a theory T_P can be said to be reduced to another theory T_F iff the laws of T_P can be deduced from the laws of T_F plus some auxiliary assumptions (which usually involve idealisations and boundary conditions). In the (common) case in which the two theories do not share their theoretical terms we need also to postulate bridge laws connecting the two vocabularies. For instance, in the context of the reduction of thermodynamics to statistical mechanics, we can derive the Boyle-Charles law from statistical mechanics' laws given a bridge law stating that 'temperature' means 'mean kinetic energy' (cf. Dizadji-Bahmani (2021), 500-1). A more refined version of this approach – put forward by Schaffner (1967) and recently by Dizadji-Bahmani et al. (2010)[19] – relaxes the derivability criterion and argues that, to ensure reduction, it is sufficient to derive laws which are approximately the same of the laws of the original theory T_P. More precisely, according to this view, T_F reduces T_P iff we can build a theory T_P^* – which is a corrected version T_P standing in a relation of 'strong analogy' with T_P – which is derivable from T_F given some appropriate auxiliary assumptions and bridge laws. Following the most recent presentations of the view, we can call this approach 'Neo-Nagelian reductionism'. The reason is that it is almost never the case that we can derive the exact laws of an upper theory (to be reduced) from a bottom theory. At most, we can recover the behaviour described from the top theory in an approximate way and just in particular situations. Finally, nothing in the account blocks us from using Nagelian reductionism to frame local reductions, e.g. we can show how a certain restricted part of a theory can be derived from a part of a lower-level theory by showing that a limited set of upper-level laws can be derived.[20]

A few other accounts about theoretical reductionism has been proposed in the literature about scientific theories, however we focus here on a specific approach which will be central to our discussion about functional reductionism. This is the so-called **structural reductionist** account, and is tied with the semantic view about theories. Indeed, we have already partially introduced it while presenting the semantic view at the beginning of the section. Within this view, reduction is primarily a relationship between models of different theories. For instance, the view has been endorsed by Suppes, who claimed that "the thesis that psychology may be reduced to physiology would be for many people appropriately established if one could show that, for any model of a psychological theory, it was possible to construct an isomorphic model within physiological theory." (Suppes 1967: 59).[21] The relation of isomorphism has been considered to be too strong in the subsequent literature, but the notion of reduction as a model-model mathemat-

[18] See van Riel and van Gulick (2019) for an overview.

[19] See also Schaffner (2012) and Dizadji-Bahmani (2021).

[20] Cf. Rosaler (2015). I discuss the combination between local reduction and Nagelian reduction also in the context of Lewisian functional reduction (section 4.2).

[21] See also Suppes (1961).

ical relation has remained the hallmark of the approach. For example, in the passage quoted above concerning the semantic view, Ladyman et al. (2007) talk about reduction as a link between mathematical structures in terms of structure-preserving mappings or 'morphisms', whereas Wallace (2021, p. 16) argues that "reduction is something like *instantiation*: the realizing by some substructure of the low-level theory's models of the structure of the higher-level theory's models". An approach to reduction focused on model-model relations is also endorsed by Rosaler (2015, 2019).[22] Notice that this view characterises reduction as a *primarily local* relation, that takes place between specific models. Global theory-to-theory reduction is thus derivative on local model-to-model reductions. Moreover, those relations of instantiation or morphism between the models will not be exact but approximate – for the same reason mentioned earlier – but approximation is here a relation between the models standing at different levels, and not between the higher-level theory and its own corrected version.

3.3 Introducing functional reductionism

Having defined functionalism and reduction, we can now move to the central topic of this paper. We take functional reductionism to denote any view that combines functionalism and reductionism in the same domain, and in particular in which reduction is functionally induced, i.e. a form of reduction is entailed by a functionalist account. Following Kim's (1998, 2005) model, provided a functional characterisations of a given x at an upper-level, we can functionally reduce it if we find something in a lower-level which plays the x-role. Kim develops this model merely in ontological terms, in order to functionally reduce properties, but we apply here functional reductionism to the context of theories as well, in the spirit of Lewis (1970). The latter view is the main focus of the essay. We can thus distinguish two brands of functional reductionism:

Definition 3.5 (**Theoretical functional reductionism**)**.** The view that a theory T can be reduced to another theory T* in virtue of the fact that the theory T* embeds theoretical elements which can play the theoretical roles of the theoretical elements belonging to T. If a theoretical element x in the reduced theory is functionally realised by a theoretical element y in the reducing theory, we can say that x is functionally reduced to y.

Definition 3.6 (**Ontological functional reductionism**)**.** The view that an entity can be reduced to another entity if the reducing entity can play – at least approximately – the material (e.g. causal) roles associated with the reduced entity.[23]

In other words, according to theoretical functional reductionism, theoretical reduction is essentially a matter of recovering the upper-level behaviour described by the reduced theory from the lower-level description stated in terms of the reducing theory.

[22] We return to Rosaler's account in section 5.

[23] The standard example of ontological functional reduction comes from the philosophy of mind, where kinds of mental states or properties (like 'pain') are argued to be reduced to brain states or properties when the latter play the right role.

Thus, functional reductionism argues that establishing reduction is a matter of finding the right lower-level realisers for the upper-level behaviour: reduction is secured if we find in the bottom-level theory some theoretical elements that play the roles described by the upper-level theory. In this sense, the core idea behind theoretical functional reductionism is that of providing a framework in which reductionism employs functionalist intuitions to characterise the reduction. It is in this sense that Robertson (2020) argues that we can reduce thermodynamical entropy S_{TD} to the statical-mechanical Gibbs entropy S_G in virtue of the fact that the latter can approximately realise the theoretical role of the former, and that this allows us to reduce the second law of thermodynamics to statistical mechanics.

Notice that, similarly to the general definition of theoretical reductionism, the definition of theoretical functional reductionism above is spelled out quite loosely. The reason is that the view can be further characterised in different ways, also depending on details such as one's view about the syntactic-semantic debate as well. For instance, the view about theories one adopts – focused on logical sentences or focused on models – heavily influences how one formalises functional roles, and this in turn influences how one defines functional reductionism in general. Spelling out these details is the central aim of the paper. In particular, we introduce two general frameworks in which theoretical functional reductionism can be articulated, and also illustrate their connection with ontological functional reductionism. Very roughly, within Syntactic Functional Reductionism, functional reductionism is embedded within a Nagelian approach to reduction formulated in syntactic terms, where functionalism plays a crucial role in the formulation of the bridge laws. On the other hand, within Semantic Functional Reductionism, functional reductionism is embedded within the semantic view and the related structuralist view of reduction, and thus functional reduction is cashed out in terms of structural relations between models. Thus, it should be stressed that functional reduction is an alternative to traditional non-functionalist account of reduction (such as Nagelian and structuralist reduction), but, at the same time – once we analyse the possible formulations of functional reduction – we can see that the view can be reformulated as a *functionalist version* of either the Nagelian view or the structuralist one. Sections 4 and 5 introduce and discuss these two views in more detail.

	Non-functional	**Functional**
Syntactic view	Nagelian reduction	Syntactic Functional Reduction
Semantic view	Structuralist reduction	Semantic Functional Reduction

4 Syntactic Functional Reductionism

The first functional reductionist framework we introduce is called 'Syntactic Functional Reductionism'. This framework is based on the functional reductionist account firstly put forward by Lewis (1970) and recently defended and improved by Butterfield and Gomes (2020a).[24] According to this approach, reduction goes as follows. The first

[24]Which is currently the most developed functional reductionist account available in the philosophy of science literature.

step is to write down the laws of the reduced theory in terms of the reducing theory. At that point, by appealing to functionalism, we derive the bridge laws between the theoretical terms of the two theories from the laws of the bottom theory alone. We are thus able to derive the upper-level laws from the bottom-level laws plus bridge laws, thereby fulfilling the criteria of Nagelian reduction. However, within this functionalist form of Nagelian reduction, bridge laws are derived from the reducing theory, and not postulated. Because of this feature, the account can be regarded as an improved version of Nagelian reduction.[25] The aim of this section is to expand the Lewisian basis – to present the most developed version of the account possible – and then provide an assessment of the framework.

4.1 The Lewisian Approach

This subsection introduces in more detail the core of Syntactic Functional Reductionism, as defended by Lewis, Butterfield and Gomes. The starting point of the view is the minimal functional reductionist thesis about theories presented in section 3.3: provided a functional characterisation of a given upper-level theoretical term x, reduction is achieved if we find a realiser of the x-role in the lower-level. What singles out this account is the way in which the functionalisation process is cashed out. According to this view, theoretical functional reduction essentially proceeds in three steps:

1. We write down the laws of theory T in logical terms, then we replace all the theoretical terms $\tau_1 ... \tau_n$ of the theory with open variables $x_1 ... x_n$, leaving just non-theoretical terms and connectives, i.e. we move from $T(\tau_1, ...\tau_n)$ to $T(x_1, ...x_n)$. We now build the Ramsey sentence of the theory by placing an existential quantifier in front of the sentence: $\exists x_1, ...x_n T(x_1, ...x_n)$. This says that there are certain xs which realise the theory. On the assumption that the theory is uniquely realised (i.e. there is only one set of $x_1 ... x_n$ that realises the theory), we can construct explicit functional definitions of the $\tau_1 ... \tau_n$ via the Ramsey sentence. These says e.g. that τ_i is 'that thing that occupies the x_i-role within the theory'.

2. We find another theory T* embedding new theoretical terms $\rho_1 ... \rho_n$. Suppose we are able to write $T^* \vdash T[\rho_1 ... \rho_n]$. $T[\rho_1 ... \rho_n]$ does not contain τ-terms, and it says that the original theory T is realized by a n-tuple $\rho_1 ... \rho_n$, taken from T^*. In case T is uniquely realised by the n-tuple $\rho_1 ... \rho_n$, Lewis shows that we can functionally define the ρ_i as the occupiers of certain x-roles in T, and those functional definitions are theorems of T*.

3. Following step (2), we can derive theoretical identifications $\rho_1 = \tau_1, \ldots, \rho_n = \tau_n$ by transitivity of identity. These are *bridge laws* and they play the role of Nagelian bridge laws in the theory derivation of T from T*.

[25] See Kim (2005), Lewis (1970), Butterfield and Gomes (2020a). See also the debate between Marras (2005) and Morris (2020) on the topic of bridge laws.

Summing up more informally, the process goes as follows: we first specify the theoretical roles of the theoretical terms within a theory T via the Ramsey sentence of a theory, i.e. we build functional definitions for the terms $\tau_1 ... \tau_n$ in the theory. Then, we find a second theory T*. This theory can realise the former theory T in terms of $\rho_1 ... \rho_n$, and so we show that it contains theoretical terms $\rho_1 ... \rho_n$ which play the roles of the entities $\tau_1 ... \tau_n$. Thus, on the assumption that the Ramsey sentence of theory T is uniquely realised, we deduce bridge laws between the two theories, i.e. we connect the vocabularies of the two theories. This happens simply because we have terms that fall under the same functional profile and then they are identified thanks to functionalism. When a term τ_i is identified to a term ρ_i in this way, we can say that τ_i is functionally reduced to ρ_i.

The Lewisian reduction is thus a special form of Nagelian reduction in which theory deduction is couched in terms of logical derivation and in which bridge laws are functionally derived and thus deduced, as opposed to postulated as additional empirical hypotheses. Given that theories are formulated as logical sentences within the account, this functional reductionist view can be said to fit naturally in the syntactic view. This is the core of the Lewisian framework as proposed by Lewis and backed up by Butterfield and Gomes.

This is clearly a form of functional reductionism about theories. However, in Lewis's account, this functional reductionism about theories, which leads to identity relations between theoretical terms, is meant to be a way to ensure functional reduction about ontology as well. Lewis makes this clear in several places, for instance:

> The T-terms have been defined as the occupants of the causal roles specified by the theory T; as *the* entities, whatever those may be, that bear certain causal relations to one another and to the referents of the O-terms. (Lewis, 1972, p. 255)

The passage from the formal mode to the material mode is quite straightforward here. On the assumption that the theoretical terms refer to actual entities, the theoretical functionalisation is just a means to codify in a scientifically accurate way the causal roles played by the worldly entities referred to by the theoretical terms.[26] That is, functional reduction of theoretical terms can be a guide to functional reduction of entities. Lewis is explicit about this. For him, the theoretical term 'electron' is meant to refer to an actual entity, as he wants to maintain a clear form of scientific realism.[27] Thus, when we functionally define a theoretical term in the upper theory and we find some other theoretical term in the bottom theory with the same role, we should believe also that there is a bottom entity (referred to by the term ρ_i) to which the upper entity denoted by τ_i is reduced to.

[26]Note that the account is neutral with respect to the notion of causality at stake – thus, for instance, we are free to adopt a deflationary notion of causality like the one defended by Lewis (1973) himself.

[27]Lewis' framework is indeed part of the so-called 'Canberra plan', a naturalistic philosophical framework (see Braddon-Mitchell and Nola (2008), Ch. 1).

Notice that this is a form of realiser functionalism, since the functionalised entity at the top is type-identified with its realiser at the bottom. This is actually a radical consequence of the account which should be stressed: on the ontological level, the Lewisian account leads us to maintain identity relations between the reduced and the reducing entity. When the entity belonging to the bottom level behaves in the right way, *that same entity* turns out to be the upper-level entity which is the target of the reduction, in virtue of the fact that it plays the role of the target entity.[28] However, notice that the step from functionalism about theory to functionalism about ontology is not automatic nor mandatory: *contra* Lewis, one is free to claim that theoretical terms should not be taken as designating actual physical entities. More on this in the next subsections.

This concludes the presentation of the core of Lewis' account. In section 4.2 we discuss two ways in which the Lewisian basis can be expanded to block possible objections. In section 4.3 we confront the framework with the case study introduced in section 2, and highlight the possible shortcomings of Syntactic Functional Reductionism.

4.2 Improving the Lewisian Account

The Lewisian account presented in the last section brings functional reductionism beyond the minimal functional reductionist thesis of section 3. It presents a formal way to spell out the notion of theoretical role (via Ramsey sentence), embeds a specific approach to reduction (Nagelian reduction), and shows a close link with a specific view about scientific theories (the syntactic one). In this subsection we present how the Lewisian core can be expanded to account for two specific issues, thereby improving the view. This improved version can be taken as the real basis of the Syntactic Functional Reductionist framework. In particular, we discuss here (1) the move from a Nagelian to a Neo-Nagelian model, and (2) the move to a more local version of functional reductionism.

The first and most natural step to take in expanding the base view concerns the implementation of Schaffner's model into the picture. As seen in section 3, according to Schaffner (1967) and Dizadji-Bahmani et al. (2010), Nagelian reduction should not directly concern the original reducing and reduced theories T* and T, but an approximate version of T. To solve the issue, we can implement their proposal into the Lewisian approach. That is, instead of functionally reducing the theoretical terms belonging to the target upper theory itself, we functionalise and then functionally reduce the terms belonging to a theory which *approximates* the original upper theory T – once provided a bottom theory which is an amended version of the original bottom-level one embedding certain auxiliary assumptions to facilitate the reduction. The result is thus a Neo-Nagelian version of the Lewisian base account. We consider this as a necessary step to save the consistency of the Lewisian account, and thus we take it for granted in the rest of the discussion about Syntactic Functional Reductionism.

Moving to the second point, we can draw an important distinction within the Syntactic Functional Reductionist framework concerning the difference between a global and

[28] Indeed, within the Lewisian account of mental states, realiser functionalism turns out to be just a peculiar version of the identity theory about mental and physical states.

a local version of the view. In fact, as mentioned in section 3, functional reduction can concern either whole theories or specific theoretical terms. In the present case, both the Lewisian basic account and the Neo-Nagelian version just introduced are primarily formulated in terms of global theory-to-theory reduction. However, they can easily be tuned into a more local form. In fact, once we have logically expressed the theory and derived the Ramsey sentence, we are actually free to functionalise either every theoretical term in the theory or just some of them. In this second case, we can provide functional definitions just for one or some 'problematic' terms, and perform a functional reduction only for them.[29] The passage from formal to material mode then goes as usual: once a specific term is functionally reduced, we can take the formal functional reduction as representing an ontological functional reduction in the actual world.

An important motivation to prefer the local version comes from Newman's objection.[30] That is, being committed to the Ramseyification of a whole theory exposes one to the objection according to which providing a set of entities which can realise the Ramsey sentence is really a trivial matter, because the Ramsey sentence (in this case) can at most constrain the cardinality of the set. One possible line of response to the challenge is to resort to one of the different strategies that have been proposed against Newman's objection itself (e.g. French and Saatsi (2006), Saunders and McKenzie (2014), Bueno and Meier (2019), Ladyman (2020)). However, it should be noted that some responses rely specifically on adopting the semantic view, and so they would likely not be available here. On the other hand, a simpler route is available, which is exactly to adopt the local version of the functional reductionist account, dissolving the problem from the outset. Given this advantage of the local version of the Syntactic Functional Reductionist approach, we take this as a natural update of the Lewisian basis, even though the more global version remains available.

To sum up, in the last two subsections we have introduced the Lewisian basis of the Syntactic approach to functional reductionism, and then presented two ways to improve the account: the move to Neo-Nagelian reduction and the passage to a local form of functional reduction. The next subsection confronts our case study with this framework.

4.3 Thermodynamics and Syntactic Functional Reductionism

Section 2 presented the reduction of the second law of thermodynamics via the functional reduction of thermodynamical entropy to Gibbs entropy. This is an example of a local functional reduction of the upper-level quantity S_{TD} to the lower-level quantity S_G. We employ here this case study to analyse Syntactic Functional Reductionism. We discuss the example both at the formal and the material levels, and raise three possible shortcomings of Syntactic Functional Reductionism: a translation issue, a challenge related to approximation, and finally an ontological problem.

Let's start from the theory-level. At the level of theoretical reduction, the aim is to functionalise the upper-level quantity S_{TD} and then to find out a bottom-level realiser

[29] This is arguably the approach adopted by Butterfield and Gomes (2020b), when discussing their case studies.

[30] See e.g. Demopoulos and Friedman (1985).

that can approximately play the relevant upper-level role in a given system. Within this framework, the functionalisation process is carried out first by the translation of the theory from mathematics to formal (second-order) logic, and then via the construction of the Ramsey sentence. In principle, this is a consistent project, but we face here two challenges. First, the whole translation process is not merely a challenging and complex task, but it could be taken to be complex in a futile or avoidable way. This aspect of the framework comes from its appeal to the syntactic approach to theories, but the translation passage could be seen more as an unnecessary attempt to shoehorn the mathematical formalism into the language of second-order logic, than as a genuinely useful step within the reduction process. Thus, an alternative functionalisation strategy that does not presuppose this passage would be preferable, other things being equal. Second, we have seen in the previous section that the higher-level theory we are meant to logically translate is not really thermodynamics, but rather an approximate version of thermodynamics, or another theory standing in a relation of strong analogy with it. Building such a theory is not a trivial task, and thus this adds an additional burden to the process of functional reduction, above the logical translation. In particular, since we are here dealing with theories as logically formulated sets of sentences, we cannot simply directly appeal to mathematical notions of approximation between models, but rather we have to rely on a syntactic-based form of approximation.[31] We shall see in section 5 that Semantic Functional Reductionism fares better than the Syntactic framework in both this latter respect and the previous one.

Moving now to the connection between theoretical functional reduction and ontology, a puzzle can be presented with respect to our case study, if one adopts a scientific realist attitude (as Lewis does). In fact, whereas S_{TD} can be interpreted as a property of an individual system, S_G is defined as a property of a probability distribution over possible micro-states, i.e. a property of an ensemble. In this sense, it is not clear if the step from formal to material is warranted. Even if we grant the success of functional reduction at the theoretical level, it is *prima facie* difficult to see how to translate the functional reduction from theoretical quantities to physical properties, since we are supposed to reduce a property of an individual system to a property of an ensemble, which looks more like a mathematical construct than a real physical property. The problem is exacerbated by the fact that the account entails type-identities between the reduced and the reducing quantities, which for Lewis reflect type-identities in the world. The puzzle is thus how a property of an individual system could be identical to an ensemble property.

This objection is specific to our particular case study,[32] but this problem is arguably symptomatic of a more general potential issue for the Syntactic Functional Reductionist framework. That is, the connection between theoretical functional reductionism and ontology is here very straightforward, but at the same time very strict. Functional roles are logically formulated using the Ramsey sentence and thus are expressed via

[31] Furthermore, if one wants to embrace scientific realism, in this case they would need to provide a notion of 'approximate truth', which is a notoriously difficult task. See Wallace (2021, pp. 8-9).

[32] For example, one could argue that when moving to the quantum domain, the canonical ensemble is represented by a density matrix and thus can represent a single quantum state.

logical predicates: x is that thing that plays a certain role, where playing a role is to satisfy certain predicates that connect that x with other kinds of theoretical terms in the network. If we adopt a scientific realist attitude, the way in which functionalism is connected with the world is very direct: the theoretical term, defined via the functional role, directly refers to the actual property that plays the roles represented by the theory. There is, allegedly, a 1:1 correspondence between theoretical terms and actual entities, which naturally matches the standard Quinean approach to the ontological commitments of theories.[33] The fact that in the case of thermodynamics this strict correspondence is challenged by the puzzle presented above is thus a reason to think that the formal-material link embedded in this framework is too strong.

In other words, the case study raises the following dilemma for Syntactic Functional Reductionism. On the one hand, one can reject scientific realism, thereby employing functional reductionism merely at the theoretical level. On the other hand, one can respond that theoretical functional reduction is a guide to ontological functional reduction, but maintain that we should not take the link as a straightforward entailment like the one pictured by Lewis. Holding up this second option would not be easy though. For instance, we need a reason why the functional reduction of a term entails a functional reduction in the world only in certain situations and not in others. And, in general, we would need a novel story about the theoretical-ontological link within the account, different from the Quinean approach which underlies the Lewisian picture.

5 Semantic Functional Reductionism

Semantic Functional Reductionism constitutes an alternative to the Syntactic framework in providing a model for functional reduction. It takes the general notion of theoretical functional reduction as introduced in section 3, and combines it with a model-based stance on scientific theories, and a structuralist conception of reduction as a relation between models, usually expressed mathematically. Reduction is so characterised in terms of the functional realisation of certain – mostly mathematical – roles in the upper-level theory's models by the theoretical elements in the lower-theory's models. This approach is arguably a (functionally-based) improved version of the structuralist account of reduction. Especially within physics, the framework takes the mathematical formalism in which theories are expressed at face value, and uses maths and mathematical models to specify the functional roles. Because of this, the view does not run into the issues raised previously against the Syntactic approach, since it (i) does not require logical translation of the mathematical formalism, (ii) accounts for approximation using a notion of approximation between models, and (iii) allows for a more flexible approach to ontology by replacing the Quinean stance on ontological commitment with a more flexible relation characterised in terms of representation between the models and the world.

[33]Cf. Quine (1948), Bricker (2016).

5.1 Introducing Semantic Functional Reductionism

The starting points of this framework are the semantic view of theories and the related account of structural reductionism. According to this approach, reduction obtains in virtue of relations of (approximate) instantiation between theoretical structures belonging to different models, defined at different levels. We start our presentation of Semantic Functional Reductionism by introducing in more detail Rosaler's structuralist account of reduction. One reason to pick this particular view is that it is arguably one of the most fully-fledged structuralist accounts of reduction available. Another reason is that, since we aim to show that the Semantic version of functional reductionism can be regarded as an improved version of the general structural reductionist view, the way in which Rosaler's account is formulated makes it particularly suitable for this goal.

Rosaler (2015, 2019) elaborates on the model-based view advanced by Ladyman et al. and Wallace, and proposes a notion of reduction as 'domain subsumption', where:

> Subsumption of the domain of some high-level description by a low-level description requires that any real behaviour that is accurately represented by the high-level description be represented more accurately and in at least as much detail by the low-level description. (Rosaler (2019), 272)

To formalise this notion he relies on model-model relations as follows:

> Theory T_h reduces$_T$ to theory T_l iff for every system K in the domain of T_h – that is, for every system K whose behavior is accurately represented by some model M_h of T_h – there exists a model M_l of T_l also representing K such that M_h reduces$_M$ to M_l (Rosaler (2015), 59),

where a low-level model reduces a high-level model if "the low-level model accounts for the success of the high-level model at tracking the behavior of the system in question". E.g. for dynamical systems reduction this boils down to finding mathematical mappings between the low-level state space and the high-level state space which approximately tracks the evolution in the high-level space from the evolution in the low-level one.[34]

For instance, to briefly present an example of structuralist-type reduction within this proposal, a semi-classical model for a point-particle system can be mathematically matched with a quantum model of the same system, under the right conditions. In fact, thanks to Ehrenfest theorem, it can be shown that we can derive Newton's law from the Schrödinger equation for the system, if the quantum state is highly localised in space. This means that, within the quantum mechanical model, the centre of the localised wavepacket has a trajectory in configuration space which is (to a very high approximation) identical to the trajectory in configuration space of a point particle of mass m within classical mechanics (in the Hamiltonian formulation). Thus, the trajectory of the wavepacket can be practically considered as a solution to the classical dynamic equation

[34] Rosaler (2019, 4.2.2) generalises this notion for non-dynamical systems as well. Notice that this approach fits particularly well within the version of the semantic view defended by Van Fraassen (1980).

for a classical particle, and we can draw a map between the quantum and the classical models defined over the respective state spaces.[35]

Taking stock of this, our aim now is to show how functional reductionism – formulated according to the Semantic Functional Reductionist framework – can build on and improve this view. To begin with, here is a statement of the Semantic Functional Reductionist thesis. In section 3 we defined theoretical functional reductionism as the view according to which a theory T can be reduced to another theory T* in virtue of the fact that theory T* embeds theoretical elements which can play the theoretical (formal) roles of the theoretical elements belonging to T. Interpreted in the Semantic Functional Reductionist way, to have theoretical functional reduction we just need to find lower-level mathematical structures, variables, or quantities playing the same theoretical roles (i.e. roles in the models) of upper-level mathematical structures, variables or quantities in the upper-level model. This is a mainly mathematically-couched way of expressing the idea that functional reduction proceeds by identifying the lower-level realiser for an upper-level role: in our case, the role is spelled out in mainly mathematical terms as a role in the models, and the realiser is a piece of mathematical structure.

Semantic Functional Reductionism thus shares Rosaler's structuralist notion of reduction as domain subsumption – which focuses on reduction as the recovery of upper-level real behaviour from the lower-level, and cashes this out in terms of model-model relations – and reformulates this in functional reductionist terms. A reason for advocating for functional reductionism here is that it gives a justification to Rosaler's structuralist approach, or at least it exposes an implicit assumption: the reason why, to reduce a classical model to a quantum model, all we need to do is providing a formal account of how the quantum model can represent the behaviour described by the classical model, is given by the functionalist thesis that 'being a classical system' just means to perform certain roles within the model of classical mechanics. Indeed, consider again the example of structural reduction state above. We have claimed that, for certain kinds of physical systems, i.e. highly localised systems, we can build mathematical mappings between the state spaces for the same system within the bottom and the upper theories' models. That account of reduction is structuralist in the sense that we provide an asymmetrical inter-levels link between the two models. But, it may be asked why this particular mapping ensures reduction, i.e. why the mapping provides a reason to believe that we can recover the classical system from the quantum one. Adopting a functional reductionist version of the structuralist approach to reduction provides us with the justification: the condition for being a classical system is to play a certain role in the classical models, and the mathematical mapping at stake shows exactly that the quantum system can indeed evolve like the classical one. It is in this sense that Semantic Functional Reductionism is a form of structuralist reduction which improves on the basic account.

[35]See Rosaler (2015, 63) for more details.

5.2 Thermodynamics and Semantic Functional Reductionism

We reconsider in this section the case study presented in section 2. In section 4.3 we analysed this case study with respect to Syntactic Functional Reductionism. We show here how the Semantic Functional Reductionist framework can accommodate the case study, in order to present in more detail how this approach works, and its differences from the Syntactic framework. In particular, we go through the same points that we have discusses in section 4.3 with respect to the Syntactic approach to functional reduction. We thus consider first how the account works at the formal level, and secondly we discuss the theoretical-ontological link within this framework.

Let's start with the formal side of the account. One stark contrast between a Semantic Functional Reductionist version of the case study and a Syntactic one concern the way in which the functional role itself is characterised within the account. With respect to the latter approach, functional reduction starts with the logical translation of the higher-level theory and the formulation of the S_{TD} theoretical role via Ramseyification in terms of formal logic. On the contrary, the core tenet of Semantic Functional Reductionism is to get rid of those steps. The privileged tool for representing scientific theories is maths, and we should just mainly stick with maths in specifying the functional roles. Thus, the approach of Semantic Functional Reductionism in dealing with our case study is to maintain that the functional roles we have to identify thermodynamic entropy with are just the mathematical roles that appear in section 2, i.e. thermodynamic entropy is that quantity which 'for thermally isolated systems, is increasing in non-quasi-static processes, but non-increasing in quasi-static processes', as formally represented by the equations of thermodynamics introduced earlier. In other words, thermodynamic entropy is that mathematical function that bears those mathematical relations within thermodynamic models of thermally isolated systems, in thus-and-so conditions. Therefore, any bottom-level piece of mathematical structure in the lower theory's models that can fulfil those mathematical relations is said to functionally realise – and reduce – the thermodynamic entropy. Of course, exact fulfilment is not required: approximate realisation of the mathematical role is enough, and that very approximation is mathematically expressed. In other words, we just need the bottom-level model to approximate the upper-level one at the mathematical level.

The upshot is that there is a crucial difference between the Syntactic and the Semantic frameworks considered here. First, there is no need for logical translation within the present account: as far as the mathematical presentation of the functional reduction carried out in section 2 is clear enough to show how reduction works, we should just take this at face value. That is, we can account here for the case study at stake simply by reading off functional reduction from the mathematical presentation provided by Robertson. This renders reduction comparatively easier to demonstrate. Second, the Semantic framework deals with approximation and approximate reduction in a straightforward way, directly exploiting the mathematical way of representing approximation which is employed by physicists. This distinguishes this framework from the Syntactic one, and makes it preferable to the latter, as it achieves the same goal in a simpler way.[36]

We move on now to the ontological aspects of the framework. We argue that the Semantic framework fares better than the Syntactic one in this respect, as it is more flexible. To recap, section 4.3 elaborated on the link between theoretical reduction and ontology within Syntactic Functional Reductionism. That account embeds a Quinean view concerning the ontological commitments of theories, which raises a potential problem when confronted with cases of functional reduction like the thermodynamics one. That account indeed predicates a direct correspondence between theoretical terms and entities, and also entails identity relations between the upper functionalised entity and the bottom entity realising its role, and thus arguably provides a too strict account of the ontological committments of functional reduction.

In contrast, Semantic Functional Reductionism allows for a more flexible approach. At the theoretical level, this framework adopts a model-based account of reduction, and frames functional reduction as a realisation relation between models: theoretical elements in the reducing theory's models, such as mathematically-formulated quantities, are taken to functionally realise certain patterns of behaviour described by the reduced theory's models. Given its reliance on models, the view is less restrictive concerning the relationship between theory and ontology, and concerning the ontological consequences of theoretical reduction, because models are merely required to *represent* the world. Adapting Wallace's (2021, 7) words to our context, we can stress that while, within the Syntactic framework, "The relations that a good theory's empirical statements have to the facts are those familiar from ordinary-language semantics: truth, reference, satisfaction", within Semantic Functional Reductionism "a theory makes contact with empirical data by modelling them. [...] The theory/world relation here is *representation*, more akin to the relation between map and territory than that between word and object". Thus, when we claim that e.g. Gibbs entropy plays the role of thermodynamic entropy, we are not committed to the claim that there's a specific property of a system denoted by thermodynamic entropy that is realised by a specific property denoted by Gibbs entropy and that they turn out to be ontologically identical properties – rather, the Semantic approach just claims that the physical systems represented by the models of statistical mechanics can be modelled accordingly to the thermodynamics models under the right conditions. Because of this, we are not committed to any specific view about the kind of relations between the physical systems – and thus we are not forced to endorse identity relations between them – and we are also free to be selectively realists about the type of entities our theories represents. In this way, we have an account which is both flexible enough to let us decide case by case, and which does so for a principled reason, and thus does not run into a dilemma like the one we raised against Syntactic Functional Reductionism.

[36]Furthermore, if we endorse scientific realism, we don't face any problem related to the fact that the models are just approximate: since the relation between models and world is just one of representation, we don't need to deploy a notion of *approximate truth* (see ft. 31).

6 Conclusion

	Syntactic Funct. Red.	**Semantic Funct. Red.**
Theories	Syntactic	Semantic
Theor. Reduction	Nagelian: Logical derivation	Structural: Model-model
Functional Roles	Logically formulated	Mainly math. formulated
Scope	Local or Global	Primarily Local

Functional reductionism is a candidate account for scientific reduction, that provides an alternative to more standard approaches like Nagelian and structuralist reductionism, and that has recently been fruitfully applied to theories like thermodynamics and general relativity. Its potential value is demonstrated by those case studies, but the view is still underdeveloped in several respects. This paper offers a thorough analysis of this approach and develops two kinds of frameworks that provide two fully-fledged alternative models of functional reductionism, that improve and clarify the basis of the view. Syntactic Functional Reductionism models the account as an improved and *sui generis* form of Nagelian reduction, cast within the syntactic approach to theories, where functional roles are logically formulated. Semantic Functional Reductionism provides a functionalist and upgraded version of structuralist reduction, where scientific theories are based on models and functional roles are accordingly (mostly) mathematically formulated.

Discussing the case study of thermodynamics has allowed for a more detailed presentation of these frameworks, and for a confrontation between them. We have seen how the Syntactic approach provides a rigorous framework which however faces potential challenges concerning the need for a logical translation of the mathematical formalism and the link between theoretical reduction and ontology. On the other hand, the Semantic approach accommodated in a simpler way the case study, providing a more mathematically-oriented framework, in which the relation between theory and ontology is more relaxed and mediated by models.

The paper thus brings the functional reductionist approach to theoretical reduction to a higher level of clarity, and provides a more complete picture of how this account works with respect to both theoretical and ontological reduction, thereby contributing to making functional reduction a viable account for reduction.

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