Idealization, Representation, and Explanation in the Sciences

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1. Introduction

A central goal of the scientific endeavor is to explain phenomena. Scientists often attempt to explain a phenomenon by way of representing it in some manner—such as with mathematical equations, models, or theory—which allows for an explanation of the phenomenon under investigation. However, in developing scientific representations, scientists typically deploy simplifications and idealizations. As a result, scientific representations provide only partial, and often distorted, accounts of the phenomenon in question. Philosophers of science have analyzed the nature and function of how scientists construct representations, deploy idealizations, and provide explanations. As such, our aim in this special issue is to bring these three pillars of research into closer contact with the contributions to it focusing on three main themes.

The first set of papers, Alan Baker (2021) and Marc Lange (2021), address mathematical explanations in science. Baker (2021), a proponent of mathematical Platonism, examines its capacity to evade the critique that the so-called Enhanced Indispensability Argument is circular. Lange (2021) examines distinctively mathematical explanations, arguing that neither Platonism nor representationalism are successful paths, and instead argues in favor of Aristotelian realism. A second theme emerging from the papers in this special issue is the impact that various conceptualizations of idealization have on our abilities to offer scientific explanations, to produce an analysis of what explanations are or should be, and to understand scientific representation. Peter Tan (2021) suggests amending inferentialist accounts of scientific representation to account for inconsistent idealizations. Michael Strevens (2021) advocates a view wherein the introduction of idealizations into a model is legitimate so long as it pertains to non-difference-making factors, arguing for a logical reading of the notion of difference-making. Natalia Carrillo and Tarja Knuuttila (2021) offer an alternative account to the idealization-as-distortion view, emphasizing instead the holistic nature of idealization.

Finally, contributions by Carrillo and Knuuttila (2021), Terzian (2021), Valente (2021), and Rodriguez (2021) illustrate how issues regarding idealization, representation, and explanation are applied to specific contexts and across various sciences. Carrillo and Knuuttila (2021) examine conceptions of idealization in the context of models of the nerve impulse. Giulia

Terzian (2021) extends the discussion of idealizations to the context of generative linguistics. Giovanni Valente (2021) examines how idealizations and evaluations of accurate representation impact capacities to explain phenomena in the context of statistical thermodynamics. Quentin Rodriguez (2021) examines the role of idealizations and analogies in various strategies to explain critical phenomena.

In what follows, we offer a brief overview of important philosophical issues connected to representation, idealization, and explanation in science. We then provide short summaries of the eight papers in this special issue.

2. Representation, Idealization, & Explanation

2.1 Representation

Science both makes use of and constructs objects that serve various representational purposes. Arguably, philosophers have predominantly focused on the question of how scientific models represent aspects of the world (see Frigg and Nguyen 2021 for an extended overview). In order to learn about the world, a scientist must often first find a way to describe or portray the object or system of interest. How a scientist chooses to represent a target system of interest can have important consequences for the scientific claims they are able to make, or kinds of explanations about the target system they can offer.

A host of philosophical questions have been raised regarding the nature and functions of scientific representation, ranging from questions of the extent to which there is a special problem of *scientific* representation (Callender and Cohen 2006; Boesch 2017) to questions concerning the various kinds of representational vehicles such as mathematical, visual, or other (Knuuttila 2005; Vorms 2011).

That said, perhaps the most attention has been given to two general problems. The first concerns the issue of what turns something into a scientific representation of something else (Frigg and Nguyen 2017). The second relates to the well-established observation that the representational relation between a model and its target is not one of perfect mirroring; rather, scientific models provide only imperfect accounts of phenomena (Frigg and Nguyen 2017).

A number of accounts have been proposed to answer how a model represents its target. Among the most prominent approaches are the structuralist conception according to which a model is a representation of its target in virtue of some sort of morphism holding between the two (see, e.g., Bartels 2006; van Fraassen 2008; Bueno and French 2011; French 2014). In addition, the similarity conception holds that a scientific model represents a target if the model and target are similar in relevant respects and degrees, given a certain purpose (Giere 1988, 2010; Godfrey-Smith 2006; Mäki 2009; Weisberg 2013). The inferentialist conception maintains that the representational relation between a model and its target is established on the basis of the former having an inferential capacity with respect to the latter (Suárez 2004; Khalifa et a.

2022). Several other accounts exist, many of which include at least some features of those mentioned above (see, e.g., Hughes 1997; Knuuttila 2011; Frigg and Nguyen 2020).

All these accounts, either explicitly or implicitly, maintain that scientific representations are never a mirror image of what they represent. While some conceptions, such as the structuralist and similarity ones, seem to be built on the premise that for a model to represent its target, a model must always at least partially capture some aspect of its target, the inferentialist and several other conceptions disentangle the question of representation from the accuracy of representation altogether, maintaining that an account of representation is fleshed out purely by the inferential capacity or by some other feature, respectively. Indeed, several authors (e.g., Suárez 2010; Shech 2015; Frigg and Nguyen 2017) have argued against rival accounts such as the similarity and structuralist conceptions on the grounds that these accounts fail to draw a distinction between the concepts of representation and accurate representation.

Most of the accounts discussed above are undergoing various modifications (and new accounts are constantly being developed), suggesting that the investigation into the nature and functions of scientific representation is an active area of philosophical research. Finally, such developments have implications for other important debates such as those about idealization and explanation.

2.2 Idealization

Many systems and natural phenomena are far too complex to study or represent in their entirety. There are numerous strategies that scientists utilize when attempting to represent a target system, one of which is idealization. Unfortunately, there is no agreed upon characterization of idealization. For example, Robert Batterman (2007) states that idealization "is a means for focusing on exactly those features that are constitutive of the regularity—those features that we see repeated at different times and in different places" (270). Margaret Morrison (2015) says that idealization "involves a process of approximation whereby the system can become less idealized by adding correction factors ..." (20). Angela Potochnick (2017) claims that idealizations "are assumptions made without regard for whether they are true, generally with the full knowledge that they are false" (2).

Following Elay Shech (2018; Forthcoming), it is useful to distinguish between *idealization broadly construed*, which refers to anything that could reasonably and intuitively be called an idealization (perhaps) because it fails to meet some veridicality or accuracy condition, and *idealization narrowly construed*, which refers to the various specific characterizations (such as the ones above) offered by philosophers with a certain aim in mind. An idealization narrowly construed may then be differentiated from similar notions such as abstraction, approximation, fiction, metaphor, thought experiment, and so on. Many, philosophers will appeal to an example of an idealization broadly construed in order to argue for a preferred notion of idealization narrowly construed (e.g., as in Carrillo & Knuuttila 2021), or else favor a narrow construal such as the distinction between idealization and abstraction in order to solve a particular problem (e.g., as in Rodrigues 2021).

An example of well-received narrow construal is associated with the suggestion from John Norton that we distinguish between an "approximation" and an "idealization" as follows:

> An approximation is an inexact description of a target system. It is propositional.
> An idealization is a real or fictitious system, distinct from the target system, some of whose properties provide an inexact description of some aspects of the target system. (Norton 2012, 209)

For instance, consider a body of unit mass falling in a weakly resisting medium. For gravitational constant g and friction coefficient k, its speed v at time t is given by: $\frac{dv}{dt} = g - kv$. Falling from rest, its speed as a function of time is given by the Taylor expansion series:

$$v(t) = \frac{g}{k}(1 - e^{-kt}) = gt - \frac{gkt^2}{2} + \frac{gk^2t^3}{6} - \cdots$$

The first term in the series expansion, $v(t) = gt$, is a good *approximation* (i.e., an inexact description) of the fall for small k and at early times. $v(t) = gt$ is also the exact velocity of a fictitious idealized system with the same mass falling under gravity in a vacuum. We can promote our approximation to an *idealization* by introducing novel reference to said system.

Additionally, there are also no accepted taxonomies of idealizations. Instead, various categorization schemes have been suggested (e.g., McMullin 1985; Nowak 2000; Shaffer 2012). For example, one recent well-received taxonomy is due to Weisberg (2013, ch. 6), who holds that the activities and justification associated with idealization give rise to three kinds of idealizations: Galilean idealizations, minimalist idealizations, and multiple-model idealizations. Briefly, Galilean idealizations simplify the treatment of a target system and render them computationally tractable. Minimalist idealizations expose the key factors that make a difference to the occurrence and character of a phenomenon of interest. Multiple-model idealizations are related but possibly incompatible models with different epistemic or pragmatic goals such as affording representations that are predictively precise, accurate or realistic, general in scope, simple, and so on (see Shech Forthcoming for further discussion and references).

2.3 Explanation

One of the fundamental goals of science is to provide explanations for natural or social phenomena. Broadly conceived, a scientific explanation attempts to provide an account of phenomena or objects of interest to scientists through a variety of means, such as natural laws or mechanisms, often aimed at describing the relationships between causes and effects or providing explanans for an explanandum. Various philosophical accounts of what constitutes a scientific explanation include the Deductive-Nomological (DN) model (Hempel 1965), statistical relevance model (Salmon 1971), causal mechanical model (Salmon 1984, 1997), and the unificationist account (Kitcher 1989), as well as more pragmatic approaches such as those offered by Peter Achinstein (1983), Woodward's counterfactual manipulationist account (2005), and Bas van Fraassen's constructive empiricism (1980) (see Woodward & Ross 2021 for a survey overview).

With respect to accounts of explanation, an emerging line of argument has related to drawing distinctions between causal and non-causal forms of explanation. While some accounts of scientific explanation place emphasis on causation, some philosophers argue there are also non-causal explanations that derive their explanatory power by other means than identifying causes or mechanisms (Lange 2016; Reutlinger 2017). Others still (such as Anderson 2018) consider ways in which causal and non-causal explanations might actually be complementary. Relatedly, some philosophers emphasize the importance of a related kind of explanation that occurs in science, namely mathematical explanations, which often reference mathematical facts as part of the explanans. For example, as discussed by Lange (2013), if trying to explain why someone could not divide 23 strawberries evenly among three people (without cutting a strawberry), the explanation involves an appeal to the mathematical fact that 15 is not evenly divisible by 2.

Connected to this is a more general theme regarding the role that mathematics plays in offering scientific explanations, either as part of explanans or as part of mathematical representations (see Mancosu 2018 for a survey overview). For instance, Morrison (2000) argues that mathematical structures "often supply little or no theoretical explanation of the physical dynamics of the unified theory" (4). Batterman (2002) on the other hand discusses what he calls asymptotic explanations, scientific explanations which heavily rely on limiting procedures and rigorous mathematical results. Morrison (2015) ultimately argues that the explanatory role of mathematics may be related to challenges connected to idealizations and representations (given representation and idealizations are often expressed via mathematics in science). Christopher Pincock (2012) also discusses how mathematics contributes to the success of our best scientific representations. Finally, cases of mathematical explanations might entail commitments to mathematical objects. Work in this special issue by Baker (2021) and Lange (2021) attempts to bridge these spaces.

3. Contributions

The enhanced indispensability argument (EIA) for mathematical Platonism states that we ought to believe in the existence of mathematical objects since this follows from two premises: (1) we ought to believe in the existence of entities that play an indispensable explanatory role in our best scientific theory: call this (IBE) since (1) is grounded in inference to the best explanation; and (2) mathematical objects play an indispensable explanatory role in science: call this (MES) for mathematical explanations in science. EIA essentially leverages scientific realist's endorsement of IBE in order to extend ontological commitment to abstracta via the identification of bona fide MES. However, various scholars have objected that EIA is circular since the explanandum in ostensible cases of MES is itself identified using mathematical concepts. As Baker (2021) notes in his contribution to the special collection, the circularity charge gets at the "heart of the nature of" MES since "it raises the question of whether there can be explanations that make essential use of mathematics and yet have explananda that are genuinely non-mathematical" (157). Ultimately, Baker defends Platonism from the circularity charge and in doing so identifies the type of ostensible MES that can non-circularly power EIAs. He claims that in considering instances with a range of properties that correspond to some concrete-physical pattern (to be explained), where each instance can be described non-mathematically, then non-circular MES are available if a mathematical theory can help explain why some mathematical property holds and said mathematical property helps explains why the patterns (manifested by said instances) holds.

An alternative to the type of mathematical Platonism endorsed by Baker (2021) is a kind of representationalism or indexing account endorsed by various nominalist, where we use mathematical terns to refer to physical properties. Interestingly, in his contribution to the special issue, Lange (2021) notes that both representationalists and Platonists tend to account for the applicability of mathematics to science in the same manner: "Where representationalism invokes morphisms between mathematical formalisms (or fictions) and aspects of the physical world, platonism invokes morphisms between mathematical abstracta and aspects of the physical world" (45). Assuming that mathematical explanations are akin to explanations by constraint, Lange then argues that such similarity renders both representationalism and Platonism incapable of accounting for how mathematical facts play a role in mathematical explanations of physical phenomena, what he calls "distinctively mathematical explanations." The basic idea is that mathematical facts cannot play the role of constraining and thereby explaining physical facts if they solely *refer* to physical or abstract properties, that is, "when there is no causal, nomological, constitutive, or similar connection between" the mathematical and physical facts. He suggests instead adopting "Aristotelian realism"—where mathematics refer to real mathematical properties possessed by physical systems—as one possible view by which mathematical facts can function as explainers in distinctively mathematical explanations.

Tan (2021) addresses the issue of what it takes for some apparatus to count as a scientific representation of a given system. According to the inferentialist account, the representational relation between a model and its target is constituted by the model's inferential capacity; or, put otherwise, it is the fact that cognitive agents can make use of a model by drawing surrogative inferences is what constitutes the model's representational status. However, as Tan argues, a serious problem (which he refers to as the problem of inconsistent idealizations) arises for the inferentialist account. The challenge amounts to the following: because many scientific models are internally inconsistent, following the rules of classical logic one may infer anything. Since anything can be inferred from a contradiction, and since what is constitutive of representation is that it is inferential, a model that is inconsistent may represent just about everything. Therefore, inferentialism about scientific representation must be amended. Tan then considers several possible options that may appear attractive for the inferetialists before deeming each unsatisfactory. As an alternative, he proposes his own solution to the issue. First, Tan draws an analogy with computer games, claiming that internally inconsistent scientific representations are akin to games with exploitable glitches: the explosive inferences much like the glitches are "allowed," but they violate the intended function of the model and the game, respectively. Second, while the intended function of any model is to provide information about the world, exploiting some features of logic to generate explosive inferences is not a part of that function. Finally, any usage of the model that violates the intended function cannot be considered as constitutive of a representational relation. Thus, inferentialism need not succumb to the challenge from inconsistently idealized models, provided that the inferentialist position is amended in the suggested way.

Another puzzling question regarding the use of idealization concerns its role in making accurate predictions. In this context, key questions are what kinds of idealizations are safe to make? Do idealizations need to be made explicitly or are implicit idealizations safe as well? Do idealizations jeopardize the reliability of a model's predictive power? Focusing on causal models, Strevens (2021) suggests that an idealization introduced into a model is safe, so long as it pertains to nondifferencemaking elements of the model. Crucially, Strevens considers two interpretations of the notion of differencemaking: one notion counterfactually driven and the other logically driven. While the counterfactual reading is rejected for exhibiting problems, the logical interpretation is championed. In particular, the logical approach to differencemaking is epistemically relativized in the following way: what makes a difference is indexed to an epistemic situation of the scientist, that is, what the scientist knows about a causal process. Based on such knowledge, the scientist then checks for what can be deleted from the model without affecting the prediction such as the occurrence of a phenomenon. Deletion here may, for example, take the form of keeping quiet on the presence of some causal factor, or making a description less specific by (for instance) replacing an exact value with a range of values. Finally, the information that has been removed may now be replaced by a misrepresentation (i.e., an idealization). For all this to work, however, the deletion, and thus also the misrepresentation, may only concern the non-differencemakers, whereas the differencemaking factors must be represented faithfully to maintain the models' predictive potential. A consequence of such treatment of the safety of idealization with respect to the predictive power of a model is that predictive and explanatory criteria start resembling one another. Strevens concludes by arguing that the logical reading of the differencemaking notion grounds both prediction and explanation in the context of causal modeling.

In "Holistic Idealization: An Artifactual Standpoint," Carrillo and Knuuttila (2021) argue for a position that does not make misrepresentation central for the analysis of idealization. The authors discuss two general approaches to understanding idealization. The first approach concerns deficiency accounts of idealization, according to which idealizations should in principle be corrected for (e.g., via de-idealization). Carrillo & Knuuttila suggest that this approach fails to account for the fact that sometimes idealizations are found to be epistemically useful without the need for them to be corrected. The second approach, benefit accounts of idealization, maintains that in many cases idealizations are necessary to facilitate explanation and understanding of phenomena. They point out that these accounts also rely on the premise that models can be decomposed into individual modeling assumptions. As such, the authors point out that both approaches assume that idealization should be understood in terms of distortion.

In the process of arguing against the idealization-as-distortion view, Carrillo and Knuuttila offer an alternative: an *artifactual* account of idealization, which construes idealization in terms of implicit or explicit assumptions that simultaneously draw on many different resources in the process of model construction, often setting the stage for whole research programs. Such assumptions exhibit a holistic nature in that they represent the system of interest as being of a certain kind. Therefore, it is misguided to focus on analyzing idealization in terms of model-world comparisons and pondering which individual assumptions misrepresent and how. The authors illustrate these claims by analyzing two models of the nerve impulse. The Hodgkin-Huxley model represents the membrane as semipermeable, and by employing the idealization of constant capacitance it builds on previous models constrained by the choice to represent the cell as a resistor-capacitor circuit, thus constituting a whole research program. In contrast, the Heimburg-Jackson model represents the membrane as an elastic material undergoing phase transitions, rendering the phenomenon of the nerve impulse as a thermodynamically reversible phenomenon; thus it constitutes a different research program.

Looking at a case in generative linguistics, Terzian (2021) broadens current philosophical discussions on idealization. A central question in the field of generative linguistics concerns how children acquire their native language given the time and input constraints. A long-held answer is the Principles and Parameters framework (P&P), which takes humans to be equipped with two types of linguistic resources: principles that are fixed and universal, and parametrized principles that take on either an "on" or "off" value. While P&P allows for explaining several phenomena, among others the observed homogeneity of language acquisition within and across linguistic communities, it fails as an actual explanation because it is inconsistent with the evolutionary account (i.e., the account of how a cognitive organ equipped to support language acquisition evolved, given the short time language has been around). Abandoning the P&P framework, as many have done, is valid so long one treats P&P as a would-be accurate, factive representation of its target. Terzian argues that P&P can and should be maintained provided it is re-interpreted in a certain way. Rather than comprising an *ad hoc* maneuver, Terzian presents reasons for justifying this move. First, Terzian identifies four idealizations characteristic of P&P. Second, he suggests that we re-conceive of P&P as a retrospective model. That is to say, rather than abandoning P&P, one may ascribe a different epistemic role to P&P by treating it as an idealized model of the target phenomenon. Given the level of idealization, Terzian proposes to view P&P as an instance of a (retrospective) toy model, serving as a vehicle of how-possibly understanding and explanation of the phenomenon of language acquisition. When viewed this way, the objection from the evolutionary account no longer poses problems since the P&P framework–interpreted along the sketched lines–is no longer treated as a would-be description of how-actually children acquire language.

The Second Law of thermodynamics is sometimes characterized as the claim that entropy never decreases in a closed system. But laws of superseded theories are idealizations of sorts. The emergence of statistical mechanics had led us to expect that an analogous theory, statistical thermodynamics, more accurately represents and explains phenomena in the macroscopic realm such as equilibrium fluctuations and entropic behavior arising from time-reversal invariant dynamics. Deduction of the laws of statistical thermodynamics from statistical mechanics thus accomplishes traditional Nagelian-Schaffner reduction. Unfortunately, a concrete candidate for statistical thermodynamics, which can be further reduced to statistical mechanics, has not been identified. In his contribution, Valente (2021) considers the prospect and limitations of a particular candidate theory for statistical thermodynamics, which is grounded in Einstein's (1909, 1910) articles on statistical thermodynamics. Specifically, Einstein introduced a fluctuation law in order to provide a probabilistic account for thermodynamical equilibrium. Valente shows that, given qualification such as considering an isolated system with fixed energy, what he calls the 'Problem of Equilibrium Fluctuations' can be solved by re-defining statistical equilibrium so that a system can fluctuate in and out of equilibrium in agreement with Boltzmannian statistical mechanics. However, he argues that what he calls the 'Problem of Irreversibility,' which concerns explaining macroscopic irreversibility from a microscopic point of view, remains unsolved and thus impedes the Nagelian-Schaffner reduction of statistical thermodynamics to statistical mechanics. In particular, "even by weakening the Second Law and the Minus First Law of classical thermodynamics, one still runs into a conflict with the time-reversal invariance of the microscopic dynamics of statistical mechanics" (183).

There are systems that are fundamentally distinct from one another and yet manifest salient commonalities via the universality of critical phenomena (CP), such as when fluids like oxygen, nitrogen, and neon, or even a ferromagnet, have order parameters that obey the same power law with the identical critical exponents. What explains the fact that such radically diverse systems manifest CP (i.e., the same critical exponents)? Typically, the standard scientific explanation concerns renormalization group RG) methods, where it is shown that under RG transformations Hamiltonians representing diverse physical systems all give rise to the same critical exponents (and thus are in the same universality class) as long as said Hamiltonians share certain symmetry and dimensionality properties. In various works (e.g. Batterman 2002 and Batterman and Rice 2014), Batterman and co-authors have argued that RG explanations of CP involve an unrecognized and importantly novel explanatory structure. In contrast, some claim that RG explanations are paradigmatic examples of a "commonality strategy" that explains why diverse systems manifest the same behavior by identifying shared properties, viz., in our case Hamiltonian symmetry and dimensionality properties. In his paper, Rodriguez (2021) aims to clarify the similarities and lack thereof of RG explanations and typical commonality explanations such as those that arise in the ideal gas model and the harmonic oscillator. He argues that while the ideal gas model concerns constitutive-physical analogy and asymptotic reasoning, and the harmonic oscillator concerns abstraction and formal analogy, the case of CP concerns both constitutive-physical and formal analogy, where the latter is justified via RG methods. He holds that the "combination of these two explanatory strategies can account for the epistemic autonomy of CP universality from underlying microscopic representations" (235) and such epistemic autonomy partially supports Batterman's novelty thesis.

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References

Achinstein, P. (1983). *The Nature of Explanation*. Oxford: Oxford University Press.

Andersen, H. (2020). "Complements, not competitors: causal and mathematical explanations." *The British Journal for the Philosophy of Science* 69(2):485–508*.*

Baker, A. (2021). "Circularity, indispensability, and mathematical explanation in science." *Studies in History and Philosophy of Science* 88: 156–63.

Bartels, A. (2006). "Defending the structural concept of representation." *THEORIA. An International Journal for Theory, History and Foundations of Science* 21: 7–19.

Batterman, R. (2002). *The Devil in the Details: Asymptotic Reasoning in Explanation, Reduction, and Emergence*. New York: Oxford University Press.

Batterman, R. (2007). "On the specialness of special functions (the nonrandom effusions of the divine nathematician)." *British Journal for Philosophy of Science* 58: 263–86.

Batterman, R. and C. Rice. (2014). "Minimal model explanations." *Philosophy of Science* 81: 349–76.

Boesch, B. (2017). "There is a special problem of scientific representation." *Philosophy of Science* 84: 970–81.

Bueno, O. and S. French (2020). "How theories represent." *The British Journal for the Philosophy of Science* 62: 857–94.

Callender, C. and J. Cohen (2006). "There is no special problem about scientific representation." *THEORIA. An International Journal for Theory, History and Foundations of Science* 21: 67–85.

Carrillo, N. and T. Knuuttila. (2022) "Holistic idealization: An artifactual standpoint." *Studies in History and Philosophy of Science* 91: 49 –59.

Einstein, A. (1909). "On the present status of the radiation problem." *Physik Z* 10: 185–93.

———. (1910). “The theory of the opalescence of homogeneous fluids and liquid mixtures near critical state.” *Annalen der Physik* 33: 1275–98.

French, S. (2014). *The Structure of the World: Metaphysics and Representation*. Oxford: Oxford University Press.

Frigg, R., and J. Nguyen (2017). “Models and representation.” In L. Magnani and T. Bertolotti (eds.), *Springer Handbook of Model-Based Science* (pp. 49–102). Dordrecht: Springer.

———. (2020). *Modelling Nature: An Opinionated Introduction to the Scientific Representation*. Springer.

———. (2021). “Scientific Representation,” The Stanford Encyclopedia of Philosophy (Winter 2021 Edition), Edward N. Zalta (ed.), URL = <https://plato.stanford.edu/archives/win2021/entries/scientific-representation/>.

Giere, R.N. (1988). *Explaining Science: A Cognitive Approach*. Chicago, IL: University of Chicago Press.

———. (2010). “An agent-based conception of models and scientific representation.” *Synthese* 172: 269–81.

Godfrey-Smith, P. (2006). “The strategy of model-based science.” *Biology & Philosophy* 21: 725–40.

Hempel, C.G. (1965). *Aspects of Scientific Explanation*. Vol. 1. New York: Free Press.

Hughes, R.I G. (1997). “Models and representation.” *Philosophy of Science* 64: S325–36.

Khalifa, K., J. Millson, and M. Risjord (2022). Scientific Representation: An Inferentialist-Expressivist Manifesto. *Philosophical Topics*.

Kitcher, P. (1989). “Explanatory Unification and the Causal Structure of the World,” In *Scientific Explanation*, edited by P. Kitcher and W. Salmon. Minneapolis: University of Minnesota Press, 410-505.

Knuuttila, T. (2005). “Models, representation, and mediation.” *Philosophy of Science* 72: 1260–71.

———. (2011). “Modelling and representing: An artefactual approach to model-based representation.” *Studies in History and Philosophy of Science* 42: 262–71.

Lange, M. (2013). “What makes a scientific explanation distinctively mathematical?” *The British Journal for the Philosophy of Science* 64: 485–511.

———. (2016). *Because Without Cause: Non-Casual Explanations in Science and Mathematics*. Oxford: Oxford University Press.

———. (2021). “What could mathematics be for it to function in distinctively mathematical scientific explanations?” *Studies in History and Philosophy of Science* 87: 44–53.

Mäki, U. (2009). “MISSing the world: Models as isolations and credible surrogate systems.” *Erkenntnis* 70: 29–43.

Mancosu, P. (2018). “Explanation in Mathematics,” The Stanford Encyclopedia of Philosophy (Summer 2018 Edition), Edward N. Zalta (ed.), URL = <https://plato.stanford.edu/archives/sum2018/entries/mathematics-explanation/>.

McMullin, E. (1985). “Galilean idealization.” *Studies in the History and Philosophy of Science* 16: 247–73.

Morrison, M. (2000). *Unifying Scientific Theories: Physical Concepts and Mathematical Structures.* Cambridge: Cambridge University Press.

———. (2015). *Reconstructing Reality: Models, Mathematics, and Simulations*. Oxford: Oxford University Press.

Norton, J.D. (2012). “Approximation and idealization: Why the difference matters.” *Philosophy of Science* 79: 207–32.

Nowak, L. (2000). “The idealizational approach to science: A new survey,” In *Idealization X: The Richness of Idealization,* edited by I. Nowakowa and L. Nowak. Amsterdam: Rodopi, 109–85.

Pincock, C. (2011). *Mathematics and Scientific Representation*. Oxford: Oxford University Press.

Potochnick, A. (2017). *Idealization and the Aim of Science.* Chicago, IL: University of Chicago Press.

Rodriguez, Q. (2021). “Idealizations and analogies: Explaining critical phenomena.” *Studies in History and Philosophy of Science* 89: 235–47.

Reutlinger, A. (2017). “Explanation beyond causation? New directions in the philosophy of scientific explanation.” *Philosophy Compass* 12(2): e12395. https://doi.org/10.1111/phc3.12395

Salmon, W.C. (1971). *Statistical Explanation and Statistical Relevance*. Pittsburgh, PA: University of Pittsburgh Press.

———. (1984). “Scientific explanation: Three basic conceptions.” In *PSA: Proceedings of the Biennial Meeting of the Philosophy of Science Association,* edited by Peter D. Asquith and Philip Kitcher. 2: 293–305.

———. (1997). “Causality and explanation: A reply to two critiques.” *Philosophy of Science* 64: 461–77.

Shaffer, M.J. (2012*). Counterfactuals and Scientific Realism*. London: Palgrave MacMillan.

Shech, E. (2015). “Scientific Misrepresentation and Guides to Ontology: The Need for Representational Code and Contents”, *Synthese*, 192(11): 3463–3485.

Shech, E. (2018). “Infinite idealizations in physics.” *Philosophy Compass* 13(9):e12514. doi: 10.1111/phc3.12514

Shech, E. (Forthcoming). *Idealizations in Physics*. Cambridge: Cambridge University Press.

Strevens, M. (2021). “Permissible idealizations for the purpose of prediction.” *Studies in History and Philosophy of Science* 85: 92–100.

Suárez, M. (2004). “An inferential conception of scientific representation.” *Philosophy of Science* 71: 767–79.

———. (2010). Scientific Representation. Philosophy Compass, 5(1), 91–101. https://doi.org/10.1111/j.1747-9991.2009.00261.x

Tan, P. (2021). “Inconsistent idealizations and inferentialism about scientific representation.” *Studies in History and Philosophy of Science* 89: 11–18.

Terzian, G. (2021). “Chomsky in the playground: Idealization in generative linguistics.” *Studies in History and Philosophy of Science* 87: 1–12.

Valente, G. (2021). “Taking up statistical thermodynamics: Equilibrium fluctuations and irreversibility.” *Studies in History and Philosophy of Science* 85: 176-84.

van Fraassen, B.C. (1980). *The Scientific Image*. Oxford: Oxford University Press.

———. (2008). *Scientific Representation: Paradoxes of Perspective*. Oxford: Oxford University Press.

Vorms, M. (2011). “Representing with imaginary models: Formats matter.” *Studies in History and Philosophy of Science* 42: 287–95.

Weisberg, M. (2013). *Simulation and Similarity: Using Models to Understand the World*. Oxford: Oxford University Press.

Woodward, J. (2005). *Making Things Happen: A Theory of Causal Explanation*. Oxford: Oxford university press.

Woodward, J. and L. Ross (2021). "Scientific Explanation", The Stanford Encyclopedia of Philosophy (Summer 2021 Edition), Edward N. Zalta (ed.), URL = <https://plato.stanford.edu/archives/sum2021/entries/scientific-explanation/>.