# For Whom the Bell Inequality Really Tolls

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**Abstract**

There is a great deal of argument over the years about what exactly Bell's theorem entails or forces us to give up, such as "locality", "local hidden variables", "local realism", etc., etc. There are many ways to characterize the assumptions of Bell's theorem and each carries serious implications for its violation, e.g., superluminal causal influences with preferred reference frames, retrocausality, superdeterminism, etc. The game in discussions of Bell's theorem is usually to argue for giving up some assumption in the proof in order to save something else such as locality. However, 58 years after Bell's publication there is still widespread disagreement on what exactly Bell's theorem entails and on what causal mechanisms or dynamical model might be responsible for the experimental violations of the inequality. One reason for the lack of consensus is that all extant causal and dynamical accounts of the experimental violations of Bell's inequality are fraught with vagueness and potentially deal-breaking consequences. Given the state of play, we suggest that it's time to consider an explanation in spacetime for the experimental violation of Bell's inequality that is fully and completely acausal and adynamical. We will argue that what is happening in quantum mechanics is much like the early history of attempts to make Maxwell's equations consistent with Galilean velocity transformation. Just as with special relativity, we will claim that what is needed is a "principle" as opposed to "constructive" account of such violations and quantum entanglement. We will provide such an adynamical and acausal explanation herein. The primary implication is that contrary to popular belief, Bell's theorem does not entail that one must give up locality, local realism, or measurement independence in any form. Indeed, neither Bell's theorem nor experimental violations of its inequality requires any formal modification of quantum mechanics or the addition of any hidden new mechanisms or dynamics.

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## 1 Introduction

There is a great deal of argument over the years about what exactly Bell's theorem entails or forces us to give up, such as the standard formalism of quantum theory itself, "locality", "local hidden variables", "local realism", etc., etc. There are many ways to characterize the assumptions of Bell's theorem and each carries serious implications for its violation, e.g., superluminal causal influences with preferred reference frames (e.g., [Goldstein, 2021, Ghirardi et al., 1986]), retrocausality (e.g., [Aharonov and Vaidman, 2008, Cramer, 2015]), superdeterminism (e.g., [Hossenfelder, 2020, 'tHooft, 2017]), etc. The game in discussions of Bell's theorem is usually to argue for giving up some assumption in the proof however explicit or tacit (e.g., statistical independence or measurement independence) to save something else (e.g., locality). Thus, 58 years after Bell's publication there is still widespread disagreement on what exactly Bell's theorem entails and on what mechanisms or dynamical model might be responsible for the experimental violations of the inequality, whether in accord with quantum mechanics (QM) or some modified version of QM.

Almost every interpretation of, account of, alternative to QM is "constructive" (broadly speaking) in Einstein's sense of the term [Einstein, 1919]. All such accounts involve causal mechanisms or dynamical laws as the fundamental explanation for Bell's theorem as well as all the experimental results over the years that go along with the theorem. Indeed, even so-called retrocausal and superdeterministic accounts of QM are constructive accounts even though they technically violate Reichenbach's original Common Cause Principle [Silberstein et al., 2021]. We call such accounts constructive because they all adhere to the idea that the explanation for the experimental violations of the inequality are causal or dynamical, though their accounts of said causal mechanisms or dynamics vary wildly. Hossenfelder calls any account of QM that violates statistical independence "superdeterministic" [Hossenfelder, 2020], whereas proponents of retrocausal accounts tend to want to emphasize the key historical differences between superdeterministic accounts and retrocausal ones [Price and Wharton, 2016]. More on this shortly, but for our purposes the point is that all such accounts provide some sort of causal or dynamical explanation of the experimental violations of Bell's inequality.

Of course, it is often noted that one could seek an explanation of Bell's theorem and its experimental violations that is not a causal or dynamical explanation, one that rejects the Common Cause Principle or any version of it altogether [Myrvold et al., 2021]. But again, most accounts of QM that allege to be providing a physical model of what's happening in spacetime give constructive explanations in terms of causal mechanisms or dynamics however weird those mechanisms and dynamics might be. Whether nonlocal, retrocausal, superdeterministic, etc., almost all such accounts require altering QM in some serious way or seeking a theory more fundamental to it from which QM can be recovered. Some constructive accounts of QM alter or add to the formalism in some way such as spontaneous collapse accounts, some simply add some new physical mechanism such as certain retrocausal accounts, and some do both such as Hossenfelder's model of superdeterminism [Hossenfelder, 2020]. None of these accounts are fully worked out theories or models of physics, all have serious flaws, and there is no consensus about which of these specific models is the most explanatory and the least odious.

Given the state of play, we suggest that it's time to consider an explanation in spacetime for the violation of Bell's inequality that is of a completely non-constructive or "principle" nature (in Einstein's sense of the word [Einstein, 1919] broadly speaking). We will argue that what is happening in QM is much like the early history of attempts to make Maxwell's equations consistent with Galilean velocity transformation, the history leading up to the creation and eventual acceptance of special relativity (SR). Roughly speaking, in a principle explanation one uses a compelling fundamental principle to explain an empirically discovered fact that dictates the kinematics constraining the dynamics.

Many theorists including Einstein himself were desperately looking for a constructive model such as "froth, jelly, and vortex" models of the aether [Goldberg, 1984] and emission theories of light [Norton, 2004]. Such a principle account will invoke neither new causal mechanisms nor new dynamics–working from neither past-to-future, future-to-past, nor time-symmetrically. We will provide such a principle account in this paper and as it turns out, the explanation for QM entanglement, for the experimental violation of Bell's inequality, is much the same explanation that Einstein hit upon in SR. We will make clear that such a principle explanation being truly adynamical and acausal, requires no violation of locality, local realism, statistical independence, etc., and requires nothing like retrocausation, superdeterminism, etc. Indeed, neither Bell's theorem nor experimental violations of its inequality requires any formal modification of quantum mechanics or the addition of any hidden new mechanisms or dynamics.

Furthermore, the adynamical and acausal principle explanation on offer unifies QM and SR explanatorily and ontologically in ways that no constructive account could. The price for such an explanation, the price for ending the never-ending and unsatisfying hunt for a constructive account of QM entanglement, just as with SR itself, is to accept that at least until further notice, the "best" explanation for QM entanglement and violations of Bell's inequality is a principle one. Here "best" explanation means the one that is most worked out, has the least baggage and problems, and most unifies SR and QM both explanatorily and in terms of the physical model of the universe in question.

In section 2 we will briefly remind the reader of the relevant history leading up to the creation and acceptance of SR, a principle account par excellence. In section 3 we will briefly explicate Bell's inequality and that will then allow us in section 4 to relate the principle explanation in SR to Bell state entanglement in QM. The primary implication is that contrary to popular belief, Bell's theorem does not entail that one must give up locality, local realism, or measurement independence in any form, nor must one embellish or replace QM with new dynamical laws or causal mechanisms.

## 2 Special Relativity Provides a Precedent

The situation in physics at the end of the nineteenth century is now long forgotten, but it was every bit as desperate as the situation today regarding violations of the Bell inequality. As [Fuchs, 2002] wrote:

> Where present-day quantum-foundation studies have stagnated in the stream of history is not so unlike where the physics of length contraction and time dilation stood before Einstein's 1905 paper on special relativity.

And [Moylan, 2022] writes:

> The point is that at the end of the nineteenth century, physics was in a terrible state of confusion. Maxwell's equations were not preserved under the Galilean transformations and most of the Maxwellian physicists of the time were ready to abandon the relativity of motion principle [3], [4]. They adopted a distinguished frame of reference which was the rest frame of the "luminiferous aether," the medium in which electromagnetic waves propagate and in which Maxwell's equations and the Lorentz force law have their usual forms. In effect they were ready to uproot Copernicus and reinstate a new form of geocentricism.

Even "Einstein was willing to sacrifice the greatest success of 19th century physics, Maxwell's theory, seeking to replace it by one conforming to an emission theory of light, as the classical, Galilean kinematics demanded" before realizing that such an emission theory would not work [Norton, 2004]. Thus, concerning his decision to produce a principle explanation instead of a constructive explanation for time dilation and length contraction, Einstein writes [Einstein, 1949]:

> By and by I despaired of the possibility of discovering the true laws by means of constructive efforts based on known facts. The longer and the more despairingly I tried, the more I came to the conviction that only the discovery of a universal formal principle could lead us to assured results.

Despite the fact that "there is no mention in relativity of exactly *how* clocks slow, or *why* meter sticks shrink" [Mainwood, 2018] (no "constructive efforts"), the principle explanation of time dilation and length contraction is so compelling that "physicists always seem so sure about the particular theory of Special Relativity, when so many others have been superseded in the meantime" [Mainwood, 2018]. This widely held conviction did not obtain immediately upon Einstein's 1905 publication of SR. It took many years for such change to take hold in the physics community. For example, questions on the Mathematical Tripos examination at Cambridge University contained reference to various "jelly, froth, and vortex" models of the aether until 1909 [Goldberg, 1984, pp. 236-240].

Today, there is still no (consensus) constructive counterpart to SR nor any widespread effort to find one. So, introductory physics textbooks introduce SR in purely principle fashion by noting that Einstein's relativity principle, "The laws of physics must be the same in all inertial reference frames" [Serway and Jewett, 2019, p. 1018] or "no preferred reference frame" (NPRF) for short, is generalized from Galileo's relativity principle, "The laws of mechanics must be the same in all inertial frames of reference" [Serway and Jewett, 2019, p. 1013]. [Serway and Jewett, 2019, p. 1018] and [Knight, 2008, p. 1149] then show that NPRF explains the light postulate of SR, i.e., that everyone measures the same speed of light $c$, regardless of their motion relative to the source. If there was only one reference frame for a source in which the speed of light equalled the prediction from Maxwell's equations ($c = \frac{1}{\sqrt{\mu_o \epsilon_o}}$), then that would certainly constitute a preferred reference frame. The mysteries of time dilation and length contraction are then understood to follow from this empirically discovered fact (light postulate), which itself follows from NPRF, rather than from any constructive fact, such as the luminiferous aether.

In summary, NPRF explains the light postulate whence the Lorentz transformations, time dilation, length contraction, and the relativity of simultaneity (kinematics of SR). Since Alice and Bob always measure the same speed of light $c$ regardless of their relative motion per NPRF, Alice says Bob's temporal and spatial measurements need to be corrected per time dilation and length contraction while Bob says the same thing about Alice's measurements. But, if NPRF is true and fundamental, then neither Alice's nor Bob's measurements need to be corrected (relativity of simultaneity). Thus, the mysteries of length contraction and time dilation in SR ultimately reside in NPRF. We will see in section 4 that the violation of Bell's inequality also ultimately resides in NPRF per Information Invariance & Continuity [Brukner and Zeilinger, 2009]. But first, we will briefly explicate Bell's inequality.

# 3 Bell's Theorem

There are by now many ways to set up Bell's theorem, but in general the proof is supposed to establish that given the probability matrix of QM confirmed by many experiments, some provisional classical assumptions such as locality, measurement independence, local realism, etc., have got to go. Herein we will basically follow the presentation in [Myrvold et al., 2021]. We will explicate the standard Bell assumptions 1 and 2 and various constructive attempts at rejecting them.

## 3.1 The Measurement Independence/Statistical Independence Assumption

This assumption says the future measurement settings are not part of what determines the time-evolution of the prepared state, i.e., what the prepared state will evolve into does not depend on the future measurement settings. Thus, this is the assumption that the preparation probability distribution be independent of experimental settings.

It is widely believed that one can reject this assumption by positing a common cause in the past that determines both experimental settings and experimental outcomes. Historically this has been called superdeterminism. The other possibility is to posit a causal influence from the future in which the settings, though free variables, causally influence the state of the system at the moment of preparation. Historically this has been called retrocausality. Hossenfelder calls both such models "superdeterminism" because for her by definition superdeterminism means a violation of statistical independence [Hossenfelder, 2020]. Again, many resist this equivalency because while both models do violate statistical independence they allegedly do so in very different ways [Price and Wharton, 2016]. Superdeterminism presumably suggests a causal or dynamical process that moves from past to future and retrocausality allegedly suggests such a process that moves from future to past.

However this simplistic picture is complicated by the fact that critiques of such accounts [Chen, 2020, Stuckey et al., 2015, Silberstein et al., 2021, Silberstein et al., 2008, Kastner, 2013, Maudlin, 2011]) call into question exactly what "move", "evolve in time", or "causally influence" could possibly mean in such spacetime or eternalist models where future outcomes already exist. In order to address such concerns many retrocausal theorists such as Wharton have taken to talking about "all at once" causal explanations, what he calls the Lagrangian approach, or falling back on purely agent-based or perspectival accounts of causation as Price does. Such moves suggest a more time-symmetric, dynamical-symmetric, or deflationary account of causation as opposed to anything like real retrocausation. For more details and critical reactions see [Silberstein et al., 2021].

Many have also raised concerns about superdeterminism in that it entails a conspiracy theory such as having to say that unlikely and inexplicable conditions at the big bang are the common cause of both experimental settings and experimental outcomes. In response to this concern [Hossenfelder, 2020] says the following:

> In a superdeterministic theory, the evolution of the prepared state depends on what the detector setting is at the time of measurement. This does not necessarily mean the hidden variables were correlated with the measurement settings at the time of preparation. This correlation may instead have come about dynamically.

As others have noted, such a move might remove the worry about conspiracy, but it raises many other concerns such as what possible naturalistic causal or dynamical mechanism could account for all such heterogeneous instantiations of Bell experiments which could involve everything from conscious humans to computers making the choices of measurement settings. Our primary point here however is that if the fundamental superdeterministic dynamics time evolving the prepared state somehow "know" about the detector settings at the time of measurement, how different is that really from retrocausation.

The bottom line for our purposes is that no matter how you taxonomize or characterize superdeterministic or retrocausal accounts, their mode of explanation is fundamentally constructive, i.e., causal or dynamical, and it is precisely this feature of such explanations that leads to trouble. Such accounts can vary in many ways such as regards dynamical determinism or stochasticity, definite values or contextuality, etc., but it in no way affects our point.

### 3.2 The Local Causality Assumption

This assumption is the conjunction of the following:

- The condition of causal locality: There exists no causal relations between events that are outside of one another's light-cones.

  It is generally believed that one can reject this assumption by invoking nonlocality (i.e., superluminal causal influences aka causal influences between spacelike separated events) via a relativistically preferred frame (as Bohmian mechanics and spontaneous collapse theories attempt to do).

- The Common Cause Principle: The aforementioned assumption that correlations between two variables that are not in a cause-effect relation to each other must be explained by some past common cause a la Reichenbach.

  It is generally believed that one can reject this by either adding something like retrocausality, temporally symmetric causal or dynamical relations as we discussed above, or by all together giving up the idea that all correlations must be explicable in terms of cause-effect or dynamical relations of any sort whatsoever, whether such relations be timelike from past to future (superdeterminism), future to past (retrocausality), time-symmetric (Lagrangian approach), or whether they be spacelike (nonlocality). The latter is the move we make in this paper. As you can see, this is far more general a move than simply rejecting Reichenbach's original Common Cause Principle. However as we noted in the last section there is a precedent in SR. That is, SR is a principle theory [Felline, 2011] and the postulates of SR are constraints offered without a corresponding constructive explanation, i.e., causal or dynamical explanation.

## 4 An Adynamical and Acausal Account of the CHSH Ineqality

As we will see in this section, in providing a fully adynamical and acausal explanation of the experimental violation of the CHSH inequality for spin-$\frac{1}{2}$ particles, contrary to accepted wisdom, we need not modify or underwrite QM in any way, give up on locality, local realism, or statistical independence. By

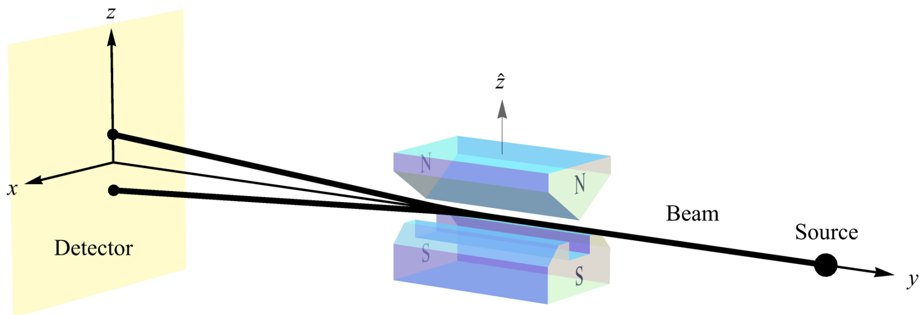


Figure 1: A Stern-Gerlach (SG) spin measurement showing the two possible outcomes, up $(+\frac{\hbar}{2})$ and down $(-\frac{\hbar}{2})$ or $+1$ and $-1$, for short. The important point to note here is that the classical analysis (Figure 2) predicts all possible deflections, not just the two that are observed. Figure reproduced from [Stuckey et al., 2022a].

the time we get to the end of this section it will be clear that just as for SR and the speed of light $c$, no preferred reference frame (NPRF) with respect to Planck's constant $h$ is the key to our principle explanation of the violation of the CHSH inequality.

## 4.1 Spin-$\frac{1}{2}$ Reference Frames

The relativity principle (NPRF) will be applied herein to reference frames related by spatial rotations. While spatial rotations plus Lorentz boosts constitute the restricted Lorentz group, spatial rotations plus Galilean boosts constitute the homogeneous Galilean group, so the use of the relativity principle here does not imply Lorentz invariance. More specifically, these spatial reference frames will be those established by mutually complementary measurements per [Brukner and Zeilinger, 1999, Brukner and Zeilinger, 2003, Brukner and Zeilinger, 2009] using the "closeness requirement: the dynamics of a single elementary system can be generated by the invariant interaction between the system and a 'macroscopic transformation device' that is itself described within the theory in the macroscopic (classical) limit" of [Dakic and Brukner, 2016, Dakic, 2021]. This is due to the fact that the measuring devices used to measure quantum systems are themselves made from quantum systems. For example, the classical magnetic field of a Stern-Gerlach (SG) magnet is used to measure the spin of spin-$\frac{1}{2}$ particles and that classical magnetic field "can be seen as a limit of a large coherent state, where a large number of spin-$\frac{1}{2}$ particles are all prepared in the same quantum state" [Brukner, 2021].

As [Weinberg, 2017] points out, measuring an electron's spin via SG magnets constitutes the measurement of "a universal constant of nature, Planck's

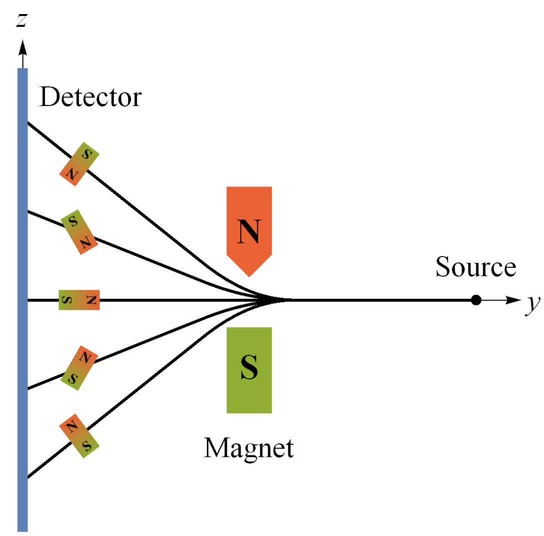


Figure 2: The classical constructive model of the Stern-Gerlach (SG) experiment. If the atoms enter with random orientations of their "intrinsic" magnetic moments (due to their "intrinsic" angular momenta), the SG magnets should produce all possible deflections, not just the two that are observed [Knight, 2008, Franklin and Perovic, 2019]. Compare with Figure 1. Figure reproduced from [Stuckey et al., 2022b].

constant" (Figure 1). So if NPRF applies here, everyone must measure the same value for Planck's constant $h$ regardless of their SG magnet orientations relative to the source, which like the light postulate is an empirically discovered fact. That means the spin-$\frac{1}{2}$ particles must always be deflected the same amount up or down relative to the magnetic field of the SG magnets, regardless of their orientation relative to the source. This prediction differs from the classical constructive model for spin-$\frac{1}{2}$ particles as shown in Figure 2.

So, per [Brukner and Zeilinger, 1999, Brukner and Zeilinger, 2003, Brukner and Zeilinger, 2009], if we identify the preparation state $|\psi\rangle = |u\rangle$ at $\hat{z}$ (Figure 3) with the reference frame of mutually complementary spin measurements $[J_x, J_y, J_z]$, then the closeness requirement means our reference frame of mutually complementary spin measurements is $[\hat{x}, \hat{y}, \hat{z}]$ in real space. Thus, [Brukner and Zeilinger, 2003] depict the Bloch sphere in that real space reference frame with associated SG magnet orientations a la Figure 4. Unless otherwise noted, we also make this association throughout, so that "the reference frame of a complete set of mutually complementary measurements" is simply "the reference frame." Accordingly, the reference frame associated with the spin measurement at $\hat{b}$ in Figure 3 is rotated by $\theta$ relative to $[\hat{x}, \hat{y}, \hat{z}]$

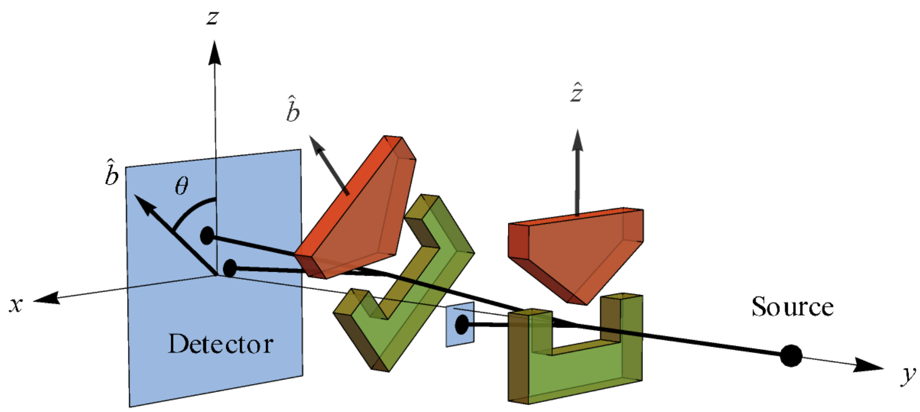


Figure 3: In this set up, the first SG magnets (oriented at $\hat{z}$) are being used to produce an initial state $|\psi\rangle = |u\rangle$ for measurement by the second SG magnets (oriented at $\hat{b}$). Figure reproduced from [Stuckey et al., 2022b].

(Figure 5).

All of this is in accord with the fundamental quantum information-theoretic principle of Information Invariance & Continuity, which states [Brukner and Zeilinger, 2009]:

> The total information of one bit is invariant under a continuous change between different complete sets of mutually complementary measurements.

And this is in accord with the first axiomatic reconstruction of quantum mechanics based on information-theoretic principles [Hardy, 2001]. As Hardy points out in that paper, if you delete just the word "continuous" from his Axiom 5, then quantum probability theory becomes classical probability theory instead. Since quantum information theorists are "reconstructing" not "revising" QM, we must be able to relate this to a more familiar concept and that concept is of course, superposition. Their reconstructions are nonetheless valuable because they show us that all of finite-dimensional QM is built upon this principle of Information Invariance & Continuity for an elementary piece of quantum information, the qubit. Therefore, we see that NPRF resides at the foundation of QM precisely as it resides at the foundation of SR [Stuckey et al., 2022b].

## 4.2 Violating the CHSH Inequality

Now we're ready to discuss a specific Bell inequality. Probably the most widely used version of Bell's inequality is the Clauser-Horne-Shimony-Holt (CHSH) inequality [Clauser et al., 1969]. We will follow Mermin's notation [Mermin, 2005] in what follows.

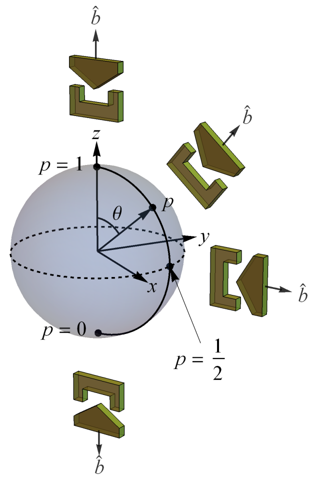


Figure 4: Probability state space for the measurement of Figure 3. Since this state space is isomorphic to 3-dimensional real space, the Bloch sphere is shown in a real space reference frame with its related Stern-Gerlach (SG) magnet orientations [Brukner and Zeilinger, 2003]. Figure reproduced from [Stuckey et al., 2022b].

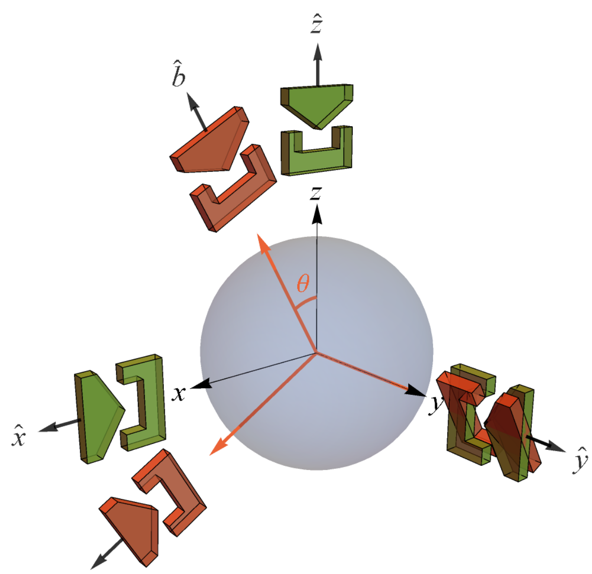


Figure 5: Reference frames for the complementary SG spin measurements associated with Figures 3 and 4 [Brukner and Zeilinger, 2003]. Figure reproduced from [Stuckey et al., 2022b].

Suppose Alice and Bob we are making spin-$\frac{1}{2}$ ("spin" for short) measurements on one of the Bell spin states given by

$$
\begin{aligned}
|\psi_-\rangle &= \frac{|u\rangle \otimes |d\rangle \, - |d\rangle \otimes |u\rangle}{\sqrt{2}} \\
|\psi_+\rangle &= \frac{|u\rangle \otimes |d\rangle + |d\rangle \otimes |u\rangle}{\sqrt{2}} \\
|\phi_-\rangle &= \frac{|u\rangle \otimes |u\rangle \, - |d\rangle \otimes |d\rangle}{\sqrt{2}} \\
|\phi_+\rangle &= \frac{|u\rangle \otimes |u\rangle + |d\rangle \otimes |d\rangle}{\sqrt{2}}
\end{aligned}
\tag{1}
$$

in the eigenbasis of $\sigma_z$. $u$ stands for the spin measurement outcome "up" (+1 numerically) and $d$ stands for the spin measurement outcome "down" (−1 numerically), as usual. The first state $|\psi_-\rangle$ is called the "Bell spin singlet state" (or "singlet state" for short) and it represents a total conserved spin angular momentum of zero ($S = 0$) for the two particles involved. The other three states are called the "Bell spin triplet states" (or "triplet states" for short) and they each represent a total conserved spin angular momentum of one ($S = 1$, in units of $\hbar = 1$) in their respective plane of symmetry [Stuckey et al., 2020]. In all four cases, the entanglement represents the conservation of spin angular momentum for the process creating the state (see for example, [Dehlinger and Mitchell, 2002]).

If Alice is making her $\hat{a}$ measurement of spin and Bob is making his $\hat{b}$ measurement of spin (Figure 6), then we write

$$
\begin{aligned}
\sigma(a) &= \hat{a} \cdot \vec{\sigma} = a_x\sigma_x + a_y\sigma_y + a_z\sigma_z \\
\sigma(b) &= \hat{b} \cdot \vec{\sigma} = b_x\sigma_x + b_y\sigma_y + b_z\sigma_z
\end{aligned}
\tag{2}
$$

Using this notation, the correlation functions for the Bell spin states are given by

$$
\begin{aligned}
\langle\psi_-|\sigma(a)\sigma(b)|\psi_-\rangle &= \quad -a_xb_x - a_yb_y - a_zb_z \\
\langle\psi_+|\sigma(a)\sigma(b)|\psi_+\rangle &= \qquad a_xb_x + a_yb_y - a_zb_z \\
\langle\phi_-|\sigma(a)\sigma(b)|\phi_-\rangle &= \quad -a_xb_x + a_yb_y + a_zb_z \\
\langle\phi_+|\sigma(a)\sigma(b)|\phi_+\rangle &= \qquad a_xb_x - a_yb_y + a_zb_z
\end{aligned}
\tag{3}
$$

As you see, the singlet state correlation function is $-\cos(\theta)$ anywhere in space where $\theta$ is the angle between $\hat{a}$ and $\hat{b}$. This means the singlet state gives opposite results ($ud$ or $du$, so that $S = 0$) when $\hat{a} = \hat{b}$. The triplet states correlation function is $\cos(\theta)$ as long as $\hat{a}$ and $\hat{b}$ reside in the appropriate symmetry plane, e.g., the $xy$ plane for $|\psi_+\rangle$. Thus, the triple states give the same results ($uu$ or $dd$, so that $S = 1$) when $\hat{a} = \hat{b}$ in the respective symmetry plane.

Consider measurements on a Bell spin triplet state in its symmetry plane. Since Alice and Bob must always measure the same outcomes when making the same spin measurements, the values of spin for each particle are coordinated in some fashion. Assuming no superluminal causal influences between Alice's(Bob's) measurement setting events and Bob's(Alice's) outcome events (Figure 7), one might use "instruction sets" to account for this correlation. That is, the particles are "scripted" to give the same outcomes for every possible spin measurement that Alice and Bob might make on them.

Suppose Alice and Bob confine themselves to making two measurements each, i.e., Alice makes measurements a and d while Bob makes measurements b and c. Let the instruction set for Alice's particle in trial $j$ of the measurement sequence be written $A_a(j)$ for her measurement a and $A_d(j)$ for her measurement d. Similarly, the instruction set for Bob's particle in trial $j$ of the measurement sequence is written $B_b(j)$ for his measurement b and $B_c(j)$ for his measurement c. Now define a quantity

$$\gamma_j := A_a(j)B_b(j) + A_a(j)B_c(j) + A_d(j)B_b(j) - A_d(j)B_c(j) \tag{4}$$

Since $A_x(j) = \pm 1$ and $B_y(j) = \pm 1$, $\gamma_j$ is either $+2$ or $-2$ for any $j$. Thus, the average of $\gamma_j$ for $N$ trials (called $\gamma$) is confined to the range $[-2, +2]$

$$\begin{aligned} -2 \le \gamma = \frac{1}{N}\sum_j \gamma_j = \frac{1}{N}\sum_j A_a(j)B_b(j) + \frac{1}{N}\sum_j A_a(j)B_c(j) + \\ \frac{1}{N}\sum_j A_d(j)B_b(j) - \frac{1}{N}\sum_j A_d(j)B_c(j) \le 2 \end{aligned} \tag{5}$$

This is the CHSH inequality. Since Alice and Bob can each only make a single measurement for each trial $j$, what we obtain experimentally is

$$\begin{aligned} \gamma_{exp} = \frac{1}{N_{ab}}\sum_{j\in X_{ab}} A_a(j)B_b(j) + \frac{1}{N_{ac}}\sum_{j\in X_{ac}} A_a(j)B_c(j) + \\ \frac{1}{N_{db}}\sum_{j\in X_{db}} A_d(j)B_b(j) - \frac{1}{N_{dc}}\sum_{j\in X_{dc}} A_d(j)B_c(j) \end{aligned} \tag{6}$$

where the sets $X_{ab}$, $X_{ac}$, $X_{db}$, and $X_{dc}$ are disjoint (this is an important point, as we will see). It is well known that QM predicts violations of the CHSH inequality, as we now show for the Bell spin states.

Suppose Alice and Bob are making their spin measurements on $|\phi_+\rangle$ in its symmetry ($xz$) plane (Figure 6). Then QM predicts

$$\begin{aligned} \gamma_{exp} = \langle\phi_+|\sigma(a)\sigma(b)|\phi_+\rangle + \langle\phi_+|\sigma(a)\sigma(c)|\phi_+\rangle + \\ \langle\phi_+|\sigma(d)\sigma(b)|\phi_+\rangle - \langle\phi_+|\sigma(d)\sigma(c)|\phi_+\rangle \end{aligned} \tag{7}$$

or

$$\gamma_{exp} = \cos(\theta_{ab}) + \cos(\theta_{ac}) + \cos(\theta_{db}) - \cos(\theta_{dc}) \tag{8}$$

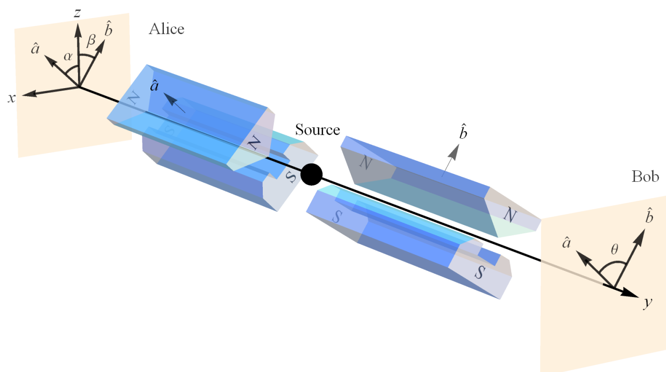


Figure 6: Alice and Bob making spin measurements on a pair of spin-entangled particles in the symmetry plane $xz$ of $|\phi_+\rangle$ with their Stern-Gerlach (SG) magnets and detectors. Figure reproduced from [Silberstein et al., 2021].

Using the measurements shown in Figure 8 where $\theta_{ab} = \theta_{ac} = \theta_{db} = 45°$ and $\theta_{dc} = 135°$, we see that QM predicts $\gamma_{exp} = 2\sqrt{2}$. Using these same measurements on the singlet state gives $\gamma_{exp} = -2\sqrt{2}$. These limits are called the Tsirelson bound and they are the largest possible violations of the CHSH inequality for QM [Cirel'son, 1980, Landau, 1987, Khalfin and Tsirelson, 1992].

### 4.3 Violating the Bell Assumptions

Of course, it is trivially possible to account for the violation of the CHSH inequality if one gives up the assumption of locality as in Figure 7, e.g., the superluminal pilot waves of Bohm. The other option, as we explained in section 3, is to give up the assumption of statistical independence as seen with superdeterminism and retrocausation. Let us see how that works with our instruction sets of section 4.2.

Since each trial $j$ only contributes to one of the disjoint sets $X_{ab}$, $X_{ac}$, $X_{db}$, and $X_{dc}$ in $\gamma_{exp}$, it is mathematically possible to produce $\gamma_{exp}$ in the range $[-4, +4]$ with our instruction sets. We show how each extreme could happen in Figure 9. For example, row 1 columns 1 and 2 tell us that for all trials when the source produces particles with the instruction set $[++++]$, Alice and Bob must never choose the measurement setting pair dc. Likewise, row 2 columns 1 and 2 tell us that for all trials when the source produces particles with the instruction set $[+++-]$, Alice and Bob must never choose the measurement setting pair ac (as they did in Figure 7). Etc. Assuming all measurement setting pairs shown in column 2 are made with equal frequency, Alice and Bob will obtain $\gamma_{exp} = 4$ and their experiment will be in accord with statistical independence as far as they know. That is, they will see no correlation between what the source produced and their (apparently) random and independent measurement setting

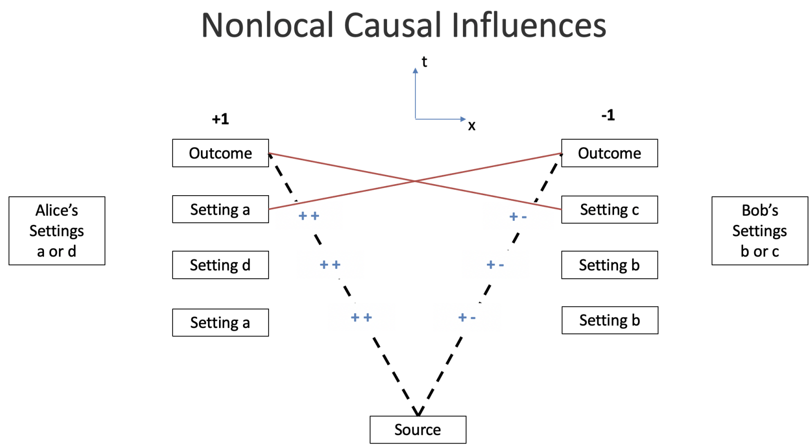


Figure 7: Alice and Bob switch constantly between their two settings until a detection event produces an outcome. In this picture, the instruction set is $[+ + + -]$ that tells us the outcomes of the measurements [a d c b], respectively. The red lines represent information about Alice's(Bob's) detector setting at Bob's(Alice's) outcome event.

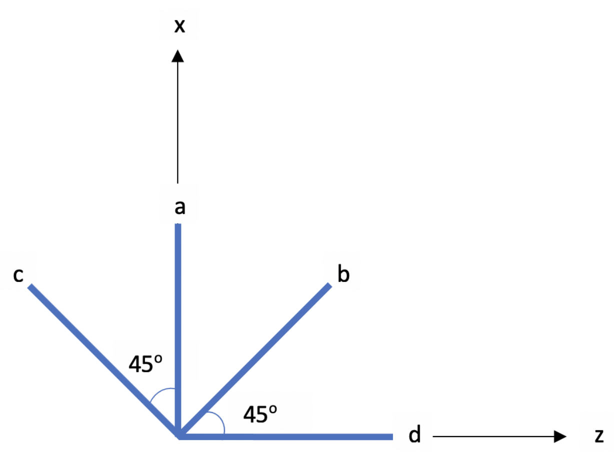


Figure 8: Alice and Bob's CHSH inequality violating measurements of the Bell spin state $|\phi_+\rangle$. The angles are half these for photons (spin-1), e.g., see Fig. 5 in [Dehlinger and Mitchell, 2002].

| a | d | b | c | $\gamma_{exp} = +4$ | $\gamma_{exp} = -4$ | $\gamma_j$ |
|---|---|---|---|---|---|---|
| + | + | + | + | ab ac db | dc | +2 |
| + | + | + | - | ab db dc | ac | +2 |
| + | + | - | + | ac | ab db dc | -2 |
| + | - | + | + | ab ac dc | db | +2 |
| - | + | + | + | db | ab ac dc | -2 |
| + | + | - | - | dc | ab ac db | -2 |
| + | - | + | - | ab | ac db dc | -2 |
| - | + | + | - | ac db dc | ab | +2 |
| + | - | - | + | ac db dc | ab | +2 |
| - | + | - | + | ab | ac db dc | -2 |
| - | - | + | + | dc | ab ac db | -2 |
| + | - | - | - | db | ab ac dc | -2 |
| - | + | - | - | ab ac dc | db | +2 |
| - | - | + | - | ac | ab db dc | -2 |
| - | - | - | + | ab db dc | ac | +2 |
| - | - | - | - | ab ac db | dc | +2 |

Figure 9: The sixteen instruction sets used for the CHSH inequality (column 1) and Alice and Bob's measurement settings for each instruction set producing $\gamma_{exp} = \pm 4$ (columns 2 and 3, respectively). Column 4 shows the value of $\gamma_j$ for each instruction set.

choices because the sixteen instruction sets are hidden. Figures 10 and 11 show how retrocausality and superdeterminism might explain such "conspiratorial" violations of statistical independence.

### 4.4 The Principle Culprit

Now let us show how NPRF is responsible for the violation of the CHSH inequality. Keep in mind, a principle explanation simply uses a compelling fundamental principle to explain an empirically discovered fact that dictates the kinematics constraining the dynamics. So, all we need to do is show how NPRF is responsible for the correlation functions in Eq. (8) and that turns out to be quite simple [Stuckey et al., 2020].

We start with our two sets of data, Alice's set and Bob's set. They were collected in $N$ pairs (data events) with Bob's(Alice's) SG magnets at $\theta$ relative to Alice's(Bob's) (Figure 6). We want to compute the correlation function for these $N$ data events which is

$$\langle \alpha, \beta \rangle = \frac{(+1)_A(-1)_B + (+1)_A(+1)_B + (-1)_A(-1)_B + ...}{N} \tag{9}$$

Now partition this into two equal subsets per Alice's equivalence relation, i.e., Alice's $+1$ results and Alice's $-1$ results, to give

$$\langle \alpha, \beta \rangle = \frac{1}{2}(+1)_A\overline{BA+} + \frac{1}{2}(-1)_A\overline{BA-} \tag{10}$$

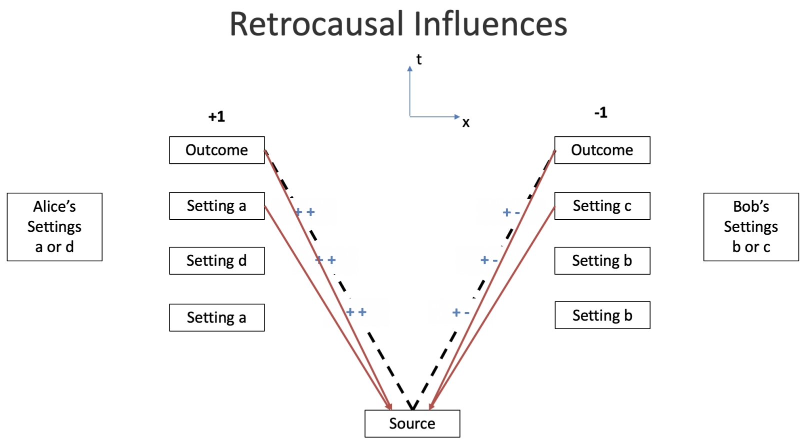


Figure 10: Alice and Bob switch constantly between their two settings until a detection event produces an outcome. In this picture, the instruction set is $[+ + + -]$ that tells us the outcomes of the measurements [a d c b], respectively. The red arrows represent information about the detector settings and their outcomes available at the source for the emission event.

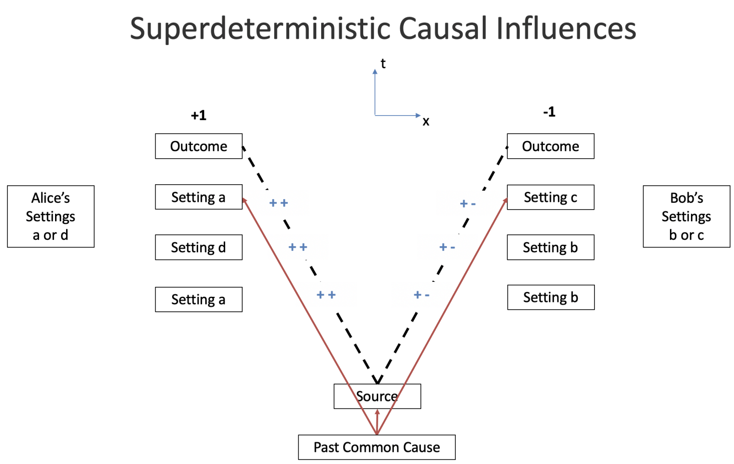


Figure 11: Alice and Bob switch constantly between their two settings until a detection event produces an outcome. In this picture, the instruction set is $[+ + + -]$ that tells us the outcomes of the measurements [a d c b], respectively. The red arrows represent causal influences from a common cause in the past on both the Source emission event and both detector setting events.

with the overline denoting average. This correlation function is independent of the formalism of QM. All we have assumed is that Alice and Bob measure $+1$ or $-1$ with equal frequency at any setting per NPRF in computing this correlation function.

We just need to know what Bob averages for all trials where Alice measured $+1$ ($\overline{BA+}$) and what he averages for all trials where Alice measured $-1$ ($\overline{BA-}$). Since Bob would have measured $\pm 1$, respectively, in all those trials if $\alpha$ had been equal to $\beta$ (Figure 6), Alice says Bob should be measuring $\pm\cos\theta$, respectively, in every trial to conserve spin angular momentum (Figure 12) per the classical constructive model (Figure 2). But, if that happened, Alice would clearly occupy a preferred reference frame, since her reference frame alone would allow for measuring the right value for Planck's constant $h$ (all other frames rotated with respect to hers would be getting some fraction of $h$). Thus, Bob's outcomes can only satisfy conservation of spin angular momentum *on average* according to Alice, i.e., his $\pm 1$ outcomes can only *average* $\pm\cos\theta$, so that he is also measuring the right value for $h$ in every trial as required by NPRF. Using $\overline{BA\pm} = \pm\cos\theta$ in Eq. (10) we obtain $\langle \alpha, \beta \rangle = \cos\theta$ in agreement with Eq. (3) giving Eq. (8). So, since NPRF requires everyone measure the value for $h$ regardless of their SG magnet orientation relative to the source, it must be possible to violate the CHSH inequality up to the Tsirelson bound [Stuckey et al., 2019].

We should point out that the trial-by-trial outcomes for this "average-only" conservation can deviate substantially from the target value required for explicit conservation per Alice or Bob's reference frame. For example, we might have Bob's $+1$ and $-1$ outcomes averaging to zero as required for the conservation of spin angular momentum per Alice's reference frame. Thus, Alice says Bob's measurement outcomes are violating the conservation of spin angular momentum as egregiously as possible on a trial-by-trial basis. However, we could also partition the data according to Bob's equivalence relation (his $\pm 1$ results), so that it is Bob who claims Alice must average her results to satisfy "average-only" conservation (Figure 13). From the perspective of Bob's reference frame, it is Alice's $+1$ and $-1$ outcomes averaging to zero that violate the conservation of spin angular momentum as egregiously as possible on a trial-by-trial basis. This is totally analogous to the relativity of simultaneity in SR. There, Alice partitions spacetime per her equivalence relation (her surfaces of simultaneity) and says Bob's meter sticks are short and his clocks run slow, while Bob can say the same thing about Alice's meter sticks and clocks per his surfaces of simultaneity.

This result clearly shows that NPRF applied to the invariant measurement of Planck's constant $h$ alone is sufficient to entail the violation of the CHSH inequality in a way that does not require or suggest any constructive explanation. Thus, as promised at the outset, we have shown that contrary to popular belief, neither Bell's theorem nor the experimental violation of its inequality entails that one must give up locality, local realism, or measurement independence in any form, nor must one embellish or replace QM with new dynamical laws or causal mechanisms. Again, the price is that one must let go of the requirement that all such explanations must be causal or dynamical in some sense.

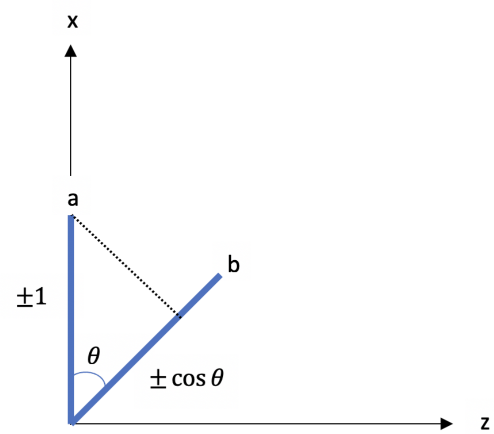


Figure 12: Per Alice, Bob should be measuring $\pm\cos\theta$ when she measures $\pm 1$.

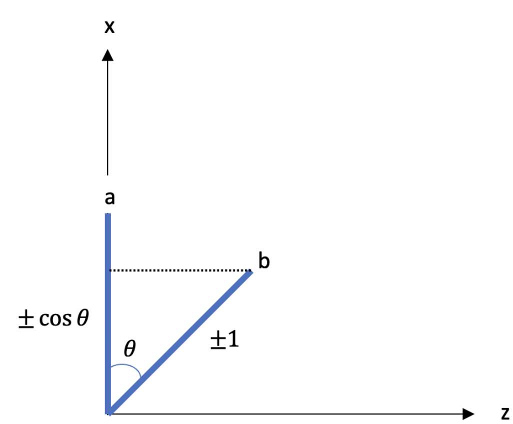


Figure 13: Per Bob, Alice should be measuring $\pm\cos\theta$ when he measures $\pm 1$.

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