

Quantum mechanics requires “conspiracy”

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Quantum states containing records of incompatible outcomes of quantum measurements are valid states in the tensor product Hilbert space. Since they contain false records, they conflict with the Born rule and with our observations. I show that excluding them requires a fine-tuning to a zero-measure subspace of the Hilbert space that seems “conspiratorial”, in the sense that

- it depends on future events, in particular of future choices of the measurement settings,
- it depends on the evolution law (normally thought to be independent of the initial conditions),
- it violates statistical independence (even in interpretations that satisfy it in the context of Bell’s theorem, like standard quantum mechanics, pilot-wave theories, collapse theories, many-worlds *etc.*).

Even the innocent assumption that there are measuring devices requires this kind of fine tuning.

These results are independent of the interpretation of quantum mechanics.

To explain away this apparent fine-tuning, I propose that an yet unknown law or superselection rule may restrict the full tensor product Hilbert space to this very special subspace.

Keywords: Born rule; fine-tuning; statistical independence; memories; Bell’s theorem; superdeterminism.

I. INTRODUCTION

Quantum mechanics, like other theories, is formulated from a God’s-eye perspective. But, as parts of the world we observe, we are limited to a worm’s-eye perspective. If in the present time we would be part of a random state of the universe, this would most likely contain incompatible records, from which we would never be able to guess the laws of quantum mechanics, in particular the Born rule.

An example of such a state is one containing n records of repeated spin measurement of the same silver atom, so that the n records of the outcomes are random values $\pm\frac{1}{2}$, and not the same value repeated n times. This state is a valid state in the tensor product Hilbert space. But the records it contains could not come from actual repeated quantum measurements. We never observe such states.

The simple fact that we exist and could discover quantum mechanics indicates that the physical law is user-friendly enough to allow our memories to form and be reliable, to reflect the evolution of our universe so that we can guess its laws, including the Born rule. We are led to a “*the universe does not mislead us*” metaprinciple:

Metaprinciple NMU (Non-Misleading Universe). The records of the experimental results and the memories of the observers reflect the actual history of the universe.

Without this, science and even life would be impossible. But Metaprinciple NMU, as we shall see, requires severe restrictions of the possible states. We will see that this fine-tuning contradicts several of our most cherished common sense beliefs. The first belief is:

Belief 1 (Universality). Quantum mechanics, including the Born rule and the results of quantum experiments, respect Metaprinciple NMU for all initial conditions.

Another belief is that of *Statistical Independence* (SI). We assume that there are enough degrees of freedom so

that any two systems separated in space can be put in independent and statistically uncorrelated states.

Definition 1. Two events A and B are *statistically independent* if $\Pr\{AB\} = \Pr\{A\}\Pr\{B\}$ ([13] p. 10). In particular, if each of two statistically independent events A and B are *possible* ($\Pr\{A\} > 0$ and $\Pr\{B\} > 0$), they are possible together ($\Pr\{AB\} > 0$). Therefore, if SI is true for the events that two subsystems are in particular states, the following should be true as well:

Belief 2 (Subsystems Independence). Let A and B be two subsystems with no common parts. If A can possibly be in the state α and B can possibly be in the state β , the combined system can possibly be in the state $\alpha \otimes \beta$.

Belief 2 is the core reason why we take as Hilbert space of a composite system the tensor product of the Hilbert spaces of each of the systems. I will show that this, and consequently SI, is contradicted, although *Bell’s weaker assumption of SI is not contradicted* (see Answer 3).

It makes sense to think that no “Laplace demon” knowing the evolution law and the future histories is needed to determine what initial conditions ensure Metaprinciple NMU. This can be stated as the following beliefs:

Belief 3 (No Input From Future). Initial conditions are independent of future events in the history, in particular of future choices of the measurement settings.

Belief 4 (No Input From Evolution Law). Initial conditions are independent of the evolution law of the system.

Another belief that will be contradicted is

Belief 5 (For-Granted Memory). In the standard tensor product Hilbert space formulation of quantum mechanics, past events leave reliable records in the present state without requiring conspiratorial fine-tuning.

In Sec. §II I show that Metaprinciple NMU contradicts these beliefs. This may seem objectionable, so in Sec. §III I discuss possible questions and implications.

II. THE PROOF

Theorem 1. *To ensure Metaprinciple NMU for the Born rule, the initial states have to belong to a zero-measure subspace of the Hilbert space, in a way that contradicts Beliefs 1, 2, 3, 4, and 5.*

Proof. Consider a closed quantum system which includes observed systems, measuring devices, and observers. This may be the entire universe. Its states are represented by unit vectors in a separable Hilbert space \mathcal{H} , and evolve governed by the Schrödinger equation with the Hamiltonian \hat{H} . In terms of the unitary evolution operator $\hat{U}_{t,t_0} := e^{-\frac{i}{\hbar}(t-t_0)\hat{H}}$ between the times t_0 and t , the evolution of an initial state vector $\Psi(t_0) \in \mathcal{H}$ at t_0 is

$$\Psi(t) = \hat{U}_{t,t_0} \Psi(t_0). \quad (1)$$

Suppose that our system contains a system to be observed S , with Hilbert space \mathcal{H}_S of finite dimension $\dim \mathcal{H}_S = n < \infty$, and a measuring device whose pointer is represented in the Hilbert space \mathcal{H}_{M_A} of dimension $n + 1$. Then $\mathcal{H} = \mathcal{H}_S \otimes \mathcal{H}_{M_A} \otimes \mathcal{H}_E$, where \mathcal{H}_E represents everything else, including the other parts of the measuring device. Let \hat{A} be a Hermitian operator on \mathcal{H}_S representing the observable of interest, with eigenbasis $(\psi_1^A, \dots, \psi_n^A)$. Let \hat{Z}^A be the pointer observable, with eigenbasis $(\zeta_0^A, \zeta_1^A, \dots, \zeta_n^A)$, where ζ_0^A represents the “ready” state of the pointer. We assume that the observable and the pointer have nondegenerate spectra, and the measurement is ideal. We work in the interaction picture, which allows us to treat the observed system as stationary, and the pointer states as stationary before and after the measurement. Let the measurement of \hat{A} take place between t_0 and $t_1 > t_0$, leading to the superposition

$$\Psi(t_1) = \hat{U}_{t_1,t_0} \psi \otimes \zeta_0^A \otimes \dots = \sum_j \langle \psi_j^A | \psi \rangle \psi_j^A \otimes \zeta_j^A \otimes \dots \quad (2)$$

To resolve the superposition from eq. (2) into definite outcomes, one usually invokes projection, objective collapse, decoherence into branches, additional hidden variables *etc.* The results from this article apply to all these options. The *Born rule* states that the probability that at t_1 the pointer is in the state ζ_j^A is $|\langle \psi_j^A | \psi \rangle|^2$.

Consider a second measurement, of an observable \hat{B} of the system S , with eigenbasis $(\psi_1^B, \dots, \psi_n^B)$. Let the pointer observable of the second apparatus be \hat{Z}^B , with eigenbasis $(\zeta_0^B, \zeta_1^B, \dots, \zeta_n^B)$, where ζ_0^B is the “ready” state. The total Hilbert space is $\mathcal{H} = \mathcal{H}_S \otimes \mathcal{H}_{M_A} \otimes \mathcal{H}_{M_B} \otimes \mathcal{H}_E$.

The measurement of \hat{B} takes place after the first measurement, between t_1 and $t_2 > t_1$. It leads to

$$\begin{aligned} \Psi(t_2) &= \hat{U}_{t_2,t_1} \hat{U}_{t_1,t_0} \psi \otimes \zeta_0^A \otimes \zeta_0^B \otimes \dots \\ &= \hat{U}_{t_2,t_1} \sum_j \langle \psi_j^A | \psi \rangle \psi_j^A \otimes \zeta_j^A \otimes \zeta_0^B \otimes \dots \\ &= \sum_{j,k} \langle \psi_j^A | \psi \rangle \langle \psi_k^B | \psi_j^A \rangle \psi_k^B \otimes \zeta_j^A \otimes \zeta_k^B \otimes \dots \end{aligned} \quad (3)$$

The probability that at t_2 the first pointer state is ζ_j^A and the second pointer state is ζ_k^B is $|\langle \psi_j^A | \psi \rangle \langle \psi_k^B | \psi_j^A \rangle|^2$. This vanishes if $\hat{A} = \hat{B}$ and $j \neq k$, and we obtain

Observation 1. The Born rule forbids orthogonal results for repeated measurements, *e.g.* if $\hat{A} = \hat{B}$, the states $\psi_k^B \otimes \zeta_j^A \otimes \zeta_k^B \otimes \dots$ with $j \neq k$ are forbidden at t_2 .

Observation 2. However, a priori, all unit vectors in \mathcal{H} are possible initial conditions at the initial time $t_i < t_0$ of the universe, including, for any j and k , the vectors

$$\Psi_{j,k}(t_i) := \hat{U}_{t_2,t_i}^\dagger \psi_k^B \otimes \zeta_j^A \otimes \zeta_k^B \otimes \dots \quad (4)$$

Moreover, the uniform probability distribution on the projective Hilbert space gives equal probabilities to all possible states $\Psi_{j,k}(t_i)$ at any time t_i , but the Born rule gives a totally different probability.

Refutation 1 (of Belief 1). This part of the analysis depends on the approach to resolve the superposition into definite outcomes. We consider first unitary approaches based on decoherence (like the *consistent histories* approach [17] and the *many-worlds interpretation* (MWI) [12]). From Observation 1, initial states $\Psi_{j,k}(t_i)$ with $\hat{A} = \hat{B}$ and $j \neq k$ are forbidden, because they evolve into forbidden states at t_2 . Such states would contradict Metaprinciple NMU. Moreover, all initial states that are not orthogonal on all forbidden states of the form $\Psi_{j,k}(t_i)$ are also forbidden, because the nonvanishing component $\Psi_{j,k}(t_i)$ leads to a forbidden branch with nonzero amplitude. Therefore, the states that are not forbidden as initial states have to be orthogonal on all initial states that would lead to forbidden states at any time. This is a Hilbert space \mathcal{H}_0 , and it is strictly included in the Hilbert space of all possible initial states. Therefore, $\mu_{\mathcal{H}}(\mathcal{P}(\mathcal{H}_0)) = 0$, where $\mu_{\mathcal{H}}$ is the invariant measure of the total projective Hilbert space $\mathcal{P}(\mathcal{H})$.

For the *pilot-wave* theory [6] and variations, the wavefunction also never collapses, but it is completed with “hidden variables”, *e.g.* point-particles with definite positions, that resolve the superposition. The initial conditions of the wavefunction are constrained exactly as in the MWI case. In addition, it is believed that to ensure the Born rule, the probability distribution of the “hidden variables” has to satisfy it at t_i (the “quantum equilibrium hypothesis”), this ensuring it at any other time [16]. This means even more fine-tuning compared to MWI.

In standard quantum mechanics (SQM), the superposition is resolved by invoking the Projection Postulate. This happens when the observation causes a macroscopic effect, usually by changing the pointer of the measuring device. Let $(\hat{P}_\alpha)_\alpha$ be a set of mutually orthogonal projectors that correspond to the macro states, so that $\sum_\alpha \hat{P}_\alpha$ is the identity operator $\hat{I}_{\mathcal{H}}$ on \mathcal{H} . In our case, these projectors are determined by the eigenstates of the pointer observables, so they are $\hat{I}_{\mathcal{H}_S} \otimes |\zeta_j^A\rangle\langle\zeta_j^A| \otimes |\zeta_k^B\rangle\langle\zeta_k^B| \otimes \hat{I}_{\mathcal{H}_E}$.

The Projection Postulate makes it possible that, for any state at t_2 , there are many possible initial states

that can lead to it by unitary evolution alternated with projections. Let us verify that any forbidden state at t_2 can be obtained like this. Suppose that the state vector is projected at a time t from Ψ_1 to Ψ_2 . Since $|\langle\Psi_2|\Psi_1\rangle|^2 = |\langle\Psi_1|\Psi_2\rangle|^2$, the probability that Ψ_1 projects to Ψ_2 in forward time evolution equals the probability that Ψ_2 projects to Ψ_1 in backward time evolution. This implies that, if we propagate a forbidden state at t_2 back in time to t_i “unprojecting” whenever is needed, we should find a set of initial states at t_i that can evolve forward in time (with projections) into the forbidden state at t_2 . All these initial states should therefore be forbidden. Since any initial state that is not orthogonal to all of them has components that can evolve into the forbidden state, these should be forbidden as well. This, again, constrains the possible initial states to a zero-measure subspace \mathcal{H}_0 orthogonal to all initial states that could evolve into forbidden states at any future time.

In *collapse theories* [14, 15], spontaneous localization is not defined in terms of projectors, but by multiplying the wavefunction with a Gaussian function centered at a random point in the configuration space. This happens at random times. Gaussian functions do not form an orthonormal basis, but they partition the identity, so the possible histories with collapses at the same moments of time also add up to the identity, and we can apply similar reasoning as in the SQM case, obtaining the same conclusion. Moreover, when the wavefunction is multiplied by a Gaussian, the result has tails, and decoherence is required to prevent those tails from interfering, because otherwise the collapse makes the state jump to a too different state, so similar constraints as in MWI are required.

Therefore, in all cases, the Born rule constrains the initial conditions to a zero-measure subspace \mathcal{H}_0 of \mathcal{H} , and Belief 1 (Universality) is contradicted.

Remark 1. In all cases, the allowed initial states are constrained to a zero-measure Hilbert subspace \mathcal{H}_0 of \mathcal{H} . Moreover, to protect the Born rule at arbitrary times in the future, the constraints of the initial conditions have to be valid at all times.

Remark 2. For the allowed future states to satisfy the Born rule, in addition to restricting the initial state to \mathcal{H}_0 , its probability distribution on \mathcal{H}_0 should be fine-tuned as well. For example, in MWI, the probabilities of initial states $\Psi_{j,k_1}(t_i)$ and $\Psi_{j,k_2}(t_i)$ should be in the right ratio given by the Born rule for $\psi_{k_1}^B \otimes \zeta_j^A \otimes \zeta_{k_1}^B \otimes \dots$ and $\psi_{k_2}^B \otimes \zeta_j^A \otimes \zeta_{k_2}^B \otimes \dots$ at t_2 . In SQM or collapse interpretations, there will be many initial states that lead at t_2 to the states $\zeta_j^A \otimes \zeta_{k_1}^B \otimes \dots$ and $\zeta_j^A \otimes \zeta_{k_2}^B \otimes \dots$, so the probabilities of these initial states should add up to the proper probabilities given by the Born rule.

Refutation 2 (of Belief 2). Consider now a factorization $\mathcal{H} = \mathcal{H}_1 \otimes \mathcal{H}_2$, obtained by dividing the total system into a subsystem S_1 and the rest of the world, S_2 . The tensor product basis cannot have all its elements in \mathcal{H}_0 , because then \mathcal{H} would be included in \mathcal{H}_0 . Therefore, there are tensor product states that are not allowed, contrary to

Belief 2. Interestingly, even if S_1 consists of a single particle, the Subsystem Independence is violated. This also contradicts Statistical Independence from Definition 1.

Refutation 3 (of Belief 3). Forbidden initial states are determined indirectly as precisely those states that can lead to states forbidden by future measurements. Therefore, the allowed states depend on input from the future.

Refutation 4 (of Belief 4). Different Hamiltonian operators \hat{H} can evolve different initial states to the forbidden states, cf. eq. (4). Therefore, the zero-measure subspace \mathcal{H}_0 of \mathcal{H} depends on the evolution law.

Refutation 5 (of Belief 5). From Observation 1, there are states that contain unreliable records. To avoid them, we have seen that fine-tuning is needed.

This concludes the proof of Theorem 1. \square

Corollary 1. *The state of any subsystem S is not completely independent of the state of the rest of the universe, in the sense that there are forbidden states of the form $\psi \otimes \varepsilon$, where ψ is the state of S and ε is the state of the rest of the universe. Therefore, the tensor product Hilbert space contains too many states.*

Proof. See Refutation 2 (of Belief 2). \square

III. DISCUSSION

Theorem 1 and Corollary 1 are in tension not only with common sense beliefs, but also with some of our assumptions about quantum mechanics. Therefore, even though I proved them mathematically, I will provide additional explanations and address possible objections.

Question 1. Why, in all known examples, standard quantum mechanics works without fine-tuning?

Answer 1. Usual scenarios of quantum experiments make assumptions that implicitly fine-tune the system:

(a) measuring devices already exist, despite their construction being complicated, necessitating precision technology, and requiring the knowledge of the Hamiltonian, in order to function as desired,

(b) before measurements, the measuring devices are in the “ready” states, like ζ_0^A and ζ_0^B in eq. (2) and eq. (3),

(c) the observed systems and measuring devices are initially in separable states, like $\psi \otimes \zeta_0^A$ in eq. (2).

All examples, in all interpretations of quantum mechanics, assume (a), (b), and (c), but this requires fine-tuning. Even MWI requires that branching is time-symmetric, which constrains the initial conditions to ensure (a). Assumption (a) requires the initial conditions to depend on the Hamiltonian. Linear combinations satisfying (b) and (c) are valid, but they form a strict subspace of the full tensor product Hilbert space \mathcal{H} .

If, in our example, $\hat{A} = \hat{B}$ and $j \neq k$, the state $\Psi_2(t_2) = \psi_k^B \otimes \zeta_j^A \otimes \zeta_k^B \otimes \dots$ cannot be reached from the state $\Psi_1(t_1) = \psi_j^A \otimes \zeta_j^A \otimes \zeta_0^B \otimes \dots$. This means that the pointer

state ζ_j^A contained in $\Psi_2(t_2)$ is an invalid record, since in the histories leading to $\Psi_2(t_2)$ there is no measurement of \hat{A} in which the observed system was found at t_1 in the state ψ_j^A and the pointer in the corresponding state ζ_j^A . Therefore, $\Psi_2(t_2)$ is forbidden simply because the state at t_1 was assumed to be $\Psi_1(t_1)$.

Even if we are not aware of this, we usually take for granted records at different times. But all that is available to us are the present time records of past events. From these records or memories, we infer laws and make predictions about future times, and experiments confirm them. And this is possible because the invalid records are already forbidden by the constraints of the initial conditions implicit in the assumptions (a), (b), and (c). \square

Question 2. We know since Boltzmann that the entropy increases because the universe was long time ago in a low-entropy state, which means the initial conditions were severely constrained. And there are reasons to believe that this is sufficient to explain the reliability of the records. Then, what does Theorem 1 bring new?

Answer 2. Theorem 1 shows that this is not enough, and the initial conditions also had to be conspiratorial.

Let us return to our example, and the discussion from Answer 1. Consider an observer at t_2 who knows the result of the first measurement but does not know yet the result of the second measurement. That observer will apply the Born rule and conclude that the state cannot be $\Psi_2(t_2)$. But how can the observer exclude the possibility that the state is $\Psi_2(t_2)$? Even if it contains an invalid pointer state and an observer with an invalid memory, $\Psi_2(t_2)$ is a valid state. Excluding such a “Boltzmann’s brain”-like situation is equivalent to excluding the initial conditions that could lead to it. To avoid invalid memories or pointer states, the same conspiratorial initial conditions from Theorem 1 are required. So if the observer can be sure at t_2 that the state at t_1 was $\Psi_1(t_1)$, it is because the initial fine-tuning took care of this. And this fine-tuning is not trivial, as one expects it to be in order to ensure the Second Law of Thermodynamics, because of Theorem 1. \square

Example 1 (EPR). The Einstein-Podolsky-Rosen (EPR) experiment [11] is a particular case of the example from the proof. At t_1 , the preparation results in a singlet state of two entangled spin 1/2 particles, with total spin 0. The preparation was done by taking as \hat{A} an observable that has the singlet state among its eigenstates. At t_2 , Alice measures the spin of the first particle along a direction \mathbf{a} , and Bob, in a different place, measures the spin of the second particle along a direction \mathbf{b} . Let $\hat{S}_{\mathbf{a}}$ and $\hat{S}_{\mathbf{b}}$ be the two spin observables. Since the two measurements are performed on different particles, they commute, $[\hat{S}_{\mathbf{a}} \otimes \hat{1}_2, \hat{1}_2 \otimes \hat{S}_{\mathbf{b}}] = 0$, and can be seen as a single measurement of the observable $\hat{B} = \hat{S}_{\mathbf{a}} \otimes \hat{S}_{\mathbf{b}}$ performed on the pair of particles. If $\mathbf{a} = \mathbf{b}$, the forbidden outcomes are those resulting in parallel spins. But the states containing pointer states that correspond to these results

are valid states. So the initial conditions that can lead to these states have to be forbidden, which means that SI from Definition 1 has to be violated. \square

Question 3. Does Corollary 1 refute Bell’s Statistical Independence assumption he used in his proof that any interpretation of quantum mechanics in which measurements have definite outcomes have to be nonlocal [3, 5]?

Answer 3. If, in Definition 1, $\Pr\{B\} > 0$, SI is equivalent to $\Pr\{A|B\} = \Pr\{A\}$, the form that appears in Bell’s theorem [5]. But Bell’s SI refers to the independence of the observed system from the measurement settings, and it is not refuted by Corollary 1. Corollary 1 allows the observed system to be in any state $\psi \in \mathcal{H}_1$, provided that the rest of the world ε is restricted to a strict subset of its Hilbert space \mathcal{H}_2 . In Bell’s SI, the only considered states $\varepsilon \in \mathcal{H}_2$ are those containing Alice and Bob’s measuring devices. This is why Corollary 1 does not contradict Bell’s SI, while still contradicting the stronger one from Definition 1. The EPR experiment fits perfectly in the situation from Answer 1, as seen in Example 1. \square

Question 4. Does Theorem 1 imply superdeterminism?

Answer 4. Theorem 1 does not refute nonlocal or multiple-outcome interpretations. However, if SI is a reason to reject superdeterministic approaches, Corollary 1 shows that all interpretations are guilty of the same. There is though a difference of degree between violating the SI from Definition 1 and violating Bell’s SI. \square

Question 5. Isn’t conspiratorial fine-tuning of initial conditions unscientific [2, 8, 19, 22] (although not everybody think so [18])? By fine-tuning you can make the theory predict anything. According to Maudlin: “*If we fail to make this sort of statistical independence assumption, empirical science can no longer be done at all.*”

Bell wrote ([4], p. 244): “*In such ‘superdeterministic’ theories the apparent free will of experimenters, and any other apparent randomness, would be illusory. Perhaps such a theory could be both locally causal and in agreement with quantum mechanical predictions. However I do not expect to see a serious theory of this kind.*”

Answer 5. Now we have a theorem showing that quantum mechanics itself requires conspiratorial fine-tuning, violating SI (Definition 1) but not Bell’s SI (Answer 3).

One cannot do science without the possibility to trust the records of the past experiments and our own memory. And this, as we have seen, requires conspiratorial fine-tuning. Does this mean that we can no longer do science?

There are proposals that try to save locality by sacrificing Bell’s SI [1, 9, 10, 20, 26]. Other proposals even try to save locality but also to maintain unitary evolution with a single world, *i.e.* without branching, and without collapse or “hidden variables” [21, 24, 25]. These approaches require very special initial conditions [23]. But, as Theorem 1 shows, so does quantum mechanics in all interpretations, and it also violates SI from Definition 1.

However, it is unfair to say that you can predict everything by fine-tuning, because unitary evolution with a single world approaches could so far only reproduce very simple quantum experiments. In addition, as explained in [21] and [24], these approaches make very strict predictions, for example, that the conservation laws hold even if their violation is allowed by SQM [7, 24] and other interpretations, including in the branches in MWI. \square

Question 6. In an infinitely large universe or in a multiverse, with an eternal history, this apparent fine-tuning happens with necessity somewhere or sometime. Since intelligent living beings like us need reliable memories to exist and survive, they can only exist in such a region. Is this not enough to explain away the fine-tuning?

Answer 6. If we would be in such a region where it just happened that the Metaprinciple NMU was respected up to a time t_1 , it would be much more likely that after t_1 NMU will be violated than not. Subsystems able to record events, like measuring devices in the “ready” state, are a limited resource. But then, we should observe already that the Born rule wears off and the world becomes flooded by unreliable records, becoming more and more inconsistent, like a dream. Also, by an indexical argument, it would be much likely that we find ourselves in a region where NMU is violated. \square

Question 7. Do you have another explanation?

Answer 7. Maybe there is an unknown law that restricts the possible states to \mathcal{H}_0 . Since subsystems are not independent (Corollary 1), the tensor product Hilbert space is too large, and should be replaced by its subspace \mathcal{H}_0 . Since the restrictions do not depend on time (Remark 1), \mathcal{H}_0 should be an invariant subspace under unitary evolution. This justifies the hypothesis that there is an yet unknown law that specifies what kind of states are allowed. This may be a superselection rule, similar to the superselection rules that forbid superpositions of systems with different electric charges or different spins [27]. If such a law or superselection rule exists for \mathcal{H}_0 , it could explain the conspiratorial behavior without fine-tuning. But such a law would have to encode the dependence of \mathcal{H}_0 on the Hamiltonian and the macro projectors $(\hat{P}_\alpha)_\alpha$.

REFERENCES

- [1] Y. Aharonov and L. Vaidman. The two-state vector formalism: an updated review. In *Time in Quantum Mechanics*, pages 399–447. Springer, 2007.
- [2] M. Araújo. Understanding Bell’s theorem part 1: the simple version. *More Quantum (blog)*, 2016.
- [3] J.S. Bell. On the Einstein-Podolsky-Rosen paradox. *Physics*, 1(3):195–200, 1964.
- [4] J.S. Bell. La nouvelle cuisine. In *Between Science and Technology*, pages 97–115. Elsevier Amsterdam, 1990.
- [5] J.S. Bell. *Speakable and unspeakable in quantum mechanics: Collected papers on quantum philosophy*. Cambridge University Press, 2004.
- [6] D. Bohm. A suggested interpretation of quantum mechanics in terms of “hidden” variables, I & II. *Phys. Rev.*, 85(2):166–193, 1952.
- [7] ME Burgos. Contradiction between conservation laws and orthodox quantum mechanics. *Journal of Modern Physics*, 1(2):137, 2010.
- [8] E.K. Chen. Bell’s theorem, quantum probabilities, and superdeterminism. In *The Routledge Companion to Philosophy of Physics*, pages 184–199. Routledge, 2021.
- [9] J.G. Cramer. The transactional interpretation of quantum mechanics. *Rev. Mod. Phys.*, 58(3):647, 1986.
- [10] O.C. de Beaugregard. Time symmetry and the Einstein paradox. *Il Nuovo Cimento B (1971-1996)*, 42(1):41–64, 1977.
- [11] A. Einstein, B. Podolsky, and N. Rosen. Can quantum-mechanical description of physical reality be considered complete? *Phys. Rev.*, 47(10):777, 1935.
- [12] H. Everett. “Relative state” formulation of quantum mechanics. *Rev. Mod. Phys.*, 29(3):454–462, Jul 1957.
- [13] R.G. Gallager. *Stochastic processes: theory for applications*. Cambridge University Press, Cambridge, UK, 2013.
- [14] G. Ghirardi. Collapse theories. In Edward N. Zalta, editor, *The Stanford Encyclopedia of Philosophy*. Metaphysics Research Lab, Stanford University, fall 2018 edition, 2018.
- [15] G.C. Ghirardi, A. Rimini, and T. Weber. Unified dynamics of microscopic and macroscopic systems. *Phys. Rev. D*, 34(2):470–491, 1986.
- [16] S. Goldstein. Bohmian mechanics. In E.N. Zalta, editor, *The Stanford Encyclopedia of Philosophy*. Spring 2013 edition, 2013. <http://plato.stanford.edu/archives/spr2013/entries/qm-bohm/>, last accessed September 27, 2022.
- [17] R.B. Griffiths. Consistent histories and the interpretation of quantum mechanics. *J. Statist. Phys.*, 36(1):219–272, 1984.
- [18] S. Hossenfelder and T. Palmer. Rethinking superdeterminism. *Frontiers in Physics*, 8:139, 2020.
- [19] T. Maudlin. Bell’s other assumption(s). *Conference “t Hooft 2019 – From Weak Force to Black Hole Thermodynamics and Beyond”*, 2019.
- [20] H. Price. Toy models for retrocausality. *Stud. Hist. Philos. Mod. Phys.*, 39(4):752–761, 2008.
- [21] L.S. Schulman. *Time’s arrows and quantum measurement*. Cambridge University Press, 1997.
- [22] A. Shimony, M.A. Horne, and J.F. Clauser. Comment on “The theory of local beables”. *Epistemological Letters*, 13(1), 1976.
- [23] O.C. Stoica. Quantum measurement and initial conditions. *Int. J. Theor. Phys.*, pages 1–15, 2015. [arXiv:quant-ph/1212.2601](https://arxiv.org/abs/1212.2601).
- [24] O.C. Stoica. The post-determined block universe. *Quantum Stud. Math. Found.*, 8(1), 2021. [arXiv:1903.07078](https://arxiv.org/abs/1903.07078).
- [25] G. ’t Hooft. *The cellular automaton interpretation of quantum mechanics*, volume 185. Springer, 2016.
- [26] K.B. Wharton and N. Argaman. Bell’s theorem and spacetime-based reformulations of quantum mechanics. *Preprint arXiv:1906.04313*, 2019.
- [27] G.C. Wick, A.S. Wightman, and E.P. Wigner. The intrinsic parity of elementary particles. *Phys. Rev.*, 88(1):101, 1952.