Properties and the Born Rule in GRW Theory

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1 Introduction

How are properties encoded in GRW theory? In this chapter I discuss an influential answer to this question, the so-called *Fuzzy Link*. Lewis (1997) argued that GRW theory, when interpreted in terms of the Fuzzy Link, implies that arithmetic does not apply to ordinary objects, an argument now known as the 'counting anomaly'. I take this argument as the starting point for a discussion of the property structure of GRW theory, and collapse interpretations of quantum mechanics in general. The main lesson to be drawn from the counting anomaly is that the property structure of these theories are more complex than that of standard quantum theories (and classical mechanics) because a seemingly plausible principle, the composition principle, fails.

2 The Fuzzy Link

The standard way to relate quantum states to physical properties is the *Eigenstate-Eigenevalue Rule* ('E-E rule' henceforth):[1]

> An observable $\hat{O}$ has a well-defined value for a quantum system S in state $|\psi\rangle$ if, and only if, $|\psi\rangle$ is an eigenstate of $\hat{O}$.

[1] A classical source for this rule is Dirac (1930, pp. 46-47).

States that are not eigenstates of $\hat{O}$ defy interpretation on the basis of this rule and are not assigned a property. Yet a measurement of $\hat{O}$ produces a definite outcome even if the system is not in an eigenstate of $\hat{O}$. To solve this problem standard quantum mechanics postulates that whenever a measurement is performed, the system's state instantaneously collapses into one of the eigenstates of $\hat{O}$ with a probability given by the Born rule. This leaves the system in a state that can be interpreted on the basis of the E-E rule. But the introduction of measurement-induced collapses brings a plethora of new problems with it. What defines a measurement? At what stage of the measurement process does the collapse take place (trigger problem)? And why should the properties of a physical system depend on observers?

Ghirardi, Rimini, and Weber (1986) proposed an ingenious way to overcome these difficulties.[2] Their approach is now known as 'GRW theory'. Rather than appealing to observers, GRW theory sees collapses an integral part of what happens in nature. It postulates that on average a collapse occurs every 10^{-16}s. There are, however, crucial differences between the collapses of standard quantum theory and those of GRW theory. In GRW theory position is privileged and the theory's mechanism induces collapses in the position basis. However, a collapse can leave a system in a proper eigenstate only if the basis is discrete and hence a system's wave function (in the position basis) cannot be arbitrarily narrow after a collapse. This fact is enshrined in GRW theory, which postulates that a collapse leaves the system not in a precise eigenstate of the position operator but in a state that is 'close' to it. Technically speaking, the original state gets multiplied by a sharply peaked Gaussian which makes the state more localised (where the variance of the Gaussian is of the order 10^{-7}m). In the context of GRW theory it is therefore more adequate to speak of a *localisation process* rather than a collapse.

However, a more localised state is still not an eigenstate of the position operator, and as far as interpreting states using the E-E rule is concerned we're back where we started. One way around the problem is to look for an alternative to the E-E rule. Common wisdom has it that 'close enough' is

[2]The theory has been put in a particularly simple form by Bell (1987). For a comprehensive review of 'GRW type' theories see Bassi and Ghirardi (2003). A semi-technical summary of the main ideas can be found in Frigg and Hoefer (2007). For a discussion of collapse theories in general see Gao (2017, Ch. 8).

good enough: for a particle to be located at x it is sufficient to say that its wave function is peaked over a narrow interval around x (see, for instance, Sakurai 1994, 42-3). Albert and Loewer (1995) give an exact formulation of this idea and say that a particle with wave function $\psi(r)$ is located in the interval R iff $\int_R |\psi(r)|^2 dr \geq 1 - \epsilon$, where ϵ is a positive real number close to zero (throughout I use 'iff' for 'if and only if'). The generalisation of this rule to a system with n degrees of freedom is straightforward and leads to the following definition:

> Consider n objects $e_1, ..., e_n$. The system $E = \{e_i, ..., e_n\}$ consisting of these n entities with wave function $\psi(r_1, \ldots, r_n)$ has the property of being in the n-dimensional interval $R_1 \times \ldots \times R_n$ iff
>
> $$\int_{R_1 \times \ldots \times R_n} |\psi(r_1, \ldots, r_n)|^2 d^n r \;\geq\; 1 - \epsilon. \qquad (1)$$

Clifton and Monton (1999) call this rule the *Fuzzy Link*, and I use the label *Fuzzy Quantum Mechanics* (FQM) to refer to any interpretation of quantum mechanics (QM) that assigns properties to states using the Fuzzy Link. Regarding GRW theory as a (version of) FQM offers a natural assignment of properties to post-localisation states.[3] For what follows it is convenient to let $P_{\epsilon, R_1 \times \ldots \times R_n}(e_1, \ldots, e_n)$ be the proposition stating that E has the property of being in the interval $R_1 \times \ldots \times R_n$. This proposition is true iff inequality (1) holds. Note that E can also consist of just one object e. In this case the definition reduces to: the entity e with wave function $\psi(r)$ has the property of being in the interval R, i.e. $P_{\epsilon,R}(e)$ is true, iff $\int_R |\psi(r)|^2 dr \geq 1 - \epsilon$.

Peter Lewis (1997) argues that the Fuzzy Link has the undesirable conclusion that arithmetic does not apply to macroscopic objects like marbles. This argument is now known as the *counting anomaly*. Consider a marble and box (large enough for the marble to fit in). The marble can be in two states, namely $|\psi_{in}\rangle$ (the marble is inside the box) and $|\psi_{out}\rangle$ (the marble is outside

[3] Ghirardi and co-workers (Ghirardi et al. 1995; Bassi and Ghirardi 1999b) favour a mass-density interpretation of GRW theory. Space constraints prevent me from discussing this approach here. However, as Clifton and Monton (2000, 156-161) point out, the problems I discuss in this paper (in particular the counting anomaly) equally arise under the mass-density interpretation. So the discussion in what follows *mutatis mutandis* carries over to an interpretation of GRW theor based on the mass-density approach.

the box). These states are mutually exclusive and therefore orthogonal. For the reasons we have encountered above, a localisation process won't leave the marble's state in position eigenstate; the best one can expect is for the localisation to leave the marble in a highly asymmetric state of the form $|\psi_m\rangle = a|\psi_{in}\rangle + b|\psi_{out}\rangle$ (or $|\psi'_m\rangle = b|\psi_{in}\rangle + a|\psi_{out}\rangle$) where $1 > |a| \gg |b| > 0$ and $|a|^2 + |b|^2 = 1$. According to the Fuzzy Link, if $|b|^2 \leq \epsilon$ then the marble is in the box: $\int_{R_{in}} |\psi_m(r)|^2 dr = |a|^2 \geq 1 - \epsilon$, where R_{in} is the region we associate with being in the box.

Now enlarge the box and put not only one but a large number n of marbles in it (assume that the box is long a slim allowing for the marbles to be placed in it side by side so that there is no interaction between the marbles). The state of the n-marble system is $|\psi_{total}\rangle = |\psi_m\rangle_1 \ldots |\psi_m\rangle_n$. When we now interpret $|\psi_{total}\rangle$ in terms of the Fuzzy Link we are faced with a paradox. While each individual marble is in the box, applying the Fuzzy Link to the state of the system yields that the system is not in the box: $\int_{R_{in} \times \ldots \times R_{in}} |\psi_{total}|^2 d^n r = |a|^{2n}$, but $|a|^{2n} \ll 1 - \epsilon$ since $|a|$ is smaller than 1.[4] So is we construct a system of n marbles each of which *individually* is in the box, we end up with an n-marble system which is not in the box. Lewis calls this the counting anomaly because ensuring (one by one) that each marble is in the box is how we count marbles. Lewis (1997, p. 320-321) calls this the *enumeration principle*. But, as we have just seen, by doing so we end up with a state in which it is false that the system of marbles is in the box. Hence counting is impossible and we must conclude that according to GRW theory (and indeed any version of FQM) arithmetic does not apply to macroscopic objects such as marbles.

This argument has sparked a heated debate between Ghirardi and Bassi on one side and Clifton and Monton on the other, centring around the questions whether the anomaly really arises in GRW, and whether it can be observed.[5] I want to take completely different line. I argue that the counting anomaly can be dissolved by an analysis of the property structure of FQM. I argue

[4]There is a limit to how close $|a|$ can be to 1 and so there will always be an n so that $|a|^{2n} \ll 1 - \epsilon$.

[5]The first reply to Lewis is Ghirardi and Bassi (1999); subsequent contributions are Bassi and Ghirardi (1999, 2001), Clifton and Monton (1999, 2000), and Lewis (2003a). For discussion of the different positions see Frigg (2003) and Lewis (2003b).

that the problem arises because of the seemingly innocuous but ultimately faulty assumption that the composition principle holds in FQM. On this reading Lewis' argument performs a different function: rather than presenting a *reductio ad absurdum* of GRW theory, it highlights that the property structure of the theory is more complex that we had assumed. This restores coherence, but it comes at the cost of a violation of common sense. Everyday experience suggests that the composition principle holds true for spatial properties (an intuition which is borne out in classical mechanics as well as in versions of QM based on the E-E rule), and we have to come to terms with the realisation the this is not the case in GRW theory.

3 The Composition Principle

The *composition principle* says that if every object e_i of an ensemble $E = \{e_1, ..., e_n\}$ has a certain property P, then the ensemble E itself also has property P, and vice versa. Formally, $P(e_1)\&...\&P(e_n)$ iff PE, where 'PE' means that the *ensemble* E *itself* has property P whereas '$P(e_1)\&...\&P(e_n)$' expresses the fact that *every member of* E has property P. To facilitate notation we refer to the latter property as '$\tilde{P}$'. The composition principle then reads: $\tilde{P}E$ iff PE. This principle holds true in many cases. The concatenation of several objects of temperature T also has temperature T, and if two object reflect light of certain wavelength then the ensemble of both objects also reflects light of the same wavelength. However, the principle does not have the status of universal law (or even a truth of logic) and it fails in certain cases. Water is wet but water molecules are not; gases have a temperature but gas molecules do not; horses have a heart but a herd of horses does not; each musician of an orchestra plays an instrument but the orchestra as a whole does not. Failures of the principle also occur in physics: the ensemble of n objects of mass m does not have mass m, and the same holds true for every additive quantity.

Examples like these highlight that asserting $\tilde{P}E$ is not the same as asserting PE. If we wish to infer $\tilde{P}E$ from PE (or *vice versa*) the composition principle has to be invoked. This principle, however, is not a universal law and its validity in a given context needs to be justified. If we fail to provide such a justification and assume, without further argument, that the composition principle holds true, we are guilty of a *fallacy of composition*.

The problem with the marbles is a problem of composition. To see how the problem emerges, we explicitly state the composition principle as regards position. Let $e_1, ..., e_n$ stand for the marbles and $E = \{e_1, \ldots, e_n\}$ for the ensemble of all marbles. Let R be the property of being in the n dimensional Interval $R_1 \times \ldots \times R_n$. $\tilde{R}E$ is the statement that all marbles individually are in the relevant intervals (i.e. e_1 is in R_1, e_2 is in R_2, etc.). We then have $\tilde{R}E \equiv P_{\epsilon,R_1}(e_1) \& \ldots \& P_{\epsilon,R_n}(e_n)$ (where '$\equiv$' is the equivalence relation between propositions). This proposition is true iff $\int_{R_i} |\psi_i(r)|^2 dr \geq 1 - \epsilon$, $i = 1, ..., n$, where $|\psi_i\rangle$ is the quantum state of the i^{th} marble and R_i the spatial interval in which it should be located. RE is the statement that the ensemble of marbles is in R and we have $RE \equiv P_{\epsilon,R_1 \times \ldots \times R_n}(e_1, \ldots, e_n)$. This statement is true iff inequality (1) holds. The *composition principle as regards position* (CPP) says $\tilde{R}E$ iff RE. The property of being in the box is a special case. Choose $R = R_{in} \times \ldots \times R_{in}$ and let 'B' stand for the property of being in the box. One can then define $\tilde{B}E$ and BE as above, and CPP says $\tilde{B}E$ iff BE.

Is CPP true? The answer is: it depends. Let us first consider the special case of $\epsilon = 0$, where the Fuzzy Link in effect reduces to the the E-E Rule. One can show that CPP holds in this case (the prove is given in the Appendix). So under the E-E rule spatial properties satisfy composition. The situation changes drastically if we move into FQM proper and assume $\epsilon > 0$. The implication in CPP that runs from left to right no longer holds and CPP is false. In fact, in FQM only the *restricted composition principle as regards position* (RCPP) holds: If RE then $\tilde{R}E$, but *not* vice versa. Applied to the marble case RCPP says that if the ensemble of all marbles is in the box (BE), then every one of its members is in the box as well ($\tilde{B}E$). The converse, however, is false: if every member of the ensemble (i.e. every individual marble) is in the box, the same need not be true for the ensemble.

Given what we have said about composition so far it should not come as a surprise that such an inference can fail. The surprise, however, is that the inference fails in the case of a spatial location. Being in the box or, more generally, being located within an interval seems to be a clear example of a property for which the composition should hold: if all members of E are located in $R_1 \times \ldots \times R_n$ then, so it seems, the ensemble E itself should be located within that interval as well. The counting anomaly shows that his expectation is wrong.

A critic might now respond that noting is gained by rephrasing the counting anomaly as a failure of CPP, because any theory in which CPP fails should be rejected. So the challenge is to make plausible that one can rationally uphold a theory in which CPP fails. To meet this challenge we need to have a closer look at the properties involved. Why is it not absurd to hold that $\tilde{B}E$ is true while BE is false?

How do we check that all marbles are in the box? Let us follow Lewis and endorse his enumeration principle. This means that we first ascertain that the first marble is in the box, then that the second marble is in the box, and so on until we reach the n^{th} marble. If each marble turns out to be in the box, then all n marbles are in the box. Given this, it is necessary and sufficient for the n marbles to be in the box that $\tilde{B}E$ is true (which, recall, is equivalent to $P_{\epsilon,R_{in}}(e_1)\& \ldots \& P_{\epsilon,R_n}(e_{in})$).

Isn't this too strong? Does doesn't BE equally describe the state of affairs of all marbles being in the box? The crucial thing to realise is that it doesn't. If we follow the procedure given in the enumeration principle, there is just no reason to assume that BE should be true. We observe one marble after the other and make sure that it is in the box, which results in the state of all marbles *individually* being in the box, but not in any property of the ensemble. One might try to resist this conclusion by arguing that it is just intuitively obvious that BE describes the state of affairs of all marbles being in the box, regardless of whether or not it squares with the enumeration principle. But an appeal to intuition does't cut the mustard. Properties of ensembles and of individuals are distinct, and one cannot infer one from the other without further argument. The required further argument in the current case is CPP, and we know that this principle doesn't hold in FQM. Hence, insisting that BE represents the same state of affairs as $\tilde{B}E$ is committing a fallacy of composition.

But if BE does not describe the state of affairs of all marbles being in the box, what then does it represent? We are instructed by RCPP that BE

implies that all marbles are in the box but not vice versa, and so BE has 'surplus content' with respect to $\tilde{B}E$. What is this surplus? I have no answer to this question, and I think we don't need one. The interest in BE is based on the belief that it reflects the 'counting property', but this is not the case. Furthermore, it is in general a mistake to think that everything we can define in the formalism represents something interesting in the world. Not every formal expression corresponds to a property that is physically relevant, and BE may well be one of those expressions.

There are two ways to push back against this conclusion. The first, mentioned by Lewis (1997, p. 320) and echoed by Clifton and Monton (2000, p. 160), appeals to the Born rule. If the system is in state $|\psi_{total}\rangle = (a|\psi_{in}\rangle + b|\psi_{out}\rangle)_1 ... (a|\psi_{in}\rangle + b|\psi_{out}\rangle)_n$, then it is unacceptable to say that all marbles are in the box. Born's rule, so the argument goes, tells us that the probability of finding the system in state $|\psi_{in}\rangle_1 ... |\psi_{in}\rangle_n$ is $|a|^{2n}$, and because $|a|^{2n} \ll 1$ there is only a vanishingly small probability of finding all the marbles in the box. But it makes no sense to say that the marbles are in the box if the probability of finding them there is almost negligible.

This argument is flawed, but it is flawed in an interesting way because it draws our attention to an issue that has not received much attention so far, namely how to calculate probabilities in FQM. Given that FQM alters the conditions for a property to obtain, the way of calculating probabilities has to be altered too. Consider a marble in state $|\psi_m\rangle = a|\psi_{in}\rangle + b|\psi_{out}\rangle$. What is the probability p of it being true that the marble is in the box? In standard QM we associate the state of being in the box with $|\psi_{in}\rangle$ and using Born's rule we get $p = |a|^2$. In FQM, however, the Fuzzy Link tells us that the marble *is* in the box if it is in state $|\psi_m\rangle$ where $|a|^2 \geq 1 - \epsilon$. Given this, it does not make sense to say that the probability of finding the marble in the box equals $|a|^2$ in $|\psi_m\rangle$. We cannot say that the proposition that the marble is in the box is true if the system is in state $|\psi_m\rangle$ and at the same time take the probability of the proposition to be smaller than one! If we allow the proposition to be true in states like $|\psi_m\rangle$ then we have to take these *same* states when using Born's rule to calculate probabilities. In the present example, one ought to say that the FQM probability of a marble in state $|\psi\rangle$

to be in the box is $|\langle\psi|\psi_m\rangle|^2$, and not $|\langle\psi|\psi_{in}\rangle$ as standard QM has it.[6]

It is now clear where the problem lies: It is true that the probability of finding the system in state $|\psi_{in}\rangle_1...|\psi_{in}\rangle_n$ is vanishingly small, but from this it does *not* follow that the probability of finding all the marbles *in the box* is equally small because these are not the same probabilities. The argument infers from the fact that the probability of finding the system in state $|\psi_{in}\rangle_1...|\psi_{in}\rangle_n$ is small that the probability of finding it in the box is equally small, and thus implicitly associates 'being in the box' with the state $|\psi_{in}\rangle_1...|\psi_{in}\rangle_n$. In doing so it carries over to FQM a way of thinking about probabilities that contradicts FQM's basic assumptions.

The second way to push back against my conclusion is to appeal to everyday-language practices. Lewis (2003, pp. 140-1; 2016; pp. 93-95) argues that an everyday language claim like 'all marbles are in the box' involves both a claim about individual marbles *and* the ensemble of marbles because everyday language does not distinguish between the two. Therefore one cannot drive a wedge between claims about individuals and claims about ensembles in the way that I suggest (Monton (2004) also endorses this argument). Lewis immediately adds that 'this is an empirical claim about everyday language, and requires justification' (2003, p. 140), but expresses confidence that such an investigation would reveal that 'we do not make the distinction in ordinary language between the claim that the ensemble of marbles is in the box and the claim that each of the marbles individually is in the box' (2016, p. 93).

Whether Lewis' claim about ordinary language is true is a factual question that ultimately can only be settled through empirical research, and the jury on this is still out. But let's assume, for the sake of argument, that Lewis is right and ordinary language does not make a distinction between claims about individual marbles and ensembles of marbles. How relevant is this for

[6]There is a question which state exactly one uses to calculate probabilities because according to the Fuzzy Link, all $|\psi_m\rangle$ with $|a|^2 \geq 1-\epsilon$ have the property at stake. A possible response is to take the state with the smallest admissible a (namely $|a|^2 = 1-\epsilon$) and stipulate that p equals $|\langle\psi|\psi_m\rangle|^2$ for all states whose coefficient of $|\psi_{in}\rangle$ is smaller than a and 1 for all states with this coefficient greater that a. The last clause is needed to prevent that a state which is closer to $|\psi_{in}\rangle$ than $|\psi_m\rangle$ is assigned a probability smaller than one of being in the box.

the question of understanding FQM? Views about the importance of ordinary language vary widely, and the judgement isn't always favourable. In fact, significant strands of analytical philosophy have been concerned with clearing up the ambiguities and inconsistencies of ordinary language in numerous domains of inquiry. So it is no anathema to replace ordinary language by a suitably regimented artificial language if this solves problems, and my claim is that arithmetic is one of those places where such a revisionary attitude is justified. The most common formal system of arithmetic, Peano arithmetic, is a first order theory (see, for instance, Machover 1996). The axioms of Peano Arithmetic capture all standard truths of arithmetic, and being first order only involve quantification over individuals. So no appeal to ensembles and ensemble properties is needed to capture the truths of arithmetic. If ordinary language creates an arithmetic anomaly by importing elements into the theory that aren't needed, then the right reaction seems to be to replace ordinary language by a suitably regimented formal language.[7] The counting anomaly shows us that that position is a more complex property than we had previously assumed and we have to update our views about it accordingly. If an air of paradox remains, it has to be dispelled in the same way in which many other fallacies and imprecisions of ordinary language have to be dispelled.

4 Conclusion

I have argued that it is sufficient for the marbles to be in the box that $\tilde{B}E$ ($\equiv P_{\epsilon,R_{in}}(e_1) and \ldots and P_{\epsilon,R_n}(e_{in})$) holds, and that nothing else is needed. Since counting is a process that is concerned with individual objects rather than with ensembles, all that is needed for counting is that the conjunction of all $P_{\epsilon,R_{in}}(e_j)$ is true. The general lesson we learn from this discussion is that the property structure of FQM is more complex than that of standard

[7] The claim is only that first order arithmetic is able to capture standard truths of arithmetic. The claim is not that no other formulations of arithmetic could be given. There are of course second order formulations of arithmetic which involve quantification over predicate variables. But these are introduced to solve problems that have nothing to do with the counting anomaly, and the existence of these theory does not force us to use them in the current context.

QM, a price we have to pay for the admission of non-eigenstates as property-bearing states.

Appendix

Note that we recovers the E-E rule if we assume $\epsilon = 0$ and replace '$\geq$' is replaced by '$=$'. A system in state $|\psi\rangle$ has the property U iff $|\langle\psi \mid e_u\rangle|^2 = 1$, where $|e_u\rangle$ is the state in the Hilbert space associated with the property U. This is equivalent to the condition $\langle\psi \mid \hat{P}_{e_u} \mid \psi\rangle = 1$, where $\hat{P}_{e_u}$ is the projection operator on $|e_u\rangle$. If there is not just one single vector, but an entire subspace S_u of the Hilbert space associated with U, the condition reads $\langle\psi \mid \hat{P}_{S_u} \mid \psi\rangle = 1$, where $\hat{P}_{S_u}$ is the projection operator on the subspace S_u. Now choose U to be 'being located within interval $R_1 \times \ldots \times R_n$'. Then this condition reads $\langle\psi \mid \hat{P}_{R_1\times\ldots\times R_n} \mid \psi\rangle = 1$. Now expand both $|\psi\rangle$ and $\hat{P}_{R_1\times\ldots\times R_n}$ in the position basis: $|\psi\rangle = \int_{-\infty}^{\infty} \ldots \int_{-\infty}^{\infty} d^n r\, \psi(r_1, ..., r_n)|r_1 ... r_n\rangle$ and $\hat{P}_{R_1\times\ldots\times R_n} = \int_{-\infty}^{\infty} \ldots \int_R d^n r\, |r_1 ... r_n\rangle\langle r_1 ... r_n|$. Plugging this into the above condition yields: $\langle\psi \mid \hat{P}_{R_1\times\ldots\times R_n} \mid \psi\rangle = \int_{R_1\times\ldots\times R_n} |\psi(r_1, ..., r_n)|^2 d^n r = 1$, which obviously is Equ. 1 with the aforementioned changes.

We are now in a position to prove that CP holds for properties thus defined. $\Rightarrow$: Assume $P_{R_1}(e_1) and P_{R_2}(e_2) and \ldots and P_{R_n}(e_n)$ holds, that is, $\int_R |\psi_i(r_i)|^2 dr_i = 1$; $i = 1, \ldots, n$. Since we built up our collective 'n-marble entity' from n non-interacting marbles the state will not be entangled and can be written as the product of the states of the individual marbles: $\psi(r_1, \ldots, r_n) = \psi_1(r_1) \ldots \psi_n(r_n)$; and since in SQM the wave functions of a well-behaved quantum state is integrable we can factorise the integral in Def. 2: $\int_{R_1\times\ldots\times R_n} |\psi_1(r_1) \ldots \psi_n(r_n)|^2 d^n r = \int_{R_1} |\psi_1(r_1)|^2 dr_1 \ldots \int_{R_n} |\psi_n(r_n)|^2 dr_n$. But by assumption all terms of this product equal one, hence $\int_{R_1\times\ldots\times R_n} |\psi_1(r_1) \ldots \psi_n(r_n)|^2 d^n r = 1$.
$\Leftarrow$: Assume $P_{R_1\times\ldots\times R_n}(e_1, \ldots, e_n)$ holds, that is, $\int_{R_1\times\ldots\times R_n} |\psi_1(r_1) \ldots \psi_n(r_n)|^2 d^n r = 1$. Factorise the integral as above: $\int_{R_1\times\ldots\times R_n} |\psi_1(r_1) \ldots \psi_n(r_n)|^2 d^n r = \int_{R_1} |\psi_1(r_1)|^2 dr_1 \ldots \int_{R_n} |\psi_n(r_n)|^2 dr_n = 1$. It is an axiom of SQM that $\int_{R_i} |\psi_i(r_i)|^2 dr_i \leq 1$ for all $i = 1, \ldots, n$. For this reason the above product can equal 1 only if $\int_{R_i} |\psi_i(r_i)|^2 dr_i = 1$ for all $i = 1, \ldots, n$. qed. This completes the proof

of CP for SQM.

It is straightforward to see that the first half of the proof no longer goes through if $\epsilon > 0$ and (1) is a proper inequality. The second part, however, is not affected by this change. For this reason, CPP fails in FQM, but RCPP holds.

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