If the global phase is real

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Abstract

The ontological meaning of the wave function has been debated for many years. A widely-discussed view is wave function realism, according to which the wave function represents a real physical field in a fundamental high-dimensional space. In this paper, I argue that wave function realism is inconsistent with the reality of the global phase. In other words, the global phase cannot be real in wave function realism, and if the global phase is real, then wave function realism will be not true. Moreover, if the global phase is real, then the wave function of an N-body system cannot represent a single physical entity, either in a high-dimensional space or in three-dimensional space, but represent N physical entities in three-dimensional space.

Recently there have been discussions about the reality of the global phase in quantum mechanics (Schroeren, 2022; Gao, 2022a; Wallace; 2022). I have also given a very short proof that the global phase is real (Gao, 2022b). In this paper, I will analyze possible implications of the reality of the global phase for the ontological meaning of the wave function.

In quantum mechanics, each isolated system at an instant can be assigned to a wave function. According to the psi-ontic view, the system has a well-defined physical state,[1] and its wave function directly represent the physical state. The question is: what physical state does the wave function

[1] I think a realist such as wave function realist cannot reject the realist assumption that an isolated system such as an isolated particle has a well-defined physical state. An isolated system by definition is independent of other systems and should have its own intrinsic properties or physical state for a realist view. If an isolated system does not have a physical state, then what system can have a physical state? Here one may admit only the whole physical state of all isolated systems as a whole. But this view will lead to a strange nonlocal effect, namely that a local unitary transformation applied to one isolated system will have an instantaneous nonlocal influence on all other isolated systems, no matter how far away they are. This nonlocal effect can hardly be explained.

represent? Is it a physical field in a high-dimensional space? or a multi-field in three-dimensional space? or something else? The global phase, if it is real, will help answer this question.

Suppose there are two isolated particles 1 and 2 being in a product state $\psi(\mathbf{r_1}) \otimes \varphi(\mathbf{r_2})$ at a given instant, where $\psi(\mathbf{r_1})$ and $\varphi(\mathbf{r_2})$ are two spatially separated (nomalized) wave functions in three-dimensional space. A local unitary transformation can be applied to add a global phase to the wave function of each particle. Consider two possible situations. One is that a local unitary transformation is applied in the region of $\psi(\mathbf{r_1})$, which adds a global phase θ to $\psi(\mathbf{r_1})$, and the state of the two particles becomes $e^{i\theta}\psi(\mathbf{r_1}) \otimes \varphi(\mathbf{r_2})$. The other is that a local unitary transformation is applied in the region of $\varphi(\mathbf{r_2})$, which adds a global phase θ to $\varphi(\mathbf{r_2})$, and the state of the two particles becomes $\psi(\mathbf{r_1}) \otimes e^{i\theta}\varphi(\mathbf{r_2})$.

If the global phase is real, then the local unitary transformation that adds a global phase to the wave function of each particle changes the physical state of the particle. Moreover, the Schrödinger equation ensures that a local unitary transformation applied to one isolated particle does not change the wave function of another isolated particle and its physical state (represented by its wave function).[2] Then, the changed physical states of the two particles in the above two situations, which are represented by the wave functions $e^{i\theta}\psi(\mathbf{r_1}) \otimes \varphi(\mathbf{r_2})$ and $\psi(\mathbf{r_1}) \otimes e^{i\theta}\varphi(\mathbf{r_2})$, will be different. In other words, if the global phase is real, the two wave functions $e^{i\theta}\psi(\mathbf{r_1}) \otimes \varphi(\mathbf{r_2})$ and $\psi(\mathbf{r_1}) \otimes e^{i\theta}\varphi(\mathbf{r_2})$ will represent different physical states.

Let us see if wave function realism is consistent with this result. According to wave function realism (Albert, 1996, 2013; Ney and Albert, 2013; Ney, 2021), the wave function represents a real physical field in a fundamental high-dimensional space. For the above two-particle system, its wave function represents a physical field in a fundamental six-dimensional space. This means that the two wave functions $e^{i\theta}\psi(\mathbf{r_1}) \otimes \varphi(\mathbf{r_2})$ and $\psi(\mathbf{r_1}) \otimes e^{i\theta}\varphi(\mathbf{r_2})$ represent the same physical field. Thus, wave function realism is not consistent with the reality of the global phase. In other words, the global phase cannot be real in wave function realism (cf. Albert, 1996, 2013), and if the global phase is real, then wave function realism will be not true.

It can be seen that the above analysis applies to all ontological interpretations of the wave function which regards the wave function as a representation of a single physical entity such as wave function realism and the multi-field interpretation of the wave function (Hubert and Romano, 2018). According to these interpretations, the two wave functions $e^{i\theta}\psi(\mathbf{r_1}) \otimes \varphi(\mathbf{r_2})$ and $\psi(\mathbf{r_1}) \otimes e^{i\theta}\varphi(\mathbf{r_2})$ represent the same physical entity, while according to the above analysis, this is inconsistent with the reality of the global phase.

[2]There may also exist other hidden variables besides the wave function, and they may change or not change under the interaction. In this paper, the psi-ontic view is assumed, and the physical state denotes the part of the physical state which is represented by the wave function.

In the final analysis, the key point is that the global phase of the product state of isolated systems does not uniquely determine the global phase of the wave function of each system. For example, there are infinitely many product states of two isolated particles whose global phases are θ, and three of them are $e^{i\theta}\psi(\mathbf{r_1}) \otimes \varphi(\mathbf{r_2})$, $\psi(\mathbf{r_1}) \otimes e^{i\theta}\varphi(\mathbf{r_2})$ and $e^{i\theta/2}\psi(\mathbf{r_1}) \otimes e^{i\theta/2}\varphi(\mathbf{r_2})$. When the global phase is not real as usually thought, this poses no problem. But when the global phase is real, this will pose serious issues for the single-physical-entity interpretations of the wave function. In this case, the global phase of the product state does not represent a unique physical state as required by the single-physical-entity interpretations of the wave function; rather, it may represent many different physical states, depending on what the global phase of each isolated system is.

A positive result of the above analysis is that if the global phase is real, then the wave function of an N-body system cannot represent a single physical entity, either in a high-dimensional space or in three-dimensional space, but represent N physical entities in three-dimensional space. It can be further argued that these physical entities cannot be fields but particles, and their motion is random and discontinuous (Gao, 2017, chap.7).

To sum up, I have argued that the single-physical-entity interpretations of the wave function such as wave function realism are inconsistent with the reality of the global phase. If the global phase is not real, then this result will be less interesting. But if the global phase is indeed real, this result may be important enough for determining the ontological meaning of the wave function.

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