Renormalization Group Methods:Which Kind of Explanation?

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Abstract

Renormalization and Renormalization Group (RG) have proven to be very powerful tools in contemporary physics, with a decisive influence on how to conceive of key physical aspects, including theories themselves. While they can be tackled from a variety of standpoints, this paper focuses on a specific philosophical issue, that is, which kind of explanation can be provided by means of RG methods. After a short, historical overview to set out the physical context, we scrutinize recent debates on the topic, with a particular focus on Morrison's seminal work. With respect to her account, where RG explanation is portrayed as mathematical, non-reductive, and non-causal, our focus is on the first aspect. Our claim is that RG theory's explanatory role cannot reside exclusively in its mathematical character, independently from a physical interpretation: mathematical and physical features intersect in a highly non-trivial way to provide an explanation of physical phenomena.

Keywords: Renormalization group, Effective theories, Critical phenomena, Non-causal explanation, Mathematical explanation

1 Introduction

In recent debates on causal *vs* non-causal explanation – an issue which has attracted much attention in the current literature on scientific explanation (e.g., Reutlinger, 2017a, 2017b; Reutlinger and Saatsi, 2018; Saatsi 2021) – it has become quite typical to turn to contemporary theoretical physics for examples of explanations not reducible to causal terms. The paradigmatic example is the use which is made of physical symmetries and, more generally, of principles acting as constraints in physics for illustrating cases of (allegedly) non-causal explanation (see, for example, Lange, 2017, 2019; French and Saatsi, 2018).[1] Other examples are usually provided by identifying the role of the *explanans* in the explanatory process in abstract, mathematical properties, structures or procedures (e.g., limiting procedures) used in physics.[2]

In the last decade, a case for non-causal explanation on which a growing literature has focused is the "renormalization group explanation of universality", as has become usual to call it, where the context is that of the physics of critical phenomena in statistical mechanics (SM) and condensed matter physics (CMP)[3] (e.g., Morrison, 2012, 2015a, 2015b, 2018, 2019; Reutlinger 2014, 2017a, 2017b; Hüttemann, Kühn and Terzidis, 2015; Saatsi and Reutlinger, 2018; Sullivan 2019).[4]

[1] Unless otherwise necessary, we limit the references to the recent, representative ones.

[2] This is well illustrated by the discussion of asymptotic explanations (Batterman, 2000), or, more generally, by the role of idealizations in science (for recent examples, see Shech, 2018; Palacios and Valente, 2021).

[3] SM is the branch of physics which adopts probability theory and statistical methods to investigate large ensembles of microscopic components – such as molecules and atoms – in their possible configurations. CMP broadly refers to the study of physical systems in their condensed states, namely solid and liquid. From now on, we will simply cite CMP, though SM represents its core theory.

[4] In this literature, the explanandum corresponds to the universality of critical exponents figuring in CMP critical phenomena. The seminal contribution for this discussion is Batterman (2002).

Renormalization group (RG) is a technical concept, with an interesting story of cross-fertization between quantum field theory (QFT) and SM from the mid-1950s through the 1970s.[5] RG and the methods based on this concept are now central, sophisticated ingredients in the actual practice of physical theorizing. It could surprise that, in order to find cases for the causal/non-causal distinction in natural sciences, philosophers choose to focus on such an advanced physical technique. In fact, there are some good reasons for philosophers to dwell on this fundamental tool in contemporary physics. First of all, for its undeniable import from both a methodological and conceptual point of view. But also because of the new perspective that RG and the related conception of physical theories as effective theories can provide on traditional issues such as inter-theory relations, reduction and emergence.[6]

Here we are interested, in particular, in framing the issue of the explanatory role of RG techniques in the general context of the relationship between mathematics and physics, i.e., more precisely, between mathematical and physical explanations. Our focus will be, accordingly, on such questions as to whether RG methods are to be portrayed as a pure mathematical technique or as a physical tool cast in mathematical terms. We will then scrutinize whether and to what extent RG theory can bear explanatory information about physical phenomena in case it is conceived as a genuine mathematical device, as claimed especially by Margaret Morrison in her influential work on the topic. In Morrison (2015a, Ch.2), for example, the declared intent is to show "that stripped of any physical adornment, mathematical techniques/frameworks are capable of generating physical information in and of themselves" (p. 51). As RG methods provide a fruiftul example to explore, the question becomes "how, as a purely mathematical technique, RG nevertheless appears able to provide us with information about the behaviour of a system at critical point" (p. 55). As we will see, Morrison takes RG methods to provide a mathematical, non-reductive, non-causal explanation. By providing an assessment of the way in which these three elements of RG methods operate and intertwine, our intent is then to problematize such a characterization, also in the light of the current discussion on RG methods in the broader context of non-causal explanations.

The paper is structured as follows. Section 2 offers a concise historical reconstruction of RG methods, while emphasizing their conceptual legacy and interpretational subtleties. The aim is to highlight what makes RG methods particularly fruitful in both QFT and CMP, as a mathematical technique applied to a physical context. Section 3 provides an overview of the most up-to-date philosophical literature on the topic, with a specific focus on the causal *vs* non-causal nature of RG methods, as well as their reductive *vs* non-reductive interpretation. In section 4 we discuss and problematize Morrison's account of RG methods, according to which the latter are portrayed as mathematical, non-reductive, non-causal. We particularly focus on the mathematical character of RG theory and claim, *contra* Morrison and others, that RG methods, while mathematical *per se*, cannot explain physical phenomena without supplementary, physical elements. We conclude that no explanation can be guaranteed by RG without a physically salient interpretation of the specific context at stake.

2 Renormalization Group Methods

Right from its early formulation, QFT has been plagued with infinities, a quite dissatisfying condition motivating to look for a proper strategy to generate finite solutions. What became known as the "old theory of renormalization" was introduced in the 1940s precisely with this aim, that is, to deal with the divergence problems arising in QFT (at the time, quantum elec-

[5] See here, section 2 for a short description. A recent, detailed overview of RG methods, starting from the seminal contributions by M. Gell-Mann and F. Low, L. Kadanoff, M. Fisher and K. Wilson, with attention also to philosophical aspects, is Williams (2021). Fraser (2020) provides a detailed reconstruction of the development of RG methods in particle physics, highlighting the role of formal analogies between classical SM and QFT.

[6] For what regards the development of the EFT idea and its relevance to the philosophical debate on inter-theory relations (and the related debates on reduction, fundamentality and emergence), see, for instance, Cao and Schweber (1993); Hartmann (2001); Castellani (2002); Butterfield and Bouatta (2015), Rosaler and Harlander (2019). A recent historical-philosophical reconstruction of Wilson's work on EFT is Rivat (2021).

trodynamics). In its original meaning, renormalization was basically a tool for removing the infinities occurring in perturbative calculations in a QFT. To do this, the conventional strategy was more or less the following: first, separate the divergent parts (high energy processes) from the finite parts (low energy processes) by introducing a cutoff Λ (threshold energy for the validity of the theory); then, absorb the divergences in some appropriate redefinition (renormalization) of the parameters of the theory (i.e., masses and coupling constants); finally, to take into account the neglected high energy effects, remove the cutoff.[7]

A crucial step towards a new understanding of renormalization theory was the introduction of the RG concept in the QFT framework during the 1950s, on the grounds of renormalization invariance: that is, an arbitrariness in the choice of the parametrization of the theory to renormalize (and the consequent introduction of RG as the group of transformations relating the different parametrizations).[8] The fundamental contribution in this direction was Gell-Mann and Low's (1954) famous paper, "one of the most important ever published in quantum field theory" according to Weinberg (1983, 1). Following Weinberg (1983, 7), this was where it was first realized that QFT has a scale invariance and that the breaking of this invariance (due to the renormalization procedure) could be accounted for by keeping track of the running coupling constant.[9]

Starting with Gell-Mann and Low's analysis, the formulation of RG methods has undergone a long and varied history throughout the 1960s and 1970s. In fact, up to the early 70s, the RG ideas of Gell-Mann and Low were substantially neglected.[10] A new understanding of renormalization theory started to take place only after the revival and extension of the application of RG methods by Kenneth Wilson between the late 60s and early 1970s.[11] By applying his previous results on the RG in field theories on a lattice to the study of critical phenomena (generalizing the 1966 Kadanoff theory of scaling near the critical point for an Ising ferromagnet),[12] Wilson in fact laid the basis for the current conception of renormalization as "an expression of the variation of the structure of physical interactions with changes in the scale of the phenomena being probed" (Gross, 1985, p. 153), where RG regulates (through the so-called RG equations) the way in which this variation occurs.

Deeply intertwined with such developments is the emergence of the effective theory approach. In general terms, an effective theory (ET) – effective field theory (EFT) in the framework of QFT – is a theory which "effectively" captures what is physically relevant in a given domain, by providing an appropriate, convenient description of the important physics in a given region of the parameter space.[13] Such a description, only applicable within a well-defined domain of validity, is thus intrinsically approximate.[14]

Historically, although "effective field theories are an old case in physics, going back to the 1936 Euler-Heisenberg nonlinear Lagrangian for photon-photon scattering and they had been used to derive soft pion theorems since 1967" (Weinberg, 1997, p. 42), it is only in the late 1970s and thanks to the development of RG methods that it was realized that "effective field theories

[7] This was achieved by letting the cutoff run to infinity, with the consequently arising problems regarding its actual meaning. See, for example, Cao and Schweber (1993, pp. 52–55).

[8] The seminal works are Stueckelberg and Petermann (1953), Gell-Mann and Low (1954), and Bogoliubov and Shirkov (1955). For detailed accounts of the early history of RG, see Weinberg (1983); Shirkov (1993); Brown (1993).

[9] More precisely, following Weinberg (1983, p. 7): the scale invariance is broken by particle masses, but these masses are negligible at very high energy (very short distances) by means of an appropriate renormalization, which is then the only cause for breaking the scale invariance.

[10] Apart from Wilson, one of the few in that period to be aware of their importance, according to Weinberg (1983, p. 10).

[11] See especially Wilson (1971, 1972). A historical reconstruction of his work is provided in Wilson (1983).

[12] Kadanoff's (1966) strategy is that of reducing the degrees of freedom of a certain physical system near a critical point by dividing the model under investigation – for instance, an Ising model in the proximity of the critical temperature T_c – into microscopically large cells encoding the magnetization as a collective variable. By a proper iteration of this procedure – which is repeated as long as the cell dimension is smaller than the correlation function, namely the distance over which the fluctuations of a microscopic variable are associated with that of another – one is able to provide a characterization of the transition from the ferromagnetic to the paramagnetic regime. For a philosophical discussion on the topic, see in particular Batterman (2017; 2021).

[13] See, for such a characterization, Georgi (1997, p. 88).

[14] Let us stress this aspect of ETs, since it will be of relevance for the discussion in section 4.

could be regarded as full-fledged dynamical theories, useful beyond the tree approximation" (*ibid.*). This led to a radical reconsideration of the nature of the QFTs of the Standard Model of particle physics, a "change in attitude" (Weinberg, 1997, p. 41) which is at the basis of the so-called modern view of QFT: that is, the view that "the most appropriate description of particle interactions in the language of QFT depends on the energy at which the interactions are studied" (Georgi, 1989, p. 446).[15] On this view, current QFTs are understood as EFTs, each EFT explicitly referring only to those particles (fields) that are actually of importance at the range of energies considered.[16] By changing the energy scale, the EFT description accordingly changes, "to reflect the changes in the relative importance of different particles and forces" (*ibid.*).

In some more detail: on the RG approach, the effect of changing the scale or rescaling ($\Lambda_0 \to \Lambda(s) = s\Lambda_0$) can in fact be absorbed into a change of the parameters, so that, for one parameter g a trajectory $g = g(s)$ is defined as $\Lambda(s)$ varies. The RG equations thus describe the flow of the parameters in a parameter space as one changes the scale. Then, typically, as one scales down to lower energies, the solutions of the RG equations approach a finite-dimensional sub-manifold in the space of possible Lagrangians:[17] in this way, an effective low energy theory, which is formulated in terms of a finite number of parameters and is largely independent of the high-energy starting situation, is defined.[18]

What is particularly intriguing about RG (and the related ET concept) is that the same ideas could be applied to apparently very different fields, such as the QFTs of particle physics, on one side, and the physics of critical phenomena, on the other. In fact, while in QFT one has to deal with short-distance (high-energy) behavior, in CMP one generally revolves around critical phenomena, and thus long-distance (low-energy) behavior.[19] Moreover, while in QFT the RG is an exact continuous symmetry *group* (whence its name), in the physics of critical phenomena (and similar cases where averaging operation are performed) the RG is a discrete *semigroup* (i.e., the transformation are not reversible).[20] Notwithstanding such (and others) physical and mathematical differences between the RGs used in physical theorizing,[21] there is a common rationale in applying the RG methods as Weinberg (1983, p. 17) points out: they can be understood as a general strategy to concentrate on the degrees of freedom that are relevant to the problem at hand.

At this point, since the philosophical discussion on RG explanations has mostly focused on the case of critical phenomena, let us go in some more detail for what regards RG methods in such field of physical investigation.[22] In CMP the main issue is that of finding a way to deal

[15] Weinberg (1976) was a crucial contribution for such "change in attitude" in particle physics (Weinberg, 1997, p. 41), leading to understand the QFTs of the Standard Model as EFTs, that is, the low-energy limit of a deeper underlying theory which may not even be a field theory. On the development of EFT, a first-person contribution is Weinberg (2021).

[16] We follow theoretical physicists' common usage in referring to "particles" as the objects of current QFTs. The controversy about what the basic QFT entities really are is not of relevance to the subject matters discussed here.

[17] Corresponding to a fixed point in the parameter space (e.g., the coupling-constant space), where a fixed point is defined by the condition that, if the parameter (e.g., the coupling constant) is put at the point, it stays there while varying the renormalization scale.

[18] More precisely, independent up to high-energy effects that are suppressed by powers of E/Λ where E is the low energy at which the effective theory is appropriate.

[19] Physical systems display critical phenomena when, under proper conditions (such as low temperature, or high pressure) they undergo specific transformations in the phenomenological features, often associated to second-order discontinuities of some parameters of the systems themselves. For a thorough discussion of the theory of critical phenomena see e.g. Binney et al. (1992).

[20] For a detailed discussion of this point, see Shirkov, (1993, pp. 178-179). Note, as pointed out by Shirkov, that RG ideas and terminology are even used in contexts where the functional iterations do not form a group.

[21] See also Weinberg (1983, pp. 13-14).

[22] The application of RG methods to CMP phenomena is highly diversified. Generally speaking, one is interested in integrating out short-distance fluctuations, which are typically defined in length terms (lattice spacing). The RG procedure can then be applied in a variety of contexts where the spacing, a, is defined according to the specific scale of interest (for instance, one proceeds in evaluating the RG flow until the spacing is comparable to the dimension L of the system under investigation). When it comes to account for critical phenomena, the aim is that of defining an algorithm to iteratively map out short-scale fluctuations while evaluating their contribution to the remaining degrees of freedom. An example is the generalization of the procedure for

with long-range characteristics, while short-scale fluctuations are those responsible for most difficulties, such as divergencies. In this context, the general strategy is that of integrating over all these short-range fluctuations, thus getting an effective theory of long-range ones. To this end, one has to arbitrarily fix a certain length scale, call it a, as the one separating among short and long wavelength fluctuations. One then proceeds by integrating out the short-range ones. This integration procedure can lead to different scenarios:

1. It might alter the algebraic structure of the long-wavelength action, thus getting to a theory which is different from the original one.
2. It might be the case that the effective action of the slow degrees of freedom is structurally similar to the starting one, the whole procedure thus leading to a different set of coupling constants.

If condition 2. holds, then the procedure is well motivated, for we have arrived at a theory which is structurally similar to the original one but for a) a changed (renormalized) set of coupling constants, b) an increased short-distance cut-off. This procedure can then be iterated until the cut-off, a^n has become comparable to the length scales that we seek to explore. More importantly, the coupling constants resulting from the renormalization procedure encode much of the important information about the long-range behaviour of the theory. The renormalization approach would be quite useless if each iterative step $a^{(n-1)} \to a^n$ had to be performed explicitly. In fact, the usefulness of the procedure relies on the fact that a single step already contains all the information about the renormalization properties of the model (Altland & Simons, 2010, pp. 409-411).

All in all, it should be evident that the RG framework is a rather generic recipe under which highly diversified methodologies can be comprehended. According to Shirkov (1993, p. 178), the development of RG methods follows at least two paths. The first corresponds to the Kadanoff-Wilson strategy, whose construction is based on a coarse-graining procedure defining a set of models for a given physical problem (such as in polymers, noncoherent transfer and percolation phenomena). The second looks for an exact RG symmetry either directly or by showing the equivalence to a QFT (such as in turbulence, plasma turbulence and phase transitions). Therefore, it is only by looking at the specificities of the problem at stake that the most suitable RG method can be developed. And this implies engaging with those measurable properties that figure as experimental outputs in the various physically salient contexts under investigation.

3 The explanatory role of RG methods

The theoretical import of RG methods has sparked much controversy from an interpretational point of view. Recent debates on the topic intersect with many others in the philosophy of science, such as explanation, causality, reduction, emergence, structuralism. Here the analysis is limited to the explanatory role of RG methods. With respect to this issue, the literature has focused on the following contraposed positions: reductive *vs* non-reductive explanation, causal *vs* non-causal explanation, mathematical *vs* non-mathematical explanation. In what follows we provide a concise survey of the main arguments on the first two contrapositions, insofar as the involved arguments have implications for the mathematical *vs* non-mathematical character of RG explanations. In this sense, this section is pivotal to the final discussion on the genuine mathematical character of RG methods. Drawing from those authors who have questioned the non-causal, and non-reductive nature of RG methods in accounting for critical phenomena, a

the one-dimensional Ising model, that is, the block spin method firstly developed by Kadanoff (1966). In this case, the spin chain gets subdivided into regular clusters of neighboring spins, the inter-cluster energy balance gets evaluated, and the procedure is then iterated to define the so-called fixed points – namely, points which are invariant under the application of the RG map. In all these applications, the lattice spacing is not arbitrarily fixed, for it corresponds to the actual (statistical) distribution of, say, spins within the system of interest. What can be defined is the number of RG iterations – which, conversely, is strictly dependent on the way in which we conceptualize the role of the thermodynamic limit.

final argument will be advanced – namely, that RG methods' explanatory import is non purely mathematical.

Starting with Batterman's (2000, 2002) seminal contribution, the philosophical literature has focused on the phenomenon of universality in critical phenomena. Batterman (2002), in particular, exploits the phenomenon of universality in physics to question the traditional deductive-nomological account of explanation. Universality accounts for the fact that in second-order phase transitions different micro-systems display the same shape of coexistence curve near a critical point.[23] This calls for an explanation, which is generally given in terms of the RG analysis of critical phenomena. Such an explanation, Batterman argues, cannot correspond to standard, Hempellian account, in that RG is a method for defining structures which are detail independent with respect to the behavior of interest (Batterman, 2002, pp. 42-43). On his view, the explanatory role of RG is thus contingent upon its ability to specify the structural stability of the system under perturbation of the underlying microscopic details (Batterman, 2002, p. 58; 2019), thus providing a non-reducible and non-causal explanation of critical phenomena. Importantly, this is also due to the fact that, according to Batterman, the thermodynamic limit plays an (indispensable) explanatory role in accounting for critical phenomena: it is only by embracing that limit that one can identify the fixed points of the RG flow within the abstract space – the latter being indispensable, together with an analysis of the neighboring topography, to evaluate the parameters of systems near criticality. Put it otherwise: infinite systems are an idealization. Yet, they are necessary to explain the behavior of real (large, but finite) systems near criticality (Batterman, 2021, p. 533).

Following a line of reasoning that resonates with Batterman's, however distinct,[24] Morrison's (2015a) explores whether and how RG methods – which she describes as a purely mathematical technique – can supply genuine physical information, where such information could not be provided by physical assumptions and originate solely from a mathematical framework (Morrison, 2015a, p. 55). On her account, RG explanation of universality provides a non-reductive account of how macroscopic behavior emerges from the microscopic one, where the latter is unable to provide the foundation for the former. Put it otherwise, the insensitiveness of the universal behavior with respect to the specificities of the underlying microphysical basis is what leads Morrison to claim that the link between the micro and the macro displays a failure of reduction. In her words (Morrison, 2015b, p. 92): "Emphasising the importance of emergence in physics is not to deny that reductionism has been successful in producing knowledge of physical systems. Rather, my claim is that as a global strategy it is not always capable of delivering the information necessary for understanding the relation between different levels and kinds of physical phenomena." At the same time, the fact that the underlying microphysical basis gets washed out via the successive iteration of the RG flow implies that it does "no longer play a role in the macro behaviour" (Morrison, 2015b, p. 111), and thus the RG flow provide a non-causal explanation of the emergent macroscopic behavior. Importantly, the non-causal nature of RG explanation is not simply contingent upon the fact that the microscopic information gets eliminated throughout the iteration. Rather, as Morrison (2018, p. 206) clarifies, "[a]n important consequence of the evolution produced by RG transformations is [...] that the explanation of universal behaviour cannot be given in terms of the system's interacting parts".

This characterization of RG methods has been contested by Hüttemann, Kühn and Terzidis (2015) on the grounds of a reductive account of structural stability, which is (also) based on the effective character of RG methods. The idea is to argue that the structural stability of finite but extremely large systems pivots on the huge number of involved components, despite the fact that the thermodynamic limit is not met. Indeed, sufficiently large systems become

[23] If one observes the behavior of various systems in the proximity of this point, what one finds is that certain dimensionless numbers, the so-called 'critical exponents', are shared by these various systems. And as the critical exponents encode the behavior of the fluid as a function of the temperature near the critical point, these systems are said to display a universal behavior in the sense that the same coexistence curve describes microscopically different systems.

[24] Morrison's treatment is different from Batterman's, insofar as she aims to show how the application of RG methods to dynamical systems is to be portrayed as non-causal, due to the structural character of the involved explanation (see, especially, Morrison 2018, p. 206).

observationally indistinguishable from infinite ones, while their finite nature blocks the anti-reductionist account *à la* Batterman. Thus, according to Hüttemann, Kühn and Terzidis, the occurrence of phase transitions and the existence of universality classes can be reductively explained in terms of the properties of and the interaction among the underlying components. Moreover, one can understand the relevance of thermodynamics even if the quantities it employs are valid only in the thermodynamic limit.[25] On this perspective, RG methods show how systems characterized by different micro-Hamiltonians, and thus by different system-specific qualities, nevertheless give rise to very similar macro-behavior.

In a similar vein, Reutlinger (2017a) argues that there are two sources of ambiguity leading to erroneously depict RG methods as non-reducible ones. First, the RG framework does not correspond to standard accounts of reducible explanations, in that it does not target system-specific micro-mechanisms which are responsible for the collective, macro-behavior of the system. Second, RG explanations are non-reducible in that not all the micro-interactions among the underlying components are explanatorily relevant (Batterman, 2000, p. 123). However, while it is true that not all the details of the microscopic environment play a role for the macro-behavior near a phase transition, this condition does not rule out the possibility of a reductive explanation, for the latter does not have to treat all micro interactions on a par. There are causal-mechanical models of explanation which purportedly ignore micro-details (Reutlinger, 2017a, p. 2999). Can then RG explanations be reductive? Yes, according to Reutlinger, provided that one accepts a broad characterization of what a reductive explanation is, namely one whose explanandum is contingent upon (certain) information about the underlying components.[26]

Following this line of reasoning, Saatsi and Reutlinger (2018) develop an argument to block the so-called anti-reductionist challenge for RG explanations: reductionists have to account for the fact that certain components figuring in RG models – such as fixed points in phase transitions – can be regarded as both indispensable and compatible with reductionism. Their argument is that, though these components play an indispensable role in RG methods, that role is merely instrumental and thus in no contradiction with reductionism. To make their case they propose a counterfactual-dependence account based on two criteria: 1. the explanandum should be inferred from the explanans, where the inference can be deductive or statistical-inductive; 2. counter-factual variations of the explanans need to produce counter-factual variations of the explanandum. The RG framework satisfies the reductive, counter-dependence account, in that it deductively warrants that systems with different micro-details behave in the same way macroscopically (condition 1) and offers a counter-factual account for why a certain system belongs to a specific universality class (condition 2).

Finally, also Franklin (2018) defends a reductive, though non-eliminativist, account of universality. According to Franklin higher-level explanations are "more parsimonious, more robust, and have a broader applicability than lower-level explanations" (Franklin, 2018, p. 1295) and should not be replaced by lower-level explanations. However, higher-level theories can be understood in lower-level terms and are thus reducible – where the notion of reducibility here implies two conditions (Franklin, 2018, p. 1296; Woodward, 2003, pp. 226-233):

- Each dependency of the higher-level explanation is described by or derived from a lower-level dependency.

[25] An objection that can be raised is that, if one embraces a reductive account of RG explanations, the interaction among the underlying components is meant to determine the features of the resulting macro-aggregate. For, as Morrison (2012, p. 156) argues, if this is not the case, then no reductive explanation is available. However, Hüttemann, Kühn and Terzidis hold that "the fact that a multiplicity of micro-states gives rise to the same macro-state is no objection to the claim that the micro-state determines the macro-state" (Hüttemann, Kühn and Terzidis, 2015, p. 189).

[26] A tangential debate is the one between the so-called common features approach *vis-à-vis* Batterman and Rice's (2014) noncausal minimal model approach. While the latter argues that a certain minimal model is able to explain why systems with different micro-components behave in the same way macroscopically without appealing to those components which are shared among these diverse systems, according to the former approach the presence of certain common features is sufficient to explain the common macroscopic behavior. For a thorough critique of Batterman and Rice's account, see Lange (2015).

- The approximations, abstractions and idealizations that are performed in order to build the higher-level explanations are justified from the bottom up.

For what regards more specifically the debate on causal *vs* non-causal explanation,[27] and the use of RG methods as a paradigm case for non-causal explanation, a recent critical analysis is Sullivan (2019). In a nutshell, Sullivan's argument is based on the claim that "simply pointing to the fact that the explanation heavily relies on abstraction procedures, such as RG transformations and Hamiltonian flow, does not preclude a causal interpretation" (Sullivan, 2019, p. 17). This is articulated by critically discussing what, according to Sullivan, are the three main motivations for non-causalists to make their case: (1) the RG procedure implies abstracting from causal (irrelevant) details; (2) RG transformations do not provide a time-asymmetry, which is required for our commonsense account of causation; (3) RG's explanatory power is contingent upon the abstract space of models, rather than a causal mapping.

This brief survey was meant to show that the current philosophical debate on the explanatory role of RG methods is mainly framed in terms of reductive *vs* non-reductive or causal *vs* non-causal explanation. However, there is a major issue in the background, implicitly assumed in the debate, on which Morrison has especially focused her attention, namely the relationship between mathematics and physics in the explanatory process involving RG. More precisely, as we will see in the following section, most of her work on the topic is devoted to unraveling the connection between the mathematical framework of RG theory and its application to the various fields of both theoretical and experimental physics. It is the purely mathematical character of RG explanation, she claims, that grants its non-causal and non-reductive nature.[28]

4 Morrison on RG explanation: A critical assessment

Morrison developed her standpoint on the nature of RG explanation and its implications on the debate on reduction and emergence in a series of recent works around what was a central concern in her research, that is, how is it possible for mathematics to apply to empirical sciences (Morrison 2012, 2015a, 2015 b, 2018, 2019). Her first contribution to the topic (2012) started precisely with the question as to how physical information can be extracted from a genuine mathematical framework. In Morrison's (2012, 2015a, 2015b, 2018, 2019), she explores whether and how RG methods – which she depicts as a purely mathematical technique – can supply genuine physical information, the latter being conceived as information that could not be provided by physical assumptions and originates solely from a mathematical framework (Morrison, 2015a, p. 55). Her intent is to show that there are mathematical strategies that are able to generate physical information even when stripped of any physical adornment. In other words, the point is whether RG methods can bring *bona fide* physical content over and above their calculational power, thus providing a *mathematical explanation* of a *physical fact*.

With regard to the current literature on mathematical explanations of physical phenomena, Morrison especially refers to discussions by Steiner (1978),[29] Baker (2009),[30], and Lange (2013; 2017).[31] Let us focus on the main points with respect to which she takes a stand in framing her discourse on RG explanation *qua* mathematical explanation.

According to Steiner (1978, p. 19) the difference between a mathematical and a physical explanation (of a physical fact) is that the former retains a truth value even when deprived of its physical content. For him, this condition is crucial in order to discriminate between

[27] It is worth mentioning that the debate concerning the causal *vs* non-causal character of RG methods and the one regarding their reductive *vs* non-reductive nature are not coincident. As a case in point, while Reutlinger (2014) maintains that RG theory can be cast in reductive terms, he nevertheless opts for a non-causal account when it comes to RG's explanatory role (see also Saatsi and Reutlinger, 2018).

[28] A reference paper on this topic is undoubtedly Batterman (2010), providing a broad analysis of the explanatory role of mathematics when it comes to physical phenomena (as well as other empirical sciences). According to him, RG methods as applied in the context of critical phenomena represent a paradigmatic instantiation of what he dubs "non-traditional idealizations".

[29] Morrison (2015a, Ch.2; 2018, p. 207).

[30] Morrison (2015a. Ch.2; 2018, pp. 207-209).

[31] Morrison (2018, pp. 209-211; 2019, pp. 721-722).

a genuine mathematical explanation and a physical explanation cast in mathematical terms: in the latter situation the role of mathematics is simply representational, whereas it is the physical information which is performing the explanatory work. On this respect, Morrison's opinion is that Steiner's criterion is perhaps too stringent. Her intent is to relax it and refine the question as to how the calculational power of RG methods provide physical understanding (Morrison, 2015a, p. 77). In other words, the general point is whether mathematics can, in and of itself, come up with genuine physical information.

Baker (2009), as well, argues that Steiner's test does not need to be passed for an explanation (of a physical or, in his case, biological fact) to be characterized as mathematical. Baker's well-known example is that of the life cycle of a North American insect (the cicada) whose period corresponds to prime numbers (Baker, 2005, pp. 229-233). This condition, which arises from a mixture of biological and mathematical considerations, is taken by Baker as a paradigmatic instantiation of the indispensable role of mathematics when it comes to explain physical phenomena.[32] In discussing this example, Morrison (2015a, Ch2; 2018, p. 209) claims that, though mathematics plays an indispensable role, it is surely not the sole explanatory element. There is indeed a causal factor which emanates from the biological information about the cycada's life cycle. And the combination between mathematical and biological information that this example displays is what, on her view, undermines its distinctively mathematical character.[33] Notably, we will get back to this point while discussing Morrison's account of RG methods as providing a purely mathematical explanation.

Especially relevant to Morrison's analysis is Lange's characterization of what counts as a "distinctively mathematical explanation", namely one which is inevitable to a "stronger degree than could result from the causal powers bestowed by the possession of various properties" (Lange (2013, p. 487).[34] According to Lange (2013; 2017, Ch. 1), the issue is not so much whether an explanation appeals to causal or non-causal contingent aspects – though he characterizes a mathematical explanation as a non-causal one – as rather whether its explanatory role is contingent upon its reference to causal facets. If the latter is the case, then the explanation cannot be qualified as distinctively mathematical. Put it otherwise, even if a distinctively mathematical explanation allows for the presence of causal features into the explanans, the link between these features and the explanandum follows from mathematical necessity rather than from a physical law (Lange, 2013, p. 497). The paradigmatic example discussed by Lange (2013, Ch. 1) is that of a mother who wants to split the available 23 strawberries as a snack for her three children. Despite the fact that the presence of the three children is what causally impedes the mother to distribute the strawberries evenly, the reason why this operation cannot be performed exceeds the specific causal contingencies. For it rests upon a mathematical truth, namely the fact that 23 cannot be divided into 3 equal portions.

According to Morrison (2018, p. 211), her discussion of RG as a case of mathematical, non-causal explanation is "similar in spirit" to Lange's analysis "in that it emphasizes very general features of systems". But it differs in that "the explanatory power comes not from the modal character of a law stated in mathematical terms but from the fact that RG is a particular type of mathematical framework used to explain structurally stable behaviour in physical systems" (*ibid.*). Taking this discussion as the background, there are several reasons Morrison puts forward to make the case that RG methods constitute a genuine mathematical explanation. Here we will focus on what appear to be the three main ones. Our plan is to analyze these arguments, by pointing out some apparent shortcomings.

a) *The indispensable explanatory role of RG methods*

One of the motivations Morrison offers to construct her case is that the critical exponents figuring in critical phenomena, and the notion of universality thereof, could not be explained

[32] Afterwards, this example has been much discussed in the literature on scientific explanation. See, for example, Saatsi (2011) for a critical analysis pointing out to an ambiguity between the explanatory and the representative role of mathematics (Saatsi, 2011, p. 145).

[33] More precisely, her argument is that, as the explanation is also contingent upon biological facets which encode causal information, then the explanation cannot be non-causal. But as she takes mathematical explanations as a sub-set of non-causal explanations, it follows that this explanation is not distinctively mathematical.

[34] See also Lange (2019).

prior to the introduction of RG theory (e.g., Morrison 2015a, p. 57; Morrison, 2015b, p. 110; Morrison, 2018, p. 216). On this aim, she discusses two issues.

The first is whether RG should be regarded as a merely calculational device. Morrison claims that RG, although a mathematical technique, is nevertheless able to epistemically justify both the irrelevance of certain microscopic parameters and the existence of collective behavior.[35] Her viewpoint is that the calculational power of RG methods can produce physical information, insofar as they show how to compute the coupling constants at different length scales, how to estimate critical exponents, and how universality follows (e.g. Morrison, 2015a, p. 78; Morrison, 2015b, p. 109; Morrison, 2018, p. 218).

The second issue has to do with the role played by the thermodynamic limit when adopting RG theory to bring out information about critical phenomena. The point is that, strictly speaking, a phase transition happens when the number of constituents diverges, but we know for a fact that phase transitions do happen in physical systems, which are composed of an admittedly huge, yet finite, number of components. The question is then how to make sense of the relation between this idealization about the constituents and the fact that RG provides us with information about phase transitions. This is a much discussed topic in the framework of the broader issue concerning the role of infinite idealizations and their implications on such questions as reduction, emergence and more generally scientific realism in physics.[36] Morrison's response is that, although phase transitions happen in finite systems, the explanation requires the limit. In her words, these systems "can be near the fixed point in the RG space and linearization around a fixed point will certainly tell you about finite systems, but the fixed point itself requires the limit" (Morrison, 2015b, p. 110).[37] Put it otherwise, Morrison specifies that the thermodynamic limit is needed for finding the fixed points of the RG transformation, thus stressing the mathematical nature of the involved explanation. On this respect, our point is not to question that, strictly speaking – namely, from a mathematical point of view – calculating fixed points of the RG transformation requires the thermodynamic limit. For this is surely uncontroversial. What is questionable is that RG methods' explanatory power of specific physical phenomena is purely contingent upon their ability to evaluate fixed points. Put it otherwise, even if the thermodynamic limit is necessary to evaluate RG flow's fixed points, RG methods' explanatory role does not rest only upon that limit.

The way in which Morrison addresses these two issues substantiates her claim that, prior to the introduction of RG methods, no theoretical basis to explain the universality of critical exponents as well as other features of statistical physics and QFT was available.[38] While the issue of the explanatory role of RG methods can be tackled from a variety of standpoints (as seen in section 3), let us discuss, here, whether one can indeed motivate the informational power of RG methods on this type of indispensability argument. There are at least two points to be discussed in this regard. One has to do with the putative purely mathematical character of RG methods, the other with the alleged indispensable character of RG explanation.

First, let us point to a difficulty arising when sharply contrasting mathematical *vs* physical import of RG theory in accounting for physical phenomena. As discussed in section 2, RG theory is a technical, general framework under which a multiplicity of methodologies are incorporated. The broad idea is the same one (i.e., that of a general strategy, based on scaling procedures, to concentrate on the degrees of freedom that are relevant to the problem at hand), but the modalities of its application significantly depend on the physical specificities of

[35]In her own words, "RG methods not only provide us with physical understanding of the behaviour of certain kinds of systems insofar as they explain how the cooperative macro behaviour emerges from the micro level, they explain the foundation for universality by showing how that behaviour is related to the existence of fixed points. In connection with this, RG also illustrates the nature of the ontological relation between macro behaviour and microphysical constituents." (Morrison, 2015, p. 74).

[36]See especially Callender (2001) and Butterfield (2011). A recent, comprehensive paper is Palacios and Valente (2021).

[37]As regards the linearization strategy, Wu (2021) uses it for dispensing with the thermodynamic limit. See Batterman (2021, p. 533) for a counter-argument.

[38]In Morrison's own words, RG methods provide "an understanding of critical phenomena in statistical physics as well as features of QFT that were not possible prior to their introduction" (Morrison, 2015a, p. 57; for a similar claim, see also, 2015a, p. 65).

the field of inquiry as well as of the phenomena studied. This clearly emerges, as highlighted in section 2, when considering the *physical* as well as the *mathematical* differences between RG methods in the analysis of critical phenomena and RG theory in QFT. Thus, we wonder whether a general idea, stripped off the technical parts needed for its concrete application, could be enough for providing an explanation of the physical phenomena at stake. In fact, even if we restrict the scope of the analysis, and consider RG theory only in the context of the physics of critical phenomena, the problem remains: in order to get the proper application in a specific physical context, one needs to supplement mathematical with physical information. No explanation is granted without an appropriate interpretation that is strictly connected to the specific problem at hand. Put it differently, we agree that adopting RG methods to explain, for instance, critical phenomena, involve mathematical features as an essential element. What we are saying is that this is not enough to speak about a purely mathematical explanation of such phenomena. In other words, RG methods involve a mixture of physical and mathematical considerations that prevent considering their explanatory import as purely mathematical. Our account of RG's explanatory power is thus hybrid: mathematical and physical features intersect in a highly non-trivial way to account, among others, the behavior of systems near criticality. On the contrary, Morrison repeatedly underlines the purely mathematical nature of RG explanations. Consider, for example, Morrisons's discussion of Baker's famous cycada argument. She argues that, though mathematics plays an indispensable role, it does not represent the sole explanatory element. For, according to her, a causal factor is added arising from biological considerations about the cycada's life cycle. Therefore, according to Morrison, one could not speak of a purely mathematical explanation in this case. Now, the same reasoning can be applied to the case of RG methods' explanation. Though RG methods involve mathematical features to explain a variety of physical phenomena, there are still physical considerations at play. Although Morrison repeatedly maintains that RG methods provide a purely mathematical explanation of critical phenomena, marking the difference with Baker's cycada explanation case, we do not see the difference between RG methods providing an explanation for physical phenomena and Baker's biological explanation about the cycada's life cycle. In both cases mathematics plays an essential role, but it is not the sole explanatory element.

Second, the fact that RG methods provide a theoretical framework for physical phenomena is not enough to claim that their role is indispensable. For this condition does not rule out other alternative, yet-to-come, explanations. This is particularly manifest in the case of RG methods as applied to critical phenomena, for they involve a coarse-graining procedure that is inherently approximate (see section 2) and may thus be eventually supplanted by a more reliable – whatever characterization one aims to offer of such a term – theory. So, not only does the putative explanatory role of RG theory have to confront with empirically observable phenomena – thus implying a coexistence of mathematical and physical considerations which would render RG methods a non-purely mathematical explanation – it is the very indispensable role of RG methods in providing an explanation of critical phenomena that can be questioned in the first place.

This intersects with the way in which the notion of effectiveness is characterized. As already discussed, the latter, born in the context of QFT, broadly refers to the limited applicability of a certain theory: a theory is thus effective if its predictions offer an appropriate description of the relevant physics in a specific domain, out of which the description may be completely inadequate. A natural extension in the context of CMP leads to a scenario in which the low energy regimes allows to practically suppress high energy degrees of freedom, thus reducing the total number of parameters necessary for an appropriate, yet effective description. In some greater detail, while a macroscopic phenomenon should be defined according to a many-body Hamiltonian accounting for the interaction among the underlying micro-components, in most relevant cases the convenient degrees of freedom to the low-energy regime can be extremely reduced, thus providing an effective description of the macroscopic behavior. The result is that, instead of dealing with the total dynamics of a system (consisting of something as 10^{23} components), the focus is on the collective degrees of freedom whereby only a restricted set of excitations is considered.

As such, this description arises out of a coarse-graining procedure which leads to information loss and is thus intrinsically approximate. On this perspective, RG theory is one of the most refined methods to extract the relevant degrees of freedom for a certain domain. Still, it corresponds to a coarse-graining procedure that reduces the total number of degrees of freedom, providing an effective description of, among other phenomena, systems near criticality. Therefore, we question the idea of constructing an indispensability argument based on RG methods. It may well be the case that, even after the definition of a more fundamental (or complete, in whatever sense) theory is established, RG methods will still be used to account for critical phenomena. What cannot be inferred is that, if they are currently used to explain critical phenomena, they should be taken as indispensable.

b) *Group vs semi-group argument*

A second argument Morrison apparently advances in support of the claim that RG methods offer a purely mathematical explanation of physical phenomena invokes the type of structure underlying RG procedures in CMP. As seen in section 2, in the context of the physics of critical phenomena the RG structure is that of a discrete *semi-group*. On this basis, Morrison argues that, since a semi-group cannot correspond to a symmetry, "as a mathematical explanation RG has a peculiar status in that its underlying structure cannot be associated with anything physical" (Morrison, 2015, p. 81). Thus, its explanatory power "can only be a product of its mathematical power" (*ibid.*).

This would seem to imply that for the RG structure to be associated with something physical it should correspond to a group, and not just a semi-group. This is a controversial assumption, and we do not think it does justice to Morrison's real intent. The reason we explicitly mention this apparent argument is that the group *vs* semi-group point will turn relevant in the next argument.

c) *The structural nature of RG explanations*

Another point discussed by Morrison in support of the mathematical character of RG explanation is its alleged structural nature. According to her, the fact that RG methods allow to explain "the structural stability of complex systems in terms of structural constraints and how they transform" (Morrison, 2018, p. 225) speaks for a structural approach to RG explanation. For the focus is "on the geometrical and topological structure of ensembles of solutions", rather than "deriving exact single solutions for a particular model" (Morrison, 2018, p. 221).

In the philosophical literature, a structural explanation is usually described as one in which the explanandum follows only from the theory's structure – where the latter typically corresponds to the formal (mathematical) apparatus which is adopted to represent specific features of interest (see for example Bokulich's, 2008, p. 149). On this view, the explanandum is explained structurally when it follows solely from this formal (mathematical) apparatus to such an extent that any additional information is spurious to account for it.[39] Lange (2017, pp. 182-183) questions this account of structural explanation in that, according to him, it fails to identify the proper explanatory priority. By claiming that the theory's structure incorporates the explanandum does not add up to the theory's explanation of the explanandum itself. In his own words "[t]he mere fact that every model of some theory incorporates the explanandum does not constitute the theory's explanation of why the explanandum holds. Rather, the theory's explanation involves the reason why every model incorporates the explanandum — that is, something about the way in which the explanandum follows from the theory" (Lange, 2017, p. 182).

Drawing from Lange's analysis,[40] it seems that the structural features of RG theory, insofar as they can be shared among a class of systems, are not sufficient *per se* to perform an explanatory role. More in general, if one is interested in detailing the way in which RG technique finds its application in the broad spectrum of both QFT and CMP, one has to seriously engage with those aspects which make a difference in these specific fields. The plasticity of RG theory resides precisely in the generic character of its description. But it is only when one

[39] Another recent discussion is, for example, Dorato and Felline (2010).

[40] From this point, Lange develops a different account, which is based on the role of constraints in explanation.

considers the difference-making factors – namely, the ones arising in specific physical contexts (such as polymers, noncoherent transfer, percolation phenomena, plasma turbulence, phase transitions, and so on) – that RG theory is able to actually provide an explanation. But as the latter requires this additional, physical, information it is hard to see how RG could explain in a bearly mathematical sense. To reinforce this point, let us note that the difference between the mathematical structures which RG theory acquires in the different domains of application – see point b) about group *vs* semi-group – seems to weaken the structural import of RG theory broadly construed. What is indeed the structure that is supposed to provide the explanatory role, given such physical and mathematical differences? In this generalized version of the issue, it seems to be even less clear where to identify the structural components from which an explanation could follow. Put it differently, what matters here is the group *vs* semi-group contrast respectively arising in QFT's and CMP's phenomena – a contrast which involves a different mathematical structure. That said, even if attention is restricted to the sole CMP case, one needs to take into account context-dependent details, where the latter are essential to understand what is going on physically and thus to provide an explanation. To make an example, though the liquid-gas transition and the ferromagnetic transition are taken as equivalent (but there are other interesting cases, such as the equivalence of models of planar magnets to two-dimensional classical Coulomb plasmas, see Altland & Simons, 2010, p. 440), in that they exhibit the same scaling behavior, there is a clear sense in which these two situations are physically distinct, thus showing different context-dependent details which are essential to explain the relevant physics.

5 Concluding Remarks

RG methods are an undoubtedly fascinating topic, from both a physical and a conceptual standpoint. Despite the differences in how they are actually applied within the various fields of physical inquiry, one key feature can be singled out, namely their ability to provide the proper set of degrees of freedom for the problem at hand. On this respect, they embody the most up-to-date mediators between different theoretical levels – the microscopic and macroscopic realms being just two paradigmatic representatives of a broader class of inter-level connections. Morrison's contribution to the conceptual assessment of such a technique is arguably unprecedented. On the one hand, she has explored the connection between the various applications of RG methods, by outlining both the similarities and differences among the diverse implementations of RG theory. On the other hand, she has indicated how RG methods fall within the more general discussion around the applicability of mathematics to science, and how this relates to the account of RG methods as providing a non-causal and non-reductive explanation of critical phenomena. She has defended an account of RG theory that depicts the latter as a mathematical, non-reductive, non-causal framework which is able to provide genuine physical information based on structural considerations. Here we have focused on the putative mathematical character of RG theory's explanatory role. We have argued that RG methods, though mathematical *per se*, cannot provide an explanation unless they seriously engage with those difference-making factors that the specific problem at hand presents. As these factors – which encode physical information – are also responsible for the success of RG theory, the explanatory role of RG methods cannot be distinctively mathematical.

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