A reductive account of “before”: deriving temporal succession

Abstract:

The aim of this paper is to propose a new, reductive account of the relation “before” between events. Rather than basing itself on causal asymmetry or thermodynamics, it instead takes the set of states of an object and exploits a characteristic asymmetry which can be found by quantifying, timelessly, over this set. It is first shown that this asymmetry is sufficient for the relation “before or simultaneous with”, and moreover, that no ordering in terms of “before” can be ascertained independently of this asymmetry appearing on at least a subset of the set of states of some object. Based on this, plus additional steps, a definition of “before” is developed which does not circularly employ temporal notions. The result is an account of “before” which not only saves the phenomena of temporal succession as experienced in everyday life, but is also consonant with physics in that “before” is a local relation which requires no global hypersurfaces of simultaneity, and moreover an emergent one which applies only to the relatively macroscopic world, not an intrinsic feature of the quantum level.

Keywords: before and after, local time, relativity theory, temporal succession, time asymmetry.

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1. Introduction

Gerald J. Whitrow, in his *Natural Philosophy of Time*, noted that the causal theorists’ attempts to define temporal succession—the ordering of events or states of affairs in terms of “before” and “after”—failed because they tacitly invoked the very temporal order to be defined. This, as Whitrow shows, is also true of one of the most ingenious attempts in this direction, Hans Reichenbach’s mark method. Whitrow concluded from this that “before” and “after” must be accepted as “elementary” concepts (Whitrow [1980, 323-327]). It is probably fair to say that the problem of giving a reductive account of “before” remains unsolved to this day. The aim of this paper is to fill this gap by providing a derivation of the relation “before” in terms which do not presuppose temporal succession, in order to then give a reductive, non-circular definition of this relation. Though the account which I will offer employs causal notions, it differs from that favoured by many causal theorists (e.g. Mellor [1998, 105-117]; Frisch [2013]) in that it does not construct “before” on causal order, dependence, or asymmetry—the ground of “before” is not the cause-effect relation. Rather, the key concepts will be that of the set of states of a physical object, and of a characteristic asymmetry which appears on this set.

This move is motivated by two observations: First, there is a need to account for “before” in a way which does without a unique, global and total order between events, since this would conflict with relativity of simultaneity. An object-centred approach can do this because it yields before orders—by which I mean orders in terms of the relation “before”—which are essentially local. Second, the main other contenders for accounting for “before” appear to run into dead ends. Briefly put: Rooting “before” in an object-independent, absolute and global passage of time leads not only to a conflict with spacetime physics, but also, as is well known, to the contradiction pointed out by McTaggart ([1993]). Nor does comparison of events or states of affairs in terms of any of their intrinsic properties seem to yield the needed asymmetry: if you see two flashes of light, their before order is independent of, say, their colour or comparative luminosity. Nor, again, do these two events need to stand in a relation of causal dependence to each other in order to succeed each other in time. Large-scale content asymmetries in the universe, such as increase in entropy, which is believed by many physicists to give time its direction, arguably only yield, to use Huw Price’s words, “asymmetries of things *in* time” ([1996], 17), but not the order *of* time (cf. also Dainton [2010], 47-50; Callender [2017], 244). It is of course beyond the scope of this article to assess whether any of these accounts could still be saved in some way.

2. Deriving “before”

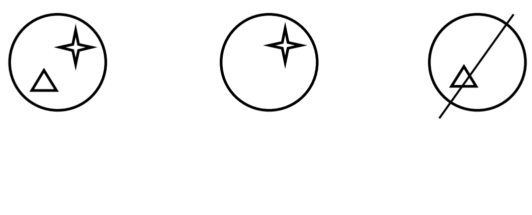
In any case, the derivation I propose starts by considering physical objects, i.e. relatively macroscopic things with “classical” identity conditions, such as, say, a rock, a bacterium, a galaxy, as opposed to, for example, a subatomic particle or a state vector (cf. Lowe [2003]). In the following, I will talk simply of “objects”.

Def. 1: For any object X, the set of its actually realized states, as opposed to merely possible ones, will be called M_X.

For example, a particular beech leaf may have a yellow and a green state, Cicero a state with knowledge of Greek, but none with knowledge of Japanese, and so on. The notion of such a set presupposes no ordering, so that we can conceptualize it like a pack of cards shuffled in random order.

Def. 2: For any object X, there is an existence asymmetry (henceforth: *EA*) on M_X if and only if there are two real intrinsic properties p and q such that there is a state of X with p, and another with p and q, but none with q and without p. Such an *EA* will be called a “pq-asymmetry” (note that the order of the variables matters), and the proposition “there is a pq-asymmetry on M_X” will be written as E_Xpq.

The *EA* is illustrated below, where the circle represents an object, the star and the triangle each symbolize a real intrinsic property of it, and the slash means “does not exist”:



For example, a particular star may have a state in which it contains helium and carbon, another with helium and without carbon, and none with carbon but no helium. Obviously, the order in which the three states are drawn does not matter, that is, given any order, a ✧△-asymmetry is satisfied. The *EA* is related to the familiar relation "before" between events or states of affairs, as known from everyday experience, in a complex way, as will spelt out in what follows.

Let *bs* mean "before or simultaneous(ly) with". Also, for any object *X* and real intrinsic properties *p* and *q* in *X*, let ***p*** denote the event "property *p* is produced in *X*", and similarly for ***q***. Then:

Theorem 1: If E_Xpq, then ***p*** is *bs* ***q***.

Proof: This is very easy to see by considering the states of the object, call it *R*, in the above illustration: Suppose the property symbolized by the triangle were produced in *R* before that symbolized by the star. Then, there would be a state (△), contradicting the assumed *EA*. The only alternative is that the star is produced in *R bs* the triangle. □

It is important to understand that theorem 1 yields an order of events, not of states. To illustrate: Suppose you have spilled wax from a candle and orange juice onto a table cloth, but you cannot remember in what order. Luckily, a friend has photographed all states of the table cloth, and hands the photos to you shuffled in random order. Going through them, you find that there is a state of the table cloth with just the juice (call it *Tj*), a state with wax and juice (*Twj*), and no state with just wax (*Tw*). Can you conclude that *Tj* must have been there before *Twj*? This is clearly a possibility, but you cannot be sure: you could also have spilled juice and wax simultaneously and then cleaned away the wax, so that *Tj* post-exists *Twj*. Theorem 1 does not state that *Tj* must pre-exist *Twj*. What is impossible, however, is that you spilled the wax (event ***w***) before the juice (event ***j***): if you had, there would be a state *Tw*. You can therefore conclude that ***j*** occurred either before, or simultaneously with ***w***. Hence, to arrive at theorem 1, we do not need to tacitly assume that properties, once produced, stay forever (which would render the account circular), nor do we need to make any assumptions as to whether or not *j* or *w* are ever removed from *T*, and if so in what order. Also, no assumptions about causality or causal asymmetry are made: the same argument would hold if, *per absurdum*, *j* and *w* had appeared on *T* acausally. Now, the definition of the *EA* is itself atemporal: it makes no reference to the temporal concepts "before" or "*bs*". Thus, we now have an atemporal sufficient condition for the temporal concept "*bs*".

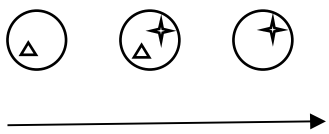
Corollary 1: Let $M(p,q)_X$ be the subset of M_X containing only and all the states of *X* with *p* or *q* (note that the order of the variables doesn't matter: $M(p,q)_X = M(q,p)_X$). Then, if the *pq*-asymmetry appears on this subset, ***p*** once again is *bs* ***q***.

The proof is identical with that given for theorem 1.

Note also that, not only does the *pq*-asymmetry on M_X imply the same asymmetry on $M(p,q)_X$, but also conversely, because the states in M_X which are not in $M(p,q)_X$ clearly have no bearing on whether the *pq*-asymmetry is on M_X. The two *EA*s are therefore equivalent, so that we can write E_Xpq to assert both of them.

Lemma 1: For any object *X*, if there is a *pq*-asymmetry on any proper subset of $M(p,q)_X$, this is not sufficient for ***p*** being *bs* ***q***.

Proof: Let *R* be the object below, and ✦, △ two properties in it. Consider the succession



where the arrow symbolizes the familiar forward direction of time. Then, there are subsets of $M(✦,△)_R$, namely the subset {(△✦),(✦)}, in addition to all other subsets obtainable from it by adding elements of $M(✦,△)_R$ other than (△), on which there is a ✦△-asymmetry. Yet **△** is by assumption before **✦** (where the boldface symbols again indicate events). □

The configuration

can be called an "existence symmetry" on the set of *R*'s states. We thus see that the situation can arise that, for some *X*, a set $M(p,q)_X$ exhibits an existence symmetry in *p* and *q*, and a *pq*-asymmetry can then be obtained by restricting this set to one of its subsets. Since such asymmetries do not imply ones on $M(p,q)_X$ they are not sufficient for ***p*** *bs* ***q***. Hence, we need to quantify over at least $M(p,q)_X$ in order for the asymmetry to be sufficient for the relation *bs*.

Premise 1: For any *X*, if M_X contains more than one state, then given any two states, the difference between them is due to a causal interaction of *X* either with its environment, or among its parts. Here, it is assumed that the combination of several causal interactions is itself a causal interaction.

It is beyond the scope of this paper to justify this premise adequately. Suffice it to point out that it is an extremely basic, and also plausible assumption which we constantly make in both everyday life and science about the world of relatively macroscopic objects in which, after all, the present account is set (cf. Illari & Russo [2014]). What matters here is that the events being ordered (symbolized by variables such as ***p***, ***q***) are causal interactions, so that their corresponding properties (*p*, *q*) in a given object can be viewed as records (just as the spot of wax is a record of the spilling event). Accordingly, objects—indeed, *all* objects, in the sense given at the beginning of this paper—are, at least potentially, recorders of events, and any two different states of a given object are distinguished from one another in virtue of records of events. Here, it is immaterial whether we think only of short, "point-like" events such as a collision, or also of extended ones such as corrosion of a metal, which can after all be decomposed into their sub-events. Note that, given premise 1, causal interactions are necessary for the orderings by the relation *bs* to get off the ground, but what constitutes these orderings is the *EA* on M_X, not a relation of causal asymmetry or dependence between the events.

Orderings of events can now be established in two ways: 1. By considering events directly involving the object in question, such as “light from supernova *S* hits object *R*”. 2. By combining the ordering obtained in (1) with information about spatial distances of events occurring in the object’s surroundings, such as “supernova *S* occurs one billion light years away from *R*”. In the first case, all the events are on the worldline of object *R*, and hence timelike-separated, yielding an invariant, local ordering. In the second case, some of the events will be spacelike-separated, yielding a non-invariant ordering, established in terms of *R*’s frame of reference, of events spread out in space.

Lemma 2: For any object *X* and real intrinsic properties *p* and *q*, $E_X pq$ is not necessary for ***p***’s being *bs* ***q***.

Proof: This too is straightforward: For example, it could be that property △ appears and then disappears on our object, whereupon ✧ appears. Then, we have no state (△✧). Or again, △ could appear simultaneously with ✧, and then both disappear, so that we have only state (△✧). In both cases, there is a *bs*-order between ✦ and ▲ without there being an *EA*. □

Now, one recorder can causally interact with another, so that, in particular, one recorder can have records of the states of another, and hence also, records of the records of another. For example, craters on the moon are records of events involving it, and a screen can function as a recorder of the moon with these records. Given this, if an *EA* fails to hold for one recorder *X*, it is possible for a corresponding *EA* to hold for another recorder *Y*. But how can we be certain that such an *ersatz-EA* gives us the right order? To answer this, it must first be borne in mind that the order of the events involving *X* is a relativistic invariant—they are all on the worldline of *X*—hence, their order cannot be reversed in *Y*’s frame of reference. Given this, two cases must be distinguished:

Case 1: In a special relativistic context and where the influence travelling between the two objects is electromagnetic radiation in a vacuum, the invariance of the order of the *X*-events also implies that the reverse order cannot be recorded locally by *Y*—that is, *Y* cannot be hit by light rays in the reverse order.[1] Hence, it is then impossible for quantification over M_Y to yield an *EA* compatible with the reverse order of *X*’s states. Such *ersatz-EA*s are therefore order-preserving.

Case 2: Where, on the other hand, there is refraction, or gravitational lensing, or influences other than electromagnetic radiation are involved, e.g. sound waves, things get more messy: one or both signals from *X* can get slowed down, so that a signal can overtake another, in which case the *ersatz-EA* are not guaranteed to be order-preserving. In this case, the correct order of the *X*-events can be found by combining the order established by quantifying over the

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[1] To see this, consider two objects *A* and *B*. Suppose that *B* changes from green to red, and that, as it does so, it rushes towards *A* at a very high speed *v*. Let *x* be the distance between *A* and *B*, according to *A’s* frame of reference, when the process of change starts and when *B* sends off a message “*B* is green”. Let $\Delta\tau$ be the proper time for the process of change as measured along *B*’s worldline, and let Δt be the time for this process according to *A*’s frame of reference. As is known from special relativity, if $\Delta\tau$ is positive, so is Δt. Let $t=0$, according to *A*’s frame, when the message “*B* is green” is sent off from *B*. This message arrives at *A* at time $t_{green} = x/c$. *B* is at a distance of $(x - v\cdot\Delta t)$, again according to *A*’s frame, when the message “*B* is red” is sent off. This message arrives at *A* at $t_{red} = \Delta t + (x - v\cdot\Delta t)/c$. Thus, $t_{red} - t_{green} = (1 - v/c)\cdot\Delta t$. This interval can be an arbitrarily small fraction of Δt, but it can never be negative, and so *A* receives the two signals in the right order.

set of states of an object *Y* with the necessary corrections for the signals' respective speeds. Thus, we can replace *Y* by a notional object *Z* whose states—and corresponding *EA*s—are those which *Y* would have under case 1 conditions. Some may object that such notional objects should not be allowed, but let it be pointed out that, for example, frames of reference—without which spacetime physics as we know it would not be possible—are likewise notional objects, not physical ones.

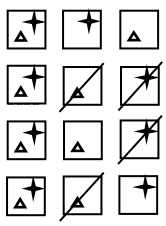
In practice, if transmission is through electromagnetic radiation, it can probably be assumed that the vast majority of cases from both everyday life and science can be considered, to a sufficient approximation, to belong to case 1. This is because the indices of refraction in the earth's atmosphere are close to unity—light is only slightly slowed down—and matter in the universe is distributed in a sufficiently scarce and homogenous way to make overtaking of light rays unlikely. Thus, though the possibility of case 2 must be borne in mind, most *ersatz-EA*s are reliable in the optical case.

Theorem 2: For any object *X*, any two events ***p*** and ***q*** involving *X*, and their records *p* and *q* in *X*: in terms of configurations of states of objects, the order "***p*** before ***q***" is definable only if there is a *Z* such that, on some subset of M_Z, there is *p'q'*-asymmetry, where *p'* and *q'* are records of *p* and *q*, respectively, in *Z*. Here, *Z* is: a. either, *X* itself, in which case *p'* is *p*, and *q'* is *q*; b. or, if the *EA* fails to hold for *X* and case 1 is approximated sufficiently well, any other object *Y*; c. or, in case 2, a notional object based on a physical object *Y*, together with the necessary corrections.

Proof: Suppose that, for some object *A*, a record ✦ appears before a record △, where "before" once again has its everyday meaning, and no ✦△-asymmetry holds for $M_{A.}$ Let the circle symbolize *A* and the square some other object *B*—physical in case 1, notional in case 2—so that, for any record ◇ in *A*, if a record of ◈ or just ◇ is in *B*, this will be written simply as [◇]. All we need to do is check through the possible configurations on subsets—proper or improper ones—of M_B :

Clearly, no before order between the two events in question is definable through subsets of M_B which have no states with ✦ or △, or which have only one of the states [✦] or [△]. Also, subsets having both these states, but not [△✦], provide no asymmetry through which the relation "before" could be defined. This leaves only subsets with [△✦], giving four possibilities:

a.
b.
c.
d.



Configurations (a) and (b) are clearly symmetric and cannot be used to define an asymmetric relation. (c), on M_B(✦,△) and its extensions, is sufficient for △ being before or simultaneous with ✦, and so cannot be necessary for the order assumed initially. (d), given the subsets just mentioned, is sufficient for ✦ being *bs* △, which in turn is a necessary condition for ✦ being strictly before △ . But (d) is simply the configuration of *B*'s states which corresponds to the missing ✦△-asymmetry on $M_{A.}$. □

In other words, an *EA* on a subset of the *M*-set of an object, or a corresponding *EA* on that of another object—again, physical in case 1, notional in case 2—is the only configuration which can be used for establishing a before order by quantifying over sets of states of objects. Though it is true that, in case 2, a *pq*-asymmetry in *X can* be replaced by a different configuration on the set of *Y*'s states, the corresponding *p'q'*-asymmetry must then hold for *Y*'s replacement *Z*. Now, it seems that such quantification over states of objects is, at least given the current state of the discussion, the only candidate available for a reductive definition of "before": as argued briefly in the introduction, no asymmetry grounded in properties of the events themselves, nor an asymmetric causal relation between them, nor rise in entropy are necessary conditions for one event being before another, and an absolute passage of time as the foundation for temporal precedence incurs grave difficulties in both logic and physics.

It could be objected that it is intuitively clearly conceivable that one event is before another without there being an *EA* on the *M*-set of any recorder, or any subset thereof, so that such an asymmetry does not deliver a necessary condition either. This objection seems very plausible. After all, it was shown in lemma 2 that, at any rate, an *EA* for a *given* recorder is not necessary for a before order. Nevertheless, the objection backfires upon closer analysis. To see why, consider, for example, a flash of blue light before one of green light: Conceiving of the order "blue before green" is possible only by either experiencing or imagining an experience of this succession. But such experience requires that, for some subset of the experiencer's states, the following conditions are satisfied:

a. There is a state with only a record of the blue light, and not the green one. Otherwise, the judgment that there is a situation where blue is "already" there, and green "not yet", is impossible. But absent "already" and "not yet", there is nothing to ground the relation "before" between events—a point made already by McTaggart who notes that there cannot be a *B* series without an *A* series ([1993], 26-27). b. There is a state with both records together. For if there are only states with each record in isolation, the experiencer can only ever be aware of one of them, and has no means of comparing the two, as occurs in living beings with defective memory. c. There is no state of the experiencer with a record of the green light alone. For given the assumed order, such a state is possible only if they have forgotten about the blue light, in which case they cannot ascertain any order between the two flashes.

But of course, conditions (a), (b), and (c) constitute an *EA* on a subset of the set of the experiencer's states. Hence, to make the above objection, the objector must tacitly presuppose the very *EA* they deemed unnecessary. In other words, we can indeed imagine a before order without an *EA*. But the reason for this, I submit, is not that "before" is primitive, and the *EA* only a derivative notion. Rather, it is because the *EA* necessary for conceiving of a before order is not explicitly analysed and recognized as such, giving to the mind the impression that "before" is a basic notion which cannot be explained in terms of anything else.

The order just considered is, of course, one of local impact events, in this case of light flashes hitting the experiencer. Obviously, the order of the flashes being emitted may not be invariant, but their order in any frame of reference can only be calculated given the order of the local impact events, which can only be established given an *EA*, together with information about the distances of the flashes' emitters. Thus, both in local impacts and in remote emissions, not only can no before order be experienced without an *EA* but, what is more, no such order is even conceivable independently of it. Given these considerations and theorem 2 we can thus write:

Thesis 1: For any object *X*, any two events ***p*** and ***q*** involving *X*, and their records *p* and *q* in *X*: ***p*** is before ***q*** only if there is a *Z* such that, on some subset of M_Z, there is *p'q'*-asymmetry, where *p'* and *q'* are records of *p* and *q*, respectively, in *Z*, and *Z* is an object as described in theorem 2.

We thus see that, even though for a *given* object an *EA* is not necessary for there to be a before order, an *EA* on a subset of the set of states of *some* object is necessary.

The *EA* plays, apparently, no role in the dominant camps in contemporary philosophy of time. Given this, many readers will think that thesis 1 cannot be true and that "before" cannot possibly be object-dependent in the way I have described. But the fact that the *EA*—which, recall, is definable independently of any before order—is sufficient for orderings by the relation *bs* (theorem 1) and arguably also necessary for the very concept of "before" shows that this concept and the *EA* are, at the very least, intimately related. Thus, the connection between the two should, in my view, no longer be ignored if an adequate understanding of temporal succession is to be reached.

However, before proposing a reductive definition of "before", two problems remain to be solved: 1. We have a necessary condition for "before", but a sufficient one only for *bs*. 2. The sufficient condition given in theorem 1 requires, by lemma 1, quantifying over at least $M(p,q)_X$. But we can confidently ascertain the before order between two events without doing this: for example, having seen a blue light followed by a green light, we do not need an overview of all our states, in particular future ones, which have a record of either of them. Rather, we can tell the before order immediately. Thus, some proper subset of $M(p,q)_X$ seems to be sufficient after all.

The answer to both difficulties can be given along the following lines: By premise 1, if any two different states of an object exist, this is due to a record of a causal interaction (where, recall, the combination of several causal interactions is itself a causal interaction). But the converse is not true: not any causal interaction or combination thereof necessarily leads to two different states, since there can also be two or more such interactions whose records yield a state no different from one of an object to which nothing occurred, that is, they cancel each other out. Examples of this are familiar: a particular material can be added to an object and removed from it (whatever the time order), a piece of iron magnetized by two opposing magnetic fields ***B*** and –***B***, and so on. Now, the occurrence of such a combination of interactions is a state of affairs really distinct from the simple absence of a record-producing interaction, even if the resulting states are not different in terms of their intrinsic properties—they may be perfectly indiscernible, although of course they need not be. I say "really distinct" because the world, as "totality of facts" (to borrow Wittgenstein's words [1995], 1.1) is clearly different if such a combination occurs than if nothing does: for example, treating the piece of iron with ***B*** and -***B*** (again, whatever the order) is different from doing nothing to it.

Now, let ◇ be a variable denoting the record of a particular causal interaction in an object, denoted again by the circle. Then, three states are possible with respect to this particular interaction:

, , and ,

where ◆ is a record cancelling ◇. Note that the distinction between these three states is independent of any assumptions about temporal order. In particular, the two states ○ and ◆⃝ must, given the above, be distinguished, although they may be intrinsically no different from each other. Furthermore, no before order between ◇ and ◆ is presupposed: all that is required is that the two combine in the way described, independently of which of the two, if any, is in the object before the other. ◇ and ◆ are, after all, only variables, so that, by interchanging them, we can rewrite the three states above as ○, ◆⃝, ◆⃝. Such combination of interactions can thus be likened to the way in which the composition of a function *f* with its inverse f^{-1} acts on a given argument. For a given object *X*, the set of all of its states, where we now distinguish between ○ and ◆⃝ for any record ◇, will be called $\mathcal{M}_X$, rather than M_X, for which said distinction was not made. It is not assumed here that any record ◇ has an inverse counterpart ◆, but only that at least some do, nor that such a counterpart is unique. Nor, clearly, do all three states ○, ◇⃝, and ◆⃝, or indeed any given one of them, need to be in $\mathcal{M}_X$, though each state in $\mathcal{M}_X$ must with respect to ◇ be one of these three, as has been said.

A further piece of terminology: For some event ◇ and some state in $\mathcal{M}_X$, the record ◇ may or may not be in that state. If it is not, we will say that the "null record of ◇" is in the state. Any two null records of two causal interactions are identical, since they correspond simply to the absence of these interactions. But given this, it is clear that the null record is in any state on $\mathcal{M}_X$. Some states of the moon, for example, have no footprint of Neil Armstrong, all of them have no footprint of Cicero, and each of these non-records is the null record. In these respects, the null record is similar to the empty set, which is a subset of any set *S* whatsoever because there is always another set *T* whose intersection with *S* has no elements, and any two empty sets obtained in this way are identical. Also, the non-occurrence of any event will be called, along the same lines, the "null event".

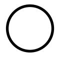
In addition, we may assume:

Premise 2: For any object *X*, if a before order exists on some subset of M_X or $\mathcal{M}_X$, then any two discernible states cannot have same place in this before order.

This premise should not be confused with its converse: on M_X, at any rate, two indiscernible states need not have the same place in the before order. The premise, as far as I can see, is not capable of strict proof, and thus has to be introduced axiomatically. But this does not stand in the way of a reductive account of "before": the premise does not assume that a before order exists on a given set, and if it does, it makes no assumptions as to what the before order between any two given states is.

Now, consider $\mathcal{M}_X$. Is there a before order on it? With respect to any given record ◇, each state is either ◆⃝ or ◇⃝ or ○, which can be written respectively as

where ○ is the null record. If all these three states exist, then by definition 2, there is now a ◇◆ -asymmetry and a ○◇-asymmetry on $\mathcal{M}_X$, and if only two exist, there are likewise *EA*s on any one of the three possible combinations of two, as is easily seen. Supposing that all three exist, applying theorem 1 yields that there is a null event *bs* the◇-event *bs* the◆-event. We can apply this order also to the states of the object and write:

This last step is possible precisely because of the distinction between  and . It could not have been made for M_X where, since this distinction was lacking, the order of events could not be applied to the order of states, as argued under theorem 1. Thus, $\mathcal{M}_X$, with respect to ◇, is totally ordered by the relation *bs*. Now, by premise 2, no two different states can have the same place in the before order. Hence, we can replace *bs* by "before" in its strict sense and write (omitting the null record):

This allows us to introduce three corresponding levels of the before order with respect to ◇, which we can call level ◇0, ◇1, ◇2. No given one of these levels needs to be occupied, but each state in $\mathcal{M}_X$ must be on one of these levels. Now, pick one of the levels, e.g. level 1. Its members constitute the set of all -states. If this set is non-empty, we can now check whether there are *EA*s with respect to any other two records ✧, △ on it. ✧ and △ may also be inverse records of each other (like ◇ and ◆) but it does not matter whether they are: it matters only whether there is an *EA* between them. If there is, this *EA* now constitutes a before order with respect to ✧, △ on level 1. In order to be confident about this order, it is not necessary to check levels ◇0 and ◇2 and see whether there are also states with ✧ or △ in them, because it has already been established through quantification over $\mathcal{M}_X$ with respect to ◇ that the states in level ◇0 are before those in level ◇1, and those in level ◇1 before those in level ◇2. We continue in this way, successively ordering each level so obtained. Since for any record, each state is on level 0, 1, or 2 with respect to it, all of $\mathcal{M}_X$ can in this way be ordered through until any two states stand in a before order. Also, it does not matter with which of the records we start this process of nested ordering: above, we started with ◇, but of course, ◇ is just a variable which can signify any given one of the records in any of the states in $\mathcal{M}_X$. Finally, having ordered $\mathcal{M}_X$ through, the events in which *X* is involved can now be ordered accordingly.

As an example of such nested ordering, consider, for example, states of the moon with Neil Armstrong's footprint, states where the footprint is erased by the crater of a meteorite, and states with neither the footprint nor the crater. The order between these three groups of states can be established through the *EA*s associated with these records in the way just described, so that, for example, the states with only the footprint are on level 1 with respect to the footprint. In a next step, we can check these states for *EA*s with respect to other records, say, traces of space probe activities, or such like. Finally, we can then use the total order of states so obtained to order the corresponding events, such as the footstep, the meteorite impact, and the space probe's scratching the moon's surface.

We thus see that the *EA* when applied to $\mathcal{M}_X$ delivers, given premise 2, the needed sufficient condition for "before", thereby solving problem 1. Problem 2 is solved as well, because $\mathcal{M}_X$ is totally ordered by *EAs*, that is, any two non-identical states are related by an *EA*, and hence, none by an existence symmetry. Contrast this with the situation discussed below lemma 1: Here, there was an existence symmetry on $M(p,q)_X$, and an *EA* arose by restricting $M(p,q)_X$ to one of its proper subsets. Hence, there was no guarantee that an *EA* on such a subset (a "false *EA*", so to speak) corresponded to one on $M(p,q)_X$ itself. But given the above, this cannot happen on $\mathcal{M}_X$. Nor can anything else go wrong: as can be easily verified by checking configurations (a) to (d) under theorem 2, only the existence symmetry (configuration (a)) can be restricted in such a way as to give rise to such a "false *EA*". Thus, an *EA* on any subset of $\mathcal{M}_X$ implies an *EA* on $\mathcal{M}_X$ itself, which in turn is sufficient for a before order.

We can check that all of this agrees with our intuition of "before" in everyday life: I experience that a blue light flashes before a green light does. This implies that there is an *EA* on a subset of $M(b,g)_{me}$ (where *b* and *g* are the records of the two flashes) as should now be clear. Of course, I do not know whether there is also an *EA* on all of $M(b,g)_{me}$: I could one day forget about the blue light, so that there is a state of me with only *g*, and $M(b,g)_{me}$ is in this case symmetric in *b* and *g*. But I also know that, should I forget, this will be due to a causal interaction, or several of them, which affect a state of my brain having *b*. But since they affect my brain, these interactions themselves leave a record, call it *c*. We can call the resulting state a *bc* state. Even though this state may be intrinsically no different from a state which lacks *b* simpliciter, we not only can, but must distinguish these two states since, again, the combination of the *b*-event and the *c*-interaction(s) is really distinct from the simple lack of the *b*-event. This, in turn, leads to the distinction between the *M*-set and the $\mathcal{M}$-set. Thus, both the *EA* and the distinction between these two sets, even though they will be unfamiliar to readers, correspond closely to our everyday experience of the succession of events in time. But note that these considerations are only a check with our temporal intuitions: the above derivation of the before order does not rely on these intuitions, but is rather based on the *EA*, a notion which does not presuppose the before order which it grounds.

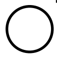
Also, notice that the before order on $\mathcal{M}_X$ is extremely simple: the more records a state has, the higher up it is in the order. This too corresponds to a familiar phenomenon of everyday life: later states of objects or persons bear traces of more interactions—though some of these traces may no longer be discernible—than earlier ones. This makes priority in terms of the before order similar to what could be called "logical priority": a later state on $\mathcal{M}_X$ has all the records which an earlier one has, plus additional ones, similar to the way in which, say, the definition of "Greek citizen" in terms of essential properties contains all those which are contained in the definition of "human being", plus additional ones, and this latter notion is logically prior to the former. "Priority" has different, but related meanings, a theme investigated already by Aristotle ([1924], 1013a).

It is now possible to synthesize the groundwork done so far and offer a reductive definition of “before”: Once again, let *X* be a variable denoting an object, *p* and *q* records in it, and ***p*** and ***q*** the corresponding events involving *X*. These can also be null records and null events, respectively. Also, let $\mathcal{E}_X pq$ denote the proposition “there is a *pq*-asymmetry on $\mathcal{M}_X$”. Then:

Def. 3: For any *X* and any ***p***, ***q*** involving *X*: ***p*** is before ***q*** if and only if $\mathcal{E}_X pq$.

This, in turn, is satisfied if there is a *pq*-asymmetry on any subset of $\mathcal{M}_X$ containing a state of *X* with *p* and one with *q*, as has been argued; and only then, since the *EA* cannot feature on $\mathcal{M}_X$ unless there is a *pq*-asymmetry on the set containing only these two states. All other subsets with these two states are merely extensions of this doublet set, and if the doublet is *pq*-asymmetric, so too are the extensions, as is clear from the answer to problem 2. Hence, it is also possible, equivalently, to define “before” in terms of an *EA* on any of these subsets.

On the left-hand side of the biconditional in definition 3, we have the familiar temporal concept of “before”, whereas on its right-hand side is a proposition asserting an *EA* on $\mathcal{M}_X$. Since neither the concept of the *EA* nor that of $\mathcal{M}_X$ make reference to a before order, the definition is indeed reductive. Causal priority, asymmetry, or dependence does not enter into it, but logical priority does, insofar as an earlier state on $\mathcal{M}_X$ can, in the sense given above, be thought of as logically prior to a later one. Once again, the before order so obtained can be used not only for local events directly involving *X*, but also—by making the necessary corrections for distance—for spatially removed events, yielding an order according to *X*’s frame of reference. In any case, the before order is not a relation between two events considered independently of objects, as is often thought. Rather, it exists in virtue of an object having records of these events.

The $\mathcal{M}$-set, unlike the *M*-set, allows establishing the order of an object’s states using only these states themselves, without the need for *ersatz*-objects. Now, the concept of the $\mathcal{M}$-set relies on the distinction between the two states  and , a distinction which has a *fundamentum in re* and must be made, as argued. But empirically, we cannot in general tell the two apart, so that we have only elements of *M*-sets available for telling which events happened, and what their order is. This means that before orders established in terms of records in concrete objects will necessarily be gappy and incomplete. But very often, the gaps can be filled by using an *ersatz*-asymmetry on the *M*-set of another object. Examples of this are familiar: We may be unable to recall a detail of a particular event which we know we have experienced—e.g. the music played at a wedding—because the record of it in our brain is erased. But we can ask a friend with a brain in which the corresponding record is intact. We may be unable to reconstruct a particular piece of the earth’s climate history through dendrochronology, because of a lack of surviving tree specimens, but we can fill this gap by using evidence from ice core samples taken from a glacier. Both the friend and the glacier can only provide us with the before order if there is an *EA* on a subset of their respective states, as has been argued in the context of theorem 2 and thesis 1 (mutual overtaking is unrealistic in the processes relevant to these two cases, so case 2 can be ignored). In other words, we can combine the gappy before orders obtained from the *M*-sets of different recorders, stitch them together, and in this way approximate the complete before order defined in terms of the $\mathcal{M}$-set of a given object. Such stitching together is done routinely in everyday life, and in the sciences dealing with the history of life, the earth, and the whole universe. In this way, the distinction between the *M*-set and the $\mathcal{M}$-set accounts for the familiar phenomenon that, even though we know the correct, complete before orders exist, they are not always easily accessible to us, but can be obtained only by combining evidence from different recorders.

3. Objections

The whole approach of quantifying over sets of states of objects is very unorthodox with respect to the various ways of thinking about temporal succession current in contemporary academic philosophy, and is often perceived as obviously wrong and implausible. Hence, it will be necessary to address at least some of the most common objections.

First, some may object that the account is manifestly circular, presupposing the very before order which it is supposed to derive: a set of different states of an object, causal interactions affecting an object in general, and in particular combining in such a way as to cancel a record—all this, the objection goes, cannot be imagined independently of a before order in the first place. To this, I answer that indeed we are unable to do so. But it must be borne in mind that we, as physical objects, are ourselves recorders: We have in our brains records of different states of other objects, we compare these states and establish a before order between them, enabling us to experience change, and any causal interaction is itself a change. But establishing the before order and conceiving of the temporal succession involved in all of this is possible only on the basis of *EA*s, as shown (theorem 2 and below). Now, neither the notion of a set of states, nor the *EA*, nor the distinction between the three possible states with respect to a given record which underlies the notion of an $\mathcal{M}$-set, nor the premises adduced required the assumption that a before order exists, and definition 3 is given entirely in terms of them. The mere inability to imagine these notions independently of time does not indicate a logical problem with the theory: many children find it impossible to imagine isotropic space without a “downward” direction, and many adults—including many physicists and philosophers—find relativity of simultaneity inconceivable even today, but neither of this indicates that a good theory of space or time must presuppose an anisotropic spatial structure or absolute simultaneity.

Second, some readers may have the impression that before orders, according to my approach, are “subjective” or “products of memory”. This too is a misconception. It is not subjective states of mind which constitute a before order, but rather *EA*s on subsets of sets of states of physical objects, of which we as conscious embodied beings are only a special case. But whether an *EA* exists on a given subset is a fact of nature, not a subjective matter like, say, musical taste or a dream is. The before order between events is an observable, just like the classical properties of the length or mass of an object, or the quantum-mechanical ones of spin or parity. None of these are definable independently of an operational procedure, i.e. an act of measurement, whether it is a conscious being carrying out the measurement or not. Such measurements are, to speak with Kant, “questions to nature” ([1787], AA 10), and nature throws at us—“ob-jects” in the etymological sense—unique answers, which are not susceptible to subjective and arbitrary interpretation. Just the same is true of measurement of the before order defined operationally in terms of the *EA*. This order, by consequent, is no more “subjective” than observables such as length, spin, and so on. Arguably, the currently dominant perception that time must either be absolute and fundamental, as Newton thought, or is otherwise “merely subjective”, and the concomitant failure of being able to think of time as an emergent phenomenon which depends on concrete physical objects, both stand in the way of progress in contemporary philosophy of time, a point argued in detail by James Harrington ([2009]).

Finally, it is often objected that the present account fits neither into the A-theory nor the B-theory, and is therefore unintelligible. But I see no reason why a new account of “before” should be obliged to be classifiable in this way. To demand this would be to rule out different approaches to temporal succession a priori. In particular, readers with A-theoretical leanings will object that it is wrong to adopt a bird’s eye view and consider the set of all states of an object which, in a timeless sense, “exist”, since to do so would presuppose the B-theory. They will point out that reality is fundamentally tensed, so that states should rather be divided up into those which have existed, exist, or will exist. To this I answer that A-theorists too will have to admit that, for a given object, only some of its conceivable states have existed, exist, or will exist: the table before me may once have had, or one day will have, a different colour than it does now, but clearly not all possible colours. There have been states of Cicero with records of Greek, but none with records of Japanese. Hence, A-theorists too cannot forgo the distinction between states which are not realized anywhen, and those which are. All the A-theorist then needs to do is to take the set of these latter, and then follow the derivation I have proposed.

In sum, it is fallacious to conclude that an account must be flawed simply because it is unorthodox or hard to intuit.

4. Temporal succession: local and emergent

To conclude, let it briefly be pointed out that the present account not only saves, in the respects described above, the phenomena associated with temporal succession in everyday life, but is also consonant with what physics teaches us about time in three crucial ways:

First, before orders are emergent, since they depend on objects with classical identity conditions allowing to distinguish between the object itself as a substrate and the records in it, as well as between different states belonging to the same object. Given this, before orders do not apply to quantum mechanical systems considered in themselves (cf. again Lowe [2003]), even though it is possible to distinguish “before” and “after” in the Schrödinger evolution of such systems in terms of measurement through a classical object. As E. J. Zimmerman writes, “‘time’ which appears in the equations [of quantum mechanics] is not a quantum mechanical observable, which would be represented in the theory by an operator, but rather a parameter external to the microscopic system” ([1981], 492). One consequence of this is that there can be no before orders in what we call the “very early” and the “very late” universe, states in which the physics does not allow classical objects to subsist, nor are such states literally “before” or “after” those more hospitable to classical objects (for the early universe, cf. Rugh & Zinkernagel [date unknown]). However, once again it is possible to view these states as “earlier” and “later” ones according to models devised by, say, the human brain or a computer, which after all are themselves classical objects.

Second, before orders are “attached to” concrete physical objects, since they are defined in terms of such objects’ $\mathcal{M}$-sets. This makes before orders essentially local, not global. On such an object-centred view, the relation “before” does not yield a total order on the set of all events occurring in all of spacetime. Rather, it gives rise to local and partial orders on this set. This means that temporal succession does not need global hypersurfaces of absolute simultaneity, which are still often defended as prerequisites for the passage of time in much contemporary literature (e.g. Unger & Smolin [2015]; Zimmerman [2013]; Craig [2001]). Such hypersurfaces, of course, conflict with both special and general relativity theory, according to which there are no global total orders encompassing all events.[2] But the lack of a global order does not make temporal succession something merely subjective or illusory. Rather, it is real, but local.

Third, temporal succession as understood in this article will be accompanied by a rise in entropy in the local vicinity of a recorder with overwhelming probability (but not absolute certainty). The reason is simple: the more records there are in a state in the $\mathcal{M}$-set of a given recorder, the higher we are up in the local before order. But each record is of a causal interaction, and each interaction tends to increase the entropy. The local rise in entropy is thus a consequence or “symptom” of an ascending before order, not the reason for the existence of this order.

[2] Cosmic time functions, which can be constructed from local comoving observers given a homogenous universe (see e.g. Whitrow [1980], 270-320; Callender [2017], 72-76), yield a total order of large-scale states of the universe, but not of all events.

References

Aristotle, *Metaphysics*, edited by W. D. Ross. Oxford: Clarendon Press (1924). <http://www.perseus.tufts.edu/hopper/collection?collection=Perseus%3Acorpus%3Aperseus%2Cwork%2CAristotle%2C%20Metaphysics>

Callender, C. *What Makes Time Special?* Oxford University Press (2017). https://doi.org/10.1093/oso/9780198797302.001.0001

Craig, W. L. *Time and the Metaphysics of Relativity*, Dordrecht: Kluwer (2001).

Dainton, B. *Time and Space*, 2nd edn, Durham: Acumen (2010). https://doi.org/10.4324/9781315539324

Frisch, M. 'Time and causation', In Bardon, A., Dyke, H. (eds.) *A Companion to the Philosophy of Time*, John Wiley & Sons (2013), pp. 282-300. <http://faculty.philosophy.umd.edu/mfrisch/papers/time-and-causation.pdf>. https://doi.org/10.1002/9781118522097.ch17

Harrington, J. 'What "becomes" in temporal becoming?', *American Philosophical Quarterly* (2009) 46(3), 249-265.

Illari, P., Russo, F. *Causality: Philosophical Theory Meets Scientific Practice*, Oxford University Press (2014).

Kant, I. *Kritik der reinen Vernunft* (2nd edn, 1787), In Preußische Akademie der Wissenschaften (ed.) *Kant's gesammelte Schriften* (1900ff.), vol. III.

Lowe, E. J. 'Individuation', In Loux, M. J., Zimmerman, D. W. (eds.) *The Oxford Handbook of Metaphysics*, Oxford University Press (2003), pp. 75-95.

McTaggart, J. M. E. 'The unreality of time', In Le Poidevin, R., MacBeath, M. (eds.) *The Philosophy of Time*, Oxford University Press (1993), pp. 23-34.

Mellor, D. H. *Real Time II*, London and New York: Routledge (1998).

Rugh, S. E., Zinkernagel, H. 'On the physical basis of cosmic time' (date unknown). <http://core.kmi.open.ac.uk/download/pdf/11921921.pdf>

Unger, R. M., Smolin, L. *The Singular Universe and the Reality of Time: A Proposal in Natural Philosophy*, Cambridge University Press (2015).

Whitrow, G. J. *The Natural Philosophy of Time*, 2nd edn. Oxford: Clarendon Press (1980).

Wittgenstein, L. *Tractatus Logico-Philosophicus*, In *Werkausgabe*, 2nd edn., vol. I. Frankfurt am Main: Suhrkamp (1995).

Zimmerman, D. W. 'Presentism and the space-time manifold', In Callender C. (ed.) *The Oxford Handbook of Philosophy of Time,* Oxford University Press (2013), pp. 163-246. https://doi.org/10.1093/oxfordhb/9780199298204.003.0008

Zimmerman, E. J. 'Time and quantum theory', In Fraser, J. T. (ed.) *The Voices of Time* (1981), 2nd edn, Amherst, Mass.: University of Massachusetts Press, pp. 479-499.