A simple proof that the global phase is real

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Abstract

It is a standard view in quantum mechanics that two wave functions that differ only in the global phase represent the same physical state. In this paper, I argue that this standard view is wrong, and the global phase is real in psi-ontic theories such as the de Broglie-Bohm theory, the many-worlds interpretation and collapse theories of quantum mechanics.

Suppose there is a superposition of two spatially separated wave packets of a particle such as a neutron $\frac{1}{\sqrt{2}}(|\psi_1\rangle + |\psi_2\rangle)$, which appears in many quantum interference experiments. A local unitary transformation can be applied to add a local phase to one branch of the superposition. For example, a local magnetic field can be introduced to rotate the spin of the neuron in one branch and add a local phase to the branch. Consider two possible situations. One is that a local unitary transformation is applied in the region of $|\psi_1\rangle$, which adds a local phase ϕ to this branch, where $\phi \in (0, 2\pi)$, and the superposition becomes $\frac{1}{\sqrt{2}}(e^{i\phi}|\psi_1\rangle + |\psi_2\rangle)$. The other is that a local unitary transformation is applied in the region of $|\psi_2\rangle$, which adds a local phase $-\phi$ to this branch, and the superposition becomes $\frac{1}{\sqrt{2}}(|\psi_1\rangle + e^{-i\phi}|\psi_2\rangle)$. We have the relation $e^{i\phi}|\psi_1\rangle + |\psi_2\rangle = e^{i\phi}(|\psi_1\rangle + e^{-i\phi}|\psi_2\rangle)$. Now if the two superpositions in these two situations, which differ by a global phase factor, correspond to two different physical states, then we can prove that the global phase is real.[1]

Consider the psi-ontic view (Pusey, Barrett and Rudolph, 2012), which says that two wave functions which differ not only in the global phase rep-

[1]Note that the global phase I discussed in this paper is not the global phase of the universal wave function, but the usual global phase of the wave function of a subsystem of the universe. For a recent discussion of the reality of the global phase see Schroeren (2022), Gao (2022) and Wallace (2022).

resent different physical states or the physical states of a single system correspond to rays in the Hilbert space. On this view, the two superpositions $\frac{1}{\sqrt{2}}(e^{i\phi}|\psi_1\rangle+|\psi_2\rangle)$ and $\frac{1}{\sqrt{2}}(|\psi_1\rangle+|\psi_2\rangle)$ correspond to different physical states, so do $\frac{1}{\sqrt{2}}(|\psi_1\rangle+e^{-i\phi}|\psi_2\rangle)$ and $\frac{1}{\sqrt{2}}(|\psi_1\rangle+|\psi_2\rangle)$. In other words, the local unitary transformation that changes the local phase of each branch of the initial superposition also changes the underlying physical state of the particle.

The next step is to prove that the changed physical states in the above two situations are different. The Schrödinger equation ensures that the local unitary transformation in one region does not change the wave function of the particle in other regions. On the psi-ontic view, this means that the local unitary transformation in one region does not change the physical state of the particle in other regions. Then, the local unitary transformation that changes the local phase of $|\psi_1\rangle$ only changes the physical state of the particle in the region of $|\psi_1\rangle$, and the local unitary transformation that changes the local phase of $|\psi_2\rangle$ only changes the physical state of the particle in the region of $|\psi_2\rangle$. Thus the changed physical states in the above two situations are different. This proves the reality of the global phase for the psi-ontic view.

The above proof implicitly assumes that the wave function of a single particle at each point in space represents a local physical property there. This is a natural assumption which is admitted by existing ontological interpretations of the wave function such as wave function realism (Albert, 2013). On this assumption, the local unitary transformation that changes one branch of a spatial superposition of a particle only changes the physical state in the region of the branch (if there is any change in the physical state). This is the basis of the above proof. Note that the wave function of a single particle at each point in space can be in principle measured by protective measurements up to a global phase (when the wave function is known) (Aharonov and Vaidman, 1993; Aharonov, Anandan and Vaidman, 1993; Gao, 2015). For example, the density and flux density of each branch of the above superposition $\frac{1}{\sqrt{2}}(|\psi_1\rangle+|\psi_2\rangle)$ can be measured locally by protective measurements. This also supports the above assumption.

The local phase ϕ in the superposition $\frac{1}{\sqrt{2}}(e^{i\phi}|\psi_1\rangle+|\psi_2\rangle)$ is often called the relative phase. This denomination seems to suggest that the local phase ϕ is a nonlocal property of the whole superposition. An argument supporting this viewpoint is that the local phase cannot be measured locally by measuring the corresponding branch, but be measured by measuring the whole superposition (Aharonov and Vaidman, 2000). I think this viewpoint is debatable. In my view, the reason why the local phase cannot be measured locally by measuring the corresponding branch is because the global phase of the wave function cannot be measured. If the global phase ϕ of the wave function $e^{i\phi}|\psi_1\rangle$ can be measured, then the local phase ϕ of the superposition $\frac{1}{\sqrt{2}}(e^{i\phi}|\psi_1\rangle+|\psi_2\rangle)$ can also be measured locally. Thus, the fact that the local phase cannot be measured locally does not imply that the local phase is a nonlocal property.

There is also another potential objection to the above proof. It is widely thought that the above superposition of a particle, $\frac{1}{\sqrt{2}}(|\psi_1\rangle + |\psi_2\rangle)$, can be rewritten in an entangled state $\frac{1}{\sqrt{2}}(|1\rangle_1 |0\rangle_2 + |0\rangle_1 |1\rangle_2)$, where $|1\rangle_1$ and $|1\rangle_2$ are the one-particle states which describe the regions 1 and 2 with one particle, and $|0\rangle_1$ and $|0\rangle_2$ are the vacuum states which describe the regions 1 and 2 without the particle. Then, each region is described not by a pure state, but by a mixed state $\frac{1}{2}(|1\rangle \langle 1| + ||0\rangle \langle 0|)$. As a result, a local phase transformation in each region does not change the local state of the region represented by this mixed state.

This seems to be a serious objection to the above proof. However, it is argubly not a valid objection. The key is to notice that the density matrix formulation contains no information about the global phase of the wave function. Not only a mixed state but also a pure density matrix such as $|1\rangle \langle 1|$ is not changed by a phase transformation. Then, if we assume that a pure density matrix is a complete representation of the physical state, we will already refute the reality of the global phase. But this assumption has not been justified. Although the density matrix formulation is enough for predictions of measurement results, it may not contain the whole truth about the ontology of quantum mechanics. In this sense, that a phase transformation does not change the density matrix of one region does not imply that it does not change the local state of the region.

To sum up, I have argued that two wave functions, which differ only in the global phase, correspond to different physical states in psi-ontic theories such as the de Broglie-Bohm theory, the many-worlds interpretation and collapse theories of quantum mechanics. In other words, if the wave function (up to the global phase) is real, then the global phase is also real.

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