Can pragmatist quantum realism explain protective measurements?

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Abstract

According to Healey's pragmatist quantum realism, the only physical properties of quantum systems are those to which the Born rule assigns probabilities. In this paper, I argue that this approach to quantum theory fails to explain the results of protective measurements.

In recent years, Healey proposed an intriguing pragmatist approach to quantum theory, which he called pragmatist quantum realism or desert pragmatism (DP) (Healey, 2012, 2017a, 2017b, 2020, 2022). DP claims to offer a realist view of quantum theory despite its denial that the wave function is an element of physical reality. What DP takes to be physically real is not the wave function, but properties of quantum systems to which the Born rule assigns probabilities. It has been debated whether DP can offer objective explanations of quantum phenomena and whether its explanations are satisfactory (Jansson, 2020; Lewis, 2020; Wallace, 2020). In this paper, I will present a new analysis of DP. In particular, I will argue that DP fails to explain the results of protective measurements.

Protective measurement (PM) is a method to measure the expectation value of an observable on a single quantum system (Aharonov and Vaidman, 1993; Aharonov, Anandan and Vaidman, 1993; Vaidman, 2009; Gao, 2015; Piacentini et al, 2017). During a PM the wave function of the measured system is protected by an appropriate procedure so that it keeps unchanged during the measurement.[1] Then, by the Schrödinger evolution, the measurement result will be directly the expectation value of the measured observable,

[1]Note that the protection requires that some information about the measured wave function should be known before a PM, and thus PM cannot measure an arbitrary unknown wave function.

even if the system is initially not in an eigenstate of the observable. By contrast, for a conventional projective measurement, the wave function of the measured system is in general changed during the measurement, and one obtains an eigenvalue of the measured observable randomly with the Born probability, and the expectation value of the observable can be obtained only as the statistical average of eigenvalues for an ensemble of identically prepared systems.

Since the wave function can be reconstructed from the expectation values of a sufficient number of observables, the wave function of a single quantum system can be measured by a series of PMs. Let the explicit form of the measured wave function at a given instant t be $\psi(x)$, and the measured observable A be (normalized) projection operators on small spatial regions V_n having volume v_n:

$$A = \begin{cases} \frac{1}{v_n}, & \text{if } x \in V_n, \\ 0, & \text{if } x \notin V_n. \end{cases} \tag{1}$$

A PM of A then yields

$$\langle A \rangle = \frac{1}{v_n} \int_{V_n} |\psi(x)|^2 dv, \tag{2}$$

which is the average of the density $\rho(x) = |\psi(x)|^2$ over the small region V_n. Similarly, we can measure another observable $B = \frac{\hbar}{2mi}(A\nabla + \nabla A)$. The measurement yields

$$\langle B \rangle = \frac{1}{v_n} \int_{V_n} \frac{\hbar}{2mi}(\psi^* \nabla \psi - \psi \nabla \psi^*) dv = \frac{1}{v_n} \int_{V_n} j(x) dv. \tag{3}$$

This is the average value of the flux density $j(x)$ in the region V_n. Then when $v_n \to 0$ and after performing measurements in sufficiently many regions V_n we can measure $\rho(x)$ and $j(x)$ everywhere in space. Since the wave function $\psi(x)$ can be uniquely expressed by $\rho(x)$ and $j(x)$ (except for an overall phase factor), the whole wave function of the measured system at a given instant can be measured by PMs.

Let's see whether Healey's pragmatist quantum realism or DP can explain the results of PMs.[2] Suppose in an isolated lab there are a quantum system with charge Q trapped in a box, a test electron and a detecting screen (see Figure 1). The quantum system is in the ground state $\psi(x)$ in the box. The test electron, whose initial state is a Gaussian wavepacket narrow in both position and momentum, is shot along a straight line near the box. The electron is detected on a screen after passing by the box. Suppose we

[2]The following analysis can be regarded as a further development of my objections to QBism (Gao, 2021).

make an adiabatic-type PM of the charge of the system in the box.[3] Then, according to the Schrödinger equation with an external Coulomb potential, the deviation of the trajectory of the electron wavepacket is determined by the charge of the system Q, as well as the distance between the electron wavepacket and the box. Moreover, the ground state of the measured system does not change during the measurement. If there were no charged quantum system in the box, the trajectory of the electron wavepacket would be a straight line as denoted by position "0" on the screen. Now, the trajectory of the electron wavepacket will be deviated by a definite amount as denoted by position "1" on the screen.

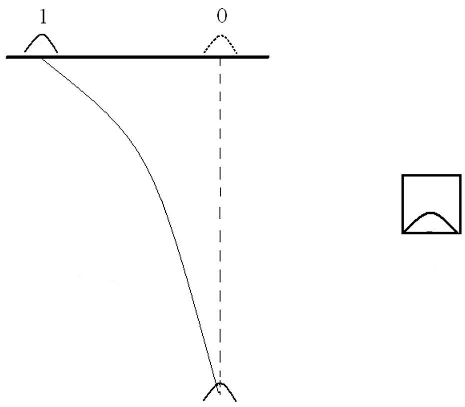

Figure 1: Scheme of a protective measurement of the charge of a quantum system in one box

This experiment can be regarded as a measurement of the charge of a quantum system being in an eigenstate of the charge. That there is no system in the box corresponds to the eigenstate of the charge with eigenvalue 0, and that there is a system with charge Q in the box corresponds to the eigenstate of the charge with eigenvalue Q.

According to DP, the only physical properties of quantum systems are those to which the Born rule assigns probabilities. As for the above experiment, this supposedly means that the physical property of the quantum system in the box is its charge, whose eigenvalues are 0 and Q; when the trajectory of the test electron wavepacket is a straight line, there is no charge in the box, while when the trajectory of the test electron wavepacket is deviated, there is a charge Q in the box, which makes the electron wavepacket

[3] The conditions for making such an adiabatic-type PM are: (1) the measuring time of the electron is long enough compared to $\hbar/\Delta E$, where ΔE is the smallest of the energy differences between the ground state and other energy eigenstates, and (2) at all times the potential energy of interaction between the electron and the system is small enough compared to ΔE (Aharonov, Anandan and Vaidman, 1993).

deviate from its free trajectory.

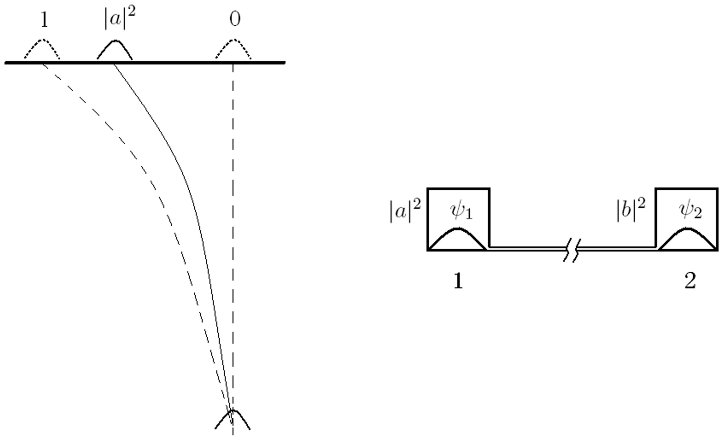

Figure 2: Scheme of a protective measurement of the charge of a quantum system in two boxes

Now consider another more interesting example of PMs. Suppose in an isolated lab there are a quantum system with charge Q, trapped in a two-box protective potential, a test electron and a detecting screen (see Figure 2). The wave function of the quantum system at an initial instant is $\psi(x) = a\psi_1(x) + b\psi_2(x)$, where $\psi_1(x)$ and $\psi_2(x)$ are two normalized wave functions respectively localized in their ground states in two small identical boxes 1 and 2, and $|a|^2 + |b|^2 = 1$. A test electron, whose initial state is a Gaussian wavepacket narrow in both position and momentum, is shot along a straight line near box 1 and perpendicular to the line of separation between the two boxes. The electron is detected on a screen after passing by box 1. Suppose the separation between the two boxes is large enough so that a charge Q in box 2 has no observable influence on the electron. Then if the system is in box 2, namely $|a|^2 = 0$, the trajectory of the electron wavepacket will be a straight line as denoted by position "0" on the screen. If the system is in box 1, namely $|a|^2 = 1$, the trajectory of the electron wavepacket will be deviated by a maximum amount as denoted by position "1" on the screen. When we make a PM of the charge of the system in box 1,[4] the trajectory of the electron wavepacket is determined by the expectation value of the charge of the system in box 1, and thus it will be deviated by an intermediate amount as denoted by position "$|a|^2$" between "0" and "1"

[4]Since the state of the system $\psi(x)$ is degenerate with its orthogonal state $\psi^{\perp}(x) = b^*\psi_1(x) - a^*\psi_2(x)$, we need a protection procedure to remove the degeneracy, e.g. joining the two boxes with a long tube whose diameter is small compared to the size of the box. By this protection $\psi(x)$ will be a nondegenerate energy eigenstate. Then we can make an adiabatic-type PM of the charge of the system in box 1 as in the first example.

on the screen.

Can DP explain the result of this PM? The answer seems negative. There cannot be no charge in box 1, since the trajectory of the test electron wavepacket is deviated. The charge in box 1 cannot be the whole charge Q either, since the trajectory of the test electron wavepacket is deviated not by the maximum amount "1", but the partial amount "$|a|^2$" between "0" and "1".

It is not beyond expectations that DP may have a potential issue of explaining PMs. According to DP, the primary target of explanation is not individual events but what Healey called probabilistic phenomena. A probabilistic phenomenon is a probabilistic data model of a statistical regularity. One explains the phenomenon by demonstrating how the probabilities of the model are a consequence of the Born rule, as applied to events that manifest the regularity. However, PM is not a probabilistic phenomenon related to the Born rule; rather, the result of a PM is definite, determined only by the deterministic Schrödinger equation and independently of the Born rule. Therefore, since DP refers only to the probabilistic phenomena which do not include PMs, it may have issues when explaining the results of PMs.

Let's see if DP can be revised or further developed to be able to explain the result of the above PM. The previous analysis, as well as the existing version of DP, does not consider the dynamics or the time evolution of the physical properties. Without the dynamics DP is incomplete at least. In the above example, we need to consider the motion of the charge Q in the two boxes. First, assume that the motion of the charge is continuous. Since the speed of continuous motion is finite, it needs a finite time for the charge Q to form an effective charge distribution $|a|^2Q$ in box 1, which can be used to explain the result of the PM. This also means that during a very short time interval there is no effective charge distribution $|a|^2Q$ in box 1. For example, during a very short time interval the charge Q has been in box 2 and there is no charge distribution in box 1. On the other hand, although the measuring time of the PM is long, the trajectory of the test electron wavepacket is deviated during any short time interval (and the rate of deviation is also proportional to $|a|^2Q$). Thus, the continuous motion of the charge or continuous dynamics cannot explain the resutls of PMs.

Next, assume that the motion of the charge is discontinuous. In this case, the motion of the charge Q may form an effective charge distribution $|a|^2Q$ in box 1 during an arbitrarily short time interval or an infinitesimal time interval (Gao, 2017). Then, the discontinuous motion of the charge or discontinuous dynamics may explain the result of the PM. However, this means that the wave function density $|a|^2$ is already a physical property of the system, which is defined during an infinitesimal time interval around an instant (like the standard velocity in classical mechanics).[5] Similarly, we

[5]It can be further argued that at a deeper level the wave function density represents an

can argue that the flux density is another physical property of the system. Since the wave function can be expressed by the density and flux density, it also represents the physical properties of the system. Therefore, although discontinuous dynamics may explain the results of PMs, it is beyond the scope of DP, since what DP takes to be physically real is not the wave function, but properties of quantum systems to which the Born rule assigns probabilities.

Finally, it is arguable that failing to explain the results of PMs is a serious issue. Although one may, like a QBist (Fuchs, 2018; Schack, 2018), assume a radical notion of quantum indeterminism that no mechanism determines the Born probabilities for the results of conventional measurements (see Myrvold, 2020a, 2020b for objections to this notion), it seems that there should exist certain mechanism which determines the *definite* results of PMs; otherwise our reasonings in other sciences and in daily life for definite causal-effect relationships will also be invalid.

To sum up, I have argued that Healey's pragmatist quantum realism, which insists that the only physical properties of quantum systems are those to which the Born rule assigns probabilities, fails to explain the results of protective measurements. Moreover, it seems that one must admit the reality of the wave function in order to explain the results of protective measurements.

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instantaneous property which determines the discontinuous jumps of the charge to form the right effective charge distribution $|a|^2Q$ in box 1, which may be called propensity (Gao, 2017).

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