On Bell's Everett (?) theory

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Abstract

Bell's Everett (?) theory is Bell's interpretation of Everett's theory, aiming to remove the picture of many worlds from the theory. In this paper, I argue that Bell's Everett (?) theory as a one-world theory contradicts quantum mechanics and experiments. Moreover, I argue that a proper understanding of this theory also leads to a picture of many worlds, and this many-worlds theory agrees with experiments.

John Bell was one of the few leading figures in quantum foundations who took Everett's theory seriously as early as 1970s. Certainly, he was an opponent, not a proponent. In 1971, Bell wrote a paper titled "On the hypothesis that the Schrödinger equation is exact" (Bell, 1971). The paper was latter published in Epistemological Letters (Bell, 1978), and a revised version of the paper was published with a more well-known title "Quantum mechanics for cosmologists" in the volume Quantum Gravity 2 (Bell, 1981). In these papers, Bell proposed his Everett (?) theory as the "final synthesis, omitting de Broglie's trajectories and Everett's other branches" (Bell, 1976). Bell thought that this theory agrees with quantum mechanics and experiments, although he did not like it due to the unreliability of an observer's memory in the theory.[1] Later, Barrett (1999) pointed out that this theory is plagued by an empirical incoherence problem, namely that even if the theory were correct, one could not have an empirical justification for accepting that it is correct. In this paper, I will present a new analysis of Bell's Everett (?) theory. I will argue that this theory as a one-world theory contradicts quantum mechanics and experiments. Moreover, a proper understanding of the theory will lead to a clearer picture of many worlds, and this many-worlds theory agrees with experiments.

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[1] Bell said that "if such a theory were taken seriously it would hardly be possible to take anything else seriously." (Bell, 1981)

According to Bell (1981), his Everett (?) theory is simply the pilot-wave theory of de Broglie and Bohm without continuous particle trajectories. In the pilot-wave theory (de Broglie, 1928; Bohm, 1952), a complete realistic description of a quantum system is provided by the configuration defined by the positions of its particles together with its wave function. The law of motion is expressed by two equations: a guiding equation for the configuration of particles and the Schrödinger equation, describing the time evolution of the wave function which enters the guiding equation. The law of motion can be formulated as follows:

$$\frac{dX(t)}{dt} = v^{\Psi(t)}(X(t)), \tag{1}$$

$$i\hbar\frac{\partial\Psi(t)}{\partial t} = H\Psi(t), \tag{2}$$

where $X(t)$ denotes the spatial configuration of particles, $\Psi(t)$ is the wave function at time t, and v equals to the velocity of probability density in standard quantum mechanics. Moreover, it is assumed that at some initial instant t_0, the epistemic probability of the configuration, $\rho(X(t_0), t_0)$, is given by the Born rule: $\rho(X(t_0), t_0) = |\Psi(X(t_0), t_0)|^2$. This is the quantum equilibrium hypothesis, which, together with the law of motion, ensures the empirical equivalence between the pilot-wave theory and standard quantum mechanics.

Bell thought that the continuous particle trajectories are not an essential part of the pilot-wave theory, and there is no need to link successive particle configurations into a continuous trajectory (Bell, 1981). Bell further argued that keeping the instantaneous configurations, but discarding the trajectories, is the essential of Everett's theory (Everett, 1957). This is Bell's Everett (?) theory (BET in brief). In Bell's own words,

> instantaneous classical configuration x are supposed to exist, and to be distributed in the comparison class of possible worlds with probability $|\psi|^2$. But no pairing of configuration at different times, as would be effected by the existence of trajectories, is supposed. (Bell, 1987, p.133)

In BET, the deterministic guiding equation of the pilot-wave theory is replaced by a random dynamics:

$$\rho(X(t), t) = |\Psi(X(t), t)|^2, \tag{3}$$

which means that at every instant the particle configuration is random, and its probability of being a given $X(t)$ is equal to the Born probability $|\Psi(X(t), t)|^2$. In other words, the particles do not move in a continuous, deterministic way, but move in a discontinuous and random way. It can be seen that the above random dynamics makes the quantum equilibrium hypothesis unnecessary, and it unifies this seemingly ad hoc hypothesis with the guiding equation in some sense.

BET, being a one-world theory, is Bell's attempt to refute the many worlds picture of Everett's theory. Bell wrote, "it seems to me that this multiplication of universes is extravagant, and serves no real purpose in the theory, and can simply be dropped without repercussions." (Bell, 1987, p.133) In the following, I will argue that Bell's attempt is not successful. The failure lies in two aspects: one is that BET is not consistent with quantum mechanics and experiments, and the other is that a proper understanding of BET also leads to a picture of many worlds.

Consider a typical z-spin measurement, in which an observer M measures the z-spin of a spin-1/2 system S being in a superposition of two different z-spins (see also Barrett, 1999, 123-6). According to the linear Schrödinger equation, the state of the composite system after the measurement will be a superposition of M recording z-spin up and S being z-spin up and M recording z-spin down and S being z-spin down:

$$\alpha \left|up\right\rangle_S \left|up\right\rangle_M + \beta \left|down\right\rangle_S \left|down\right\rangle_M , \tag{4}$$

where α and β are not zero and satisfy the normalization condition $|\alpha|^2 + |\beta|^2 = 1$.

According to BET, the positions of the particles representing the measurement record of M are definite at each instant. Moreover, these particles randomly jump between the two states $\left|up\right\rangle_M$ and $\left|down\right\rangle_M$ over time, and the probability of they being in these two states are $|\alpha|^2$ and $|\beta|^2$, respectively. Then the observer M will obtain a definite record corresponding to one of the two terms in the above superposition at each instant. Moreover, which record M obtains is randomly determined, and the probability of M getting a particular record is equal to the modulus squared of the wave function associated with the record, namely the probability of M recording z-spin up is $|\alpha|^2$ and the probability of M recording z-spin down is $|\beta|^2$. This is consistent with the Born rule.

Obviously, due to the essential randomness of the motion of particles, the observer M's measurement record will change in a random way over time and thus be unreliable as a record of what actually happened. As Bell (1981) argued, however, that there is no association of the particular present with any particular past does not matter. "For we have no access to the past. We have only our 'memories' and 'records'. But these memories and records are in fact present phenomena. The theory should account for the present correlations between these present phenomena. And in this respect we have seen it to agree with ordinary quantum mechanics, in so far as the latter is unambiguous." (Bell, 1987, 135-6)

Here is a more detailed explanation of Bell's idea as given by Barrett

(1999, 123-5). Suppose the observer M gets the result z-spin up for her first measurement. When she repeats her measurement, the state of the composite system after this second measurement will be

$$\alpha \left|up\right\rangle_S \left|up, up\right\rangle_M + \beta \left|down\right\rangle_S \left|down, down\right\rangle_M \tag{5}$$

by the linear Schrödinger evolution. Now, according to BET, there is a probability of $|\beta|^2$ that M will end up with a configuration recording z-spin down for the second result even though he recorded z-spin up for the first result. Thus it appears that there is a probability of $|\beta|^2$ that M's two measurement results will disagree. However, if M does get z-spin down for her second measurement, her configuration will be the one associated with the second term of the above state. This means that M's actual memory configuration will record z-spin down for her first result, and thus for M the two measurements in fact yield the same result. Therefore, for repeated measurements, BET still agrees with quantum mechanics.

What the above analysis shows is the consistency of the results of repeated measurements on a single quantum system in BET. According to the theory, if M records z-spin up for her first result, there is still a non-zero probability $|\beta|^2$ that he will get z-spin down for her second measurement. This prediction itself contradicts quantum mechanics, according to which if M's first result is z-spin up, then her second result must be also z-spin up. But according to BET, when we actually test this prediction, a strange thing happens: M's second measurement, whose result is z-spin down, will change her memory of the first result and make it be also z-spin down. In this case, M will think her first result is also z-spin down, and thus her second result is the same as her first result as quantum mechanics predicts.

Now consider an ensemble of the above spin one-half systems being in a superposition of two different z-spins, for which M's first measurement result is z-spin up. These results are recorded. This is preparation of the z-spin up state in experiments. Then, M makes her second measurements on these systems. Quantum mechanics will predict and experiments will also show that M's second measurement results will be all z-spin up, the same as her first recorded results. This is verification of the prepared z-spin up state. These two procedures are common in usual quantum experiments. However, BET will predict that M's second measurement results will be partly z-spin up and partly z-spin down. Certainly, according to this theory, M will not know that her first measurement results are all z-spin up; rather, she will think her first measurement results are also partly z-spin up and partly z-spin down, and for each system her second result is still the same as her first result. Thus, BET contradicts quantum mechanics and experimental observations for this experiment, and for the preparation and verification of quantum states in general.

Another way to see the above contradiction is to notice that in BET we cannot prepare an ensemble of quantum systems on which the results of the measurements of an observable are all the same in general. In other words, in BET we cannot prepare a quantum state which is equivalent to an eigenstate of an observable in standard quantum mechanics. Since BET is a unitary quantum theory, the state of a quantum system is in general a superposition of the eigenstates of an observable. Then due to the random jumps of particles, the measurement result for each system is always random, being one of the eigenvalues of the observable, and the results will be a Born probability distribution over all eigenvalues. This is not consistent with experiments. By comparison, in other unitary quantum theories such as the pilot-wave theory or the many-worlds interpretation of quantum mechanics, we can prepare an ensemble of quantum systems for which the results of the measurements of an observable on these systems are all the same. This is consistent with experiments.

An interesting question then arises: where does BET go wrong in theory? In my view, it fails to identify the observers in different result branches as different observers. It is arguable that two persons with different memories about the past events such as the result of a measurement should be regarded as two different persons. In the previous example of z-spin measurement, although the observers in the two result branches are composed of the same particles and they also have the same history of random motion (which may make some people think they are the same person), they have different particle configurations which contain different recorded histories or memories due to the linear Schrödinger evolution (see (4) and (5)). Then, if it is required that both the actual history and the recordable history must be the same in order that two persons are qualified to be the same person, then the two observers in different result branches should be regarded as two different observers.[2]

A similar analysis can also be given for other systems such as measuring devices. The key point is that the (typical) particle configuration that represents a system in each result branch undergoes different, independent time evolution due to the linearity of the Schrödinger equation, and the effects of the interactions of the system with the environment are accumulated only in each result branch, not between different result branches. Thus, the systems in different result branches will have different behaviors and recordable histories, and they should be regarded as different systems, not different states of the same system.

Then, a proper understanding of BET will lead to a picture of many worlds (see also Duerr and Ehmann, 2021 for an excellent exposition of a similar idea). This is against Bell's original expectation. In this picture, the

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[2] In the final analysis, it is the law of motion, namely the Schrödinger equation that leads to the difference between the actual history and the recordable history. If the law of motion permited that the observer can record the whole history of her random jumps such as "up, down", then there would indeed exist only one observer as BET assumes.

random jumps of the particles among different result branches provides a way of time division multiplexing for the existence of many worlds, which may be called time division multiverse (Gao, 2021). It can be seen that in this many-worlds theory, the memory of each observer is reliable, and the theory also agrees with experiments.

It has been widely argued that the locality of interactions and the resulting environmental decoherence yield the stability of worlds in which objects are well localized (Vaidman, 2021). However, as can be seen from the above analysis, decoherence is not the essential reason for the appearance of many worlds in the above theory. Moreover, the structure and pattern of the wave function, which is a necessary element for defining worlds in Wallace's (2012) formulation of the many-worlds interpretation, is also replaced by the clearer particle configuration in three-dimensional physical space in the above many-worlds theory.

Finally, it is worth noting that the picture of random motion of particles in BET may have a firm basis. It has been argued that a quantum system is composed of particles with mass and charge which undergo random discontinuous motion (RDM) in three-dimensional space, and the wave function represents the propensities of these particles which determine their RDM, and as a result, the state of RDM of particles is also described by the wave function (Gao, 2017, 2020).[3] If this interpretation of the wave function in terms of RDM of particles is true, then no additional ontologies and postulates are introduced in the above many-worlds theory. Moreover, the RDM of particles provides a natural solution to the two thorny problems of the many-worlds interpretation, namely the problems of ontology and probability (Wallace, 2012; Maudlin, 2014). A more detailed analysis of this new many-worlds theory will be given in another paper.[4]

To sum up, I have argued that Bell's Everett (?) theory as a one-world theory contradicts quantum mechanics and experiments. Moreover, a proper understanding of this theory will lead to a clearer picture of many worlds. This many-worlds theory of random discontinuous motion of particles agrees with experiments, and it deserves further investigation.

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[3] By comparison, the wave function has been widely regarded as a real physical field in a high-dimensional space. It is arguable that Bell also held this view of the wave function in his Everett (?) theory (Bell, 1976, 1981).

[4] An initial, mainly critical analysis of this many-worlds theory was given in Gao (2017, chap.8).

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