

On the ontologies of quantum theories

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Abstract

What is the ontology of a realist quantum theory such as Bohmian mechanics? This has been an important but debated issue in the foundations of quantum mechanics. In this paper, I present a new result which may help examine the ontology of a realist physical theory and make it more complete. It is that when different values of a physical quantity lead to different evolution of the assumed ontic state of an isolated system in a theory, this physical quantity also represents something in the ontology of the theory. Moreover, I use this result to analyze the ontologies of several realist quantum theories. It is argued that in Bohmian mechanics and collapse theories such as GRWm and GRWf, the wave function should be included in the ontology of the theory. In addition, when admitting the reality of the wave function, mass, charge and spin should also be taken as the properties of a quantum system.

Key words: quantum theory; ontology; wave function; particle; mass; charge

1 Introduction

It has been debated what the ontology of a realist quantum theory is. Bohmian mechanics or the pilot-wave theory of de Broglie and Bohm is a typical example (de Broglie, 1928; Bohm, 1952). According to some authors, the universal wave function is not ontic, representing a concrete physical entity, but nomological, like a law of nature (Dürr et al, 1992; Allori et al, 2008; Goldstein and Zanghì, 2013; Esfeld et al, 2014; Goldstein, 2021). On this view, there are only particles in the ontology of Bohmian mechanics. While according to others (Bohm and Hiley, 1993; Holland, 1993; Gao, 2017; Hubert and Romano, 2018; Valentini, 2020), the ontology of Bohmian

mechanics includes both particles and the wave function. In this paper, I will present a new result which may help examine the ontology of a realist physical theory and make it more complete. Moreover, I will use this result to analyze the ontologies of several realist quantum theories. It will be argued that in Bohmian mechanics and collapse theories such as GRWm and GRWf, the wave function should be included in the ontology of the theory. In addition, when admitting the reality of the wave function, mass, charge and spin should also be taken as the properties of a quantum system.

2 A new result

Suppose there are two free (uncorrelated) particles that have the same properties. Moreover, they have the same state of motion at a given instant, and the law of motion is deterministic for them. The question is: will they have the same state at later instants? If the laws of motion are the same for the two particles, then they will have the same state at later instants. On the other hand, if the laws of motion are different for the two particles, then they may not have the same state at later instants. But this is an impossible situation; since the two particles have exactly the same properties and they cannot be distinguished, the laws of motion must be the same for the two particles.

This impossibility can be used to derive a more rigorous result. Consider a deterministic realist physical theory, which assumes that each isolated system has an ontic state, and the law of motion that governs the time evolution of the ontic state is deterministic. If different values of a physical quantity lead to different evolution of the ontic state of an isolated system, then this physical quantity should represent something in the ontology of the theory, which is either a property of this system or a property of another system. This result can be proved as follows. Suppose there is an isolated system whose ontic state is $\lambda(0)$ at an initial instant. Moreover, the time evolution of the ontic state is affected by a physical quantity A , and two different values of A , such as a_1 and a_2 , lead to different evolution of the ontic state, namely we have $\lambda(t, a_1) \neq \lambda(t, a_2)$ for some later instants t . Now if the physical quantity A does not represent anything in the ontology of the theory, then the two situations, in which the physical quantity A assumes two different values, a_1 and a_2 , will be exactly the same in ontology for the system at the initial instant; there is an initial ontic state $\lambda(0)$ in both situations. Since the two situations (which are the same in ontology) cannot be distinguished, the law of motion must be the same for the two situations. Then we must have the relation $\lambda(t, a_1) = \lambda(t, a_2)$ for all later instants t by the deterministic law of motion. This is inconsistent with our presupposition that $\lambda(t, a_1) \neq \lambda(t, a_2)$ for some later instants t . Therefore, the physical quantity A must represent something in the ontology of the theory, being

either a property of the studied system or a property of another different system.

3 Bohmian mechanics

The above result can be used to analyze the ontology of a realist physical theory and make the ontology more complete. Let's first consider Bohmian mechanics. In Bohmian mechanics, a complete realistic description of a quantum system is provided by the configuration defined by the positions of its particles together with its wave function. The law of motion is expressed by two equations: a guiding equation for the configuration of particles and the Schrödinger equation, describing the time evolution of the wave function which enters the guiding equation. The law of motion can be formulated as follows:

$$\frac{dX(t)}{dt} = v^{\Psi(t)}(X(t)), \quad (1)$$

$$i\hbar \frac{\partial \Psi(t)}{\partial t} = H\Psi(t), \quad (2)$$

where $X(t)$ denotes the spatial configuration of particles, $\Psi(t)$ is the wave function at time t , and v equals to the velocity of probability density in standard quantum mechanics.

Suppose there are only particles in the ontology of Bohmian mechanics, and these particles have only positions and velocities as their properties. Consider a free particle such as a free electron.¹ Its ontic state at each instant is represented by the position and velocity of its Bohmian particle at the instant. By space translation invariance, the difference between two ontic states of the particle lies only in the difference between the velocities of its Bohmian particle. According to the guiding equation, the velocity of the Bohmian particle at each instant t is

$$v(x, t) = \frac{1}{m} \nabla S(x, t), \quad (3)$$

where $S(x, t)$ is the phase of the wave function of the particle. Moreover, the acceleration of the Bohmian particle is:

$$\frac{\partial v(x, t)}{\partial t} = \frac{1}{m} \nabla Q(x, t), \quad (4)$$

where $Q(x, t) = \frac{\hbar^2}{2m} \frac{\nabla^2 R(x, t)}{R(x, t)}$ is the so-called quantum potential, and $R(x, t)$ is the amplitude of the wave function of the particle. It can be seen that

¹Since the following analysis concerns only one particle, it also applies to Albert's marvellous point ontology where there is only one particle.

different $S(x, t)$ and different $R(x, t)$ or different wave functions $\psi(x, t) = R(x, t)e^{iS(x, t)/\hbar}$ will in general lead to different evolution of the velocity of the Bohmian particle or different evolution of the ontic state of the free particle.² Then according to the previous result, the wave function must represent something in the ontology of Bohmian mechanics.³ Since the wave function is nonlocal and the Bohmian particles are local, the wave function will represent the properties of another physical entity, which is different from the Bohmian particles.

Similarly, we can argue that mass should also be included in the ontology of Bohmian mechanics. Different masses will lead to different evolution of the ontic state of a free particle according to the guiding equation. In addition, by considering the velocity of the Bohmian particle of a spin- s particle with mass m and charge Q and magnetic moment μ_s in an external electromagnetic field:

$$v(x, t) = \frac{1}{2m} \left[\frac{(\psi^*(x, t)\hat{p}\psi(x, t) - \psi(x, t)\hat{p}\psi^*(x, t))}{\psi^*(x, t)\psi(x, t)} - 2QA(x, t) \right] + \frac{\mu_s}{s} \frac{\nabla \times (\psi^*(x, t)\hat{S}\psi(x, t))}{\psi^*(x, t)\psi(x, t)}, \quad (5)$$

where $\psi(x, t)$ is the wave function of the particle, \hat{p} is the momentum operator, $A(x, t)$ is the magnetic vector potential, and \hat{S} is the spin operator, we can argue that charge and spin should be also included in the ontology of Bohmian mechanics. It has been argued that mass, charge and spin are the properties of the wave function, not the properties of the Bohmian particles (Brown et al, 1995; Gao, 2017, ch.6).

²There may be a deeper reason of why the evolution of the velocity of a Bohmian particle is affected by its wave function. If this were not true, the theory would disagree with quantum mechanics. For example, consider a particle being in the ground state in a box. The Bohmian particle is at rest in one position in the box. When the walls of the box move, the wave function of the particle will change. In this case, if the velocity of the Bohmian particle does not change with the wave function or the Bohmian particle is still at rest in its original position, then the results of position measurements on this particle for the two wave functions (before and after the walls moving) will have the same probability distribution, since the ontic states of the particle in these two situations are the same. But this obviously violates the Born rule.

³This analysis is independent of whether the universal wave function is a product state or an entangled state. If the universal wave function is a product state, then there will be truly isolated systems in the universe. While if the universal wave function is an entangled state, then there are only effective isolated systems whose wave functions are effective wave functions in Bohmian mechanics. In this case, the Bohmian particles of an isolated system are not influenced by the Bohmian particles in the environment either (Gao, 2017, ch.3), and thus the above analysis is still valid.

4 Stochastic Bohmian mechanics

There are also stochastic variants of Bohmian mechanics such as the Bohm-Bell-Vink dynamics (Bell, 1984; Vink, 1993; Barrett, 1999, p.203). Can a stochastic law of motion avoid the above result? Let's have a look at Vink's discrete dynamics for the position of a one-particle system. The continuity equation in the discrete position representation $|x_n\rangle$ is:

$$\hbar\partial P_n(t)/\partial t = \sum_m J_{nm}(t), \quad (6)$$

where

$$P_n(t) = |\langle x_n | \psi(t) \rangle|^2, \quad (7)$$

$$J_{nm}(t) = 2\text{Im}(\langle \psi(t) | x_n \rangle \langle x_n | H | x_m \rangle \langle x_m | \psi(t) \rangle), \quad (8)$$

where $|\psi(t)\rangle$ is the wave function of the system, and H is the Hamiltonian of the system.

In Vink's dynamics, the position jumps of the Bohmian particle of the system are governed by a transition probability $T_{mn}dt$ which gives the probability to go from position x_n to x_m . The transition matrix T gives rise to a time-dependent probability distribution x_n (for an ensemble of identically prepared systems), $P_n(t)$, which has to satisfy the master equation:

$$\partial P_n(t)/\partial t = \sum_m (T_{nm}P_m - T_{mn}P_n). \quad (9)$$

Then when the transition matrix T satisfies the following equation:

$$J_{nm}/\hbar = \sum_m (T_{nm}P_m - T_{mn}P_n). \quad (10)$$

the above continuity equation can be satisfied.

Vink (1993) showed that when choosing Bell's simple solution where for $n \neq m$ ⁴

$$T_{nm} = \begin{cases} J_{nm}/\hbar P_m, & J_{nm} \geq 0 \\ 0, & J_{nm} < 0, \end{cases} \quad (11)$$

the dynamics reduces to the guiding equation of Bohmian mechanics in the continuum limit.

In this case, when two Bohmian particles with different wave functions have the same initial position, they may generally have different positions at a later instant. But this difference may not result from the difference of the wave functions, but be merely a result of different random jumps, and

⁴The probability $T_{nn}dt$ follows from the normalization relation $\sum_m T_{nm}dt = 1$.

thus it cannot be used to derive the reality of the wave function as in the deterministic case. On the other hand, in this stochastic theory, we also need to consider the transition probability of a Bohmian particle at each instant, which gives the probability for the Bohmian particle to go from its position at the instant to another position, and it should be taken as one part of the ontic state of the Bohmian particle. Then, since the law of motion for the transition probability is deterministic and different wave functions will lead to different evolution of the transition probability, the wave function must also represent something in the ontology of the theory according to the previous result.

5 Collapse theories and many worlds

Let's now consider collapse theories. In the GRWm theory, the ontology is assumed to be a mass density field $m(x, t)$ in three-dimensional space, and in the GRWf theory, the ontology is assumed to be flashes or space-time points that correspond to the localization events (i.e. collapses of the wave function) in three-dimensional space (Allori et al, 2008). In both theories, different wave functions will lead to different evolution of the ontic state (the mass density field or the flashes), and thus the wave function must be included in the ontology of these theories according to the previous result. Note that although the law of motion in a collapse theory is not deterministic but stochastic, when the stochastic effects are small compared with the influences of the wave function (e.g. for microscopic systems), the previous result is still valid.

Finally, consider the many-worlds interpretation of quantum mechanics (MWI). In MWI, the wave function is clearly included in the ontology of the theory. Here the previous result further requires that mass, charge and spin should be also included in the ontology of the theory, since different values of these quantities will lead to different evolution of the wave function. This is also true for wave function realism (Albert, 1996, 2013). It remains to be seen how mass and charge can be put in the ontology for wave function realism and space-time state realism (Wallace and Timpson, 2010).

6 Further discussion

There have been worries about the reality of the wave function, since it is defined in a high-dimensional space, not in our three-dimensional space. This is also a main reason of why some Bohmians remove the wave function from the ontology of Bohmian mechanics (see, e.g. Esfeld et al, 2014). However, the wave function does not necessarily represent a physical entity in a high-dimensional space, and there are also ontological interpretations of the wave function in three-dimensional space such as the multi-field interpreta-

tion (Hubert and Romano, 2018) and the RDM of particles interpretation (Gao, 2017, 2020). Thus, there are other possible pictures of quantum reality in three-dimensional space besides the pictures of Bohmian particles, mass density fields and flashes.

Compared with the multi-field interpretation, the RDM of particles interpretation can accommodate mass and charge in the ontology more directly. According to this interpretation, a quantum system is composed of particles with mass and charge which undergo random discontinuous motion (RDM) in three-dimensional space, and the wave function represents the propensities of these particles which determine their random discontinuous motion, and as a result, the state of motion of these particles is also described by the wave function.⁵ At each instant all particles have a definite position, while during an infinitesimal time interval around each instant they move throughout the whole space where the wave function is nonzero in a random and discontinuous way, and the probability density that they appear in every possible group of positions in space is given by the modulus squared of the wave function there. Visually speaking, the RDM of each particle will form a mass and charge cloud in space (during an infinitesimal time interval around each instant), and the RDM of many particles being in an entangled state will form many entangled mass and charge clouds in space. It has been suggested that one can solve the measurement problem based on this interpretation of the wave function in two ways: one is to resort to the dynamical collapse of the wave function (Gao, 2017, ch.8), and the other is to formulate it as a theory of many worlds (Gao, 2022).

7 Conclusion

It has been debated what the ontology of a realist quantum theory is, e.g. whether the ontology of Bohmian mechanics should include the wave function. In this paper, I present a new result which may help examine the ontology of a realist physical theory and make it more complete. It is that when different values of a physical quantity lead to different evolution of the assumed ontic state of an isolated system in a theory, this physical quantity also represents something in the ontology of the theory. Moreover, I use this result to analyze the ontologies of several realist quantum theories. It is argued that in Bohmian mechanics, as well as in collapse theories such as GRWm and GRWf, the wave function should be included in the ontology of the theory. In addition, when admitting the reality of the wave function as in these theories and the many-worlds interpretation of quantum mechanics,

⁵Note that there is also a picture of random discontinuous motion of particles in Bell's Everett (?) theory (Bell, 1981). In that theory, however, the wave function is regarded as a real physical field in configuration space, and the random discontinuous motion of particles is not aimed to provide an ontological interpretation of the wave function.

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References

- [1] Albert, D. Z. (1996). Elementary Quantum Metaphysics. In J. Cushing, A. Fine and S. Goldstein (eds.), *Bohmian Mechanics and Quantum Theory: An Appraisal*. Dordrecht: Kluwer, 277-284.
- [2] Albert, D. Z. (2013). Wave function realism. In Ney, A. and D. Z. Albert (eds.), *The Wave Function: Essays on the Metaphysics of Quantum Mechanics*. Oxford: Oxford University Press. pp. 52-57.
- [3] Allori, V., S. Goldstein, R. Tumulka, and N. Zanghì (2008). On the common structure of Bohmian mechanics and the Ghirardi-Rimini-Weber theory, *British Journal for the Philosophy of Science* 59 (3), 353-389.
- [4] Barrett, J. A. (1999). *The Quantum Mechanics of Minds and Worlds*. Oxford: Oxford University Press.
- [5] Bell, J. S. (1981). Quantum mechanics for cosmologists. In *Quantum Gravity 2*, (eds.) C. Isham, R. Penrose, and D. Sciama. Oxford, Clarendon Press. pp. 611-637.
- [6] Bell, J. S. (1984). Beables for quantum field theory. In Bell, J. S. (1987). *Speakable and Unspeakable in Quantum Mechanics*. Cambridge: Cambridge University Press. pp.173-180.
- [7] Bohm, D. (1952). A suggested interpretation of quantum theory in terms of "hidden" variables, I and II. *Physical Review* 85, 166-193.
- [8] Bohm, D. and B. J. Hiley (1993). *The Undivided Universe: An Ontological Interpretation of Quantum Theory*. London: Routledge.
- [9] Brown, H. R., C. Dewdney, and G. Horton (1995). Bohm particles and their detection in the light of neutron interferometry. *Foundations of Physics* 25, 329.
- [10] de Broglie, L. (1928). La nouvelle dynamique des Fields Quanta. In J. Bordet (eds.), *Electrons et photons: Rapports et discussions du cinquieme Conseil de Physique*. Paris: Gauthier-Villars. pp.105-132. English translation: The new dynamics of quanta, in Bacciagaluppi and Valentini (2009), pp.341-371.
- [11] Dürr, D., S. Goldstein, and N. Zanghì (1992). Quantum equilibrium and the origin of absolute uncertainty. *Journal of Statistical Physics* 67, 843-907.

- [12] Esfeld, M., Lazarovici, D., Hubert, M. and Dürr, D. (2014). The ontology of Bohmian mechanics. *British Journal for the Philosophy of Science*. 65 (4), 773-796.
- [13] Gao, S. (2017). *The Meaning of the Wave Function: In Search of the Ontology of Quantum Mechanics*. Cambridge: Cambridge University Press. pp. 26-28.
- [14] Gao, S. (2022). On Bell's Everett (?) Theory. *Foundations of Physics*, 52, 89.
- [15] Goldstein, S. (2021). Bohmian Mechanics, *The Stanford Encyclopedia of Philosophy* (Fall 2021 Edition), Edward N. Zalta (ed.), <https://plato.stanford.edu/archives/fall2021/entries/qm-bohm/>.
- [16] Goldstein, S. and N. Zanghì (2013). Reality and the Role of the Wave Function in Quantum Theory. In *Quantum Physics Without Quantum Philosophy*, pp. 263-278. Berlin, Springer.
- [17] Holland, P. R. (1993). *The Quantum Theory of Motion: An Account of the de Broglie-Bohm Causal Interpretation of Quantum Mechanics*. Cambridge: Cambridge University Press.
- [18] Hubert, M. and Romano, D. (2018). The wave-function as a multi-field. *European Journal for Philosophy of Science* 8, 521-537.
- [19] Valentini, A. (2020). Foundations of statistical mechanics and the status of the Born rule in de Broglie-Bohm pilot-wave theory. In V. Allori (ed.), *Statistical Mechanics and Scientific Explanation: Determinism, Indeterminism and Laws of Nature*. World Scientific. pp. 423-477.
- [20] Vink, J. C. (1993). Quantum mechanics in terms of discrete beables. *Physical Review A* 48, 1808.
- [21] Wallace, D. and C. G. Timpson (2010). Quantum Mechanics on Spacetime I: Spacetime State Realism. *The British Journal for the Philosophy of Science* 61, pp. 697-727.