COUNTING 3D-SPACES
Classicality and probability in standard and many-worlds quantum mechanics from quantum-gravitational background-freedom

Ovidiu Cristinel Stoica

Dept. of Theoretical Physics, NIPNE—HH, Bucharest, Romania.
Email: cristi.stoica@theory.nipne.ro, holotronix@gmail.com
(Dated: October 10, 2022)

I argue that background freedom in quantum gravity automatically leads to a dissociation of the quantum state into states having a classical 3d-space. That is, interference is not completely well-defined for states with different 3d-space geometries, even if their linear combination is.

The dissociation into 3d-space geometries still allows for interference at small scales, but precludes it at macro scales. It grants the possibility of classical-looking macroscopic objects, including measuring devices. Counting the 3d-space geometries automatically gives the Born rule.

But the wavefunction collapse turns out to be even more ad-hoc. Fortunately, the dissociation entails a kind of absolute decoherence, making the wavefunction collapse unnecessary. This naturally leads to a new version of the many-worlds interpretation, while solving some of its problems:

  1) the classical-3d-space states form an absolute preferred basis,

  2) at any time, the resulting micro-branches look like classical worlds, with objects in the 3d-space,

  3) macro-branches stop interfering, even though micro-branches can interfere,

  4) the 3d-space geometries converge at the Big-Bang, favoring macro-branching towards the future,

  5) the wavefunctional becomes real by absorbing the phases in the global U(1) gauge,

  6) ontologically, the wavefunctional consists of many gauged 3d-space states, each of them counting as a world by having local beables (the 3d-space geometry and the classical fields),

  7) the density of the classical-3d-space states automatically obeys the Born rule.

Keywords: Everett's many-worlds interpretation; Born rule; quantum gravity; background-independence; many-spacetimes interpretation.

I. INTRODUCTION

I argue that background free approaches to quantum gravity prevent most quantum state vectors from having physically meaningful superpositions. Interference effects require a way to relate the positions in space among different state vectors, but background freedom limits this possibility. Linear combinations exist mathematically, but interference effects are suppressed in most situations.

This leads to a new explanation of the emergence of classicality at the macro level, and to a natural derivation of the Born rule by counting states with definite classical 3d-space. The resulting approach to understand quantum mechanics works less naturally with the wavefunction collapse, but very well with the many-worlds interpretation, solving some of its main problems.

In Sec. §II I sketch the generic features of wavefunctional formulations of background-free quantum gravity. This leads to the notion of classical 3d-space states, having a definite classical 3d-space (or other structure assumed to be more fundamental than the 3d manifold).

In Sec. §III I explain how background freedom makes the state vector dissociate into classical 3d-space states, by limiting their ability to interfere.

In Sec. §IV I show how counting the 3d-space states into which the state vector dissociates gives the Born rule. Each 3d-space state either is absent from the wavefunctional, or it appears in it with equal amplitude but varying density (see Fig. 2). The density can be made
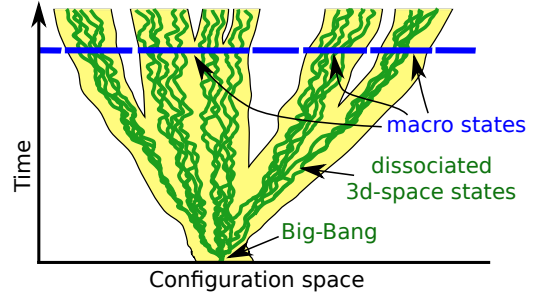

FIG. 1. **Wavefunctional dissociation due to quantum-gravitational background freedom.** The micro-states (in green) are 3d-space states. They are very similar at the Big-Bang, then background freedom makes them dissociate and form a branching structure like in the many-worlds interpretation. The dissociation is reversible at micro scales, allowing interference, but it becomes irreversible when it manifests at macro scales. The branching structure (in yellow) corresponds to the macro-states (in blue). Counting the 3d-space states for each macro-state or branch gives the Born rule.

real, by absorbing the phases into the U(1) gauge of the classical fields defining the 3d-space states.

In Sec. §V I argue that the 3d-space states approach works less well with the collapse postulate, but it works naturally with the many-worlds interpretation, resulting in a version of it named here the *many-spacetimes interpretation* of quantum mechanics.

In Sec. §VI I explain how the many-spacetimes inter-

pretation contributes to solving some of the main problems of the many-worlds interpretation (Fig. 1), such as the existence of a preferred basis, the emergence of quasi-classical macro worlds, the existence of familiar, classical-looking objects in the 3d-space, the time-asymmetry of the branching structure, probabilities by counting states, the appearance of complex numbers in quantum mechanics, and the ontology, including the local beables, which justify counting each 3d-space state as a world.

Sec. §VII concludes the article with a discussion.

II.   3D-SPACE STATES IN QUANTUM GRAVITY

A.   Classical 3d-space states

We do not have yet a final theory of quantum gravity, and even less so one that includes the other fields. But I will assume that such a theory is possible.

Many of the various currently known approaches to quantum gravity admit wavefunctional formulations.

The Wheeler-de Witt equation

$$\widehat{\mathsf{H}}\underline{\Psi} = 0 \tag{1}$$

involves a wavefunctional $\underline{\Psi} = \underline{\Psi}[\gamma_{ab}]$ on the space Riem(Σ) of all possible Riemannian geometries (Σ, γ_{ab}), where γ_{ab} is the intrinsic metric tensor on a three-dimensional manifold Σ. Equation (1) was obtained [18] by quantizing the Hamiltonian formulation of classical general relativity by Arnowitt, Deser, and Misner (ADM) [3], where (Σ, γ_{ab}) is a spacelike 3d-slice of the spacetime manifold $M = (\Sigma \times \mathbb{R}, g_{\mu\nu})$.

The quantization replaces the classical 3d metric γ_{ab} and its conjugate momentum π_γ^{cd} by operators,

$$\begin{cases} \widehat{\gamma}_{ab}(\boldsymbol{x})\underline{\Psi}[\gamma_{ab}] = \gamma_{ab}(\boldsymbol{x})\underline{\Psi}[\gamma_{ab}], \\ \widehat{\pi}_\gamma^{cd}(\boldsymbol{x})\underline{\Psi}[\gamma_{ab}] = \dfrac{\hbar}{i}\dfrac{\delta\underline{\Psi}[\gamma_{ab}]}{\delta\gamma_{cd}(\boldsymbol{x})}, \end{cases} \tag{2}$$

subject to the canonical commutation relations

$$\begin{cases} \left[\widehat{\gamma}_{ab}(\boldsymbol{x}), \widehat{\pi}_\gamma^{cd}(\boldsymbol{y})\right] = i\hbar\delta_{(a}^c\delta_{b)}^d(\boldsymbol{x}, \boldsymbol{y}), \\ \left[\widehat{\gamma}_{ab}(\boldsymbol{x}), \widehat{\gamma}_{cd}(\boldsymbol{y})\right] = \left[\widehat{\pi}_\gamma^{ab}(\boldsymbol{x}), \widehat{\pi}_\gamma^{cd}(\boldsymbol{y})\right] = 0, \end{cases} \tag{3}$$

where $\boldsymbol{x}, \boldsymbol{y} \in \Sigma$ and $\delta/\delta\gamma_{cd}(\boldsymbol{x})$ is the functional derivative.

The Wheeler-de Witt equation is a constraint equation, not an evolution equation, despite de Witt initially calling it the *Einstein-Schrödinger equation*. It is complemented by three other constraint equations that factor out the space diffeomorphisms. The wavefunctional $\underline{\Psi}$ is a timeless solution. A proposal to decode a dynamical solution, made by Page and Wootters [38], consists of interpreting it as a quantum system $|\psi(\tau)\rangle$ entangled with a clock $|\tau\rangle$, $\underline{\Psi} = \int_{\mathbb{R}} |\tau\rangle|\psi(\tau)\rangle d\tau$. This, and other proposals to recover a dynamical solution, were assessed critically in [25, 29]. According to Page and Wootters,

we can consider that the state of the universe at the time t is represented by the vector $\Psi(t) := |t\rangle|\psi(t)\rangle$.

In the following we will assume the existence of a quantum theory of gravity based on time-dependent states.

Ashtekar's Hamiltonian formulation of classical general relativity [4] is similar to ADM, except that instead of γ and π_γ, its variables are an su(2) connection, whose conjugate variable is a densitized frame field on Σ. At the classical level the ADM formalism and the Ashtekar variables are equivalent. When quantized, the resulting operators satisfy commutation relations similar to (3) [27]. Its quantization was interpreted by Rovelli and Smolin in terms of *loop variables* [45].

We do not know with certainty that spacetime is continuous. Various approaches to quantum gravity are discrete, being based in general on structures that can be represented as graphs or hypergraphs that may have attached numbers at their vertices and (hyper-)edges. For example, in the *causal sets* approach [50], the vertices of the graph represent events from spacetime, and oriented edges join pairs of events in causal relation, in the sense that the first event is in the past lightcone of the second one. The *Regge calculus* [44] is based on triangulations of spacetime into 4-simplices further approximated as flat. Distances are attached to the edges, and the spacetime curvature is concentrated at 2-faces, and expressed in terms of deficit angles *etc.* The *causal dynamical triangulation* approach is similar, but with fixed-length edges [30]. *Loop quantum gravity* can be formulated in terms of spin networks and spin foams. *Spin networks* are graphs with the edges labeled by half-integer numbers corresponding to irreducible representations of su(2) [5, 39, 46]. Two spin networks at different times are joined by a *spin foam*, a hypergraph used in the path integral formulation of loop quantum gravity.

All these graph or hypergraph structures are background-independent. They can also be seen as equivalence classes of (hyper)graphs embedded in the 3d-space Σ or in the spacetime M, where two such embedded structures are equivalent if a diffeomorphism of the background manifold can transform one into another.

Many of these discrete approaches use Feynman's path integral quantization, but at the end a complex coefficient is associated to each classical basis state, so it is likely that a wavefunctional representation always exists.

I will assume that quantum gravity can be described by a theory admitting a wavefunctional representation.

Let $\mathcal{C}_S$ be the set of classical 3d-space configurations. These may be the diffeomorphism equivalence classes of Riemannian geometries (Σ, γ), or more fundamental structures approximated by such geometries at low energies. For example, if quantum gravity is one of the discrete theories whose classical configurations are labeled (hyper)graphs, these will be the elements of $\mathcal{C}_S$.

While much of the following works well with both continuous and discrete spacetimes, we will see that continuous spacetimes have some advantages.

I will assume that there is a Schrödinger formulation

of quantum gravity in terms of wavefunctionals over $\mathcal{C}_S$ endowed with a measure μ_S on $\mathcal{C}_S$. We assume that problems like the nonexistence of an infinite-dimensional Lebesgue measure are solved or avoided. The states of the universe are represented by unit vectors Ψ in the Hilbert space $\mathcal{H}_S$ spanned by states $|\gamma\rangle$, where $\gamma \in \mathcal{C}_S$ stands for (Σ, γ_{ab}) or the (hyper-)graph structure if the underlying quantum gravity theory is based on such a structure, and $\Psi[\gamma] := \langle \gamma | \Psi \rangle$, with the Hermitian scalar product

$$\langle \Psi | \Psi' \rangle := \int_{\mathcal{C}_S} \Psi^*[\gamma] \Psi'[\gamma] \mathcal{D}\mu_S[\gamma]. \tag{4}$$

For matter quantum fields I will assume, like in the quantum field theory on the Minkowski spacetime, that there is a formulation in terms of wavefunctionals on the classical configuration space of classical fields on Σ. The classical fields include bosonic fields, which commute, and fermionic fields, which are expressed using Grassmann numbers because they anticommute at equal times, see *e.g.* [23, 26]. Any other variables needed to specify how the 3d geometries integrate into 4d manifolds, for example the shift and lapse variables, will be included as well in the configuration space of classical fields. If the 3d-space is a (hyper)graph $\gamma \in \mathcal{C}_S$, I assume that matter can be described, in principle, by attaching various quantities or other structures to the elements of γ.

Although ultimately the quantum field equations are those to be obeyed, each classical matter field has to be defined on an underlying classical 3d-space γ. To express this dependence, I will denote classical matter fields by ϕ_γ. In quantum field theory on flat space the Fock basis consists of linear combinations of classical states $|\phi\rangle$, where ϕ is a classical field on the 3d-space $\mathbb{R}^3$ [23]. Let us assume, for each fixed $\gamma \in \mathcal{C}_S$, the choice of such a basis $(|\phi_\gamma\rangle)_{\phi_\gamma \in \mathcal{C}_M^\gamma}$, where the elements of $\mathcal{C}_M^\gamma$ label the basis in a way dependent on $\gamma \in \mathcal{C}_S$.

Let us summarize all of the above into the following

Assumption 1. The complete state of the universe is represented by a wavefunctional on a configuration space

$$\mathcal{C} := \bigcup_{\gamma \in \mathcal{C}_S} \{\gamma\} \times \mathcal{C}_M^\gamma, \tag{5}$$

endowed with the measure μ related to the measures μ_S on $\mathcal{C}_S$ and μ_M^γ on $\mathcal{C}_M^\gamma$ by $\mathcal{D}\mu[\gamma, \phi_\gamma] = \mathcal{D}\mu_S[\gamma]\mathcal{D}\mu_M^\gamma[\phi_\gamma]$. Let the Hilbert space of such wavefunctionals be $\mathcal{H} \cong \mathcal{H}_S \otimes \mathcal{H}_M$, with the Hermitian scalar product

$$\langle \Psi | \Psi' \rangle := \int_{\mathcal{C}} \Psi^*[\gamma, \phi_\gamma] \Psi'[\gamma, \phi_\gamma] \mathcal{D}\mu[\gamma, \phi_\gamma], \tag{6}$$

where $\Psi[\gamma, \phi_\gamma] = \langle \gamma, \phi_\gamma | \Psi \rangle$.

B. Macro-states and ontic micro-states

Macro-states correspond to equivalence classes of micro-states. Since different macro-states are distinguishable, there is a set of commuting projectors $(\widehat{\mathsf{P}}_\alpha)_{\alpha \in \mathcal{A}}$ on $\mathcal{H}$, so that $[\widehat{\mathsf{P}}_\alpha, \widehat{\mathsf{P}}_\beta] = 0$ for any $\alpha \neq \beta \in \mathcal{A}$, and $\bigoplus_{\alpha \in \mathcal{A}} \widehat{\mathsf{P}}_\alpha \mathcal{H} = \mathcal{H}$. Any *macro-state* is represented by a subspace of the form $\widehat{\mathsf{P}}_\alpha \mathcal{H}$. We call *quasi-classical* the states belonging to macro-states $\widehat{\mathsf{P}}_\alpha \mathcal{H}$.

Since all projectors $\widehat{\mathsf{P}}_\alpha$ commute, there are bases of $\mathcal{H}$ consisting of common eigenvectors of $(\widehat{\mathsf{P}}_\alpha)_{\alpha \in \mathcal{A}}$. Since both γ and ϕ_γ are classical, it makes sense to assume that the states $|\gamma, \phi_\gamma\rangle$ are quasi-classical, *i.e.* $|\gamma, \phi_\gamma\rangle \in \widehat{\mathsf{P}}_\alpha \mathcal{H}$ for some α. This expresses the idea that, at the macro level, the world looks like the classical world.

Assumption 2. The basis $(|\gamma, \phi_\gamma\rangle)_{(\gamma, \phi_\gamma) \in \mathcal{C}}$ consists of quasi-classical states, *i.e.* for any $|\gamma, \phi_\gamma\rangle$ there is an $\alpha \in \mathcal{A}$ so that $|\gamma, \phi_\gamma\rangle \in \widehat{\mathsf{P}}_\alpha \mathcal{H}$.

The existence of a basis of quasi-classical states is guaranteed by the existence of the macro-projectors. Assumption 2 specifies that such a basis can consist of states with classical 3d-geometry and classical matter fields.

Definition 1. States of the form $|\gamma, \phi_\gamma\rangle$ will be called *3d-space states*, and also *ontic states* for reasons that will be explained later.

We expect that a 3d-space state immediately evolves into a linear combination of 3d-space states. Dissociation and reassociation happen continuously. However, at the macro level, the state may remain quasi-classical under unitary evolution for finite time intervals. This accounts for the fact that macroscopic systems do not evolve all the time into linear combinations of macro-states like the Schrödinger cat, although unitary evolution may lead to such linear combinations during quantum measurements.

C. 3d-space states are fundamental

Just because physicists first discovered classical physics, and later quantum theory, and formulated the latter by quantizing the former, it does not mean that quantum theory requires classical physics to exist. The universe is what it is, and it is fundamentally quantum.

However, the Hilbert space is too symmetric as it is, and without the existence of preferred structures that break its symmetry, there would be no relation between Hilbert space vectors and physical reality, nor between Hermitian operators and physical observables. Physical properties cannot simply emerge from the abstract state vector, even if the Hamiltonian is known, because if they would, infinitely many entities with the very same properties, but able to represent completely different physical realities, would emerge as well [61, 65]. Therefore, the basis $(|\gamma, \phi_\gamma\rangle)_{(\gamma, \phi_\gamma) \in \mathcal{C}}$ is special among the others, because of its physical meaning. This justifies

Assumption 3. The 3d-space states are fundamental, in the sense that, by their physical meaning, they are special among the other states represented by $\mathcal{H}$.

As explained earlier, the states γ are not necessarily Riemannian geometries, they can be other structures approximated at low energies by such geometries. What is important is that they have a special physical meaning, in the same sense in which, in nonrelativistic quantum mechanics, the position operators and their eigenvectors have a special physical meaning compared to other operators or vectors in the Hilbert space. Similar considerations hold for the matter fields.

III. DISSOCIATION INTO 3D-SPACE STATES

A. Background freedom

To construct the configuration space $\mathcal{C}$, we eliminated the unphysical degrees of freedom due to diffeomorphisms and global gauge transformations. For example, two metric tensor fields on Σ may look different, but a coordinate transformation, which corresponds to a diffeomorphism of Σ, may be able to map them into one another, showing that they are isometric. For this reason, we took as classical states the equivalence classes of metrics on Σ under diffeomorphisms. Matter fields also have to be background-free, achieved by fixing a gauge, as usually.

Similarly if the 3d-space is a discrete structure like the ones that can be represented by graphs or hypergraphs from §II A, we took the configuration space consisting of such structures based on their internal relations, not as particular embeddings in a 3d manifold. But let us state this explicitly, since it will be central in the article:

Assumption 4. Our theory is background-free.

The case for background freedom was made for example by Smolin [49]. General relativity already shows that the structures have to be relational: we use coordinates, but they are not absolute, they are just ways to assign numbers to points in space or spacetime. The *hole argument* [37, 51] shows that taking the points of the underlying manifold as having an independent reality from the intrinsic relations introduced by the metric tensor leads to indeterminacy.

This is why many of the approaches to quantum gravity seem to require background freedom, or even have it built-in. This is true for the formulation based on the Wheeler-de Witt equation (1), the discrete approaches based on (hyper)graphs discussed earlier, like causal sets, Regge calculus, causal dynamical triangulations, loop quantum gravity *etc.* For a discussion of background independence in *string theory* see Witten [74].

B. Background freedom and dissociation

In general, we make no difference between the concepts of linear combination and superposition, except maybe that a linear combination is understood as the mathematical expression of a superposition, which is a physical concept related to the position in the 3d-space and phenomena like interference. And they usually coincide.

In nonrelativistic quantum mechanics, any two wavefunctions can be superposed in the 3d-space, because the underlying geometry is the same, and the reference frames are the same. In the wavefunctional formulation of quantum field theory, the local information about the wavefunctional of a scalar field is obtained by using local operators at $\boldsymbol{x} \in \Sigma = \mathbb{R}^3$, definable in function of the operators $\widehat{\varphi}(\boldsymbol{x})$ and $\widehat{\pi}_\varphi(\boldsymbol{x})$ (to be rigorous, one uses operator-valued distributions, applied to a sequence of test functions that converge uniformly to the Dirac distribution $\delta_{\boldsymbol{x}}$).

In background-dependent theories of quantum gravity we can define local operators in a similar way, in function of the operators $\widehat{\gamma}(\boldsymbol{x})$ and $\widehat{\pi}_\gamma(\boldsymbol{x})$, and $\widehat{\varphi}_\alpha(\boldsymbol{x})$ and $\widehat{\pi}_{\varphi_\alpha}(\boldsymbol{x})$ for each matter field φ_α, where α stands for the spin and the internal degrees of freedom.

But in background-free quantum gravity local operations on the 3d-space, and therefore superpositions, do not always make sense, even if the linear combinations are always defined. If the theory is background-free, a difference appears when we apply local operators to linear combinations. Any local operator $\widehat{\mathsf{A}}(\boldsymbol{x})$ depends on $\boldsymbol{x}$, but background freedom prevents uniquely matching points from Σ for $|\gamma, \phi_\gamma\rangle$ to points from Σ for $|\gamma', \phi'_{\gamma'}\rangle$, because in general $\gamma \neq \gamma'$. The situation is even more visible in background-free theories where (Σ, γ) is replaced by a labeled (hyper)graph, because in this case correspondences between the vertices of different (hyper)graphs are not even possible in general.

A correspondence between the points of Σ for $|\gamma, \phi_\gamma\rangle$ and those of Σ for $|\gamma', \phi'_{\gamma'}\rangle$ requires (Σ, γ) and (Σ, γ') to be isometric. Sometimes such a correspondence exists only between some open regions of Σ. So the dissociation is not always ensured, and we will see that this is essential. Because local operators and superpositions are not completely well-defined in the absence of a common background, we arrived at the following:

Key observation 1. Background freedom implies the dissociation of the wavefunctional into 3d-space states.

The dissociation is not necessarily complete, and various cases are captured in the following definition.

Definition 2. Two state vectors $|\gamma\rangle|\Psi_M\rangle$ and $|\gamma'\rangle|\Psi'_M\rangle$ are *locally associable* if there exist two open subsets $U, U' \subseteq \Sigma$ and an isometry between (U, γ) and (U', γ'). In case that $U = U' = \Sigma$, they are *globally associable*.

Two state vectors are *dissociated* if they are not globally associable. They are *partially dissociated* if they are locally but not globally associable. They are *completely dissociated* if they are not associable (globally or locally).

In the discrete case, in Definition 2, (local) isometries are replaced by (local) isomorphisms between the labeled (hyper)graphs γ and γ'.

It is possible for the 3d-space states to reassociate, at least partially. This can be seen by using the path in-

tegral formulation. This allows quantum interference to exist at micro scales. This is the key to understanding why our quantum world looks quantum at small scales, and classical at macro scales.

When the system evolves into associable states, the association is allowed only if it is consistent with the dynamics, which is given by a local Hamiltonian [23].

It is interesting to compare dissociation induced by background freedom with Penrose's *gravitational decoherence*. According to Penrose [41], "I envisage that nature feels uncomfortable about 'linearly superposing' space-time geometries which differ significantly from one another and, instead, prefers to settle for (i.e. to 'reduce' to) essentially just one of the geometries involved." Then, he proposes that this requires a nonlinear collapse due to the gravitational entropy. Later he justifies the collapse by invoking the existence of different time parameters for the superposed spacetimes [42].

By contrast, the dissociation induced by background freedom proposed in this article is simply due to the absence of a correspondence between the points in different 3d-spaces in the absence of common background. The dissociation is accepted, and no change of the Schrödinger wavefunctional equation is postulated.

For 4d-spacetimes, background freedom may lead to an even stronger dissociation than for 3d-spaces. However, because we start with Hamiltonian approaches to quantum gravity, and it is expected that 3d-space states evolve into linear combinations of 3d-space states, dissociation of 3d-space states is sufficient here.

IV. PROBABILITIES FROM COUNTING 3D-SPACE STATES

A. Taking dissociation seriously

Every vector $|\Psi\rangle$ from $\mathcal{H}$ has the form

$$|\Psi\rangle = \int_{\mathcal{C}} c_{\gamma,\phi_\gamma}|\gamma,\phi_\gamma\rangle \mathcal{D}\mu[\gamma,\phi_\gamma], \qquad (7)$$

where $c_{\gamma,\phi_\gamma} = \Psi[\gamma,\phi_\gamma] = \langle\gamma,\phi_\gamma|\Psi\rangle$, $\gamma \in \mathcal{C}_S$, and $|\phi_\gamma\rangle$ belongs to the basis $(|\phi_\gamma\rangle)_{\phi_\gamma \in \mathcal{C}_M^\gamma}$ which may depend on the 3d-geometry γ.

We may be tempted to simply proclaim the Born rule, asserting that the probability density is

$$P[\gamma,\phi_\gamma] = |c_{\gamma,\phi_\gamma}|^2. \qquad (8)$$

But let us resist this for a while, and explore the consequences of the dissociation. If we explore the consequences of a physical principle, we should do it in its own terms, and if the result contradicts the observations, we should drop the starting principle.

The dissociation into 3d-space states suggests:

Principle 1. Each 3d-space state is either not present in $|\Psi(t)\rangle$, or it is present once (*i.e.* it cannot be "half-present", even if eq. (7) may suggest this possibility).

This "either-or" is vague at this point, but it will be clarified later in this Section. At any rate, it may seem to contradict everything we know. However, we will get quantum theory back, with the familiar complex numbers, which will receive a geometric meaning in terms of a global gauge, and, if $\mathcal{C}$ is continuous, with the Born rule as we know it, but resulting from counting the 3d-space states.

B. Making the wavefunctional real

Background freedom implies that the quantum state dissociates automatically into 3d-space states, but since the coefficients c_{γ,ϕ_γ} from eq. (7) are complex numbers, we need to understand their meaning.

First, while $|\gamma,\phi_\gamma\rangle$ is an ontic state, which may even be classical if ϕ_γ is a classical field, $c_{\gamma,\phi_\gamma}|\gamma,\phi_\gamma\rangle$ is not. Let us for the moment ignore γ, and consider that ϕ_γ is a scalar field. In general, $c|\phi_\gamma\rangle \neq |c\phi_\gamma\rangle$. Even if $c \in \mathbb{R}$, if $c \neq 1$, $|c\phi_\gamma\rangle$ represents a classical field $c\phi_\gamma$ different from ϕ_γ, so $|c\phi_\gamma\rangle$ and $|\phi_\gamma\rangle$ are orthogonal.

But if ϕ_γ represents the electromagnetic potential and $\theta \in \mathbb{R}$, $e^{i\theta}\phi_\gamma$ represents a global gauge transformation of ϕ_γ. The classical fields ϕ_γ and $e^{i\theta}\phi_\gamma$ are physically equivalent. The state vectors $|\phi_\gamma\rangle$ and $e^{i\theta}|\phi_\gamma\rangle$ are distinct vectors, but they represent the same physical state. This suggests the following interpretation:

Key observation 2. If the matter fields admit an U(1) gauge symmetry, for any $\theta \in \mathbb{R}$,

$$e^{i\theta}|\gamma,\phi_\gamma\rangle \equiv |\gamma,e^{i\theta}\phi_\gamma\rangle. \qquad (9)$$

This accounts for the fact that the physical equivalence of the classical fields ϕ_γ and $e^{i\theta}\phi_\gamma$ corresponds to the physical equivalence of the state vectors $|\phi_\gamma\rangle$ and $e^{i\theta}|\phi_\gamma\rangle$.

This works for fields that admit an U(1) symmetry, like charged fields, gauge potentials, and spinor fields.

Let us express the complex coefficients c_{γ,ϕ_γ} from eq. (7) in the polar form

$$c_{\gamma,\phi_\gamma} = r[\gamma,\phi_\gamma]e^{i\theta[\gamma,\phi_\gamma]}, \qquad (10)$$

with $r[\gamma,\phi_\gamma] \geq 0$. Then, eq. (7) becomes

$$|\Psi\rangle = \int_{\mathcal{C}} r[\gamma,\phi_\gamma]|\gamma,e^{i\theta[\gamma,\phi_\gamma]}\phi_\gamma\rangle \mathcal{D}\mu[\gamma,\phi_\gamma], \qquad (11)$$

We see that, whenever a physical classical field contributes to $|\Psi\rangle$, it contributes only once, with a uniquely determined gauge $e^{i\theta[\gamma,\phi_\gamma]}$ and real coefficient $r[\gamma,\phi_\gamma]$. As $|\Psi\rangle$ evolves in time, the gauge and $r[\gamma,\phi_\gamma]$ can change.

It remains to explain the relation between $r[\gamma,\phi_\gamma]$ and the probability density of ontic states, in accord with Principle 1.

C. Emergence of the Born rule

Now that we have seen that gauge freedom allows the coefficients in the linear combination (7) to be real numbers, let us see what their meaning is and how it relates to probabilities.

I will assume that the configuration space $\mathcal{C}$ is continuous. This likely requires that Σ is a 3d manifold. I show that, under this assumption, the Born rule emerges by counting the 3d-space states. A more general discussion can be found in [63].

Let us choose all fields ϕ_γ so that $\theta[\gamma, \phi_\gamma] = 0$ in eq. (10). We denote for simplicity $\xi := (\gamma, \phi_\gamma)$.

First, we notice that a state vector of the form $|\Psi\rangle = \frac{1}{\sqrt{n}} \sum_{k=1}^{n} |\xi_k\rangle$, where $(|\xi_k\rangle)_{k \in \{1,...,n\}}$ are distinct basis vectors, leads to the Born rule. If $\widehat{P}_\alpha$ is a macro projector and n_α basis vectors composing $|\Psi\rangle$ belong to $\widehat{P}_\alpha \mathcal{H}$, then $\langle \Psi | \widehat{P}_\alpha | \Psi \rangle = n_\alpha/n$. Therefore, the Born rule simply coincides with the usual counting rule "probability is the ratio of the number of favorable outcomes to the total number of possible outcomes". But only a small subset of the possible state vectors have this form, so this idea fails if the basis is discrete.

However, this idea works in the continuous case, since the basis vectors can be distributed with nonuniform density. More precisely, if $r[\xi] := r[\gamma, \phi_\gamma]$ from eq. (11) is μ-measurable, we can define a new measure

$$\mathcal{D}\widetilde{\mu}[\xi] := r[\xi] \mathcal{D}\mu[\xi], \tag{12}$$

and obtain

$$|\Psi\rangle = \int_{\mathcal{C}} |\xi\rangle \mathcal{D}\widetilde{\mu}[\xi]. \tag{13}$$

Since $r[\xi]$ is μ-measurable, the measure $\widetilde{\mu}$ is absolutely continuous with respect to μ.

Consider a macro projector $\widehat{P}_\alpha$ so that the macro-state $\widehat{P}_\alpha \mathcal{H}$ is the closure of a subspace spanned by $(|\xi\rangle)_{\xi \in \mathcal{C}_\alpha}$, where the set $\mathcal{C}_\alpha$ is μ-measurable. Then,

$$\left| \int_{\mathcal{C}_\alpha} |\xi\rangle \mathcal{D}\widetilde{\mu}[\xi] \right|^2 = \left(\int_{\mathcal{C}_\alpha} \langle\xi| \mathcal{D}\widetilde{\mu}[\xi] \right) \left(\int_{\mathcal{C}_\alpha} |\xi'\rangle \mathcal{D}\widetilde{\mu}[\xi'] \right)$$
$$= \int_{\mathcal{C}_\alpha} \left(\int_{\mathcal{C}_\alpha} \langle\xi|\xi'\rangle \mathcal{D}\widetilde{\mu}[\xi'] \right) \mathcal{D}\widetilde{\mu}[\xi]$$
$$= \int_{\mathcal{C}_\alpha} \left(\int_{\mathcal{C}_\alpha} \langle\xi|\xi'\rangle r[\xi'] \mathcal{D}\mu[\xi'] \right) \mathcal{D}\widetilde{\mu}[\xi]$$
$$= \int_{\mathcal{C}_\alpha} r[\xi] \mathcal{D}\widetilde{\mu}[\xi] = \int_{\mathcal{C}_\alpha} r^2[\xi] \mathcal{D}\mu[\xi]$$
$$= \langle \Psi | \widehat{P}_\alpha | \Psi \rangle. \tag{14}$$

Therefore, state counting gives the Born rule, in accord to Principle 1 (Fig. 2).

That's all. The role of the new measure is just to show that the states satisfy Principle 1, it does not change the original measure, it only expresses it differently, so that the probabilities become apparent.

Note that it is not necessary to prove the Born rule for individual particles or subsystems, it is sufficient to prove it for macro-branches or macro-projectors, and micro-branch counting per macro-branch gives the Born rule.
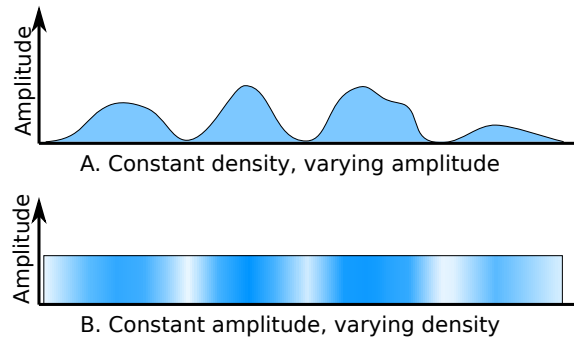

FIG. 2. **The Born rule from counting** 3d-**space states.**
A. The usual interpretation of a wavefunction as a linear combination of basis state vectors of different norms.
B. The interpretation of the wavefunction in terms of constant norm basis state vectors, but with inhomogeneous density.

Therefore, the numbers from eq. (11) have a direct meaning: Principle 1 combined with the gauge freedom allows the interpretation of the states $|\Psi\rangle$ as consisting of 3d-space states that are either present or not. We obtained the Born rule from counting 3d-space states.

Key observation 3. If $\mathcal{C}$ is continuous, any state vector $|\Psi\rangle \in \mathcal{H}$ consists of mutually orthogonal 3d-space states whose density is $|\Psi[\gamma, \phi_\gamma]| \mathcal{D}\mu[\gamma, \phi_\gamma]$. Then, for the macro-projectors $(\widehat{P}_\alpha)_{\alpha \in \mathcal{A}}$, the Born rule results by 3d-space states counting based on the density.

Remark 1. Note that the derivation of the Born rule from this Section is not limited to the case when the basis states are 3d-space states [63]. What is important is that the basis is continuous, and that the basis vectors belong to macro-states. In quantum field theory in the Schrödinger wavefunctional representation, one can use the classical field configurations to obtain the basis. But the 3d-space states have the advantage of dissociating in a natural way, and of including gravity. Moreover, the 3d-space states are the only ones consisting of local beables, which are γ and ϕ_γ (see Sec. §VI G). This justifies counting these states to get the Born rule. □

V. COLLAPSE POSTULATE OR MANY-WORLDS?

Let us see how dissociation into 3d-space states works with quantum measurements, and whether it works better by assuming the collapse postulate or with the many-worlds interpretation.

A measuring device is a quantum system in a quasi-classical state. When interacting with the observed system, assumed to be microscopic in the sense that it is

not directly observable, the combined system evolves into a linear combination of macroscopically distinct states. Each of these states contains the observed system in a different state, and the pointer of the measuring device indicating that state. So the Schrödinger equation predicts that two or more stories describing the measurement are simultaneously true. But we never observe such linear combinations: after the measurement, the pointer state is always in a definite macro-state.

QM Problem 1. Why do all linear combinations appear to be possible at micro-scales, but not at macro-scale?

To resolve this problem, in standard quantum mechanics one invokes the *collapse postulate* [70], which simply states that quantum measurements suspend the Schrödinger evolution, so that from the linear combination we keep only the term that corresponds to one of the possible pointer states, removing the others.

In doing this, standard quantum mechanics assumes, without explaining it, the pre-existence of measuring devices in quasi-classical states. But, since most quantum states are linear combinations of quasi-classical states, we have the following problem:

QM Problem 2. Why is the measuring device already in a quasi-classical state?

The collapse postulate purports to solve QM Problem 1 by assuming implicitly that QM Problem 2 is already solved. But both problems can be solved simultaneously, by extending the collapse postulate to apply not only to measurements, but whenever a quantum system becomes a linear combination of states that belong to distinct macro-states. Then, when any quantum system is no longer quasi-classical, the collapse postulate is triggered and only one of the macroscopic possibilities remains [60].

A problem with the collapse postulate is that the Schrödinger equation is considered valid in some situations, and suspended in other situations.

There seems to be a double standard here. On one hand, linear combinations and entangled states appear and evolve in parallel as long as no observation is made, and the experiments are consistent with this. On the other hand, if we measure them, since we do not observe more parallel sets of outcomes simultaneously, we allow only one of the histories, and censor all the other ones, by appealing to the collapse postulate.

We can try to use the 3d-space states approach to solve QM Problems 1 and 2 at once, by reformulating the collapse postulate in the following way:

Tentative Postulate 1 (3d-space states Collapse Postulate)**.** During the evolution of the system, the 3d-space states may become irreversibly dissociated into two or more sets of 3d-space states, determined by the macro projectors. Let us call these sets macro-branches. When this happens, only one of the macro-branches remains, and the others disappear. The probability is given by counting the ratio between the 3d-space states in the remaining macro-branch to the total number of 3d-space states before the collapse.

This Tentative Postulate seems to provide a basis to explain macro systems, including measuring devices. If so, it can solve both QM Problems 1 and 2 at once.

But dissociation and reassociation happen all the time. Reassociation allows interference effects, but when dissociation is irreversible, these effects are suppressed automatically. Therefore, the collapse is still arbitrary, there is still no clear rule when it should be invoked. When no measurement is made, multiple 3d-space states are allowed to coexist, dissociate and associate in interference patterns in the wavefunctional. But when a measurement is made, only some of the 3d-space states seem to remain. Some linear combinations of 3d-space states seem to be "more equal" than others. Dissociation makes this arbitrariness more evident, because the micro-branching is well-defined by the geometry compared with the usual superpositions, yet the 3d-space states are kept or removed as it is more convenient.

Remark 2. If we assume collapse and try to explain the Born rule by counting 3d-space states as in Sec. §IV, we will have to accept that the wavefunction consists of many micro-states that exist simultaneously, and part of them are eliminated by every collapse. But this would make quantum mechanics with the collapse postulate a strange version of the many-worlds interpretation, in which some of the micro-branches are removed with every collapse, and others are kept. Moreover, those that are kept have to belong to the same macro-state, one cannot just keep 3d-space states from different macro-states. □

These remarks immediately prompt the following:

Key observation 4. Tentative Postulate 1 is unnecessary, because once the dissociation becomes irreversible, the macro-branches evolve independently and no longer interfere, and no 3d-space states from different macro-branches associate again.

Therefore, since once dissociation becomes irreversible at macro scales the macro-branches no longer reassociate anyway, the 3d-space states approach works more naturally with the many-worlds interpretation (MWI) rather than with the wavefunction collapse.

The key idea of MWI is to take the Schrödinger equation seriously, without introducing any ad-hoc rule that applies only to macro scales. This implies that all possible components of the total wavefunction continue to exist after the measurement, but thanks to decoherence, they no longer "see" each other. The linearity of the Schrödinger equation allows the macroscopically distinct states that result from a quantum measurement by unitary evolution to be independent, but in addition, they no longer interfere. The wavefunction is branching so that the different branches occupy different regions in

the configuration space. Interference is suppressed because the copy of any measuring device in one branch is unable to detect anything from another branch, so the branches no longer "know" about one another. And the branches become macroscopically distinct, in the sense that they correspond to projections of the state vector on different macro-states $\widehat{\mathsf{P}}_{\alpha_1}\mathcal{H}, \ldots, \widehat{\mathsf{P}}_{\alpha_n}\mathcal{H}$.

Decoherence into macro-branches seems to explain the existence of measuring devices and solve the measurement problem without violating the Schrödinger equation by invoking an ad-hoc wavefunction collapse.

There are several problems that are not solved, at least not in a generally accepted way or so that it does not require a complete reinterpretation of well-established concepts like probabilities. They will be discussed in Sec. §VI, where I will propose that these problems are solved, or at least alleviated, by the dissociation into 3d-space states, which provides an absolute form of decoherence.

VI. THE MANY-SPACETIMES INTERPRETATION

We think that we are forced to suspend the Schrödinger equation as a result of measurements, because we observe only one of the stories that the Schrödinger equation describes as taking place in parallel. But could we observe more than one of these stories at once? The Schrödinger equation predicts that even the observers would be "multiplied", each of its instances participates in one of the stories and not in the others, of which they are oblivious.

Everett noticed the perfect symmetry of the situation, and saw no reason to favor the story in which one gets an outcome against the competing stories. He proposed to trust the Schrödinger equation and accept that all stories continue to happen independently, once they are separated [19, 20]. Schrödinger himself proposed earlier, along the same lines, something that he worried may "seem lunatic" [7, 17, 48].

The result of Everett's realization is the *many-worlds interpretation* (MWI) of quantum mechanics. But there are still open questions in MWI. Various proposals were made to solve them, and some researchers think they are solved. Others think that they cannot be solved and even that MWI does not deserve to be taken seriously.

In this Section I argue that the 3d-space states approach solves some of these problems, or provides a more natural way to solve them. This leads to a variant of the many-worlds interpretation, which may be called "the many-3d-space states interpretation", but I will call it *the many-spacetimes interpretation* (MSTI).

A. Preferred basis: 3d-space states

Let us start with a problem whose solution is the key to solving other problems.

MWI Problem 1 (Preferred basis)**.** In what basis does the branching take place, so that the worlds appear classical at the macro level?

Presumably, decoherence makes a preferred basis emerge [31]. However, there has to be more to the preferred basis than that it simply "emerges". Otherwise, if a preferred basis emerges, either for the entire universe, or for a subsystem, infinitely many others emerge [61].

In nonrelativistic MWI, it is expected that the preferred basis is related to the positions in the configuration space, or the positions and momenta in the phase space. Accordingly, the branches no longer interfere because they no longer overlap in the configuration space.

The MSTI answer is based on the form of "absolute decoherence" provided by the dissociation into 3d-space states enforced by background freedom:

MSTI Answer 1 (Preferred basis)**.** The dissociation of the state vector automatically selects as the preferred basis the ontic 3d-space states basis.

B. Objects in 3d-space

Another problem is that the wavefunction is not defined on the 3d-space, but on the much larger configuration space. This disturbed Schrödinger [6], Lorentz ([43], p. 44), Einstein [21, 24], Heisenberg, Bohm [12] *etc*. This is true for the wavefunction of any state vector in the total Hilbert space.

MWI Problem 2 (Objects in 3d-space)**.** Given that the wavefunction is defined on the high-dimensional configuration space, how do familiar, classical-looking objects localized in space emerge from the wavefunction?

The wavefunction, being an element of a representation of the Galilei or the Poincaré group [73], is intrinsically associated to space or spacetime. Therefore, properly analyzed, it satisfies all expectations of standard geometric objects in space or spacetime [62]. Moreover, if one thinks that this is not sufficient, and the wavefunction should be expressed as classical-like fields in space or spacetime, this is also possible, albeit in an inaesthetic way that at least serves as a proof of concept [59].

But even if the wavefunction is, in the sense of group theory or as fields, an object in space, in general it does not look like the familiar, classical-looking objects we see.

Decoherence might lead to branches that look like familiar, classical-looking objects localized in space. Wallace [72] thinks that the branches form patterns in the sense of Dennett [15], but are these patterns classical-looking enough?

Observation 1 (Schrödinger atoms in 3d-space)**.** Let us start with a simple atom, consisting of n electrons, n protons, and $n' \approx n$ neutrons. Each of the nucleons consist of three quarks, but the things are more complicated, since there is a practically infinite number of

virtual quarks and gluons involved, and the interactions between nucleons involve exchange of mesons. But, due to the quarks' large masses compared to the electron's mass, and to their confinement to the short range of the strong interaction, the nucleus of the atom can be very well localized in space. Let us consider an atom in a state that is a solution of the stationary Schrödinger equation. Decompose the wavefunction as eigenfunctions of the particle number operator with integer eigenvalue k. These components are very well localized around a point $(x, y, z)^k$ in the configuration space of k particles. The electrons are spread on a wider region than the nucleus particles, with an amplitude that vanishes very quickly as we move away from $(x, y, z)^k$. But they are still very peaked around $(x, y, z)^k$. Since the solution is stationary, the atom is very well localized. Its position (x, y, z) decouples from the other quantum numbers, so it behaves like a one-particle wavefunction with many "internal" degrees of freedom defining a "rigid object" consisting mainly of the orbitals.

This picture can form the basis for describing molecules and larger objects as localized objects in the 3d-space. The wavefunction, even without decoherence, is a superposition of products of such bound states and free particle states. Decoherence is expected to decompose the wavefunction into branches that look quasi-classically in this way (Problem 3), and nothing else is needed. □

Maudlin [32–34] and Norsen [36] think that Problem 2 is not solved, and that it is hard to solve it even if Problems 1 and 3 would be. They contrast this with the pilot-wave theory (PWT) [11], which includes, along with the wavefunction, point-particles at definite positions in space, and with the Ghirardi-Rimini-Weber (GRW) interpretation [22], where the wavefunction collapses around definite points of the configuration space, appearing nearly classical.

Their arguments can be seen as relying on the idea that the primitive ontologies of the GRW interpretation – the *mass density ontology* (GRWm) but especially Bell's *flash ontology* (GRWf) [10] – and PWT are very similar to the classical ones. This similarity also seems to help solving the other problems of the PWT and GRW interpretations. But for PWT to work, the wavefunction governing the motion of the particles has to be itself well localized around the points of the configuration space, otherwise the macroscopic objects like chairs and tables, expected to be stable, can explode or transform into completely different configurations. Also, in the GRW interpretation, to avoid the tails of the wavefunction (which gives the probability density for the spontaneous localization) from allowing the next collapses to lead to abrupt changes in the way the world appears at the macro level, the wavefunction has to decohere as well and its branches have to become well localized in the configuration space. Therefore, the MWI Problem 2 applies to PWT and GRW as well. Decoherence and the localized description of the atom from Observation 1 are needed by PWT and GRW as much as it is needed by MWI.

But an important lesson that can be learned from their arguments is that classical physics is clearer, and so any interpretation of quantum mechanics that is closer to classical physics has an important advantage.

This suggests the following heuristic rule

Rule of Thumb 1. If a solution is considered to work without problems in classical physics, and if it can be applied to an interpretation of quantum mechanics, it should also be considered to work without problems in that interpretation of quantum mechanics.

We can see that the MSTI Answer 1 already aligns MWI to this Rule of Thumb, except that the classical objects are fields and not point-particles. But it is hard to see how the beables localized to points in space from PWT and GRWf continue to remain as such once we advance these interpretations to quantum fields.

It is therefore desirable to have a solution of Problem 2 along the Rule of Thumb 1 as in the PWT and GRW interpretations, but without adding more structure. Background freedom automatically makes this possible.

The representation of the wavefunction in terms of fields on the 3d-space given in [59] is too dependent on nonrelativistic quantum mechanics, even if it works for the Fock space of quantum field theory too. In the case of quantum gravity, it works only if the theory is background-dependent.

But the wavefunctional formulation allows for a simpler and more adequate answer:

Observation 2. On each 3d-space (Σ, γ), since $|\Psi\rangle_\gamma := \langle\gamma|\Psi\rangle = \int_{\mathcal{C}_M^\gamma} c_{\phi_\gamma} |\phi_\gamma\rangle \mathcal{D}\mu_M^\gamma[\phi_\gamma]$, it automatically consists of many classical fields $\phi_\gamma \in \mathcal{C}_M^\gamma$, each of them having attached a complex number c_{ϕ_γ}. But since in the polar form $c_{\phi_\gamma} r[\phi_\gamma] e^{i\theta[\phi_\gamma]}$ the factor $e^{i\theta[\phi_\gamma]}$ can be removed by replacing ϕ_γ with its gauge transformed $e^{i\theta[\phi_\gamma]}\phi_\gamma$ (§IV B), and since $r[\phi_\gamma]$ is just a density factor (§IV C), what we have is just a collection of independent classical fields. States like those containing atoms, molecules, or larger quasi-classical objects are linear combinations of this form, because the Fock space basis can be obtained from the wavefunctional classical basis [23].

Observation 3. In the wavefunctional formulation, the Hamiltonian is explicitly local on Σ [23].

MSTI Answer 2 (Objects in space)**.** The 3d-space states consist of classical fields on the 3d-space. The dynamics is local. Linear combinations of such states form MWI worlds only if their dissociation is not manifest at the macro level.

C. Macro world. Quasi-classicality as classicality

Another important problem is the following

MWI Problem 3 (Macro world)**.** How does the classical-looking macroscopic world emerge from the wavefunction?

Often, Problem 3 is considered solved by decoherence [28, 31, 75], which appeared in the first place to solve it.

Without denying its importance, the process of dissociation strengthens the idea of decoherence, by introducing a notion of "absolute decoherence", and solves the problem in accord with the Rule of Thumb 1.

MSTI Answer 3 (Macro world). Each macro world corresponds to multiple ontic 3d-space states that belong to the same macro-state, because they are not distinguishable at the macro level.

Multiple 3d-space states can form macro states (Assumption 2). Since each 3d-space state is also quasi-classical, and since they are not distinguished by the macro projectors, they can account for the macro world.

D. Branching asymmetry from Big-Bang symmetry

Another problem is the following

MWI Problem 4 (Branching asymmetry). Why is the branching happening only towards the future, and why do the branches remain separated?

This is also often claimed to be solved by decoherence, but since the Schrödinger equation is time-symmetric, without very fine-tuned initial conditions of the universe, decoherence would equally predict branching towards the past. Interference with branches previously separated would affect the probabilities and violate the Born rule.

In the standard framework of the many-worlds interpretation, Wallace acknowledged the problem of branching asymmetry, analyzed it, and concluded that it correlates with the *thermodynamic arrow of time* [72]. For the importance of the thermodynamic arrow of time in relation to branching asymmetry, but also for unforeseen complications, see [64]. But we do not have an explanation for the thermodynamic arrow of time either, although the second law of thermodynamics is a well-established empirical fact.

The dissociation into 3d-space states allows us to make progress, by relating branching asymmetry with the *cosmological arrow of time*. The cosmological arrow of time points from the Big-Bang to the direction of time in which the universe expands. The closer the state of the universe is to the Big-Bang, the more homogeneous and isotropic the universe is. Moreover, as the singularity is approached, the 3d-space contracts.

A possible assumption about this is that it contracts to a point, which is the singularity. This would be problematic, since if Σ is a lower-dimensional set at $t = 0$, we will need to explain how it evolves into a 3d manifold. The singularity has to be removed from spacetime.

An alternative option is that the 3d-space components of the metric tensor tend to 0 as $t \searrow 0$, but the topology of space does not contract to a point, it is still the 3d manifold $(\Sigma, \gamma_{ab}(\boldsymbol{x}) \equiv 0)$. By avoiding the assumption that the topology derives from distance, we can obtain

equations for general relativity that continue to be valid under more general conditions. For this we need an alternative formulation of semi-Riemannian geometry and Einstein's general relativity, which is equivalent to these ones outside the singularity, but well-defined and free of infinities at the singularity. This was achieved and shown to work in many situations in which non-singular semi-Riemannian geometry is not defined [52, 54, 57]. Moreover, when it is valid at the Big-Bang, it automatically satisfies Penrose's *Weyl curvature hypothesis*, whose motivation was in the first place to connect the cosmological and the thermodynamic arrows of time [40, 53].

Then, there is only one possible 3d-space state at the Big-Bang singularity, $(\Sigma, \gamma_{ab}(\boldsymbol{x}) \equiv 0)$. The condition of validity of *singular general relativity* from [54] also requires that the matter fields are homogeneous in it [55].

As $t \searrow 0$ the system may be chaotic, as in the Mixmaster model [35] or the Belinski–Khalatnikov–Lifshitz model [9]. Then, while at the singularity there is still only one possible 3d-space state, it can be approached in different ways as $t \searrow 0$. However, the limit $\gamma \to 0$ forces the solutions to depend on a small number of parameters as they converge to the unique 3d-space $(\Sigma, \gamma_{ab}(\boldsymbol{x}) \equiv 0)$.

This severe constraint of the initial conditions for (Σ, γ_{ab}) implies that the branching structure of the wavefunctional is extremely asymmetric in time. This suggests a possible reason why, at macro scales, branching happens only towards the future.

MSTI Answer 4 (Branching asymmetry). Branching happens only towards the future because at the Big-Bang the 3d-space states have a very small number of degrees of freedom and all converge to the same initial 3d-space state $(\Sigma, \gamma_{ab}(\boldsymbol{x}) \equiv 0)$.

This answer is of course incomplete. We do not know why the initial state had to be the Big-Bang, and it is not even sure that there was a singularity, many researchers think that quantum gravity will be able to remove it. But singular general relativity suggests that singularities are not necessarily ill-behaved, they can even solve notorious problems in quantum gravity [56, 58].

E. Probabilities from continuity

When a quantum measurement is made, the probability to obtain a certain outcome is given by the Born rule to be the square of the projection of the state vector. Different outcomes may therefore have different probabilities. However, in MWI, there is only one branch for each of these outcomes. A direct counting argument implies that all outcomes should be obtained with the same probability, contrary to the Born rule. Everett proposed that somehow the squared amplitude of the branch gives the probability that the observer ends out being the observer from that branch.

MWI Problem 5 (Probabilities). Why are the probabilities proportional to the squared amplitudes of the branches?

There are various proposed solutions, based on many-minds [1], decision theory [16, 71], measure of existence [66] *etc.* For a review see [69]. Proposals that somehow the amplitude of a branch yields probability have merits and led to interesting insights into the nature of probability [72]. But if probabilities could be obtained in the old-fashioned way, for example by branch counting (Saunders advocates this [47]) or ideally as *the ratio of the number of favorable outcomes to the total number of possible outcomes*, the result would be more palatable, without necessarily contradicting other proposals.

These proposals are well justified, but it may help to have a solution based on micro-state or micro-branch counting, according to the Rule of Thumb 1. A micro-branch counting method would, preferable, rely on an absolute notion of micro-branches. MSTI does just this:

MSTI Answer 5 (Probabilities). "Counting" 3d-space states allowed to have an inhomogeneous density gives probabilities proportional to the squared amplitudes. Counting each 3d-space state is justified by the fact that only those states have local beables, see §VI G.

This may provide a concrete realization of other proposals, without necessarily rejecting them.

F.   Real wavefunction

The Rule of Thumb 1 also suggests the following

MWI Problem 6 (Real-number-based probabilities). It is true that the norm of the (complex) wavefunction is real. But is there a deeper reason why we get real probabilities?

MSTI suggests a solution according to the Rule of Thumb 1 for this too:

MSTI Answer 6 (Real-number-based probabilities). The wavefunctional is real, and the phases only represent a global U(1) gauge choice for the classical fields in the 3d-space states. The real coefficients are the densities of the 3d-space states composing the wavefunctional.

G.   Ontology: a real wavefunction

Sometimes it is said that the wavefunction does not have "enough ontology", that it lacks local beables (in the 3d-space).

MWI Problem 7 (Ontology). What is the ontology of MWI? What are the local beables?

A primitive ontology similar to that of GRWm (see §VI B) can work for MWI [2], by using as local beables

the mass or charge density, solution originally proposed by Schrödinger to interpret the wavefunction. But it can be argued that this adds new ontology besides the wavefunction, and that the same can be said if we take the center of mass of the atoms as the local beables in MWI.

Some researchers consider that the abstract state vector and the Hamiltonian are sufficient to specify the ontology of MWI, and from it one can derive an essentially unique 3d-space, the tensor product structure, the preferred basis, and all there is to be known about the universe [14]. But if any of these structures can be derived from the state vector and the Hamiltonian, infinitely many other physically distinct solutions exist [61]. So a preferred basis or another structure should exist.

And indeed, other researchers consider that not merely the abstract state vector is needed, but the wavefunction, as a state vector expressed in the 3d-space, and this is sufficient to specify the complete ontology [67, 68].

Despite this, researchers like Maudlin [32–34] and Norsen [36] consider that MWI does not have a primitive ontology in terms of local beables.

But in every micro-world in MSTI there are local beables, just like in classical physics.

MSTI Answer 7 (Ontology). The 3d-space states are the *ontic states* of the theory. Each of them consists of a 3d-space (Σ, γ), on which classical fields ϕ_γ are defined. Each ontic state appears at most once in the composition of the wavefunctional, with a nonuniform density. The distribution gives the real wavefunctional, and a global gauge gives the phase of each term in the wavefunctional. The local beables are the classical fields ϕ_γ and γ defined on the 3d manifold Σ, so they are defined only for 3d-space states. The 3d-space states, having definite local beables, correspond to (micro-)worlds. This justifies counting them to obtain the probabilities.

Therefore, local beables exist, and the Rule of Thumb 1 was followed. What can be more classical than the classical itself?

VII.   DISCUSSION

It is uncommon to use the wavefunctional formulation of quantum field theory in the interpretation of quantum mechanics. It is more common to take nonrelativistic quantum mechanics as a benchmark for these interpretations. But the wavefunctional formulation is natural too, if not even more natural, and it is more realistic.

Observation 4. When we perform a quantum measurement of a smaller system, we never observe directly its state, only the pointer state of the apparatus, which is macroscopic. A measuring device is dedicated to a particular location and type of quantum field (or subsystem in general), not to a particular particle (or subsystem). The result of any measurement translates into a change in the macro-state of the universe. All these are described adequately by the wavefunctional of the entire universe.

Wheeler and Everett considered MWI as *the* interpretation of quantum mechanics that is suitable for quantum gravity [8, 13]. According to DeWitt [18], p. 1141:

> Everett's view of the world is a very natural one to adopt in the quantum theory of gravity, where one is accustomed to speak without embarrassment of the 'wave function of the universe.' It is possible that Everett's view is not only natural but essential.

Here, we have seen that background free quantum gravity solves some foundational problems of quantum mechanics, and especially of MWI. It even suggests a version of MWI, which is MSTI, as the more natural interpretation of quantum mechanics. The relation between quantum gravity and MWI is therefore reciprocal.

Finally, I argued that MSTI solves some of the main problems of standard quantum mechanics and MWI. The strategy to make this interpretation more palatable was to highlight similarities with classical physics, based on the Rule of Thumb 1. It turns out that, except for the existence of a multiplicity of worlds, MSTI is a more classical-like version of MWI, with respect to the appearance of classicality, the existence of local beables, the probabilities, and even the understanding of the complex numbers inherent to the theory.

REFERENCES

[1] D.Z. Albert and B. Loewer. Interpreting the many worlds interpretation. *Synthese*, pages 195–213, 1988.

[2] V Allori, S Goldstein, R Tumulka, and N Zanghì. Many worlds and Schrödinger's first quantum theory. *British Journal for the Philosophy of Science*, 62(1):1–27, 2010.

[3] R. Arnowitt, S. Deser, and C. W. Misner. The dynamics of general relativity. In L. Witten, editor, *Gravitation: An Introduction to Current Research*, pages 227–264, New York, U.S.A., 1962. Wiley, New York.

[4] A. Ashtekar. New variables for classical and quantum gravity. *Phys. Rev. Lett.*, 57(18):2244–2247, Nov 1986.

[5] Abhay Ashtekar and Eugenio Bianchi. A short review of loop quantum gravity. *Rep. Prog. Phys.*, 84(4):042001, 2021.

[6] G. Bacciagaluppi and A. Valentini. *Quantum theory at the crossroads: reconsidering the 1927 Solvay Conference.* Cambridge University Press, 2009.

[7] J.A. Barrett. *The quantum mechanics of minds and worlds.* Oxford University Press, Oxford, 1999.

[8] J.A. Barrett and P. Byrne. *The Everett interpretation of quantum mechanics: Collected works 1955-1980 with commentary.* Princeton University Press, 2012.

[9] Vladimir A Belinskii, Isaac M Khalatnikov, and Evgeny M Lifshitz. Oscillatory approach to a singular point in the relativistic cosmology. *Advances in Physics*, 19(80):525–573, 1970.

[10] J.S. Bell. Are there quantum jumps? In *Speakable and Unspeakable in Quantum Mechanics*, pages 201–212. Cambridge University Press, 2004.

[11] D. Bohm. A suggested interpretation of quantum mechanics in terms of "hidden" variables, I & II. *Phys. Rev.*, 85(2):166–193, 1952.

[12] D. Bohm. *Causality and chance in modern physics.* Routledge, London, 2004.

[13] P. Byrne. *The many worlds of Hugh Everett III: Multiple universes, mutual assured destruction, and the meltdown of a nuclear family.* OUP Oxford, UK, 2010.

[14] S.M. Carroll and A. Singh. Mad-dog Everettianism: Quantum Mechanics at its most minimal. In A. Aguirre, B. Foster, and Z. Merali, editors, *What is Fundamental?*, pages 95–104. Springer, 2019.

[15] D.C. Dennett. Real patterns. *The Journal of Philosophy*, 88(1):27–51, 1991.

[16] D. Deutsch. Quantum theory of probability and decisions. *P. Roy. Soc. A-Math. Phy.*, 455(1988):3129–3137, 1999.

[17] D. Deutsch. Apart from universes. In S. Saunders, J. Barrett, A. Kent, and D. Wallace, editors, *Many worlds?: Everett, quantum theory, & reality*, pages 542–552, Oxford, 2010. Oxford University Press.

[18] B.S. DeWitt. Quantum theory of gravity. I. The canonical theory. *Phys. Rev.*, 160(5):1113, 1967.

[19] H. Everett. "Relative state" formulation of quantum mechanics. *Rev. Mod. Phys.*, 29(3):454–462, Jul 1957.

[20] H. Everett. The Theory of the Universal Wave Function. In *The Many-Worlds Hypothesis of Quantum Mechanics*, pages 3–137. Princeton University Press, 1973.

[21] A. Fine and H.R. Brown. The shaky game: Einstein, realism and the quantum theory. *Am. J. Phys.*, 56:571, 1988.

[22] G.C. Ghirardi, A. Rimini, and T. Weber. Unified dynamics of microscopic and macroscopic systems. *Phys. Rev. D*, 34(2):470–491, 1986.

[23] Brian Hatfield. *Quantum field theory of point particles and strings.* CRC Press, 2018.

[24] D. Howard. "Nicht Sein Kann was Nicht Sein Darf," or the Prehistory of EPR, 1909–1935: Einstein's early worries about the quantum mechanics of composite systems. In *Sixty-two years of uncertainty*, pages 61–111. Springer, 1990.

[25] C.J. Isham. Canonical quantum gravity and the problem of time. In A. Ibort and M.A. Rodríguez, editors, *Integrable systems, quantum groups, and quantum field theories*, volume 409, pages 157–287. Kluwer Academic Publishers, 1993.

[26] R. Jackiw. Analysis on infinite-dimensional manifolds – Schrödinger representation for quantized fields. In O.J.P Éboli, M. Gomes, and A. Santoro, editors, *Field theory and particle physics*, Singapore, 1988. World Scientific.

[27] C. Kiefer. *Quantum gravity.* Oxford University Press, Oxford, UK, 3rd ed. edition, 2012.

[28] C. Kiefer, editor. *From Quantum to Classical: Essays in Honour of H.-Dieter Zeh*, volume 204, Cham, Switzerland, 2022. Springer Nature.

[29] K.V. Kuchař. Time and interpretations of quantum gravity. *Int. J. Mod. Phys. D.*, 20(supp01):3–86, 2011.

[30] R. Loll. Quantum gravity from causal dynamical triangulations: a review. *Class. Quant. Grav.*, 37(1):013002, 2019.

[31] M. M. Schlosshauer. *Decoherence and the Quantum-To-*

Classical Transition. Springer, Berlin, 2007.

[32] T. Maudlin. Can the world be only wavefunction. In *Many worlds?: Everett, quantum theory, & reality*, pages 121–143. Oxford University Press, Oxford, 2010.

[33] T. Maudlin. Critical study: David Wallace, The emergent multiverse: Quantum theory according to the Everett Interpretation. *Noûs*, 48(4):794–808, 2014.

[34] T. Maudlin. *Philosophy of physics: Quantum Theory*, volume 33 of *Princeton Foundations of Contemporary Philosophy*. Princeton University Press, Priceton, 2019.

[35] C.W. Misner. Mixmaster universe. *Phys. Rev. Lett.*, 22(20):1071, 1969.

[36] T. Norsen. *Foundations of Quantum Mechanics*. Springer, Switzerland, 2017.

[37] J. Norton. The hole argument. In *PSA: Proceedings of the biennial meeting of the Philosophy of Science Association*, pages 56–64. Cambridge University Press, 1988.

[38] D.N. Page and W.K. Wootters. Evolution without evolution: Dynamics described by stationary observables. *Phys. Rev. D*, 27(12):2885, 1983.

[39] R. Penrose. Angular momentum: an approach to combinatorial space-time. In *Quantum theory and beyond*, pages 151–180, Cambridge, 1971. Cambridge University Press, Cambridge.

[40] R. Penrose. Singularities and time-asymmetry. In S.W. Hawking and W. Israel, editors, *General relativity: an Einstein centenary survey*, pages 581–638. Cambridge University Press, 1979.

[41] R. Penrose. Gravity and state vector reduction. In R. Penrose and C.J. Isham, editors, *Quantum concepts in space and time*, pages 129–146, Oxford, 1986. Clarendon Press.

[42] R. Penrose. On gravity's role in quantum state reduction. *Gen. Relat. Grav.*, 28(5):581–600, 1996.

[43] Przibram, K. (ed) and Klein, M.J. (trans). *Letters on Wave Mechanics: Schrödinger, Plank, Einstein, Lorentz*. Philosophical Library, New York, 1967.

[44] T. Regge. General relativity without coordinates. *Il Nuovo Cimento (1955-1965)*, 19(3):558–571, 1961.

[45] C. Rovelli and L. Smolin. Loop space representation of quantum general relativity. *Nucl. Phys. B*, 331(1):80–152, 1990.

[46] C. Rovelli and L. Smolin. Spin networks and quantum gravity. *Phy. Rev. D*, 52(10):5743, 1995.

[47] S. Saunders. Branch-counting in the Everett interpretation of quantum mechanics. *Proc. Roy. Soc. London Ser. A*, 477(2255):20210600, 2021.

[48] E. Schrödinger. *The Interpretation of Quantum Mechanics: Dublin Seminars (1949-1955) and Other Unpublished Essays*. Ox Bow Press, Woodbridge, Connecticut, USA, 1995.

[49] Lee Smolin. The case for background independence. In D. Rickles, S. French, and J.T. Saatsi, editors, *The structural foundations of quantum gravity*, pages 196–239, Oxford, 2006. Clarendon Press.

[50] R.D. Sorkin. Spacetime and causal sets. *Relativity and gravitation: Classical and quantum*, pages 150–173, 1990.

[51] J. Stachel. The hole argument and some physical and philosophical implications. *Living Reviews in Relativity*, 17(1):1–66, 2014.

[52] O.C. Stoica. Beyond the Friedmann-Lemaître-Robertson-Walker Big Bang singularity. *Commun. Theor. Phys.*, 58(4):613–616, 2012.

[53] O.C. Stoica. On the Weyl curvature hypothesis. *Ann. Phys.*, 338:186–194, 2013. arXiv:gr-qc/1203.3382.

[54] O.C. Stoica. *Singular General Relativity – Ph.D. Thesis*. Minkowski Institute Press, 2013. arXiv:math.DG/1301.2231.

[55] O.C. Stoica. Gauge theory at singularities. *Preprint arXiv:1408.3812*, 2014.

[56] O.C. Stoica. Metric dimensional reduction at singularities with implications to quantum gravity. *Ann. Phys.*, 347(C):74–91, 2014.

[57] O.C. Stoica. The Friedmann-Lemaître-Robertson-Walker big bang singularities are well behaved. *Int. J. Theor. Phys.*, 55(1):71–80, 2016.

[58] O.C. Stoica. Revisiting the black hole entropy and the information paradox. *AHEP*, 2018, 2018.

[59] O.C. Stoica. Representation of the wave function on the three-dimensional space. *Phys. Rev. A*, 100:042115, 10 2019.

[60] O.C. Stoica. Standard quantum mechanics without observers. *Preprint arXiv:2008.04930*, 2020.

[61] O.C. Stoica. 3d-space and the preferred basis cannot uniquely emerge from the quantum structure. *Preprint arXiv:2102.08620*, pages 1–18, 2021.

[62] O.C. Stoica. Why the wavefunction already is an object on space. *Preprint arXiv:2111.14604*, pages 1–9, 2021.

[63] O.C. Stoica. Conservation laws + unitarity = one-worlds. *To appear on arXiv*, 2022.

[64] O.C. Stoica. Does quantum mechanics requires "conspiracy"? *Preprint arXiv:2209.13275*, 2022.

[65] O.C. Stoica. Versatility of translational quantum dynamics. *Preprint arXiv:2204.01426*, 2022.

[66] L. Vaidman. Probability in the many-worlds interpretation of quantum mechanics. In Y. Ben-Menahem and M. Hemmo, editors, *Probability in physics*, volume XII, pages 299–311. Springer, 2012.

[67] L. Vaidman. All is ψ. In *J. Phys. Conf. Ser.*, volume 701, page 012020, 2016.

[68] L. Vaidman. Many-Worlds Interpretation of Quantum Mechanics. In E.N. Zalta, editor, *The Stanford Encyclopedia of Philosophy*. Metaphysics Research Lab, Stanford University, 2018.

[69] L. Vaidman. Derivations of the Born rule. In M. Hemmo and O. Shenker, editors, *Quantum, Probability, Logic: The Work and Influence of Itamar Pitowsky*, pages 567–584. Springer, 2020.

[70] J. von Neumann. *Mathematical Foundations of Quantum Mechanics*. Princeton University Press, 1955.

[71] D. Wallace. Quantum probability and decision theory, revisited. *Preprint arXiv:quant-ph/0211104*, 2002.

[72] D. Wallace. *The emergent multiverse: Quantum theory according to the Everett interpretation*. Oxford University Press, 2012.

[73] E.P. Wigner. *Group Theory and its Application to the Quantum Mechanics of Atomic Spectra*. Academic Press, New York, 1959.

[74] E. Witten. Quantum background independence in string theory. *Preprint arXiv:hep-th/9306122*, 1993.

[75] W.H. Zurek. Emergence of the classical world from within our quantum universe. In Kiefer [28], pages 23–44.