# The Causal Axioms of Algebraic Quantum Field Theory: A Diagnostic

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**Abstract**

This paper examines the axioms of algebraic quantum field theory (AQFT) that aim to characterize the theory as one that implements relativistic causation. I suggest that the spectrum condition (SC), microcausality (MC), and primitive causality axioms (PC), taken individually, fall short of fulfilling this goal against what some philosophers have claimed. Instead, I will show that the "local primitive causality" (LPC) condition captures each axiom's advantages. However, this is only the case because SC, MC, and PC, taken together, imply LPC, as I will show from a construction by Haag and Schroer (1962).

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## Introduction

As put by one of the standard textbooks on the subject (Peskin and Schroeder, 1995), quantum field theory (QFT) combines three major "themes": the concept of field, quantum mechanics (QM), and special relativity (SR). One of the main formulations of QFT, algebraic QFT (AQFT), has often been highlighted by philosophers due to its mathematically precise, axiomatic framework. The theory's core is to assign to every open, bounded region of spacetime $\mathcal{O}$ a set of observables $\mathcal{A}(\mathcal{O})$ describing the physics in said region. For this paper, every $\mathcal{A}(\mathcal{O})$ is a von Neumann algebra that, in particular cases, can be isomorphic to $\mathcal{B}(\mathcal{H})$, the bounded, linear operators on some Hilbert space $\mathcal{H}$—a structure more familiar from ordinary QM. Although there has been much debate on which formulation we should use to address foundational, philosophical, and interpretive questions (Fraser, 2009; Wallace, 2011), one could argue that the fruitfulness of choosing AQFT lies in the rich and robust framework that its axioms provide. However, as AQFT's advocates know, these axioms are very demanding from a mathematical and physical point of view. The clearest evidence of this restrictiveness is the fact that the attempts to go beyond free fields have proven to be unsuccessful or require certain modifications (Fredenhagen and Rejzner, 2015; Buchholz and Fredenhagen, 2020).

Another troublesome instance of the worry of restrictiveness is the multiplicity of axioms of AQFT that have a causal gloss. Is it not overkill to lay down multiple axioms striving to do the same job—namely, characterizing the causal structure of a theory that we know is too demanding? The tendency in the literature on relativistic causation in QFT has been to single out the "most direct expression of the prohibition of spacelike processes" (Butterfield, 2007, p. 303). Jeremy Butterfield has argued in favor of the **spectrum condition**, John Earman and Giovanni Valente (2014) have criticized Butterfield and highlighted **primitive causality** in its stead, and **microcausality** is still the most famous constraint among physicists. In this paper, I will show in a minimally technical way that none of these **causal axioms** *fully* explains the notion of causation appropriate for AQFT. There are two kinds of shortcomings for the spectrum condition (SC), microcausality (MC), and primitive causality (PC) conditions. First, they only capture some (and not all) of the desiderata for relativistic causation that I will state in a moment. Second, it is often unclear how it is that each axiom implements its respective desideratum. In this way, I argue against the strategy in the literature to rivalize the axioms and privilege one among them.

Additionally, I will show that a fourth condition, **local primitive causality** (LPC) (also called the **diamond property** in older literature), *does* fully characterize relativistic causation in the sense of fulfilling all the relevant desiderata. Instead of making LPC my own favorite axiom, I will show a construction from Haag and Schroer (1962) with which we will see that it only encompasses the virtues of the other axioms because it is implied by them. I thus dismiss the initial worry of redundancy in the axioms or a lack of parsimony in the theory while simultaneously holding, via LPC, that they are a package deal for characterizing the causal structure of AQFT.

Before moving on, I will specify what I will mean by **relativistic causation** and clarify the methodology of this paper. The physics literature has multiple facets of the notion of "causation". First, as a form of "action," relativistic causation attempts to capture a notion of (i) locality and of (ii) a finite speed of propagation of events:

> For the relative independence of spatially distanced things (A and B), this idea is characteristic: external influences from A have no *immediate* influence on B; this is known as the "principle of proximal action [*Nahewirkung*]," which is only used consistently in field theory. The complete abolition of this principle would make impossible...the formulation of empirically testable laws in our familiar sense. (Einstein, 1948, p. 321-322, my translation)

The worry about the *immediacy* of the propagation of this kind of influence encapsulates (i) and (ii) into the preclusion of superluminal signaling, arguably the most famous notion of relativistic causation. Events are *independent* in a relevant causal sense if we cannot connect them with a light signal. That is, we have merely specified what causal connections *cannot* be. "Relativistic causation" also includes a notion of causal *dependence*, not merely *in*dependence (Earman and Valente, 2014, p. 4). Here is an account from another renowned physicist:

> In physics, causal description...rests on the assumption that the knowledge of the state of a material system at a given time permits the prediction of its state at any subsequent time. (Bohr, 1948, p. 312)

Therefore, (relativistic) causation should also capture an idea of (iii) a deterministic connection between events. Finally, I will add (iv) the metric structure of Minkowski spacetime as a fourth desideratum since Geroch (2011) showed that superluminal signaling is not incompatible with SR. Additionally, some of the axioms only require that the spacetime is globally hyperbolic, not that it is specifically the Minkowski one. These are the **four desiderata** for relativistic causation with which I will diagnose the axioms.

This brings me to my final clarification about methodology. I will devote three sections of this paper to analyzing each causal axiom. My *diagnostic* scrutinizes what it means to be a *causal* axiom in the sense of capturing the four features of relativistic causation mentioned before. The main tools at our disposal are the three "themes" from Peskin and Schroeder: an axiom of AQFT should implement the notion of field (or, in other words, to make clear what is **local** in the sense of situating phenomena in delimited patches of spacetime, as opposed to **global**) and have a clear input from QM and SR. This analysis will not, however, take these axioms as members of a formal system but rather as something more similar to what Rédei and Stöltzner (2006) have called "soft" axioms: mathematically rigorous but mostly physically intuitive and hermeneutically rich constraints on a physical theory. As such, my worries driving this paper are very similar to those of the creators of AQFT: beyond their mathematical consistency, are these physically meaningful and well-motivated requirements? Are they interdependent, and if so, do they codify compatible or rival features of the theory? In short, my diagnostic aims to take the "wide frame allowed by the general principles and, narrowing the...frame step by step, to look for the neuralgic spots." (Haag, 2010, p. 243).

# 1 The Spectrum Condition

The first axiom we will consider is the spectrum condition (SC):

**SC**: (a) The joint spectrum of the infinitesimal generators of translations in

> Minkowski spacetime is confined to the forward lightcone. (b) There is a unique (up to phase factors) **vacuum state**, which is translationally invariant and has zero energy-momentum.

I will argue in this section that the role of SC in forbidding superluminal signaling (if it has one) is unclear. Of the remaining desiderata, this axiom captures the structure of Minkowski spacetime but is fairly silent about the rest.

An initial problem with SC is that it does not hold for every QFT since the energy (density) of a *quantum* field need not be positive (Epstein et al., 1965). On top of that, the energy-momentum operators do not necessarily coincide with the energy-momentum (tensor) of the quantum field in question (Earman and Valente, 2014). Now, it is possible to formulate classical SC-analogs that have the advantage of being generalizable (in a non-unique way) to curved spacetimes (Curiel, 2017) and study superluminal signaling in more general fields. The key idea behind this strategy is to generalize the restriction that we will only consider states with positive energy-momentum (plus the vacuum), which is the usual way in which SC is interpreted.

However, there are two problems with this strategy relevant to my diagnostic. First, SC is inherently tied to the global symmetries of Minkowski spacetime. It is usually supplemented with a local "counterpart," the **axiom of covariance**, which dictates how the algebra of a given region transforms under the unitary representations of Lorentz transformations. Since SC and the axiom of covariance cover the demands of the Poincaré group of SR, including both Lorentz transformations and spacetime translations, the axiom of covariance is usually taken as one of the causal axioms of the theory. Now, from this perspective of symmetries, they are both straightforward applications of Wigner's theorem: to obtain the quantum analog of the symmetry group of a classical theory (in this case, the Poincaré group), take the group's unitary representations. Thanks to Stone's theorem, those unitary representations can be further expressed in terms of their infinitesimal generators, which, for the translation subgroup, are the key ingredient for SC. So the way in which SC implements QM and SR only codifies the geometry of Minkowski spacetime. Although this is a virtue from SC in being explicitly compliant with SR and in straightforwardly implementing the fourth desideratum of relativistic causation I laid out, it is a shortcoming of SC as a causal axiom in the sense that we surely want to be able to claim that there is some form of "relativistic causation" in AQFT when the theory extends to curved spacetimes. The other problem with the classical SC-analogs is that they are not sufficient conditions to prohibit superluminal signaling (Earman, 2014, pp. 103-105). Moreover, quantum fields obtained by quantizing classical relativistic fields exhibit no superluminal signaling (Earman and Valente, 2014, pp. 17-18). So whatever the culprit is in ruling out superluminal signaling in quantum theories, SC does not seem to be the key contributor.

So far, SC (a) does not fulfill the goal we would want for *a single* causal axiom to have, or to have in the most exemplary way. What about SC (b)? SC (b) is sometimes taken not as a characterization of relativistic causation in AQFT but, ironically, as one of the ingredients needed to question AQFT's status as a relativistic theory. One of the consequences of SC (when supplemented with a condition of "additivity" saying that observables can be expressed in terms of observables of arbitrarily small regions) is the **Reeh-Schlieder theorem**. The theorem claims that if we act on the vacuum defined by SC (b) with an element of $\mathcal{A}(\mathcal{O})$ for some region $\mathcal{O}$, we can generate any other state. This result is unsettling because it seems that we can measure the energy of the vacuum in a lab and yet approximate any other quantum state in any other region of spacetime, even states that *do* have energy! There are interpretive strategies and more sophisticated mathematical tools to get out of this problem. Still, they do not change that SC (b) does not implement the prohibition of superluminal signaling, even if we manage to show that it does not impede it.

## 2 Microcausality

The second axiom we will consider is the microcausality (MC), Einstein causality, or local commutativity condition:

**MC**: $\mathcal{A}(\mathcal{O}_1)$ commutes with $\mathcal{A}(\mathcal{O}_2)$ if $\mathcal{O}_1$ and $\mathcal{O}_2$ are space-like separated.

Let us unpack this condition: since no physical process can occur along a spacelike trajectory, no measurement in $\mathcal{O}_1$ can disturb the outcomes of a measurement carried out in $\mathcal{O}_2$, and vice-versa. Even though we disambiguated in the introduction the metric structure of Minkowski spacetime (with which we classify the separation of events as spacelike), MC is often rephrased to say that there would be statistical correlations if we could connect $\mathcal{O}_1$ and $\mathcal{O}_2$ with a superluminal signal. However we want to read MC, it is evident that it is a transcription of the passage I cited from Einstein in the introduction to the language of operator algebras. For that reason, MC is taken to be a condition of **independence** or **separability** between quantum systems in regions $\mathcal{O}_1$ and $\mathcal{O}_2$. Prima facie, MC is a straightforward implementation of relativistic causation in the sense of no superluminal signaling. That is, MC attempts to capture the first two desiderata of relativistic causation that I had laid out. Aside from only capturing some of the desiderata for relativistic causation in AQFT, this section will show that MC's attempt to prohibit superluminal signaling comes with a "morass of recalcitrant interpretational issues." (Earman and Valente, 2014, p. 16).

Since the way in which this axiom implements its causal desiderata comes off badly from my diagnostic, I want to stress its importance for QFT in general before moving on. First, MC can be easily corroborated for the most common field theories, or it is a crucial assumption in their construction. Consider the simplest relativistic free field: a Klein-Gordon (KG) field $\Phi$ of mass $m$. Its dynamics are described by

$$(\partial_\mu \partial^\mu + m^2)\Phi = 0 \tag{1}$$

After a canonical quantization procedure, it is easy to show that the commutator $[\Phi(x), \Phi(y)]$ of the field operator at spacetime points $x$ and $y$ is given by

$$\begin{gathered} i\Delta_0(x-y;m) = [\Phi(x), \Phi(y)], \\ \Delta_0(x-y;m) := -\frac{i}{(2\pi)^3} \int \frac{\mathrm{d}\vec{p}}{2\sqrt{m^2 + ||\vec{p}||^2}} \left( e^{-ip(x-y)} - e^{ip(x-y)} \right) |_{p^0 = \sqrt{m^2 + ||\vec{p}||^2}}, \end{gathered} \tag{2}$$

where $\Delta_0(x-y;m)$ is called the **propagator** or **causal distribution**. $\Delta_0(x-y;m)$ vanishes for equal times $x^0$ and $y^0$ (for any spatial separation $\vec{x} - \vec{y}$) and for spacelike values of $x - y$. Second, MC can be adapted and modified to include fields with spin or charge via the spin-statistics theorem. It is even a sufficient condition to define a Lorentz-covariant scattering matrix since it is connected to important analyticity properties of the fields (Weinberg, 1995; Duncan, 2012). It is, then, no surprise that physicists follow Einstein's suit in putting MC at the heart of relativistic causation in QFT.

However, MC is not as straightforward as it seems. For it to be a QM-informed axiom, we must now bite the bullet of not knowing what "measurement" means in a quantum setting (Earman and Valente, 2014, p. 11). Whatever "measurement" means, we need observables and states to obtain expectation values. One way to obtain the theory's observables is through a quantization procedure of a classical field theory.[1] Then, we need a state $|\Psi\rangle$ such that $AB\,|\Psi\rangle = BA\,|\Psi\rangle$, where $A \in \mathcal{A}(\mathcal{O}_1)$ and $B \in \mathcal{A}(\mathcal{O}_2)$. However, since $\mathcal{O}_1$ and $\mathcal{O}_2$ are spacelike separated, and since **isotony** tells us that $\mathcal{A}(\mathcal{O}_1), \mathcal{A}(\mathcal{O}_2) \subseteq \mathcal{A}(\mathcal{M})$ (where $\mathcal{M}$ is Minkowski spacetime) because $\mathcal{O}_1, \mathcal{O}_2 \subseteq \mathcal{M}$, this requires that $\mathcal{A}(\mathcal{M})$ admits a state. However, it would be absurd that local operations like $A\,|\Psi\rangle$ or $B\,|\Psi\rangle$ would alter a physical state for all space and time (Ruetsche, 2011, p. 110).

Despite these problems, MC still seems to be a reasonable axiom for a field theory in implementing some "separability" of physical subsystems, which, additionally, has testable consequences in terms of the measurements we can perform in each subsystem. A reason to probe this further is that, following Einstein's quotation, the separability of physical subsystems and the possibility of communicating them are closely connected, so we need a better grasp of that connection before diagnosing MC as a causal axiom. I will address those topics separately.

[1] Since the fields are operator-valued distributions, we also need a sufficiently smooth test function to smear them before using them in calculations.

## 2.1 Separability/Independence Conditions

This subsection aims to nail down some interpretive difficulties of diagnosing MC as an axiom in AQFT. Since MC strives to make the separability of quantum field theoretical systems compatible with the prohibition of superluminal signaling between them, this subsection sets the stage for my discussion of causation in the following subsection.

Consider $\mathcal{A}(\mathcal{O}_1) \vee \mathcal{A}(\mathcal{O}_2)$, the algebra generated by $\mathcal{A}(\mathcal{O}_1)$ and $\mathcal{A}(\mathcal{O}_2)$, where $\mathcal{O}_1$ and $\mathcal{O}_2$ are spacelike separated. A state $\omega$ in AQFT is a linear, positive-definite functional over the operator algebra. For our purposes, $\omega(A)$ is the expectation value of the observable $A$ belonging to some algebra $\mathcal{A}$. Now, consider a pair of states $\omega_j$ acting on $\mathcal{A}(\mathcal{O}_j)$ for $j = 1, 2$. Then, we should be able to find a state $\omega$ acting on $\mathcal{A}(\mathcal{O}_1) \vee \mathcal{A}(\mathcal{O}_2)$ such that its restriction to $\mathcal{A}(\mathcal{O}_j)$ is just $\omega_j$. To give an illustration more familiar to readers coming from QM, if the algebras are matrix algebras, the composition $\vee$ can be equivalent to a tensor product, and the expectation value $\omega(A)$ can be calculated using the trace prescription for a density matrix $\rho_\omega$. In that case, the "restriction" from the global state to the local states would be taking the partial trace over either one of the subsystems.

The most famous form[2] of independence in the spirit of my last paragraph is **statistical independence**, which is merely taking the expected values of $\omega_1$ and $\omega_2$ as probabilistically independent, that is, $\omega(C) = \omega_1(C)\omega_2(C)$ for some observable $C \in \mathcal{A}(\mathcal{O}_1) \vee \mathcal{A}(\mathcal{O}_2)$. There is no resulting statistical mixture of the individual states

[2]Earman and Valente (2014, §4-5) survey other ways to formulate independence conditions.

because they are localized in disjoint regions. This form of independence looks like a straightforward rendition of what we would want the compatibility of measurements carried out in spacelike separated regions to be since we are breaking potential correlations between the measurements in both regions. However, for statistical independence to hold, we need an additional assumption about the nature of the algebras called the **split property**, which in turn depends on the fields' energy densities having certain properties (Fewster, 2016) different from SC that, though reasonable, are not always fulfilled. That is, it can be shown that statistical independence can be derived from MC, but it needs additional assumptions. On their own, MC and statistical independence are logically independent (Earman and Valente, 2014, p. 11). Moreover, more general forms of independence that avoid superluminal signaling do not imply MC (Halvorson, 2007, pp. 753-755).

Still, MC does seem to implement some "separability" of physical subsystems, even if it is not because of their independent statistical predictions. One suggestion is to see MC more as a form of mereology of the structure of the algebras than a claim about locality (Ruetsche, 2011, pp. 112-113); the operation $\vee$ in $\mathcal{A}(\mathcal{O}_1) \vee \mathcal{A}(\mathcal{O}_2)$ should incorporate and respect MC. However, consider the composition of physical subsystems of standard QM: take two Hilbert spaces $\mathcal{H}_1$ and $\mathcal{H}_2$, and construct their tensor product, $\mathcal{H}_1 \otimes \mathcal{H}_2$. Then the algebras $\mathcal{B}(\mathcal{H}_1) \otimes \mathrm{Id}_2$ and $\mathrm{Id}_1 \otimes \mathcal{B}(\mathcal{H}_2)$ commute, and yet QM is not a relativistic theory. This example shows that the composition of bigger subsystems from commuting algebras is not a feature of relativistic theories (Halvorson, 2007, p. 752). So even if MC seems reasonable, the way in which it combines QM and SR brings to light multiple difficulties.

## 2.2 MC and causation

Aside from statistical independence, the **no-signaling theorem** is a result closely connected to MC, but that shifts our focus from separability to relativistic causation. In this subsection, I will finally show that the problems of separability and measurements in the two previous ones disrupt the possibility that MC fully characterizes relativistic causation in AQFT.

Of the many formulations of the no-signaling theorem, I will follow (Earman and Valente, 2014, §4.2) to avoid further technicalities. Consider an observable $A \in \mathcal{A}(\mathcal{O}_1)$ with a (countable) spectral decomposition $A = \sum_i a_i E_i^A$, where the $E_i$s are projectors and the $a_i$s $A$'s eigenvalues. We can then define a map $T^A(\cdot) = \sum_i E_i^A \cdot E_i^A$ over $\mathcal{A}(\mathcal{O}_1) \vee \mathcal{A}(\mathcal{O}_2)$ that is explicitly related to an $A$-measurement. Then, for a state $\omega$ and an observable $B \in \mathcal{A}(\mathcal{O}_2)$, we have:

$$\omega(T^A(B)) = \omega(B)$$

This means that states acting on the algebras of spacelike separated regions are unaffected by measurements in the other region. The moral is clear: MC allows the statistical predictions of one of the subsystems to be preserved even after performing measurements in the other.

However, a few complications stop us from proclaiming victory over superluminal signaling using MC. First, the preservation of the statistics of the outcomes of a measurement may not require commutativity after all (Rédei and Valente, 2010), and the no-signaling theorem only works for a restricted class of operations (Ruetsche, 2011, p. 111). Second, even if we have no-signaling theorems, the statistical independence of subsystems can get bypassed by measurement protocols like the following one:

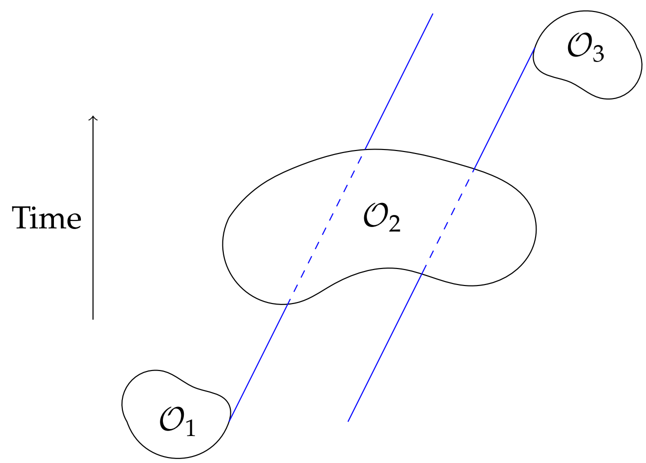


Figure 1: $\mathcal{O}_1$ and $\mathcal{O}_3$ are spacelike separated.

For this setup, successive observations of the same state in $\mathcal{O}_1$, $\mathcal{O}_2$, and $\mathcal{O}_3$ exhibit correlations between the measurement carried out in $\mathcal{O}_1$ and that in $\mathcal{O}_3$ (Sorkin, 1993). This result seems to imply that there is some superluminal signaling in QFT. Recent papers claim to have solved this problem (Bostelmann et al., 2021), but whether their solution is satisfactory or not, they had to do much more legwork than merely assuming MC.

In conclusion, although MC seems to shed some light on relativistic causation in AQFT, multiple difficulties impede us from claiming we have achieved a complete, clear characterization of it. First, as an axiom stated in terms of quantum measurements, we fall into the difficulties of understanding what "measurement" means and, especially, what it means for spacelike measurements to be compatible.

We tried to implement MC as a form of separability for this purpose. Still, statistical independence needs more than MC to work, and some other forms of independence are logically independent of MC or satisfied in non-relativistic cases. Even having failed to characterize what form of separability MC implies, the no-signaling theorem made us hopeful that it would still prohibit superluminal signaling. However, the theorem only works for a particular case of measurements, it can be proved without MC, and it can be bypassed setups like Sorkin's. The takeaway is that we still do not have a good characterization of relativistic causation in AQFT. Every time MC seemed to lead us in the right direction, interpretive difficulties or other pitfalls stopped us from clearly seeing the role of MC in prohibiting superluminal signaling.

# 3 Primitive Causality

From my initial characterization of relativistic causation, SC and MC have aimed at implementing the role of locality, subluminality, and the metric structure of Minkowski spacetime. However, I have not talked about determinism. Primitive causality (PC), or the time slice axiom, will fill this gap. First, we need some definitions. Consider two spacetime regions $\mathcal{O}_1$ and $\mathcal{O}_2$. $\mathcal{O}_2$ **depends causally** on $\mathcal{O}_1$ if every light ray in the backward (or forward) light cone originating from any point in $\mathcal{O}_2$ intersects $\mathcal{O}_1$.

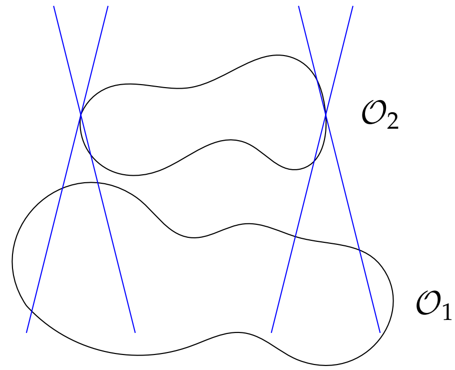


Figure 2: $\mathcal{O}_2$ depends causally on $\mathcal{O}_1$.

Then we have:

> **PC**: If $\mathcal{O}_2$ depends causally on $\mathcal{O}_1$, $\mathcal{A}(\mathcal{O}_2) \subseteq \mathcal{A}(\mathcal{O}_1)$.

Notice that, if $\mathcal{O}_2 \subset \mathcal{O}_1$, $\mathcal{O}_2$ depends causally on $\mathcal{O}_1$, so the property of isotony can be derived from PC. Although I will not use the term "local primitive causality" to refer to PC, it is often called that way to contrast it with the following condition, which is how Haag et al. usually present primitive causality (1962):

> Consider a **time slice** in Minkowski spacetime.[3] That is, a region infinitely extended in space but restricted to a time interval of size $\tau$:
> $\mathcal{O}_{t,\tau} := \{x \in \mathcal{M} : |x^0 - t| < \tau\}$. Then $\mathcal{A}(\mathcal{O}_{t,\tau}) = \mathcal{A}(\mathcal{M})\ \forall \tau$.

$\mathcal{M}$ depends causally on any time-slice, and any time-slice is isotonically included in $\mathcal{M}$, so both versions end up being equivalent. However, this version of the axiom makes determinism à la Bohr even more salient since the standard interpretation of

[3]More generally, the spacetime manifold should be globally hyperbolic and the time slice is some Cauchy surface.

PC is that there should be a "dynamical law" that allows us to determine the values of the fields at any given time with the values of the fields at a time slice (Haag, 1996, p. 48). This claim can be re-stated in the following way: the time-slice provides a region of evaluation of an initial value problem, and the "dynamical law" is deterministic (Bogolubov et al., 1990). Clearly, the exact nature of the law depends on the dynamics of the fields (Earman and Valente, 2014, p. 21). In any case, Earman and Valente take PC as the "most direct" expression of relativistic causation in AQFT because deterministic laws have causal-like behavior. This section will argue that PC's determinism is not enough to characterize relativistic causation in AQFT.

As I mentioned before, PC is local, and its time-slice version is global. Aside from this point, I will treat both versions of PC interchangeably. Dimock proved the relevance of PC for QFT since PC holds for the Klein-Gordon field (1980) and the Dirac field (1982). However, as an initial value problem, it is trivially fulfilled. The Lagrangians of field theories are formulated in terms of the fields and their derivatives at a time slice, and if their derivatives are of a sufficiently higher order, we can recover the field values in all spacetime (Bogolubov et al., 1990, pp. 330-331). As such, PC is an entirely reasonable assumption for AQFT to make, but only because this form of "determinism" is a truism for QFT.[4] PC is not, then, a feature of relativistic causation *in AQFT* but in field theories in general.

Now, the QFTs considered by Dimock should satisfy PC since their equations of

[4]Within AQFT, there are technical difficulties that complicate claiming that we can restrict the global dynamics to those of a time-slice. So if it is a truism in Lagrangian QFT, it is hardly obvious that it holds in AQFT. Thanks to Noel Swanson for this remark.

motion are hyperbolic partial differential equations (PDEs), which are characterized by having unique solutions within their domain of dependence (Geroch, 2011) and by having a finite propagation speed (Bär et al., 2007). The notion of **domain of dependence** has a precise definition within the realm of PDEs, but, in our case, it is merely the locus of causally dependent points to some region $\mathcal{O}$, denoted by $D(\mathcal{O})$. Another example of a hyperbolic PDE is the wave equation describing every undulatory phenomenon. From this point of view, PC is not a requirement from SR since it appears in non-relativistic phenomena, nor QM since it appears in classical ones. It is also not a sufficient characterization of relativistic causation since Klein-Gordon or similar wave equations would only be relativistic because their finite speed of propagation is the speed of light, but that does not rule out any higher speeds! These are the same reasons why PC is not a good causal axiom: the claim that the dynamics of the field are deterministic does not mean that the theory is causal in the sense that the dynamics should *also* respect the light cone structure of the underlying spacetime (Hofer-Szabó and Vecsernyés, 2018, p. 19).

Additionally, as Earman (2014) has claimed, determinism should not merely apply to the time evolution of the field observables but to that of the states. If states $\omega_1$ and $\omega_2$ give the same predictions with the observables in $\mathcal{A}(\mathcal{O})$, they should also provide the same ones with the observables in $\mathcal{A}(D(\mathcal{O}))$. Although I agree with Earman's suggestion, he bases it on taking $\mathcal{A}(\mathcal{O}) = \mathcal{A}(D(\mathcal{O}))$ as a result of Haag and Schroer (1962). However, this equation is not valid in general. We only obtain $\mathcal{A}(\mathcal{O}) \subseteq \mathcal{A}(D(\mathcal{O}))$ thanks to PC, and that is not what Haag and Schroer prove. They claim the local primitive causality that I will discuss in the next section.

# 4 Local Primitive Causality

In the past sections, we have seen the shortcomings and interpretive problems with the three causal axioms. Some of their drawbacks stem from not implementing QM, SR, and locality in a way that is amenable to codifying the desiderata of relativistic causation I laid out in the introduction, as we saw in the different diagnostics, or from implementing them in ways that imported interpretive difficulties from each of those themes. Moreover, of our desiderata for relativistic causation, SC focused on the metric structure of Minkowski spacetime, MC on locality and subluminality, and PC on determinism. It is only reasonable to look for a condition that puts all those advantages in one place. I will show that local primitive causality (LPC) does that for us, and I will sketch how it does in virtue of being implied by the conjunction of SC, MC, and PC.

Defining the **causal complement** of a region $\mathcal{O}$, $\mathcal{O}'$, as the set of points outside $D(\mathcal{O})$, we have:

> **LPC**: For a region $\mathcal{O} \subset \mathcal{M}$,[5] $\mathcal{A}(\mathcal{O}) = \mathcal{A}((\mathcal{O}')')$.

This condition is local since its dependence on spacetime regions is explicit; it is a crucial ingredient in showing that concrete experimental setups exhibit no superluminal signaling (Buchholz and Yngvason, 1994; Yngvason, 2005); it is the key assumption in theories of local observables that attempt to ensure that the determinism of PC is compatible with the temporal evolution of field values in the

[5](Hofer-Szabó and Vecsernyés, 2018, p. 19) give a more general definition that I can give here. See also their footnote 2.

light cone of their regions (Hofer-Szabó and Vecsernyés, 2018); and it emphasizes the metric structure of $\mathcal{M}$ underlying the definition of "causal complement." In summary, it passes the diagnostic of a causal axiom! It has two further advantages. First, LPC can be generalized to curved spacetimes and used in concrete measurement protocols (Fewster and Verch, 2020), thus alleviating some of the worries I had claimed about SC and MC.[6] Second, in keeping this paper minimally technical, I have avoided any talk on the details of the mathematical structure of observable algebras for space reasons and because my arguments in this paper are independent of these considerations. However, these details are vital for relativistic causation. For example, superluminal signaling is unavoidable if the algebras are the matrix algebras of standard QM (Hegerfeldt, 1994). From this point of view, Buchholz and Yngvason (1994; 2005) have shown how LPC and the specific type of von Neumann algebra relevant for AQFT come together in implementing relativistic

[6]I should clarify that I am not claiming that LPC solves problems like Sorkin's paradox since that would require a different conception of measurements in QFT from the one I have been assuming in this paper. The advantages I have presented from Buchholz, Yngvason, Fewster, and Verch, rely on specific measurement protocols or experimental setups. Fewster and Verch do not assume LPC explicitly, but they obtain it for free (and with it, its advantages) because their framework relies heavily on *causally convex* (i. e., diamond-like) regions, for which LPC is trivially true. However, their version of LPC is more general than the one I presented here because they work with local $^*$-algebras, not von Neumann algebras, which gives them less topological structure than the one the derivation of LPC from the other axioms that I cite in the main text can exploit.

causation.

At this point, I could claim that LPC, not one of the other three axioms, should be the most direct expression of relativistic causation in AQFT since it captures all the desiderata I had stated above. This attempt at privileging one condition would be, however, too rash. Consider the following construction of (Haag and Schroer, 1962):

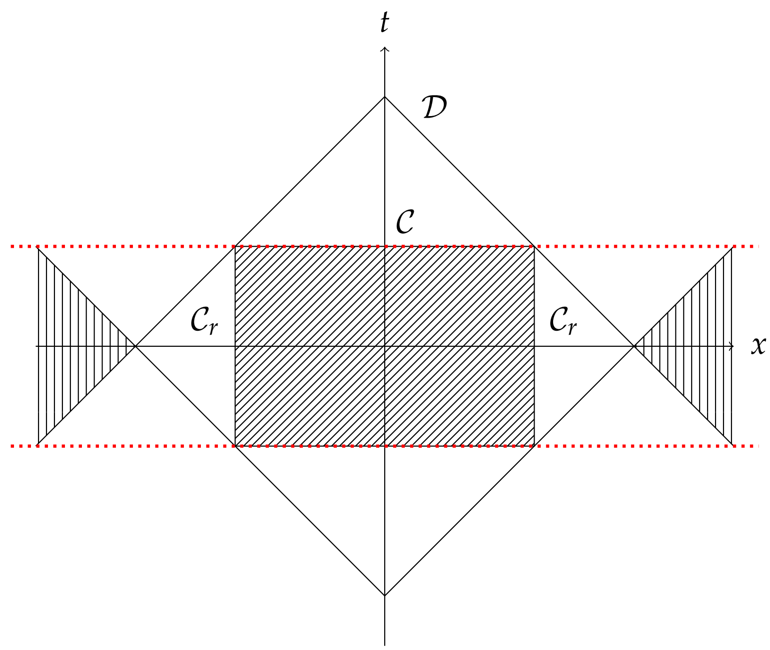


Figure 3: The region shaded with vertical lines is $\mathcal{C}'$, the causal complement of $\mathcal{C}$. The red dotted lines extend spatially to form a time-slice of the temporal size of $\mathcal{C}$. Call $\mathcal{C}_r$ the two caps outside of $\mathcal{C}$ that extend in the spatial direction.

It is now possible to prove that $\mathcal{A}(\mathcal{D}) = \mathcal{A}(\mathcal{C})$, which is exactly LPC since the diamond $\mathcal{D}$ satisfies $\mathcal{D} = (\mathcal{C}')'$. All the pieces of the proof have been put together in (Calderón, 2019, pp. 22-31),[7] but here, I can only sketch the main steps. From SC, one

[7]That proof appeals to **Haag Duality**, which is not satisfied by some of our best field theories and only works in particular spacetime regions (Haag, 1996, pp. 145-147). For-

can prove that the domain of analyticity of the fields in $\mathcal{C}$ can be extended to $\mathcal{C} \cup \mathcal{C}_r$ (Borchers, 1961). From PC and MC, we can build a time-slice from $\mathcal{C} \cup \mathcal{C}_r$ and $\mathcal{C}'$, where $\mathcal{C}$ and $\mathcal{C}'$ localize separated subsystems. The fact that the observables at a time slice generate those in all spacetime gives some nice closure properties that display the intimate connection between the local algebras and the regions in which they are supported. Finally, from MC, the observables from $\mathcal{C}'$ commute with those of $\mathcal{C}$ but also with those of $\mathcal{C}''(= \mathcal{D})$. The takeaway from this construction is that even if SC, MC, and PC individually failed to characterize relativistic causation in AQFT, LPC captures all the desiderata of relativistic causation in AQFT *because* it is a fruit of their conjunction.

# 5 Outlook

Throughout this paper, we saw that SC, MC, and PC only give partial characterizations of relativistic causation. Even if this is unsatisfactory for those casting a vote for an individual axiom as fulfilling that role (or fulfilling it in the most exemplary way), the need for so many axioms should not be startling anymore precisely because they highlight different aspects of the causal framework of the theory. Additionally, LPC encapsulates each axiom's advantages for the most widely used regions and theories in AQFT. Assuming LPC has become a widespread move in more technical literature in AQFT. Still, its motivations are rarely stated, its role as a causal axiom is left uninterpreted, and its interdependence with the other causal

tunately, **essential duality** (Halvorson, 2007, p. 841), a less problematic assumption, can be used instead. Thanks to Noel Swanson for this suggestion.

axioms is ignored.

However, many additional worries emerged in analyzing this wide variety of axioms. First, an appropriate interpretation of MC would require tackling the measurement problem in QFT. One way to address this problem is to introduce a more sophisticated account of local operations in AQFT, which is where some of the more technical literature has been heading (Okamura and Ozawa, 2016; Drago and Moretti, 2020). However, from an interpretational point of view, it is unclear whether considering a more widespread class of measurements would solve the problem of answering what "measurement" means. Even if looking for an account of measurements in QFT seems orthogonal to a project on relativistic causation, I do think they are connected through the worries that philosophers have raised about the "operationalist" views of the founders of AQFT and the "algebraic imperialist" (Ruetsche, 2011) tendencies of the mathematical physicists working in this formulation of QFT. Additionally, revising the connections between causation and measurements would require further examining the local algebras' mathematical structure and the states that can act on them. The need to appeal to those tools can be seen not only in the problems of measurements in e. g. MC but in the interplay between the different formulations of PC as a requirement demanding that we have a compatible notion of local dynamics and global determinism. That is one of the problems that even promoting LPC to an additional assumption in AQFT leaves unsettled.

The second problem is that any account of relativistic causation in AQFT is challenged by entanglement, which is ubiquitous and maximal in AQFT, even in spacelike separated regions (Summers and Werner, 1988). Entanglement *itself* is not something we should worry about since it is just a key feature of QM and QFT. Instead, the origin of the uneasiness about entanglement is that it is often conceived as a form of non-locality or of non-separability, which conflicts with e. g. conceiving MC as a form of independence between spacelike separated subsystems. The pull of entanglement as a problem for causation in AQFT relies heavily on assumptions outside of the theory's axiomatic framework about measurements and operations in QFT (Ruetsche, 2021) and, as such, it is connected to my first remaining worry. These problems are, to the best of my knowledge, unsettled. I hope this diagnostic points to some of the issues that need to be addressed and the importance of doing so.

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