No Quantum Threat to Special Relativity.

Abstract

Following Jarrett, Shimony, Hellman etc., an important school of interpretation of Bell-type theorems is that parameter independence (PI) is a locality condition while outcome independence (OI), and hence factorizability, are not. A recent series of articles (e.g. Butterfield, 2007; Norsen, 2009; Norsen, 2011; Seevinck, 2010; Seevinck and Uffink, 2011) challenges this interpretation by appealing to Bell's own analysis of "local causality". In this article, a defense of the mainstream interpretation is offered. It is argued that Bell's "local causality" is inadequate as a condition of locality in the stochastic case, and a proper condition is provided instead. The analysis of Stochastic Einstein Locality offered in Butterfield (2007) is also critically assessed. It is shown that the space-time version of PI is necessary, but not sufficient for locality. As for the space-time version of OI, it is argued that it is a version of the principle of common cause, and that it is not a locality condition.

Keywords

Quantum Mechanics; Locality; Bell's Theorem; Factorizability; Local Causality; Stochastic Einstein Locality; Parameter Independence; Outcome Independence; Bell's Theorem; Principle of Common Cause

1 Introduction

Most Bell-type theorems follow the general scheme: from a set of requirements that candidate theories are assumed to satisfy, one derives some inequalities (the Bell Inequalities – henceforth the BI), which are shown to be violated by some quantum statistical predictions. Furthermore, it is widely agreed that there is strong empirical evidence that such predictions are accurate, and that the BI are violated by some phenomena. The upshot follows by a simple modus tollens: no candidate theory or model satisfying the requirements used for the derivation can give an account of all quantum phenomena.

The interpretations of the experimental violation of the BI mostly vary with the set of requirements considered in the derivation. This article focuses on Bell-type theorems in which the BI are derived from the condition of factorizability on the probability distributions of the outcome-events. Factorizability is a condition on determinate[1] probabilistic models of the outcomes of a Bell-type experiment. Considering the case of a spin measurement, let λ stand for the "determinate state"[2], i (respectively j) for the setting of the apparatus **A** (respectively **B**) measuring a particular component of spin of one of the systems[3], and a (respectively b) for the possible outcomes of measurements by apparatus **A** (respectively **B**) of the spin component, then factorizability is defined as follows:

Definition 1 – ***Factorizability***

A probabilistic model for Bell-type situations is factorizable if and only if:

$$p(a, b|i, j, \lambda) = p_(a|i, \lambda)p(b|j, \lambda).$$

Factorizability[4] thus requires that the probabilities of outcomes at each end of the experiment be statistically independent, each conditional on the complete specification of the state and measurement protocol at its respective end of the experiment.

Factorizability was introduced by Bell as a consequence of "local causality", which he takes to be a locality condition for determinate probabilistic models (Bell, 1990, 243). If this is correct, then the conclusion of the Bell-type modus tollens is that no local determinate model can give an account of all quantum phenomena. This is troubling because it seems to be in conflict with relativity theory.[5] As Bell (1987, 172) puts it:

[1] A theory or model is determinate if and only if the measurement outcomes are determinate, namely each experiment will have definite results which may be fixed deterministically or stochastically by the state of the system (and the measurement context) before the measurement. Note that determinateness implies neither determinism nor value definiteness.

[2] The "determinate state" is the state that the candidate determinate theory will ascribe to the system and which yields probability distributions for the outcomes. We will avoid using the term "hidden state" as it is at best confusing, at worse misleading. Note also that whether λ is a single variable or a set of variables does not matter in the context of the following.

[3] The possibility that the apparatuses might also have hidden states has important implications, but not for our results. See Jones and Clifton (1993) for details.

[4] In our formulation of factorizability, we take apparatus settings i and j as well as the hidden state λ as a variables on which we conditionalize, and not as a parameter which appears as an index. In the deterministic case, the settings of the apparatuses have to be taken as parameters on pain of contradiction (see Wüthrich (2004, p.61-72) for more details). Since we confine ourselves with the stochastic case, we are not constrained to define a new probability measure where i and j are fixed. We shall indeed consider in general that the settings of the apparatuses are the result of a stochastic physical process, and not the result of the free choice of the experimenters. Now, whether the determinate state λ should be taken as a parameter or a variable depends on whether or not we assume that we can assign a probability to it. We shall assume throughout that probabilities can be assigned both to λ and to i, j; therefore we shall consistently use the framework of conditional probabilities. The extent to which this stands in conflict with the views of Butterfield (2007) and Seevinck and Uffink (2011) will be discussed later on.

[5] This might hinge on a certain interpretation of relativity theory. For a serious discussion concerning

> For me then this is the real problem with quantum theory: the apparently essential conflict between any sharp formulation of fundamental relativity. That is to say, we have an apparent incompatibility, at the deepest level, between the two fundamental pillars of contemporary theory ...

The discussion concerning the interpretations of the experimental violation of the BI by quantum phenomena have focused on the issue of whether such a conflict exists. An influential result in this regard was (independently) shown by Jarrett (1984) and Shimony (1986). They showed that factorizability in the conjunction of two distinct conditions on the probability distributions over outcome-events: Outcome and Parameter Independence.[6]

Outcome Independence (henceforth OI) expresses the requirement that the outcome of the measurement on one subsystem be independent of the outcome of the measurement on the other one:

Definition 2 – ***Outcome Independence***

A probabilistic model for Bell-type situations satisfies Outcome Independence if and only if for all a, b, i, j, and λ:

$$p(a|b,i,j,\lambda) = p(a|i,j,\lambda) \quad \textit{and} \quad p(b|a,i,j,\lambda) = p(b|i,j,\lambda);$$

or, equivalently:

$$p(a,b|i,j,\lambda) = p(a|i,j,\lambda)p(b|i,j,\lambda).$$

OI is a condition of statistical independence of the outcome-events. In addition to mere statistical independence, what is needed to obtain Factorizability is Parameter Independence (henceforth PI):

Definition 3 – ***Parameter Independence***

A probabilistic model for Bell-type situations satisfies Parameter independence if and only if for all a, b, i, i', j, j' and λ :

$$p(a|i,j,\lambda) = p(a|i,j',\lambda) \quad \textit{and} \quad p(b|i,j,\lambda) = p(b|i',j,\lambda).$$

Quantum Mechanics, Bell-type theorems, and the theory of relativity, see Maudlin (1994) and references therein.

[6]The distinction had been made before, but Jarrett and Shimony were the first to claim its relevance to the issue whether the experimental violation of the BI stands in conflict with relativity theory. For more details, see also Jarrett (1989) and Shimony (1989).

Note also that the phrases ‘parameter independence’ and ‘outcome independence’ are Shimony’s. Jarrett calls similar conditions respectively ‘locality’ and ‘completeness’. Shimony’s and Jarrett’s conditions are not identical, but the differences are not relevant to the argument presented here.

that is, the requirement that the outcome of the measurement on one subsystem not depend upon the setting of the apparatus measuring the other one.

Jarrett and Shimony argued that, while any theory violating PI would stand in conflict with relativity theory, a theory violating only OI could stand in, as Shimony (1978) puts it, "peaceful coexistence" with it. The core of their argument is that while a failure of PI could in principle lead to superluminal signaling, it is not the case for a failure of OI.

Following Jarrett and Shimony, it has been widely accepted that, the violation of the BI do not necessarily conflict with the locality constraints that many take relativistic theories to impose on physical processes. To put it simply, this school of interpretation holds that:

Definition 4 – ***(PI-LOC)***:

Parameter independence (PI) is a locality condition while outcome independence (OI) is not.

If true, then the BI are not derived from a principle of locality *alone*.

Criticisms have been recurrently leveled against this interpretation in the literature (e.g. Berkovitz, 1998a; Berkovitz, 1998b; Jones and Clifton, 1993; Maudlin, 1994). Most of the debate has focused on the questions of whether Jarrett's and Shimony's arguments in terms of signaling are sound, or of whether the distinction between outcome and parameter independence is relevant to various issues of interpretation.

A more recent line of criticism (e.g. Butterfield, 2007; Norsen, 2009; Norsen, 2011; Albert and Galchen, 2009; Seevinck, 2010; Seevinck and Uffink, 2011) consists in insisting that the BI should be seen as derived from *locality alone*. That is to say, the experimental violation of the BI would constitute a full blow to locality. For example, Albert and Galchen (2009) quite dramatically write:

> And so the actual physical world is nonlocal. Period.

Critics following this trend often argue that this corresponds to Bell's actual interpretation of his own theorem, and that this has been largely misunderstood by the protagonists of the traditional debate.

This article provides a defense of $PI - LOC$ against these most recent attacks.[7] It is argued that Bell's "local causality" is an inadequate notion of locality in the stochastic context. It is also argued that Butterfield's attempt at deriving the space-time version of outcome independence from Stochastic Einstein Locality fails. Thus the space-time version of outcome independence does not qualify as a locality condition, and $PI - LOC$ remains unchallenged.

In Section 2, a space-time structure is specified and the question of how Bell-type situations

[7]For a recent take on these attacks from a very different point of view, see Friederich (2015).

can be embedded in it is discussed. To this aim, we follow Earman (1986)[8] and define locality as Einstein Locality. With this in hand, formulations for the space-time (ST) versions of Factorizability, OI and PI (STFAC, STOI and STPI, respectively) are proposed and defended.

Section 3 deals with the issue of whether or not Einstein Locality entails STPI and/or STOI. The question of how to construe Einstein Locality in the stochastic context is discussed. It is argued (contra Norsen, 2011; Norsen, 2009; Seevinck, 2010; Seevinck and Uffink, 2011) that Bell's "local causality" is not an acceptable locality condition. A natural generalization of Einstein Locality to the stochastic case, or Stochastic Einstein Locality (SEL) is defended instead. Then, the question of whether our formulation of SEL implies STPI and/or STOI is discussed. It is shown that STPI is a locality condition, in the sense that failure of STPI is a case of non-locality. It is also shown, however, that the converse does not hold, that is: STPI's holding is not sufficient for SEL to be secured. The upshot is thus that STPI is a necessary condition for locality, but not a sufficient one. Note, however, that this is all that the advocate of the mainstream interpretation needs for a modus tollens in the interpretation of Bell-type experiments. As for STOI, our analysis shows that it is not necessary for locality to hold. This last result contradicts Butterfield (2007)'s claim. It is shown that Butterfield's argument rests on a false assumption, and thus fails to show that STOI (or, in his terms, SELD1) is a locality condition.

Finally, Section 4 addresses what STOI represents (since it is not a locality condition). It is argued that STOI, when applied to Bell-type situations, is just a space-time version of the principle of common cause (STPCC). The question of the appropriate space-time version of the principle of common cause is discussed. It is argued that failure of STOI is equivalent to failure of STPCC. Depending on how one interprets failure of the principle of common cause, the significance of failure of STOI may change. This issue is not fully resolved here, but one interpretation of failure of the principle of common cause is rejected; namely, that it amounts to failure of locality.

2 Probabilistic conditions in space-time

2.1 A space-time framework

To provide an assessment of whether failure of Factorizability, PI, and OI threatens locality or not, we need a specification of how events are to be embedded within space-time. Towards this end, this section offers a discussion of locality in conjunction with a fixed space-time framework.

[8]In that paper, Earman argues that Bell's factorizability condition is not a locality condition. We extend Earman's argument in two ways. First, Earman focusses only on Bell's factorizability condition: our analysis deals with parameter and outcome independence. Second, Earman's argument is mostly confined to the deterministic context: we generalize the discussion to the stochastic case.

A minimal space-time framework should provide a notion of localization of events within the space-time structure that is considered. In particular, if the notion of spatial separation and spatially separated systems are to be precisely defined, it should be possible to localize the systems under consideration within space-time.

This requirement is difficult to fulfill in standard quantum mechanics (henceforth SQM), because SQM is not a space-time theory. In effect, SQM provides no representation for quantum magnitudes as space-time quantities. So, it is not obvious how to discuss locality issues rigorously.[9]

We shall suppose that a space-time perspective can be taken on quantum events. Within ordinary relativistic space-time, quantum events are taken to be localizable in extended space-time regions. We propose that an event e in region R can be represented by the total physical state over R.[10] We will not consider any space-time point-events in order to avoid singularities.[11] Whenever we shall refer to events embedded in space-time, we shall specify the regions on which these events are localized. We denote event e located in region R by e_R.

What is at issue is whether certain probabilistic relations between events or failure thereof is indicative of non-local interactions between events. In order to address that issue we must examine what counts as a local interaction in relativistic space-time.[12]

We first need the following notions: domain of influence, time slices, domain of dependence, and Cauchy surface. Let us begin with domains of influence. Let $(\mathcal{M}, g)$ stand for a space-time, with g a metric on a manifold $\mathcal{M}$. The domains of influence of a point p of $\mathcal{M}$ is defined as follows:

Definition 5 – ***Domain of influence***

The future (respectively past) domain of influence $I^+(p)$ $(I^-(p))$ of a point $p \in \mathcal{M}$ is the set of all the points q of $\mathcal{M}$ such that there exists a future(past)-directed timelike curve which begins at p and ends at q.

One contrasts domains of influence with domains of dependence. In order to define domains of dependence, we need the notion of time slice:

[9] That said, it should not be impossible either. One possibility is to endorse the Copenhagen interpretation in a version that takes the apparatuses to be classical. In this way, one can say that the apparatuses are located in classical space-time. This is however an unattractive option for assessing the mainstream interpretation, which is meant to be independent of any interpretation of quantum mechanics.

[10] One worry arises. If the total physical state over R is fixed, then events are fixed, but the converse might not hold. An event might then have to be represented by a class of physical states over R. Such details do not affect our argument.

[11] Note that Reichenbach considers only point events, as opposed to Butterfield (compare Reichenbach (1978) and Butterfield (1989), Butterfield (2007)).

[12] We restrict discussion to space-times in which Cauchy surfaces can be defined. See below for the definition of a Cauchy surface.

Definition 6 – ***Time slice***

A time slice S of $\mathcal{M}$ is defined to be a spacelike surface $S \subset \mathcal{M}$ without edges.

With this in hand, one can define the past and future domains of dependence of a time slice S in the following way:

Definition 7 ***Domain of dependence***

The future *(respectively* past*)* domain of dependence $D^+(S)$ *(*$D^-(S)$*) of a timeslice $S \subset \mathcal{M}$ consists of all those points $p \in \mathcal{M}$ such that every past (future) directed timelike curve that passes through p and has no past (future) endpoint intersects S. The total domain of dependence of S is $D(S) \equiv D^+(S) \cup D^-(S)$.*

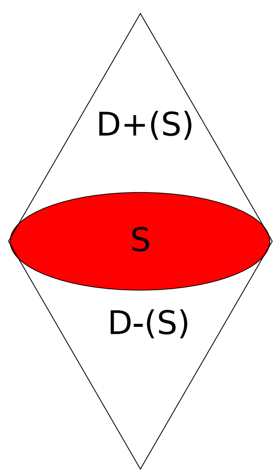

Figure 1: Domains of dependence

This is illustrated by Figure 1. With this in hand, we can finally introduce the notions of (future and past) Cauchy surface:

Definition 8 – ***Cauchy surface***

A timeslice S is said to be a future (respectively past) Cauchy surface for $(\mathcal{M}, g)$ just in the case that all the points of $\mathcal{M}$ to the future (past) side of S are in $D^+(S)$ ($D^-(S)$).

S is a Cauchy surface simpliciter just in case it is both past and future Cauchy.

The notion of Cauchy Surface is closely related to the notion of determinism, as well as to our favorite notion of locality: Einstein Locality. In short, Einstein Locality simply states that an event localized within a certain region R of space-time should be determined by an appropriate slice S across its backward light cone. But let us use Earman's (1986, 462) definition of Einstein Locality.[13]

To do so, we need to introduce the notion of global Laplacian determinism:

Definition 9 – ***Laplacian Determinism***

A theory T satisfies Laplacian Determinism if and only if there do not exist two different models M_1,M_2 of T such that M_1 and M_2 agree on relevant data on a Cauchy surface, but disagree on relevant data anywhere else.

It is also useful to introduce the notion of local Laplacian determinism:

Definition 10 – ***Local Laplacian Determinism***

A theory T satisfies Local Laplacian Determinism if and only if there do not exist two different models M_1, M_2 of T such that M_1 and M_2 agree on relevant data on a spacelike surface S such that $S \subset I^-(R)$ and $R \subset D^+(S)$, but disagree on relevant data on R.

This is illustrated by Figure 2.

Finally, Einstein locality (EL) can be formulated:

Definition 11 – ***Einstein Locality – EL***

A theory T satisfies Einstein Locality if and only if, when T is Laplacian deterministic, it is locally so.

We now have the minimal space-time structure needed for the connection between probability conditions and locality to be properly discussed.

2.2 Factorizability, PI and OI in space-time

We shall now propose formulations of Factorizability, Outcome and Parameter Independence within our space-time framework. To this aim, let us consider a Bell-type situation. What we are given are the outcome-events a and b (for variables A and B). In agreement with the

[13]For another formulation, see Hellman (1982).

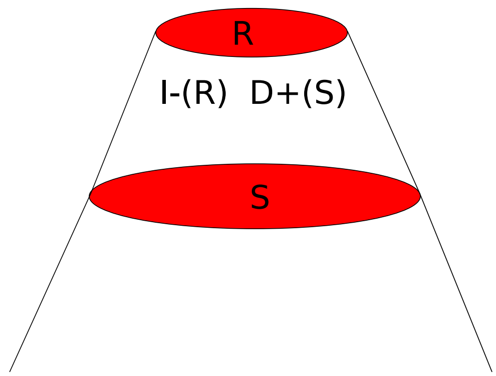

Figure 2: Local Laplacian Determinism: The state on R is determined by a time slice S such that S $S \subset I^{-}(R)$ and $R \subset D^{+}(S)$.

above, we consider that a and b are located on two regions of space-time R_1 and R_2. Now, in order to be able to assess whether the correlations between A and B are the result of a non-local underlying physical process, we need to make R_1 and R_2 spacelike separated.

The relativistic space-time structure then gives us the past and future domains of influence, or backward and forward light cones of R_1 and R_2. Let S stand for the whole time slice across the backward light cones of R_1 and R_2, before they get separated, and such that $S \subset I^{-}(R_1 \cup R_2)$ and $(R_1 \cup R_2) \subset D^{+}(S)$. Let S_1, S_2 and S_3 be space-time regions constituting a partition of S, so that S_1 (respectively S_2) correspond to the part of the slice within the past light cone of R_1 (respectively R_2) which is not within the past light cone of R_2 (respectively R_1), and S_3 corresponding to the part of the slice lying in the intersection, as in Figure 3.

It is worth emphasizing that, in order to keep the analysis as general as possible, S is here situated *before* the backward light cones of R_1 and R_2 get separated. This stands in contrast with the way Norsen (2009), Norsen (2011), Seevinck (2010), and Seevinck and Uffink (2011) propose to frame Bell-type situations in space-time. According to these authors, it is crucial

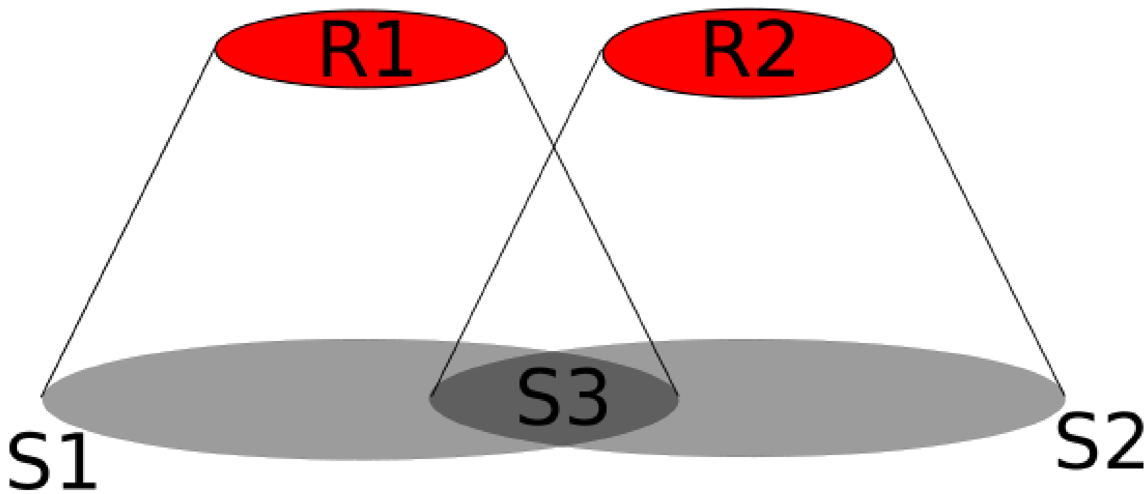

Figure 3: A Bell-type situation in space-time: the outcome-events are located on regions R_1 and R_2, S is a time slice across the backward light cones of R_1 and R_2, such that $S \subset I^-(R_1 \cup R_2)$ and $(R_1 \cup R_2) \subset D^+(S)$.

to consider a surface S that shields off R_1 (and R_2) from the overlapping of the backward light cones of R_1 and R_2. It will become clear that this has no impact on the validity of our argument in what follows (in particular in Section 4).

Now we need to identify and locate the various causal factors and causal paths that could give rise to the observed correlations between the outcome-events respectively located on R_1 and R_2. Only then will we be able to assess whether or not these causal paths correspond to non-local processes. We thus need to connect the space-time regions described above with the various causal factors possibly influencing the phenomena. Typically, one considers the following potential factors: the determinate state λ and the parameters i and j. On which space-time regions can these factors be localized?[14]

Let us consider first the time slice S. Let σ be the total physical state over S. σ includes the determinate state λ. Now, the question of how λ is to be localized on S is a little tricky.[15] λ certainly depends on some parts of S_3. That said, the state of S_3 and λ do not necessarily coincide. Let S_3^* be the region on which λ is located. On the one hand, there is no a priori reason to consider that λ depends entirely on S_3. In particular, nothing in the ordinary relativistic space-time framework implies that the determinate state of the quantum systems involved in Bell-type experiments be localized on the intersection of the backward

[14]Strictly speaking this language is misleading. States and variables are mathematical objects and are not "located" anywhere. What we mean by localizing states and variables is localizing the properties of systems on which the states and values of variables supervene. For brevity though, we will continue to use the language of localizing states and variables.

[15]For a different, but, as far as we can tell, compatible, analysis of the localization of λ on the timeslice of choice, see Seevinck and Uffink (2011, 5, (vi))

light cones.[16] Thus λ depends on the state of S_3, but perhaps also on the state of S_1 and S_2. Accordingly, S_3^* overlaps with S_3 but may also extend outside of it. On the other hand, it is not clear for now that λ exhausts the state of S_3. The localization of the other relevant factors needs to be discussed in order to settle that matter.

The total physical state σ on S also includes the states of the apparatuses. To localize these within our framework is again not easy, but a little more can be said about the states of the apparatuses than about the determinate state λ of the system under consideration. We can legitimately localize the states of the two apparatuses on some parts of S_1 and S_2 respectively, so that they stay spacelike separated.[17] It seems physically possible to make sure that the two apparatuses do not interact with one another. That said, there is no a priori reason to consider that i and j exhaust S_1 and S_2 respectively, especially when the fact the λ may well depend on some part of S_1 and S_2 is taken into account. Let S_1^* and S_2^* be the regions on which i and j are located. S_1^* and S_2^* are included in, but do not exhaust S_1 and S_2, respectively.

What else does σ include that matters? Arguably, if we consider that $\{\lambda, i, j\}$ are the only factors that are *relevant* to Bell-type experiments, then whatever else is included on σ does not matter. If so, then one can safely assume that $\{\lambda, i, j\}$ exhaust whatever within is relevant to Bell-type situations, and S_1^*, S_2^* and S_3^* together exhaust S. Note however that there is no reason to hold that the regions of S_1 on which i and λ depend be mutually exclusive. Similarly for j and λ on S_2. In both cases, a tight localization of the system may be difficult to obtain – especially if the systems are field-like. Since we do not need such localization for our argument, we allow for some overlap. We only require that the regions on which i and j depend do not overlap and be spacelike separated.

The upshot is as follows: S_1^*, S_2^*, and S_3^* exhaust S; S_3^* may extend outside of S_3; S_1^* and S_2^* may overlap with S_3^* but may not overlap with one another and must stay spacelike separated. This is illustrated in Figure 4.[18] With this space-time representation of Bell-type experiments in hand, we can formulate the space-time versions of Outcome Independence and Parameter Independence (STOI and STPI).

Remember that OI is the requirement that the outcome-events on R_1 and R_2 be statistically independent from each other, conditional on the hidden states of the system undergoing measurement and the parameters of the measurement devices. Within our space-time framework, this translates into: STOI prescribes that the outcome-events a and b, localized on

[16]We shall clarify this statement later on in Section 4 by explaining why Einstein locality does not imply that two events localized on spacelike separated regions should be determined by the state over a region of the intersection of their backward light cones.

[17]Note that this is again compatible with the analysis offered by Seevinck and Uffink (2011, 5, Note 13).

[18]One could object that a representation of Bell-type experiments in space-time should take into account whether $\{\lambda, i, j\}$ will evolve, either deterministically or stochastically, between S and R_1, R_2. If so, the causal factors should be located on the extended regions stretching from S to just below R_1 and R_2, as Butterfield (1989) proposes to do. We believe we do not need to do so in the context of this paper. The definition proposed for SEL in Section 3.3 obviates this problem.

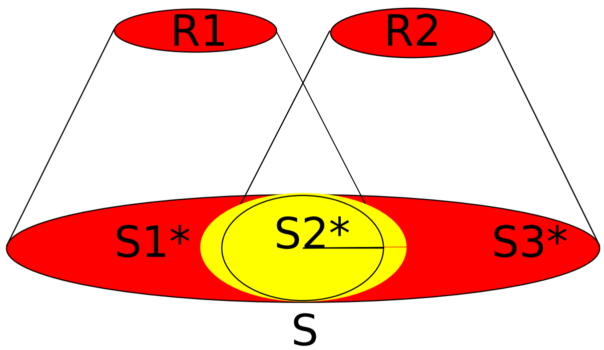

Figure 4: A Bell-type situation in space-time: localization of the potential causal factors.

the two spacelike separated regions R_1 and R_2, are statistically independent, conditional on the total physical state on an appropriate slice across the backward light cones. That is to say:

Definition 12 – ***Space-Time Outcome Independence*** – ***STOI***

A probabilistic model for Bell-type situations satisfies STOI if and only if for all a_{R_1}, b_{R_2}, $i_{S_1^*}$, $j_{S_2^*}$, $\lambda_{S_3^*}$:

$$p(a_{R_1}, b_{R_2}|\lambda_{S_3^*}, i_{S_1^*}, j_{S_2^*}) = p(a_{R_1}|\lambda_{S_3^*}, i_{S_1^*}, j_{S_2^*}).p(b_{R_2}|\lambda_{S_3^*}, i_{S_1^*}, j_{S_2^*})$$

where the subscripts indicate on which space-time regions these states, settings, and outcomes depend or occur.

On the other hand, Parameter Independence requires that an outcome-event at one end be determined only by the apparatus parameters on its end, and not by the parameters on the other end. Translating within our framework, STPI requires that the probability of an outcome-event a, localized on a region of space-time R_1, be determined by the total physical state on an appropriate slice across the backward lightcone of R_1. That is to say:

Definition 13 – ***Space-Time Parameter Independence***

A probabilistic model for Bell-type situations satisfies STPI if and only if for all a_{R_1}, b_{R_2}, $i_{S_1^*}$, $j_{S_2^*}$, $\lambda_{S_3^*}$*:*

$$p(a_{R_1}|\lambda_{S_3^*}, i_{S_1^*}, j_{S_2^*}) = p(a_{R_1}|\lambda_{S_3^*}, i_{S_1^*}, j'_{S_2^*}); \tag{1}$$

$$p(b_{R_2}|\lambda_{S_3^*}, i_{S_1^*}, j_{S_2^*}) = p(b_{R_2}|\lambda_{S_3^*}, i'_{S_1^*}, j_{S_2^*}). \tag{2}$$

The space-time version of Factorizability (STFAC) follows straightforwardly from the above:

Definition 14 – ***Factorizability: space-time version – STFAC***

A probabilistic model for Bell-type situations satisfies STFAC if and only if for all a_{R_1}, b_{R_2}, $i_{S_1^*}$, $j_{S_2^*}$, $\lambda_{S_3^*}$*:*

$$p(a_{R_1}, b_{R_2}|\lambda_{S_3^*}, i_{S_1^*}, j_{S_2^*}) = p(a_{R_1}|\lambda_{S_3^*}, i_{S_1^*}).p(b_{R_2}|\lambda_{S_3^*}, j_{S_2^*}).$$

We are thus provided with a rigorous space-time framework, and with space-time versions of the various probability conditions about which we want to assess whether or not they are locality conditions. We shall begin with STPI. That said, before we can assess whether STPI is a locality condition or not, we need to discuss the issue of how to construe Einstein Locality in the stochastic case.

3 Parameter Independence: a necessary but not sufficient condition of stochastic Einstein locality

In this section, we shall argue that STPI follows from a version of Einstein Locality in the stochastic context within our space-time framework. We shall first discuss the issue of how to construe locality in the stochastic case. In Subsection 3.1, Bell's proposal, i.e. "local causality", is presented. Contra Norsen (2009), Norsen (2011), Seevinck (2010), and Seevinck and Uffink (2011), it is argued that it is not a reasonable locality condition in the stochastic context. A more satisfactory formulation of the stochastic version of Einstein Locality, SEL, is proposed. On the basis of our formulation of SEL, we show that STPI is entailed by SEL, when the latter is applied to Bell-type situations, but also that the converse does not hold. Since our result contradicts some his findings, we'll wrap up the section with a critical assessment of the analysis given by Butterfield (2007).

3.1 Bell's Local Causality

Recent criticisms of the mainstream interpretation of the experimental violation of the BI crucially rely on the claim that Bell's "local causality" (LC) is an adequate formulation of locality in the stochastic context. Since factorizability can be derived from LC, if it is correct that LC is a locality condition, it follows that factorizability is a necessary condition of locality. Hence, the argument goes, violations of factorizability would be necessarily violations of locality, and the mainstream interpretation is undermined.

Advocates of LC all seem to agree that its best formulation is found in Bell (1990). There Bell first roughly characterizes the "principle of local causality" in the following way:

> The direct causes (and effects) are near by, and even the indirect causes (and effects) are no further away than permitted by the velocity of light" (Bell, 1990, 239)

Such a characterization is then made more precise ("sharp and clean") as follows:

> A theory will be said to be locally causal if the probabilities attached to the values of local beables in a space-time region 1 are unaltered by specification of the values of local beables in a space-like separated region 2, when what happens in the backward light cone of 1 is already sufficiently specified, for example by a full specification of local beables in a space-time region 3...." (Bell, 1990, 239-40)

LC is thus that given a sufficiently complete specification[19] of the backward light cone of a given event e, the probability of e should not be modified by information on events located outside the backward light cone of e. This translates in the following way in terms of conditional probabilities:

Definition 15 – ***Local Causality***

Let e be an event located on a region R_e of space-time. Let σ, stand for the physical state over a time slice S of space-time across the past light cone of R_e. Let n stand for any event located outside of R_e and its backward light cone. Then LC requires that the probability of the event e, lying over the region of space-time R_e, be independent of n, conditional on σ:

$$p(e_{R_e}|n,\sigma) = p(e_{R_e}|\sigma).$$

Admittedly, this condition allows for the derivation of the factorization condition.[20] This is because n is interpreted as representing *any* event outside R_e and its past light cone. Indeed, consider two regions of space-time R_1 and R_2, on which are respectively located the a and

[19] See Norsen (2009), Norsen (2011), Seevinck (2010), and most of all Seevinck and Uffink (2011) for interesting discussions about how the notions of "sufficiency" and "completeness" must be interpreted. This issue falls beyond the scope of this paper.

[20] For an explicit derivation of factorizability from LC, see Norsen (2011, VI, A).

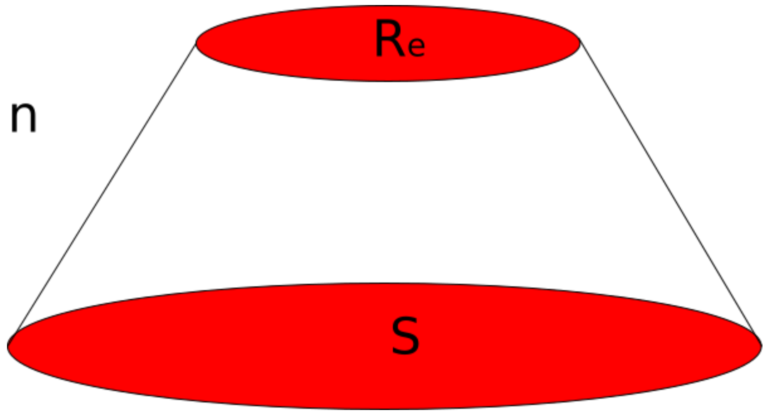

Figure 5: Bell's condition of locality in the stochastic case: $p(e|\sigma)$ must be independent of n.

b outcome-events. Let λ stand for the physical state over a time slice across the past light cones of R_1 and R_2. Let n, (respectively m), represent any event outside the past light cone of R_1 (respectively R_2). Since n (respectively m) can be any event outside R_1 (respectively R_2) and its backward light cone, it can be, for each side of the experiment, the choice of settings or the outcome-event on the other side. Hence, outside of the backward light cone of R_1 are b and j; while, outside of the backward light cone of R_2 are a and i. We then obtain, from LC, that[21]:

$$p(a|\lambda, i, j, b) = p(a|\lambda, i) \quad and \quad p(b|\lambda, j, i, a) = p(b|\lambda, j), \tag{3}$$

from which in turn one can get the factorization condition. Thus, if LC is the correct construal of locality within the stochastic context, given that factorizability is derivable from it, factorizability also qualify as a necessary condition of locality.

Note that, if this was correct, then the mainstream interpretation, as characterized by (PI-LOC) above, would turn out false. Indeed, since factorizability is equivalent to the conjunction of Parameter and Outcome Independence together, a failure of Parameter Independence alone *or* of Outcome Independence alone is sufficient to entail a failure of factorizability. This means that, if a failure of factorizability counted as a case of non-locality, then a failure of Parameter Independence alone would count as a case of non-locality, as well as a failure of Outcome Independence alone. In this case, failure of Outcome Independence alone would not be less problematic than failure of Parameter Independence alone in so far as locality is concerned. Thus, the mainstream interpretation is not supported if one follows Bell on the formulation of locality for the stochastic case.

In what follows, it is argued that, when assessed within the space-time framework and the definition of Einstein Locality proposed in Section 2, LC does not qualify as an adequate

[21] We omit the specification of the regions of space-time on which the events are localized.

condition of locality in the stochastic context.

3.2 Stochastic Locality: Bell's proposal rejected

However natural and intuitive it may look like, LC appears, under closer scrutiny, to be too strong a condition. Suspicion of this comes from the fact that relativistic quantum field theories do not satisfy LC, which Bell himself admits.[22]

The example that Bell gives to the effect that quantum theories, ordinary or relativistic, violate LC is symptomatic of a more serious problem for LC. Bell mentions an experimental set up in which a radioactive nucleus can emit a single α-particle, and several α-particle detectors are scattered around the nucleus. As long as none of the counters has detected that α-particle, there is a nonzero probability that, say, counter C_i detects the α-particle. That probability becomes zero if one adds the information that, say C_j, has actually detected the α-particle. Bell's locality condition is violated in this case.

The problem that the example uncovers is that Bell's condition somewhat trivializes the notion of non locality because it makes it ubiquitous. Moreover the example shows that the non local correlations identified as such by Bell's condition are not generally those appropriately explainable by a non local physical process between events. So, Bell's condition seems inappropriate to the analysis of Bell-type situations.[23] A new locality condition for the stochastic case is required.

3.3 Stochastic Einstein Locality: a proper formulation

Recall Einstein Locality in the deterministic case: it requires that theories that are Laplacian deterministic be locally so. Laplacian Determinism requires the existence of a Cauchy surface, the state on which determines the outcome-events to the future of the Cauchy surface. Local Laplacian Determinism requires that events on a region R of space-time be determined by the state on the intersection of a Cauchy surface to the past of R and the backward light cone of R.

Let us propose a formulation of Einstein Locality in the stochastic case by forming natural adaptations of Laplacian Determinism and Local Laplacian Determinism to the stochastic case. The idea is that instead of demanding that the states be determined, we require that probability distributions be determined. We arrive at the following definitions.

Definition 16 – ***Stochastic Laplacianism***

[22]See Bell (1976) reprinted in Bell (1987, 55). Norsen (2011, 17) also admit that Bell's particular formulation of locality "should not necessarily be regarded as definitive", given that it seems directly incompatible with some theories (e.g. non-Markovian or Bohmian theories).

[23]See Hellman (1982) for a similar criticism.

A theory T satisfies Stochastic Laplacianism if and only if there do not exist two different models M_1, M_2 of T such that M_1 and M_2 agree on relevant data on a Cauchy surface, but disagree on the probability distributions of events anywhere to the future of the Cauchy surface.

Definition 17 – ***Local Stochastic Laplacianism***

A theory T satisfies Local Stochastic Laplacianism if and only if there do not exist two different models M_1, M_2 of T such that M_1 and M_2 agree on relevant data on a spacelike surface S such that $S \subset I^-(R)$ and $R \subset D^+(S)$ but disagree on the probability distributions over events on R.[24]

Definition 18 – ***Stochastic Einstein Locality***

A theory T satisfies SEL if, when T is Stochastic Laplacian deterministic, it is locally so.

Note that the above formulation of Stochastic Einstein Locality SEL does not entail Bell's Local Causality LC. This is because it states that, of all possible determining factors for the probabilities on R, only some are in fact relevant. *It says nothing about whether there are any correlations between events in R and other events to the future of the Cauchy surface.* Again, because SEL as formulated above says nothing about correlations, it does not entail Factorizability, which we make explicit in the following subsections. In particular, we'll critically assess the analysis offered by Butterfield (2007) in the last subsection.

3.4 Parameter Independence, but not Outcome Independence, is entailed by Stochastic Einstein Locality

SEL prescribes that the probability distributions of events localized on a given region R of space-time be determined by the physical state of an appropriate slice S across the backward light cone of R. How is this applied in the case of Bell-type situations? Consider the same notation as in Section 2: a_{R_1} and b_{R_2} for the outcome-events localized on R_1 and R_2, respectively, R_1 and R_2 being two spacelike separated regions of space-time. Consider then the past domains of influence of R_1 and R_2, and a time slice S across it. Consider then that the physical states i, j, and λ are localized on S_1^*, S_2^* and S_3^*, respectively.

Concerning a_{R_1}, SEL prescribes that its probability assignment be determined by the physical state on $S_1 \cup S_3$. Since we are assuming that the only possible causal influences are λ, i, and j, this region of dependence can be restricted to the subregion $S_1^* \cup S_3^* \setminus S_2$. Similarly, concerning b_{R_2}, SEL prescribes that it be determined by the physical state on $S_2 \cup S_3$, but this

[24]Note that, in the above definitions, a time asymmetry has been introduced that was not in Laplacian Determinism nor Local Laplacian Determinism, which is natural because we are considering the stochastic case. Note also that, when the probabilities are trivial (0 or 1), Stochastic Laplacianism, respectively global or local, recovers *future* Laplacian Determinism, global or local, respectively.

can be restricted to the subregion $S_2^* \cup S_3^* \setminus S_1$. So what SEL prescribes in Bell-type situations is that probability assignments on $R_1 \cup R_2$ be determined by $S_1^* \cup S_2^* \cup S_3^*$. *But SEL does not imply that correlated events on $R_1 \cup R_2$ should be made independent by conditionalization on the total physical state on S.*[25] In other words, SEL does not entail STOI as defined in Section 2, Definition 12.[26]

If SEL does not prescribe that a be uncorrelated with any event outside the past domain of dependence of R_1 as LC would have it, it still prescribes, once one considers the appropriate time slice S, that a is not determined by the physical state over S_2. And similarly for b. So, in the end, we can say that SEL, applied to Bell-type situations, becomes:

$$p(a_{R_1}|\lambda_{S_3^*}, i_{S_1^*}, j_{S_2^*}) = p(a_{R_1}|\lambda_{S_3^* \setminus S_2}, i_{S_1^*}), and \tag{4}$$
$$p(b_{R_2}|\lambda_{S_3^*}, i_{S_1^*}, j_{S_2^*}) = p(b_{R_2}|\lambda_{S_3^* \setminus S_1}, j_{S_2^*}), \tag{5}$$

from which STPI, as defined in Section 2, Definition 13, follows.[27] This should be straightforward when one considers that the equations above are stronger than STPI: $p(a_{R_1})$ ($p(b_{R_2})$) is made independent of the state on $S_3^* \setminus S_2$ ($S_3^* \setminus S_1$) instead of the state on S_3^* alone. Indeed, STPI is:

$$p(a_{R_1}|\lambda_{S_3^*}, i_{S_1^*}, j_{S_2^*}) = p(a_{R_1}|\lambda_{S_3^*}, i_{S_1^*}, j'_{S_2^*}), and: \tag{6}$$
$$p(b_{R_2}|\lambda_{S_3^*}, i_{S_1^*}, j_{S_2^*}) = p(b_{R_2}|\lambda_{S_3^*}, i'_{S_1^*}, j_{S_2^*}). \tag{7}$$

Thus, we have shown that the appropriate version of SEL in our space-time framework implies STPI. As such, STPI counts as a necessary condition for locality. In this sense, part of (PI-LOC) can be supported: failure of STPI amounts to a case of non-locality. That said, it also follows from our argument that one cannot support the converse, that is: that STPI holds does not imply that locality does – in the sense of SEL , and STPI does not qualify as a sufficient condition for locality. The advocate of the mainstream interpretation could content herself with this though, since the claim that STPI is sufficient to secure locality is not needed for the interpretation of the experimental violation of the BI by modus tollens. What is needed in this latter case is that *failure* of STPI be sufficient for *non-locality*.

[25] Neither does it imply that correlated events on $R_1 \cup R_2$ should be made independent by conditionalization on the "sufficiently complete" state on S in the sense that Norsen, Seevink and Ufffink give to this term.

[26] This argument is consistent with the derivation of an approximate form of factorizability of the relevant joint probabilities given in Hellman (1992). A crucial assumption for this derivation is that outcome probabilities at a given wing be treated as physical propensities. This illustrates our point about how outcome independence and locality relate to one another. Thanks to an anonymous reviewer for pointing that out.

[27] One might be worried that $\lambda_{S_3 \setminus S_2}$ may not be well defined since we explicitly left open that it may depend on regions in S_2. In this case, we can simply conditionalize on all λs compatible with the properties fixed in $S_3 \setminus S_2$.

Concerning STOI, we have shown that it is not entailed by our definition of locality within our space-time framework. Now, at least the two following questions arise:

1. Is there no other rationale for taking failure of STOI as failure of locality in a rigorous space-time framework?
2. If not a locality condition, what is STOI?

Question 2. above will be covered in Section 4. Questions 1. is (at least partially) addressed through an assessment of the arguments offered in Butterfield (2007) in what immediately follows.

3.5 Why Butterfield's argument fails

In his (2007), Butterfield addresses the issue of Stochastic Einstein Locality, its various possible formulations, and the relationships between these. To do so, he uses a framework involving the set W of possible worlds w and their histories: one considers an event e occuring in spacetime $\mathcal{M}$ and the probability of e on a hypersurface t in various possible worlds w in W characterized by their histories, i.e. "the collection or conjunction of all events up to a given hypersurface" (Butterfield, 2007, 809-13). As Butterfield himself recognizes, this framework easily translates into a framework à la Helllman used in this article, i.e the framework involving physical theories and models (Butterfield, 2007, p. 813).

With that framework in hand, Butterfield offers three different formulations of SEL: SELS (Stochastic Einstein Locality – Single), SELD1 (Stochastic Einstein Locality – Double – Version 1), and SELD2 (Stochastic Einstein Locality – Double – Version 2). As it will become clear, SELS corresponds to our SEL, SELD1 corresponds to our STOI, while SELD2 is a novel notion that will play a crucial role in the argument. Before we get to that point, let us look at the definitions (adapted to our notation, in which upper-case letters are used for space-time regions and lower-case letters for the events associated with these regions):

Definition 19 – ***SELS (Butterfield, 2007, 814):***

Let worlds $w, w' \in W$ match in their history in $I^-(e) \cap I^-(t)$. Then they match in their probability at t of e:

$$p_{t,w}(e) = p_{t,w'}(e).$$

Definition 20 – ***SELD1 (Butterfield, 2007, 816)***

For any world w, for any hypersurface t earlier than the event e and dividing $I^-(E)$, and any event f future to t and spacelike to e and such that $I^-(E) \cap I^-(F) \cap I^+(t) = \oslash$:

$$p_{t,w}(e, f) = p_{t,w}(e).pr_{t,w}(f).$$

As mentioned above, SELS corresponds to our SEL, and SELD1 is roughly our STOI. Now let us turn to SELD2, in which the event f is moved in the past of the hypersurface t:

Definition 21 – ***SELD2 (Butterfield, 2007, 817)***

For any world w, for any hypersurface t earlier than the event e and dividing $I^{-}(E)$, and any possible event f in the difference $I^{-}(t) - I^{-}(E)$:

$$p_{H,w}(e, f) = p_{H,w}(e).p_{H,w}(f),$$

where H stands for the history up to hypersurface t and within $I^{-}(E)$.[28] *SELD2 is represented in Figure 6.*

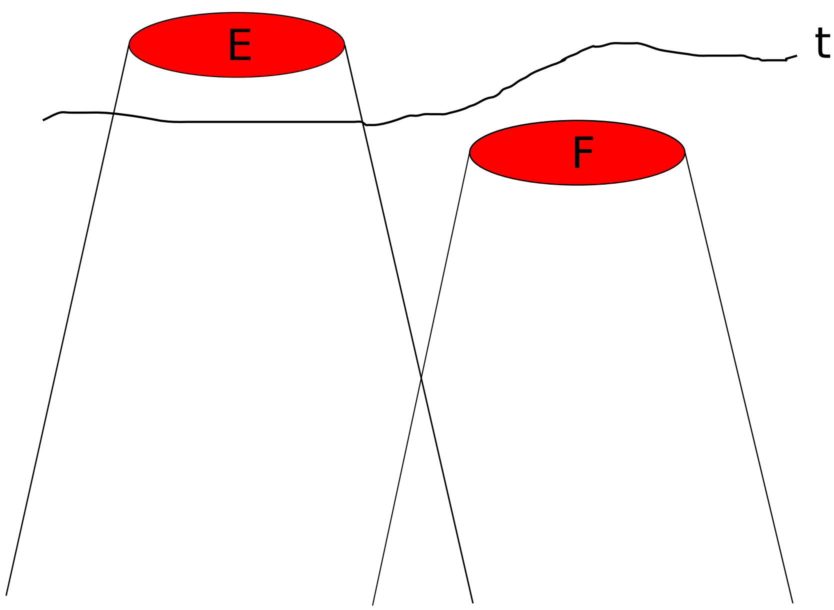

Figure 6: SELD2.

Butterfield's main claim is that SELS and SELD1 are equivalent. His argument uses SELD2 as an intermediary, i.e. he shows:

- first that, under certain conditions, SELD1 and SELD2 are equivalent;
- second that, under certain conditions, SELD2 and SELS are equivalent.

If Butterfield is correct, then Stochastic Einstein Locality is equivalent to Outcome Independence, and both are locality conditions, i.e. failure of outcome independence is a failure

[28] $pr_{H,w}$ is defined by reference to $H = I^{-}(t) \cap I^{-}(E)$ alone because $pr_{t,w}(f)$ is trivially 1 whenever f is past to t.

of locality, which contradicts our result. In what follows, it will be argued that Butterfield's argument is faulty. More precisely, it is argued that his derivation of $SELD2 \rightarrow SELD1$ is problematic, because one of the assumptions it requires is false. In what follows, a counter-example to that assumption is offered. If we are correct, then Butterfield's argument has failed to prove that SELS implies SELD1, in other words, it has failed to prove that outcome independence is a locality condition.

In order to show that SELD2 implies SELD1, Butterfield begins with defining two events E and F, a hypersurface t earlier than E and dividing $C^{-}(E)$, as well as H_E, H_F, and H^* as follows:

$$H_E = I^{-}(t) \cap I^{-}(E)$$
$$H_F = I^{-}(t) \cap I^{-}(F)$$
$$H^* = I^{-}(t) - I^{-}(E)$$

Applying SELD2 to this context (see Figure 6), one obtains (the world index will be dropped from now on):

$$p_{H_E}(e, f) = p_{H_E}(e).p_{H_E}(f) \tag{8}$$

From there, Butterfield seeks to derive that SELD1 obtains, i.e.:

$$p_t(e, f) = p_t(e).p_t(f) \tag{9}$$

The argument uses two main assumptions. The first one is about how probabilities evolve in space-time, and it is that, for any event x:

$$p_t(x) = p_H(x|H^*)$$

In other words, Butterfield assumes that the probability of x on t is the probability of x on H (or the hypersurface δH that is constituted by the boundary of H) conditional on H^*, i.e. the history outside of H_E and past to t.

This first assumption seems quite reasonable. On the basis of this assumption, and applying SELD2 again, Butterfield deduces that:

$$p_t(e) = p_{H_E}(e|H^*) = p_{H_E}(e); and \tag{10}$$
$$p_t(f) = p_{H_F}(f|H^*) = p_{H_F}(f). \tag{11}$$

Accordingly, what need to be shown, Equation 9, is now:

$$p_t(e, f) = p_{H_E}(e).p_{H_F}(f) \tag{12}$$

In order to derive this last equation from Equation 8, Butterfield assumes that the following ratios of probabilities are equal:

$$\frac{p_t(e, f)}{p_{H_F}(f)} = \frac{p_{H_E}(e, f)}{p_{H_E}(f)}. \tag{13}$$

Or, equivalently:

$$\frac{p_t(e, f)}{p_{H_E}(e, f)} = \frac{p_{H_F}(f)}{p_{H_E}(f)}. \tag{14}$$

Butterfield gives an informal argument in favor of the claim that the assumption is plausible. In what follows, we provide a counterexample, showing that the assumption is false.

To start, Equation 14 can be rewritten as follows:

$$p_t(e, f) = p_{H_E}(e|f).p_{H_F}(f). \tag{15}$$

We also know that, in general:

$$p_t(e, f) = p_t(e|f).p_t(f). \tag{16}$$

Now, consider three events e, f, and g, such that E and F are spacelike and future to the hypersurface t as usual, and G is in $I^-(t) - (H_E \cup H_F)$ (Figure 8). Let us assume that e, f, and g are the outcome-events associated with measurement of the spin along the z direction for the system efg in the state:

$$|\psi_{efg}\rangle = \frac{1}{\sqrt{3}}(|100\rangle + |010\rangle + |001\rangle)$$

.

Let e be the outcome-event such that a measurement of the spin in the z direction is 1. Let f be the outcome-event such that a measurement of the spin is 0. Let g be the outcome-event such that the spin is 0.

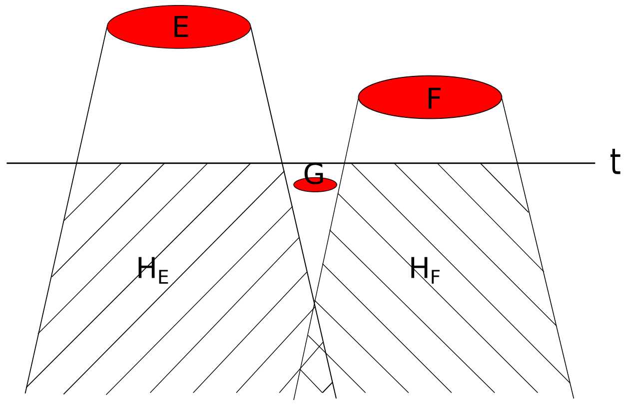

Figure 7: A counterexample to Butterfield's assumption.

In these conditions, we obtain:

$$p_{H_E}(e|f) = 1/2, \text{and} \tag{17}$$
$$p_{H_F}(f) = 2/3. \tag{18}$$

So, in this context, Equation 15 becomes:

$$p_t(e, f) = 1/3 \tag{19}$$

By contrast, examining Equation 16 now, we obtain (given that G happens before t):

$$p_t(e|f) = 1, \text{and} \tag{20}$$
$$p_t(f) = 1/2, \text{so:} \tag{21}$$
$$p_t(e, f) = 1/2, \tag{22}$$

which contradicts Equation 19, and thus shows that Butterfield's assumption (Equation 14) is false. It follows from this that Butterfield's derivation from SELD2 to SELD1 relies on a faulty assumption, and hence his argument fails to show that SELD1 is a locality condition.[29]

In what follows, we offer an alternative interpretation for SELD1, or our STOI.

[29]Note that this result is consistent with Butterfield's analysis of the quantum field theory case.

4 Outcome Independence in space-time is equivalent to the PCC formulated in space-time

In this section, we show that the requirement that STOI holds is equivalent to the demand that the space-time version of Reichenbach's Principle of Common Cause (STPCC) holds when applied to Bell-type situations. To this aim, we shall present first the PCC (Subsection 4.1). We shall then discuss the appropriate space-time formulation of the PCC (STPCC) and make its relation to STOI clear. We shall end this section by showing that STPCC is neither a necessary nor sufficient condition for locality, so that STOI is not either.

4.1 The principle of common cause

The PCC is the requirement that, given some correlated events, neither of which causes the other, one should be able to recover statistical independence of the events by conditionalization on a common cause. Or, in other words, the dependence between the events should be screened off by a common cause.

Let us illustrate this by way of example. Consider the following example. Two of your friends, Jules and Jim, and who do not know each other, are both cyclothymic.[30] While Jules and Jim display a random distribution of cheerful and crabby days, they are perfectly anti-correlated, one being always crabby when the other is cheerful and vice versa. One could make sense of such correlations if 1. both Jules and Jim have opposite taste concerning the proper baking stage of croissants, 2. they both buy croissants for breakfast every morning in the same bakery, 3. the baker randomly over- or under-bakes croissants, and 4. Jules and Jim's moods are directly determined by the level of satisfaction they take from their breakfast. Such a situation is a typical example of a common cause pattern: the perfect correlations between two events, otherwise apparently distributed randomly, and none of which causes the other, is explained in terms of a common cause. That is to say, the correlation is screened off when conditionalizing upon the croissants' baking time in the common bakery. In other word, the Principle of Common Cause is satisfied.

A rigorous statement of the PCC is the following:[31]

Definition 22 – ***The Principle of Common Cause***

If coincidences of two events A and B occur more frequently than would correspond to their independent occurrence, that is, if the events satisfy the relation:

$$P(AB) > P(A).P(B),$$

[30]It should be stressed that Reichenbach's own examples are of the type of our example: macroscopic examples taken from ordinary life situations rather than complicated physical phenomena.

[31]I am here quoting (van Fraassen, 1980, 28). For the original version by Reichenbach, see his (1956, 158-9).

then there exists a common cause C for these events such that the fork ABC is conjunctive*, that is, satisfies the relations below:*

$$P(AB|C) = P(A|C).P(B|C) \tag{23}$$
$$P(AB|\overline{C}) = P(A|\overline{C}).P(B|\overline{C}), \tag{24}$$
$$P(A|C) > P(A,\overline{C}) \tag{25}$$
$$P(B|C) > P(B,\overline{C}) \tag{26}$$

Conditions (23) and (24) imply that the events A, B and C satisfy the two relations below, called screening off conditions:

$$P(A|B,C) = P(A|C); \tag{27}$$
$$P(A|B,\overline{C}) = P(A|\overline{C}); \tag{28}$$

and similarly for A and B interchanged. Reichenbach shows that the conditions (23) to (26) above together imply that $P(AB) > P(A).P(B)$ (Reichenbach, 1956, 160-1).

The PCC is highly controversial.[32] Many cases of correlations can be cited which do not satisfy it. Further, the four conditions above do not properly characterize common causes. Some examples can be found in which a factor meets all of the above requirements but is not a common cause.[33] Still, what the PCC requires is so commonsensical that we seem to use it as a guide (quite successfully) both in everyday life and in scientific research.[34]

What is the PCC supposed to useful for? Reichenbach hoped that the PCC could be part of the characterization of what he calls "normal causality".[35] He characterized normal causality as the requirement that causal effects spread continuously through time.[36] That the PCC be satisfied, that is, that correlations between events which do not cause each other must be explained in terms of a common cause, is, according to Reichenbach, required by "normal causality". By contrast, if there is no such common cause while there are correlated events, then a "causal anomaly" occurs. Normal causality and the requirements associated with

[32]See Arntzenius (2005) and references therein for a good overview of the issue.

[33]See Arntzenius (2005, note 1) for such an example.

[34]That the PCC is too strong a requirement is at the origin of the work by the Budapest school on common cause systems, and the distinction between common causes, common common causes systems, and separate common causes systems. For more details on this research program, see Hofer-Szabó (2008) and references therein. See also their discussion with the Bern school in Grasshoff, Portmann and Wüthrich (2005) and Hofer-Szabó (2011).

[35]We wish to express our gratitude to Alexis Bienvenue for useful discussions about Reichenbach's works and thought.

[36]See, for example, Reichenbach (1958, 65).

it are normative prescriptions for a satisfactory causal picture of the world.[37] Correlations that cannot be explained in terms of a common cause do not satisfy such prescriptions. In other words, the PCC deems pure chance an unsatisfactory explanation for correlations. As said before, the interpretation and status of the PCC are controversial. That all correlations are or should be explainable in terms of a common cause is highly debatable. We shall not further discuss these issues in this article.

For now, what is important to note in the above is that the PCC is formulated without mention of locality. As explained before, probability conditions do not by themselves indicate how the various events and factors are embedded in the space-time structure. In particular, nothing in the formalism above tells us that the common cause C occurs earlier than its effects A and B. Along the same lines, nothing in the formalism above precludes that we consider the probabilities of C, conditional on A and/or B. That Reichenbach had to add the temporal characteristics of the common causes as restrictions on the above formalism is symptomatic of this problem (Reichenbach, 1956, p.162).

Common causes and their effects thus have to be explicitly embedded within time, independently of the above formalism. Similarly, nothing in the formalism above tells us about the spatial location of A, B and C. Again, the spatial characteristics of common causes and their effects have to be specified independently of the condition in terms of probability distributions. Whether or not the PCC is a locality condition can be assessed only under the condition that such a specification has been made.

Finally, note that, to the best of our knowledge, there is no statement in Reichenbach's own work about the PCC being a locality condition. This historical consideration is in support of the thesis that the PCC, however useful it may be as a methodological principle, is not essentially a locality condition. Before we can produce a rigorous argument to this effect though, we need to discuss the appropriate formulation of a space-time version of the PCC.

4.2 The space-time version of the PCC and its relation with STOI

The PCC prescribes that, whenever there are correlations between events which do not cause each other, there exists a common cause, say c, by conditionalization on which the correlations are screened off. How can such a Principle be appropriately applied to our space-time structure? Let us consider two correlated events e and f. Consistent with Section 2, these two events can be located on extended regions of space-time, say R_e and R_f, respectively. Now, can we choose R_e and R_f such that e and f do not cause each other on

[37]Note that the PCC and normal causality can be meant to give metaphysical or only epistemological prescriptions, that is, the issue whether they are meant to describe either the true causal structure of the world or only one of our favorite frameworks when trying to make sense of the world, remains open in Reichenbach's writings. We shall not dig into Reichenbach's work any further in this article.

the basis of what SEL prescribes? More precisely, does the PCC apply to correlated events located on spacelike separated regions of a space-time structure in which SEL holds?

Some clarifications concerning our vocabulary are needed here. It will be recalled from Section 3 that SEL prescribes that the probability distribution over events located on a given region R of space-time be determined by the total physical state depending on an appropriate time slice across the backward light cone of R. More generally, Einstein Locality, whether deterministic or stochastic, imposes some constraints on which regions of space time can *determine* each other. It is common in the literature to refer to such constraints in terms of which regions of space-time can *cause* each other. On the other hand, the PCC makes use of the term "cause" as well. The so-called "common cause", however, is only characterized by a series of four probability conditions (see Definition 22), without any reference to the notion of physical determination. The term "cause", when used both in the context of the PCC's common cause and of the structure of ordinary relativistic space-time, is thus ambiguous.

Here is how we shall clarify this situation. First, we shall avoid talking about common causes altogether. Instead, we shall focus on screening off events. Granted, the full characterization of the common cause includes more than its screening off role. That the common cause satisfies the screening off condition is a consequence of its full characterization. We can focus on this consequence rather than considering a full version of the PCC because it is sufficient for our argument. With this in hand, our formulation of what we still call – with slight abuse of language – the PCC is: there must exist a screening off event for any two correlated events that do not "cause" each other. Now, what "cause" refers to in this formulation of the PCC remains to be filled according to the context in which the PCC is used. In our case, the question arises of how to fill it in the context of a space-time structure in which SEL holds.

The problem is that it is not all clear whether SEL prescribes anything about causation. This might depend on which theory of causation is considered as well as on which interpretation of relativistic space-time one holds. In particular, it is not clear that SEL prescribes that events located on spacelike separated events do not cause each other. Since we do not want to take any specific stance either on theories of causation, or on the interpretation of relativistic space-times, let us consider the two options. If one considers that SEL does not impose any constraint on causation, then there is no way to apply the PCC to our space-time structure. In particular, one cannot argue that the PCC applies to events located on two spacelike separated regions. In this case, the formulation of STOI is simply unmotivated, let alone motivated by locality conditions.

Let us consider now that the case where the constraints imposed by SEL on which regions of space-time can determine each other amount to constraints on causation. In this case, SEL is taken to prescribe that events located on spacelike separated regions cannot cause each other. Hence, the PCC applies to any two correlated events e and f located on two spacelike separated regions R_e and R_f. Further, the PCC prescribes that there be a screening off event, by conditionalization on which e and f can be made independent.

Now, what does SEL imply regarding a screening off event? A rather common view is that SEL prescribes that the screening off event be located within the intersection of the backward light cones of R_e and R_f. In the case of R_e for example, SEL prescribes that the probability of e be determined by the physical state on an appropriate time slice, say S_e across its backward light cone. Similarly for R_f and S_f. We maintain with Earman (1986) that such a view is not supported by consideration of relativistic space-times.

It will be recalled from Subsection 3.4 that, concerning the probability distribution over events on $R_e \cup R_f$, SEL only implies that it is determined by the state on an appropriate slice across the *union* of the backward light cones, $S_e \cup S_f$, and not across the intersection of these, I. In particular, SEL does not imply that the state on a time slice I across the intersection of the backward light cones of R_e and R_f alone determine possible correlations between events on R_e and R_f. First, consistent with SEL, the future domain of dependence of I is trivial: strictly speaking, it does not determine anything but itself (See Figure 7). Second, a synchronization of some events on I is not sufficient to guarantee that events on R_e and R_f will be correlated because the correlations could be canceled out by influences which do not register on I, but only on $(S_e \cup S_f) \setminus I$. Third, that we have correlation between events on R_e and R_f does not imply that a synchronization took place between events on I – that is to say, it does not imply that the screening off event lies on I: for I could contain synchronized events itself, and the correlations could propagate without the synchronizing event lying on I. Thus, consistent with SEL, it could be the case that a screening off event does not lie within a time slice across the intersection of the backward light cones of the regions on which the two correlated events are located.

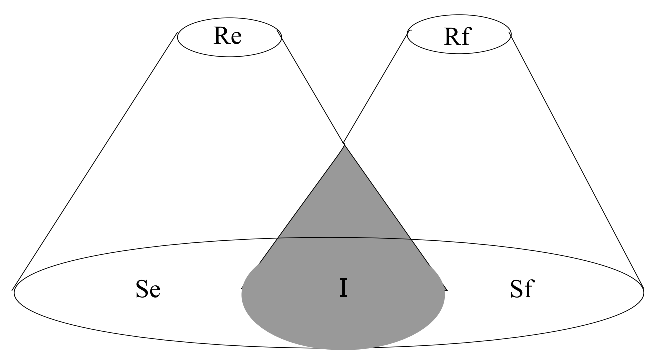

Figure 8: The future domain of dependence of the intersection of the backward light cones is trivial.

Note that this is not saying that were some correlations explained in terms of a screening off event lying within their common past, SEL would be violated. Nor is it saying that I cannot be relevant to correlated events on R_e and R_f. In particular, it is not denied above that, if S_e and S_f were fixed, SEL prescribes that variations of the probability distributions on R_e and R_f be determined by variations registering on I. However, the naive space-time interpretation of the PCC requires more than this determination by I, S_e and S_f being fixed: indeed, it demands that *whatever* S_e and S_f, variations on I *alone* account for synchronization of events on R_e and R_f. This is an empirical question about our world. And it is much more than what SEL can rigorously justify. By contrast, the only statement concerning probability distributions over events on R_e and R_f that SEL by itself implies is that they are to be determined by the total physical state on $S_e \cup S_f$.[38]

In the end then, we can formulate the space-time version of the PCC:

Definition 23 – ***The Principle of Common Cause: space-time version – STPCC***

Let e and f be two events located on spacelike regions R_e and R_f of ordinary relativistic space-time. It is required that, if e and f are statistically dependent, then there is a screening off event c, lying on an appropriate slice across the union of the backward light cones of R_e and R_f, by conditionalization on which e and f are made statistically independent.

$$p(e_{R_e} f_{R_f} | c_{D^-(R_e) \cup D^-(R_f)}) = p(e_{R_e} | c_{D^-(R_e) \cup D^-(R_f)}) p(f_{R_f} | c_{D^-(R_e) \cup D^-(R_f)})$$

Considering the definition above, one realizes that the PCC applied to our space-time structure (henceforth STPCC) corresponds to STOI. More precisely, in the case of Bell-type correlations, the requirement above is equivalent to the requirement that STOI holds. Recall Bell-type situations as described in Section 2: a_{R_1} and b_{R_2} are correlated outcome-events located on two regions of space-time R_1 and R_2. The experiment can be set up so that R_1 and R_2 are spacelike separated. We consider then the backward light cones of R_1 and R_2, and a time slice S across the union of these backward light cones. SEL prescribes that the total physical state σ_S on S determines the probability distributions over events on R_1 and R_2. If there is a screening off event for the correlations between a_{R_1} and b_{R_2}, then it has to be located on S, and conditionalizing on σ_S will guarantee screening off. Conversely, if conditionalizing on σ_S screens off the outcome-events from each other, then there is a screening off event. So, in the case of Bell-type correlations, STPCC becomes the requirement that:

$$p(a_{R_1}, b_{R_2} | \sigma_S) = p(a_{R_1} | \sigma_S).p(b_{R_2} | \sigma_S). \tag{29}$$

In the context we are considering, we have been assuming that the only factors on S that can influence probabilities of a_{R_1} and b_{R_2} are $\lambda_{S_3^*}, i_{S_1^*}$, and $j_{S_2^*}$. On this assumption, and in

[38] Note that our analysis also suggests that the question of whether or not the Principle of Common Cause – in its full-blown or weak versions as defined in Rédei and Summers (2002) – is valid in Local Quantum Field Theories is non trivial. See Rédei and Summers (2002), Hofer-Szabó and Vecsernyés (2012), and Hofer-Szabó and Vecsernyés (unpublished) for more details on this recent debate.

this situation then, STOI as defined in Definition 12, is equivalent to STPCC, which can be seen by substituting $\lambda_{S_3^*}, i_{S_1^*}, j_{S_2^*}$ for σ_S.

So, we have proved that the requirement that STOI holds is equivalent to the requirement of STPCC in the case of Bell-type situations. From this, one could object to our conclusion from Section 3 that STOI is not a locality condition in arguing that the PCC is a locality condition, and hence, that STOI is as well in the case of Bell-type situations. The following subsection is devoted to undermining such an objection.

4.3 STPCC is not a locality condition

Before we present our objections to the claim that the PCC is a locality condition, it will prove useful to analyze the common argument which is given in favor of that claim. Roughly speaking, the connection between the PCC and locality is made by saying that the PCC, when applied to correlations between relatively spacelike separated events, becomes a condition of locality. In Bell-type situations in particular, it seems quite common to believe that, on the one hand, locality is violated because of the failure of common cause models, and, on the other hand, were we to find a common cause, locality would be secured. However natural this idea might seem at first, the argument does not stand when laid out and analyzed in detail.

Here is how the argument goes. Consider two relatively spacelike separated events. Introducing the framework of relativity, note then that, under a rather common interpretation of relativistic constraints, these two events cannot cause each other: spacelike separation forbids any direct causal link. According to the PCC, if there is any correlation between two given events, this has to be explained in terms of a common cause. But, the argument goes, more can be said about such a common cause. In accordance with relativistic constraints, nothing but events lying within the backward light cone of an event E can have a causal influence on E. If then the common cause is so conceived as to have a causal influence on both of the correlated events, it has to lie within their common past.[39] However, if there is a correlation between relatively spacelike separated events, but there is no common cause in the common past, then a specific form of causal anomaly would occur, namely: non-locality. That is to say, locality would be preserved if and only if such a common cause could be found. If the above is true, then the PCC, when construed in space-time terms and applied to spacelike separated events is a locality condition.

Anyone would have recognized that the argument above looks very similar to our own analysis in the previous subsection. But in fact, the argument above goes beyond. Let us for now

[39]It is very likely that anyone formulating this argument would take the "common past" to be within the intersection of the backward light cones. We have already criticized this part of the argument in the previous subsection, and that will not be the point at issue in this subsection. Thus, it is not important what is meant here by "common past". In particular, it does not matter whether or not the common past is taken to be located "above" the intersection of the backward light cones.

make clear the logic of both arguments, our own in the previous section and the one above, in more detail.

Leaving aside the issues over rigorous definitions and vocabulary, insofar as relativistic space-times are assumed to rule out determination of probability distributions other than from appropriate slices across that past light cones, the argument up to (and including) "a specific form of causal anomaly occurs", seems fine. From the requirements of the PCC *and* of Locality the claim is made that given spacelike correlated events, a common cause must exist in the common past of these events. The problem lies in the very last step of the argument, which states that such a causal anomaly, i.e. a lack of a common cause, amounts to a form of non-locality. This last step simply does not follow.

To make this point more clear, let us consider the following notation:

- PCC stands for the requirement that any two correlated events that do not cause each other be screened off from each other by a screening off event;
- SEL stands for the requirement that probability distributions over events located on a region R of space-time be determined by the state on an appropriate time slice across the backward light cone of R;
- $STPCC$ stands for the requirement that any two correlated events located on spacelike separated regions of space-time be screened off from each other by a screening off event lying within the common past of the correlated events.

Now, the argument above amounts to saying that *if* one takes the PCC to be true, and *if* one wants to apply it to events embedded a space-time structure where Locality holds, then one can conclude that the PCC prescribes that such events be screened off from each other by a screening off condition which lies within their common past. This is perfectly fine and is essentially our own argument in the previous subsection. If, as it has been argued, SEL is the correct notion of locality in our space-time structure, the argument has the form:

$$PCC + SEL \rightarrow STPCC. \tag{30}$$

However, it should be clear enough that, from this, it does not follow that the PCC, nor STPCC is a locality condition. That is to say, it does not follow from this either that (a) violations of STPCC are violations of SEL, or that (b) SEL is secured whenever STPCC is satisfied.

What would be needed to draw Conclusion (a) is that SEL prescribes by itself that correlations between relatively spacelike events be screened off by a screening off event lying in the common past of the correlated events. That is to say, one would have to argue for the following independent premise (with the question marks indicating that the proposition following is not claimed to be true):

$$SEL \xrightarrow{???} STPCC. \tag{31}$$

Only with this in hand could someone argue that a violation of STPCC amounts to a form of non-locality.

What would be needed to draw Conclusion (b), is that SEL cannot fail whenever the STPCC is satisfied, that is to say, one would need to argue in support of the following independent premise:

$$STPCC \xrightarrow{???} SEL, \tag{32}$$

Of course, the trouble is that neither (31) nor (32) follow from (30). From this of course, it does not follow that either (31) or (32) is false. That said, it follows that the argument above alone does not support the thesis that STPCC is a locality condition. What needs to be assessed is whether (31) and (32) hold, that is to say, whether STPCC is either a necessary, or a sufficient condition for SEL (or both).

Earman (1986) shows that satisfaction of the PCC is neither necessary nor sufficient for Einstein Locality to be preserved in the deterministic case. We can generalize his arguments to the stochastic case, and show that STPCC is neither necessary or sufficient for SEL. First, it is not necessary. The argument here is very similar to our argument that SEL does not entail STOI in Section 3. Indeed, SEL prescribes that the probabilities for the given events be determined by the state on S, but it does not say what form these probabilities should take. In particular, it says nothing about whether the events should be correlated or not. So, STPCC has consequences that are not implied by SEL.

Nor is it the case that STPCC is sufficient for SEL. Earman's counter-example in this case is that the first half of STPCC "can be satisfied in action-at-a-distance particle theories that allow an indefinite number of particles" (Earman, 1986, 461). We can flesh out this in the following way by appeal to Bohm's theory. That it recovers what is generally taken to be a satisfactory scheme of explanation for Bell-type correlations is a great achievement of Bohm's theory. That it violates Einstein Locality, whether deterministic or stochastic, however, is one of its most well known drawbacks. Thus, Bohm's theory is a non-local theory in which STPCC is secured. Thus, it is simply not the case that STPCC entails SEL.

The upshot is thus that STPCC neither entails, nor is entailed by SEL. Since STOI is simply the application of STPCC to Bell-type situations, we can now deduce that the argument to the effect that STOI is a locality condition on the basis of its relation to the STPCC does not stand. The upshot is thus that STOI is not a locality condition, but a requirement equivalent to Reichenbach's Principle of Common Cause applied to Bell-type situations. Hence, failure of STOI is failure of STPCC. The issue of how to interpret failure of the STPCC remains open.[40]

[40]In particular, we shall leave open here the question whether or not failure of STPCC are to be interpreted

5 Conclusion

We have provided a rigorous space-time framework, within which we have formulated space-time versions of Factorizability, Outcome Independence and Parameter Independence (STFAC, STOI and STPI, respectively). We have argued that Bell's "Local Causality" LC is not an adequate locality condition in the stochastic case. We have defined instead a notion of stochastic locality by generalization of Einstein Locality defined in the deterministic case (Stochastic Einstein Locality, SEL). We have shown that SEL implies the space-time version of parameter independence, when applied to Bell-type situations. That said, we have also shown that the converse does not hold. The space-time version of parameter independence is a necessary but not a sufficient condition for stochastic einstein locality to hold.

Further, we have shown that SEL does not entail the space-time version of outcome independence STOI, so that failure of STOI does not imply non-locality. We have argued that Butterfield's argument to the contrary relies on a faulty assumption.

We closed our analysis by showing that the requirement that STOI holds is equivalent to the demands of the space-time version of the Principle of Common Cause (STPCC) when applied to Bell-type situations. We have shown that this relation between STOI and STPCC cannot provide a rationale in order to take STOI as a locality condition. This is because STPCC is neither necessary nor sufficient for SEL to hold. Whatever the principle of common cause is about, it is *not* about locality.

At the end of the day, we have thus provided a space-time framework in which (PI-LOC), i.e. one of the main claims of the mainstream interpretation of Bell-type theorems and experiments, can be rigorously assessed. Our result is that it (mostly) holds: failure of parameter independence alone amounts to a case of non-locality, while failure of outcome independence does not. This, of course, does not preclude that other definitions of the space-time structure considered, or of the versions of the various conditions considered, could not yield alternative results. That said, since Bell's Local Causality is criticized as an adequate condition of locality in the stochastic case, and Butterfield's argument is shown to be faulty, our analysis undermines the most likely candidates for supporting criticisms against the mainstream interpretation. Peaceful coexistence is maintained.

References

Albert, D. Z. and Galchen, R. (2009). A quantum threat to special relativity. (cover story), *Scientific American* **300**(3): 32–9.

in terms of holism à la Howard (1985) and his followers.

Arntzenius, F. (2005). Reichenbach's common cause principle, *in* E. N. Zalta (ed.), *The Stanford Encyclopedia of Philosophy.*
URL: *http://plato.stanford.edu/archives/spr2005/entries/physics-Rpcc/*

Bell, J. (1990). La nouvelle cuisine, *in* A. Sarlemijn and P. Kroes (eds), *Between Science and Technology*, Elsevier (North Holland), pp. 97–115.

Bell, J. S. (1976). The theory of local beables, *Epistemological Letters* **March**: 52–62.

Bell, J. S. (1987). *Speakable and Unspeakable in Quantum Mechanics*, Cambridge University Press, Cambridge.

Berkovitz, J. (1998a). Aspects of quantum non-locality I: Superluminal signaling, action-at-a-distance, non-separability and holism, *Studies in the History and Philosophy of Modern Physics* **29**(2): 183–222.

Berkovitz, J. (1998b). Aspects of quantum non-locality II: Superluminal causation and relativity, *Studies in the History and Philosophy of Modern Physics* **29**(4): 509–45.

Butterfield, J. (1989). A space-time approach to the Bell inequality, *in* J. T. Cushing and E. McMullin (eds), *Philosophical Consequences of Quantum Theory*, University of Notre Dame Press, Notre Dame, pp. 114–44.

Butterfield, J. (2007). Stochastic Einstein-locality revisited, *British Journal for the Philosophy of Science* **58**: 805–867.

Earman, J. (1986). Locality, non-locality and action at a distance: A skeptical review of some philosophical dogmas, *in* P. Achinstein and R. Kargon (eds), *Theoretical Physics in the 100 Years since Kelvin's Baltimore Lectures*, MIT Press, Cambridge.

Friederich, S. (2015). Re-thinking local causality, *Synthese* **192**(1): 221–240.
URL: *http://dx.doi.org/10.1007/s11229-014-0563-6*

Grasshoff, G., Portmann, S. and Wüthrich, A. (2005). Minimal assumption derivation of a bell-type inequality, *British Journal for the Philosophy of Science* **56**: 663–680.

Hellman, G. (1982). Stochastic Einstein-locality and the Bell theorems, *Synthese* **53**: 461–504.

Hellman, G. (1992). Bell-type inequalities in the nonideal case: Proof of a conjecture of Bell, *Foundations of physics* **22**(6): 807–17.

Hofer-Szabó, G. (2008). Separate- versus common-common-cause-type derivations of the Bell inequalities, *Synthese* **163**(2): 199–215.

Hofer-Szabó, G. (2011). Bell(δ) inequalities derived from separate common causal explanation of almost perfect EPR anticorrelations, *Foundations of Physics* **41**: 1398–413.

Hofer-Szabó, G. and Vecsernyés, P. (2012). Reichenbach's common cause principle in algebraic quantum field theory with locally finite degrees of freedom, *Foundations of Physics* **42**: 241–55.

Hofer-Szabó, G. and Vecsernyés, P. (unpublished). Bell inequality and common causal explanation in algebraic quantum field theory. (available at http://philsci-archive.pitt.edu/9101/).

Howard, D. (1985). Einstein on locality and separability, *Studies in History and Philosophy of Science* **16**: 171–201.

Jarrett, J. P. (1984). On the physical significance of the locality conditions in the Bell arguments, *Noûs* **18**: 569–89.

Jarrett, J. P. (1989). Bell's theorem: A guide to the implications, *in* J. T. Cushing and E. McMullin (eds), *Philosophical Consequences of Quantum Theory*, University of Notre Dame Press, Notre Dame, pp. 61–79.

Jones, M. R. and Clifton, R. K. (1993). Against experimental metaphysics, *Midwest Studies in Philosophy* **XVIII**: 295–316.

Maudlin, T. (1994). *Quantum Non-Locality and Relativity, Metaphysical Intimations of Modern Physics*, Blackwell, Oxford.

Norsen, T. (2009). Local causality and completeness: Bell vs Jarrett, *Foundations of Physics* **39**: 273–94.

Norsen, T. (2011). John S. Bell's concept of local causality, *American Journal of Physics* **79**(12): 1261–1275.

Rédei, M. and Summers, S. (2002). Local primitive causality and the common cause principle in quantum field theory, *Foundations of Physics* **32**: 335–55.

Reichenbach, H. (1956). *The Direction of Time*, University of California Press, Berkeley, Los Angeles.

Reichenbach, H. (1958). *The Philosophy of Space and Time*, Dover, New York.

Reichenbach, H. (1978). The causal structure of the world and the difference between past and future, *in* M. Reichenbach and R. S. Cohen (eds), *Selected Writings 1909-1953*, Vol. 2, Reidel, pp. 81–117.

Seevinck, M. (2010). Can quantum theory and special relativity peacefully coexist?, *Invited white paper for Quantum Physics and the Nature of Reality, John Polkinghorne 80th Birthday Conference. St Annes College, Oxford.*

Seevinck, M. and Uffink, J. (2011). Not throwing out the baby with the bathwater: Bell's condition of local causality mathematically 'sharp and clean', *in* D. Dieks, W. Gonzalez, S. Hartmann, T. Uebel and M. Weber (eds), *Explanation, prediction and Confirmation.*

New trends and old ones reconsidered., Springer Science & Business Media, Dortrecht, Neth., pp. 425–50.

Shimony, A. (1978). Metaphysical problems in the foundation of quantum mechanics, *International Philosophical Quaterly* **18**: 3–17.

Shimony, A. (1986). Events and processes in the quantum world, *in* R. Penrose and C. Isham (eds), *Quantum Concepts in Space and Time*, Oxford University Press, Oxford, pp. 182–203.

Shimony, A. (1989). Search for a worldview which can accomodate our knowledge of microphysics, *in* J. T. Cushing and E. McMullin (eds), *Philosophical Consequences of Quantum Theory*, University of Notre Dame Press, Notre Dame, pp. 25–37.

van Fraassen, B. C. (1980). *The Scientific Image*, Oxford University Press.

Wüthrich, A. (2004). *Quantum Correlations and Common Causes*, Bern Studies in the History and Philosophy of Science.