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What spacetime does: ideal observers and (Earman's) symmetry principles

Adán Sus

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Abstract The interpretation and justification of Earman's symmetry principles (stating that any spacetime symmetry should be a dynamical symmetry and vice-versa) are controversial. This is directly connected to the question of how certain structures in physical theories acquire a spatio-temporal character. In this paper I address these issues from a perspective (arguably functionalist) that relates the classical discussion about the measurement and geometrical determination of space with a characterization of the notion of dynamical symmetry in which its application to subsystems that act as measuring devices plays an essential role. I argue that in order to reformulate and justify Earman's principles, and to provide a general account of the chronogeometrical character of some structures, the existence of a coordination between two notions of congruence, one mathematical and one dynamical, must be assumed for the interpretation of physical theories. This coordination provides the basis on which we can understand spacetime in physical theories as the codification (representation) of certain features of the access ideal observers have to experience.

1 Introduction

Many recent discussions of spatio-temporality in physical theories consider the idea of spacetime not being ontologically fundamental. Moreover, some recent proposals take a functionalist perspective and regard structures as spatio-temporal in virtue of them playing certain roles in our physical theories.

Adán Sus
Dept. of Philosophy
University of Valladolid
Tel.: Fax:
E-mail: adansus@uva.es

These are often appealed to when considering questions about the emergence of spacetime and are also referred to in discussions on the interpretation of symmetries.[1] If spacetime is as spacetime does, as the functionalist mantra is sometimes put,[2] the question is: What does spacetime do? Or to be more explicit: What roles do spatio-temporal notions play in our physical theorizing and what consequences can we extract from those roles for specific questions regarding the status of spacetime symmetries, in particular, and the interpretation of spacetime theories in general? Different answers to this central question are possible, with Knox' inertial frame functionalism probably being the most commonly discussed, and there seems to be a general feeling that there is no single function that is apt to be used to identify spacetime generally.[3] In this paper, I defend a particular answer to this central question which is linked to a general strategy that allows us to dispel some problems that have worried philosophers of physics for the last few decades: those related to the origin of the relation between spacetime and dynamical symmetries.

Functionalism can be seen as a way of framing one of the most important questions in the interpretation of physical theories: the question about the conditions/criteria for certain (mathematical) structures to be considered spatio-temporal. This, ultimately, in a more ontological fashion is the question about how space and time are represented in physics. A general (natural) scheme adopted to tackle the problem consists of thinking that the relation must come from some common element present both in how the metric of spacetime (and any other spatio-temporal structures) is determined and in some general conditions for the formulation of the laws that describe the dynamics. A traditional answer has to do with noting the fact that the chrono-geometry of the metric of spacetime is determined through the operations of measuring physical/dynamical systems like rods and clocks. This hint (as Weatherall notes [40]) is also the original inspiration for the so-called dynamical approach to relativity.

The general perspective I have just alluded to, which embraces an interpretive core according to which determination of the metric (the chronogeometrical significance of the metric) is dynamical, seems to have become obscured at times in recent debates. Nonetheless, it is always there, lurking in the wings. Take, for instance, the recent debate concerning the two primary perspectives on the relation between spacetime and dynamics in relativity theory: the geometrical approach (GA) and the dynamical approach (DA). Although some efforts have been made to play down the differences in this dispute ([40], [29]), it is often understood in an extremely stylized and highly formalistic fashion. In such interpretations of the dispute, the GA is seen as assuming that

[1] For a general introduction to the different contexts of spacetime functionalism see [9]. In that same volume, different authors engage with spacetime functionalism in relativity and quantum gravity.

[2] See [22].

[3] See [20] and [21] for the original proposal; [31] for a critical appraisal; [3] also offers a criticism of Knox' kind of functionalism and defends the idea that spacetime should be treated as a cluster concept.

some structures are primitively spatio-temporal and that they somehow constrain the dynamics, while the DA takes certain features of the dynamical laws to be primitive and it is these that eventually define some structures as being spatio-temporal. Undoubtedly, in any reasonable understanding of the two perspectives, in the characterization of their different starting positions, a common reference to the role of rods and clocks must be acknowledged. But despite the fact that this common ground can be seen as containing the seeds of their mutual relation, the discussion tends to forget this dimension. An example of this, different from the aprioristic version of the GA I have just given is the defence of the DA that takes the coincidence of the symmetries of all the matter laws as a brute fact (a 'miracle') and considers their relation to spacetime symmetries to be analytical or definitional.[4] What happens in both cases, it can be argued, is that certain relevant features of spacetime and dynamics are first separated from their physical origin and then a question about how one of them explains or can be reduced to the other is posed. We encounter this together with a tendency to frame the discussion only in terms of the formal structures of theories without explicitly considering how such structures, according to some assumed interpretation of the theory, are supposed to come into contact with actual experience.

Let me focus on why I think that the DA, even if correctly embracing the dynamical origin of spacetime structures in physical theories, falls short of providing a fully satisfactory account of the relation between spacetime chronogeometry and dynamics. Put simply, I maintain that the declared aim of the DA, "to account for the chronogeometry of metric structure..." ([8], p. 9), cannot be achieved within a version of the approach in which the coincidence of the spacetime symmetries and the dynamical symmetries of a theory is taken to be analytical or definitional. This analytical version of the DA[5] may be a simplification that does not do justice to a more sophisticated version, but it does indicate that we need some account of how, starting from the assumption of certain symmetries of the dynamical laws, we arrive at spacetime symmetries. This is usually completed in the DA by appealing to the strong equivalence principle (SEP), which imposes the condition that the local symmetries of matter laws must be such and such, together with a functionalist perspective (in particular, inertial frame functionalism) that would identify some features of certain structures—those that determine the local inertial frames—with spacetime.

This is problematic for different reasons. Weatherall ([40]) mentions the difficulty of arriving at a formulation of the SEP that would allow us to identify which are the relevant symmetries of the equations, and also the question of whether this is sufficient for us to recover spacetime as we understand it. In any case, what the DA seems to be lacking is an elucidation—not some postulation—of the connection between the dynamical symmetries (again,

[4] For a defence of the 'miracles' view, see [30], [28], [29]; for a critical perspective on it: [36], [40].

[5] Myrvold [25] provides an explicit defence of this perspective, while [2], [21], [30] can also be taken as partially endorsing such a view. See [36] for a critical take on this.

which symmetries?) and what we call spacetime symmetries. This problem also affects Knox' functionalist extension of the DA: if you define spacetime by the role that structures play in determining local inertial frames, and assume a version of SEP that declares that locally the symmetries of the laws of matter coincide with those of the metric thus determining local inertial frames, then you entrust all the functionalist work to SEP. But then you certainly have a problem: you are assuming that the notion of dynamical symmetry can be defined uncontroversially without any previous determination of spatio-temporal structures and that you can then define spacetime symmetries from those dynamical symmetries. This might be all right if an independent account of dynamical symmetry is provided. But what may well actually be going on in such approaches, is that the notion of spacetime in sneaked in through the back door via some implicit reference to rods and clocks. What is needed is an explicit connection between the notion of rods and clocks and dynamical symmetry. The use of such ellipsis must stop at some point!

We need then criteria to identify which dynamical symmetries define spacetime ones. My general (functionalist) perspective is based on the following: it is precisely because some structures play the role of codifying the ways in which we (ideal observers) gain access to empirical content, which is implicit in using certain systems to probe spacetime, that we identify some dynamical symmetries as spacetime symmetries and therefore some structures as spatio-temporal.[6]

What this initial take assumes is that the spatio-temporal character of some structures in a physical theory is derived from the fact that we can interpret them as encoding the structural (formal) characteristics of the way observers gain access to the empirical world. However, the approach that I adopt in this paper can be read in a more down-to-earth way. I will demonstrate that some features in the characterization of (systems that act as) measuring devices are such that they can be (and have been) naturally interpreted as being spatio-temporal. This being the case, it is not too far removed to take a general characterization of measuring devices as a putative abstract representation of an ideal observer, and then to see the spacetime role as the codification of some general features of idealized observers. Wherever one starts, the key point of my analysis is the connection between certain general features of the behaviour of measuring devices and some structures that can be taken to be part of the formal determination of spacetime.

So, the plan for the rest of this paper is as follows. In Section 2, I present a general overview of the framework of my proposal. Then I introduce the so-called problem of space (Section 3) and the discussion of the interpretation

[6] This can be read as a functionalist extension of the DA, in line with what [21] proposes. But apart from the question regarding its functionalist character, in order to give a definite answer to the question regarding the relation between the two types of symmetries, the DA must provide an account that goes beyond the general claim that spacetime symmetries are dynamical. Be that as it may, the aim of this paper is not to develop a spacetime functionalism alternative to Knox' version (I leave that for a different paper) but to offer a precise strategy that enables us to justify the relation between spacetime symmetries and dynamical symmetries.

of dynamical symmetries in which the treatment of subsystems plays a central role (Section 4). After that I bring the two discussions together to offer a justification of Earman's principles (Section 5), including (Section 5.1) a discussion of the consequences that this understanding of those principles has for the interpretation of SR and its differences with Lorentz' theory. I finish with some conclusions (Section 6).

2 Spacetime symmetries, dynamical symmetries and observers

What is the origin of the relation between spacetime symmetries and the symmetries of dynamical laws? If one rejects the possibility of it being a simple definitional relation, and in the previous section I have provided reasons to do so, then this question is in need of an answer. In this paper I propose one. A simple way to state the underlying general motivation for my response is the following: the justification for such a relation is connected to how, according to a given interpretation, a physical theory is taken to represent ideal observers. However, an explicit representation of observers is nowhere to be found in spacetime theories, so it might initially seem to be a dubious strategy to refer to one in order to justify the relation between two, in principle, uncontroversial features of the theory: its spacetime and dynamical symmetries. Perhaps the claim I support can be understood in these less contentious terms: certain elements in the formulation of spacetime theories, in particular those that allow us to interpret some structures as spatio-temporal and some symmetries as dynamical in a physically relevant way, can be understood as traces of the implicit representation of ideal observers. Spacetime, then, from this perspective, will be identified with certain structures that can be interpreted as playing the role of codifying formal features of the access observers have to experience.[7] The expected benefits of this approach are that, understood in this manner, we have a natural justification for the relation between spacetime and dynamical symmetries. Precisely the extent to which this is a faithful presentation of things can only be decided after an explication of such a relation has been given in detail.

Let me advance the general features to be developed in the rest of the paper. As mentioned above, there is a venerable approach to the nature of spacetime in physical theories that links its determination to the standard operation of rods and clocks. In general, one can say that the empirical determination of physical chronogeometry will always involve some procedures governed by certain dynamics and therefore constrained by some principles. On the other hand, we also have physical principles that are implicit in the codification of some specific processes through which it is assumed that empirical content is

[7] I am sure it will not escape the reader that this general approach has many precedents in the history of philosophy and its spirit can be linked to (neo-)Kantian approaches to spacetime. Without wanting to deny these links, I think that it is important to evaluate the merits of the specific proposal I defend in this paper without recourse to its historical connections. Such an evaluation should depend only on how the proposal deals with the question of the relations between spacetime and dynamical symmetries.

acquired or, in other words, in the descriptions of measuring apparatus. And finally, we may consider that this characterization also imposes constraints on the dynamics of matter, as described by the theory. So, the basic assumption here is that the dynamics of these two processes (determination of physical geometry and the empirical content that is evidence for a theory) can be taken to be the same if we interpret certain features of the theories as somehow codifying the role of idealized observers. From here, I will argue, we can derive a relation between spacetime symmetries and dynamical symmetries. It must be clear that this is not a version of the GA view in which geometry is taken to explain dynamics, but neither does it involve a reduction of spacetime symmetries to dynamical symmetries. Geometry, in this approach, is dynamically constructed, but at the same time it is recognized that this construction involves some principles which contain or imply general restrictions on matter dynamics. The existence of these constraints on the formulation of dynamics is a consequence of interpreting the constructions as being derived from the physical description of measuring devices (which might be interpreted as part of the codification of the empirical receptivity of idealized observers).

The key notion that technically bridges the two types of symmetries is that of congruence, which originated in geometry and has been ever-present in the debates about the true geometry of physical space motivated initially by the discovery of non-Euclidean geometries and then later by the eruption of relativity theory. I will argue that the same transformations (motions) that are part of the definition of the notion of congruence, and therefore can be interpreted as spacetime symmetries, from the point of view of the description of the dynamics of subsystems are symmetry transformations with features that make them ideal for the formulation of a dynamical notion of congruence. In particular, these transformations are unobservable from the interior of the subsystem but detectable because they change some quantities that encode relations between subsystems. Through the use of some technical machinery introduced by David Wallace,[8] this will become the basis for establishing the connection between spacetime symmetries and symmetries of the dynamics in physical theories (my main claim). It will also allow us to make the limits and conditions of such a relation explicit, and to tackle such a relation in the context of particular theories (a claim that would involve correcting some ideas about how to interpret the situation in some paradigmatic cases).

Through developing the details of this schematic presentation, I will also bring together two much discussed themes in spacetime theories. One is the determination of physical geometry that I have already mentioned, the so-called problem of space (PoS). The other is the observability of dynamical symmetries.

[8] This framework for the treatment of issues related to the interpretation of symmetries is developed by David Wallace in [37], [38].

3 The problem of space: the Helmholtz–Lie–Weyl theorem.

The question concerning which geometrical structures are suitable to be used to describe physical space and what justification can be given for this is generally referred in the literature as the problem of space (PoS). Even if it is obvious that formulated in this way the question only fully makes sense after the discovery of non-Euclidean geometries, much of the reflection that has occurred during the search for responses has its roots in the Kantian analysis of space and time as forms of intuition. Irrespective of whether Kant was aware of the challenge that the new geometries posed, his analysis of the notions of space and time has been highly influential in the different formulations of the PoS due to the fact that he placed the question of how to give an account of the physical/empirical validity of geometry centre stage. Furthermore, we must distinguish two stages in the history of the discussion: the classical pre-relativistic era, mainly carried out by mathematicians like Riemann, Helmholtz, Lie and Poincaré; and the relativistic stage, formulated mainly by Hermann Weyl.

There are a fair number of presentations of the history of the PoS in the literature.[9] My intention is not to repeat the story; although we will need a brief account to be able to focus on some aspects of the problem that I think are essential for my discussion and that perhaps have not been sufficiently stressed to date.

Even if Riemann can be considered the initiator of the classical formulation of the PoS, I will take some features of Helmholtz' approach as a reference to understand the dynamical dimension of the problem. The basic question that Helmholtz was trying to answer is: How can the geometry of physical space be determined? His answer is based on the idea that the measure of spatial geometry requires a notion of congruence for physical bodies and this, in turn, is made possible by the condition of free mobility of bodies. The notion of free mobility, as it is generally recognized, plays a central role in Helmholtz' conceptualization of the PoS. From this condition, Helmholtz claimed to derive the notion that the geometries that are able to represent physical space are those of constant curvature (although he originally excluded the Lobachevskian geometry). This result was rigorously derived later, through applying group theory, by Sophus Lie.[10]

The mathematical derivation of the conditions that the geometries (the metric) of physical spaces must satisfy if one assumes free mobility is one side of the problem. In fact, this comprises the purely mathematical part of the question: starting from a notion of congruence, which must be specified through the formulation of a number of axioms, one extracts the consequences for the geometries that are compatible with it. This part is what Lie perfects. But, one can argue, this makes up only half of the problem, at least as it seems to be understood by Helmholtz and, more importantly, if one wants to fully

[9] See [12], [33], [34] as some examples of recent philosophical approaches to this historical discussion.

[10] See [33] for an summary of Lie's contribution.

answer the question of the physical validity of geometry. So, in this case, one must also ask about the consequences of the attribution of a certain spatial (and temporal) geometry for the formulation of dynamical laws. To tackle this, it is necessary to reflect on the status of free mobility as a physical condition, in addition to the derivation of the formal restrictions on the metric.

We find the first seeds of this kind of reflection in [16]. There, Helmholtz' discussion about how the axioms of physical geometry are based on the notion of congruence, presupposes the possibility of moving solids without deformation. At the end of these considerations, he explicitly refers to the question of the mechanical principles that must be conjoined to the geometrical propositions in order for them to be more than mere definitions without empirical validity. He eloquently adds that without presuming such mechanical principles, the answer to the question regarding the geometry of physical space hides the presumption of a pre-established harmony between form and reality.[11]

Let me reformulate the core of Helmholtz' position. This can be done in the following way: in order to claim that the geometry of space is such and such, some mechanical principles are necessarily involved and these are involved in the functioning of the systems through which we gain empirical access to the geometry. In Helmholtz' case, the relevant physical systems are rigid bodies and the principles concern the independence of the mechanical properties of bodies and their interactions under certain physical operations (translations, rotations and so forth). The reason for this choice is that these are the systems that are involved in the empirical determination of spatial geometry. To go beyond these specific systems, we need to deepen and generalize the principle.

It is evident that Helmholtz' particular formulation of the PoS, linked to the notion of the free mobility of rigid bodies conceived as a procedure that measures spatial geometry, cannot withstand the progression to a relativistic context. To have a general scheme that is applicable to physical theories in this new scenario, two generalizations would be needed: the problem would need to be formulated in a way that can be interpreted as referring to measurements of spacetime metric; and it would need to be detached from the narrow, finite notion of a rigid body in a way that extends its validity to the infinitesimal domain. Weyl addresses this task in his development of a purely infinitesimal geometry around 1920. Although his reformulation of the PoS passes through different stages,[12] it seems clear that he understands that his approach is partly a generalization of the Helmholtz–Lie strategy that is now compatible with the theory of general relativity. In a stylized manner, we can present its main points as follows. The fundamental question that guides the enquiry is how to justify the notion that the metric which describes spacetime has a certain general form; in particular, the Pythagorean form. The strategy adopted to arrive at an answer consists of starting from a notion analogous to the congruence by free mobility in the Helmholtz–Lie problem, which is given by the definition of infinitesimal congruences at each point and for displace-

[11] [16], p. 17.
[12] See [34] for an account of this.

ments between infinitesimally close points.[13] Weyl realizes that it is necessary to define the congruence of displacements by introducing a metric connection that sets the standard of comparison between close points. The conditions that define such an infinitesimal notion of congruence are expressed in two postulates, named by Weyl the Principle of Freedom and the Principle of Coherence. The former can be understood as a principle of free mobility at each point, while the latter expresses the condition of compatibility between the metric connection and the affine connection. Finally, Weyl is able to prove a result which constrains the form of the metric. Glossing over many difficulties and subtleties, we can say that he arrives at the result (see [33]) that a metric satisfying the conditions of infinitesimal congruence,for which the metric connection uniquely determines the affine connection, has the form of a Weylian metric (a Riemmannian metric of fixed signature plus a metric connection) with Pithagorean line element.

We have here a general formal scheme that connects a mathematical notion of congruence with certain restrictions on the metric, which furthermore can be formulated in terms of a group of symmetry transformations. In a sense, these symmetry transformations can be interpreted as providing the definition of a notion of congruence through the specification of a mathematical group. (The reference to the infinitesimal structure in the case of Weyl's characterization introduces some problematic features that must be treated separately.) Now, in order for this metric to be considered a property of physical space(time), we should be able to interpret the congruence transformations as motions of physical systems which—despite the fact that in idealized form they are defined merely by the mathematical notion of congruence—insofar as they are taken as valid surveyors of the spacetime metric, must be governed by dynamical laws that satisfy certain constraints. This perspective thus has two questions at its core which must be answered in order to say something specific about spacetime and its relation to dynamics: Which chronogeometrical structure is determined by the assumptions of the idealized systems; and what constraints does such an idealization impose on the dynamics of the systems?

The first question, the mathematical part, is answered in the classical problem of space by Helmholtz and Lie through the proofs that free mobility, mathematically defined in a certain way, constrains the metric in such a way that it has to be of constant curvature. And for the infinitesimal case, it is answered by Weyl's generalization.

The second, dynamical part is more conspicuous in the problem of the physical validity of geometry. That it must always be taken into account is revealed by this simple fact: without it, we only have the definition of a mathematical structure with no claim concerning its physical relevance. Only by assuming that there are physical systems that fit the mathematical axioms, is this applicability endorsed. But the question that is rarely brought to the fore concerns the consequences that this has for the formulation of dynamics.

[13] [33], [12] provide very competent discussions of Weyl's position as developed at different stages but concentrating especially on the mature presentation delivered in his Barcelona lectures ([41]).

Helmholtz suggests, rightly I think, that these consequences can be formulated in terms of some symmetry principles that the dynamics must satisfy. Nonetheless, this demands a precise formulation. My intention is to provide this through the ensuing discussion of the notion of dynamical symmetry as applied to subsystems.

4 Dynamical symmetries and subsystems

A central aspect of the present approach to the issue of the relation between spacetime symmetries and dynamical symmetries is how, in a given theory, the procedures through which we acquire empirical content (that confirms/refutes the theory) are reflected. I assume that every physical theory, even if it does not have the resources to model measuring devices explicitly, must at least include some features whose interpretation can be linked to measuring procedures performed by ideal observers. This seems unavoidable when the models of a physical theory are taken to represent parts of the world that we experience. So, I must now turn to the question of how these measuring procedures are encoded in features of the formalism of the theory and what consequences this has for its symmetries.

As my starting point, I take a basic, minimal characterization of measuring as a physical process in which two different subsystems interact, with the result that the final state of one of them—the measuring device—can be taken as providing information on the state of the other—the target system—just before the measuring took place.[14] As I hope to show, from this extremely schematic characterization it is already possible to extract some general consequences for the definition of dynamical symmetries and their relation to spacetime symmetries for a theory whose interpretation incorporates such minimal modelling of measuring devices.

In order to do this, I must delve into the discussion about whether quantities that are variant under symmetry transformations are observable. A perspective on this issue developed by David Wallace that takes the role played by subsystems as central, will prove essential. In a series of works,[15] Wallace emphasizes that the answer to questions concerning the observability of symmetries are always linked to how the symmetry transformations behave when interpreted as being applied to subsystems. He develops a powerful framework to tackle the main problems in the interpretation of symmetries. I fully agree with this perspective. Wallace argues that the preponderance given to the behaviour of subsystems for the interpretation of symmetries stems from the usual treatment that physicist afford them. Moreover, I would add that the special role that subsystems play in the characterization of measuring de-

[14] I borrow this characterization from [37]. There might be questions about whether this characterization is fully general and includes measuring processes in quantum mechanics. If it is not and does not, then it would restrict us to the classical context. But this in itself is not bad insofar as we are able to state the limits of its application clearly.

[15] [37], [38], [39].

vices explains why the notions of symmetries that matter most in physics are connected to their interpretation in terms of the behaviour of subsystems.

So we have a general strategy to tackle some of the main issues concerning the interpretation of symmetries which, starting from a general formal characterization of the notion of dynamical symmetry (basically, a transformation that takes solutions to solutions), complements this sparse definition, which by itself seems unable to provide answers to questions about the representational capacity or the observability of symmetries, with the idea that such issues must be interpreted in the context of the application of symmetries to subsystems. In particular, in order to decide whether certain quantities that are variant under symmetry transformations (and therefore usually considered to be unobservable) are observable, one must look at how the symmetry extends from its application to a given subsystem to the interaction between that subsystem and its environment. Only in cases in which a symmetry transformation of a given subsystem is also a symmetry of the composition subsystem-plus-environment (and it is, using Wallace' terminology, extendible), can some variant quantities be observed despite the 'common wisdom' that only invariant quantities are observable.[16] Let me sketch Wallace' argument, as it introduces some elements that are extremely fruitful with regard to the relation between the characterization of measuring devices and judgements about the dynamical symmetries of a theory.

Wallace starts from the aforementioned notion of dynamical symmetry and assumes a physical description of a measuring device: a system that has a ready state that is independent of the target system and which, after interacting with it, ends up in a state that is a function of the pre-measurement state of that target system ([37], pp. 8-9). From this, it follows that a measurement that is internal to the system cannot detect whether a dynamical symmetry transformation has been performed. This is proof of what Wallace calls the Unobservability Thesis. Now, the interesting question is what happens to the measurements of quantities for systems that can be considered as external to the subsystem in which the measuring device is placed (that is, measurements external to the subsystem). This involves considering the device itself as a subsystem interacting with a target system that can vary independently of it. Wallace introduces the the following notation to express the combined state of the two subsystems: $(O, g; O', g')$, where O and O' are orbits of equivalent states under symmetry transformations of the target and measuring systems, respectively.[17] If the symmetry is extendible and global, it is possible to define the invariant quantity: $g'^{-1}g$. Now, assuming that the primed system is a measuring device as characterized above, and in particular that it meets the

[16] For the discussion on how to solve the puzzle of observability of variant quantities, apart from the cited work of Wallace, see [32], [11].

[17] This notation needs some clarification. We are assuming that we can define a state space S for each system. Dynamical symmetries can be defined as transformations such that $gx(t)$ is a solution of the equations of motions iff $x(t)$ is a solution. These transformations form a group and an orbit will be the equivalence class of states connected by symmetry transformations. By specifying an orbit and an element of the symmetry group, we can therefore identify the state of the system. See [37], p. 6.

condition of having a dynamics that is independent of the target system, then we can fix the quantity g' and realize that, because $g'^{-1}g$ covaries with g, it is possible to measure g. So, this amounts to an account of how a quantity that is variant under a dynamical symmetry transformation can be measured, if we are ready to interpret it as a relational quantity expressing some kind of target–device relation. The synopsis of this argumentation is that, for subsystem-global symmetries,[18] globally variant quantities are observable via measuring devices outside the system, but such observations can always be reinterpreted as observations of an invariant relation between system and measuring device (at least, this is so in the context of the theories that Wallace considers).

My intention now, as preparation for the next section, is to reverse Wallace' argument: instead of starting by assuming a given dynamics with a certain type of symmetry, as Wallace' does for the case of Newtonian particle mechanics, I will explore what can be inferred about the relation between the internal dynamics of a subsystem that acts as a measuring device and the dynamics of target systems measured by it, if one starts from just the general characterization of a device that measures some quantities of external target systems.

5 Earman's principles

Famously, Earman [14] explicitly expresses two heuristic principles for the formulation of theories of motion declaring the equality of spacetime symmetries and dynamical symmetries. My aim in this section, through making use of the analysis in the two previous sections, is to address the question of the foundation of Earman's principles and, in general, to discuss the possibility of formulating principles that relate spacetime symmetries and dynamical symmetries. This must necessarily involve a discussion of the motivation behind the definitions of the symmetries that the principles interrelate. In particular, because it is usually taken for granted, I am especially interested in discussing the notion of spacetime symmetry.

First, we need to consider what kind of principles Earman's principles are. For this we must make explicit what definitions of symmetry they presuppose. Let me start with the notion of spacetime symmetry. Earman's discussion assumes that a formulation of a physical theory (a theory of motion) incorporates the identification of certain structures as spatio-temporal. If this is the case, then we can define spacetime symmetries as transformations that leave these structures invariant. From this posit, Earman's principles are understood as providing criteria to establish which formulations of a given theory are preferable in virtue of their not containing spacetime structures whose symmetries do not coincide with the dynamical symmetries. Dynamical symmetries, on the other hand, are defined in the standard way (see the previous section). This

[18] A subsystem-global symmetry group for two interacting systems, in Wallace' terminology, is a symmetry group whose action is a symmetry of each subsystem and for which the combined action is a symmetry of the combined system.

is consistent with Earman's understanding of the principles as heuristic: one begins with some posit on what the spacetime symmetries—implicitly encoded in a given interpretation of a theory—are and attempts to refine it by recourse to the principles. To avoid circularity, though, the justification of the principles must be independent of the definition of what a spacetime symmetry is. This is why Earman stresses that these are not principles of meaning (they are not analytical) and he invokes some epistemic considerations, allegedly related to the general notion of spacetime, to try to provide a justification for the principles. I think that the two central ideas in Earman's discussion of the principles are right: that the principles should not be taken as analytical and that their force derives from the connection between the notion of spacetime and its epistemic role in physical theories. Nonetheless, keeping the nominal definition of the notion of spacetime symmetries (i.e., as symmetries of spacetime structures), it is easy to fall into one of two interpretive traps (that excessively burden the discussion). The first consists of taking the nominal definition as substantive and thinking that what the justification of the principles would determine is that the dynamics is adapted to spacetime structures that are not dynamically determined. The second, partly motivated by dissatisfaction with the first, is to think that dynamics, transparently and without presupposing any further epistemic input, dictates what the spacetime symmetries are. To avoid these extremes, it is advisable to note from the beginning that in the determination of which structures are spatio-temporal, and therefore what spacetime symmetries are, epistemic considerations of a dynamical character must be taken into account. This is why I propose to make it explicit from the start that in the determination/definition of spacetime symmetries, general conditions that can be interpreted as proceeding from the characterization of measuring devices are essential, and the precise sense in which they are. These are the epistemic considerations of a dynamical character that might also be taken as providing content for a definition of the notion of spacetime symmetry that goes beyond the nominal definition.

Let me try to make all of this more precise. The connection between the determination of geometry and dynamical conditions was at the centre of the responses to the PoS. The link, in those frameworks, was provided by coordinating a notion of congruence (finite, in the Helmholtz–Lie classical response; infinitesimal, in Weyl's version) with some transformations that are taken to be the correlate of the motions of physical systems that would measure physical geometry, expressed as the condition of free mobility and its translation to the relativistic context. The general assumption here is that a determination of physical geometry is always going to be through the identification of certain transformations that can be interpreted as defining a notion of congruence (some kind of relation of equivalence for physical systems that meet certain criteria that permits us to interpret them as congruences). These transformations of congruence are the natural candidates for providing the definition of spacetime symmetries. So far, they are the transformations that can be used to define a geometry, in line with Klein's Erlangen programme, from the structures which are invariant under them. The connection to (physical) spacetime, in the PoS approaches, comes from assuming that such transformations can represent physical motions of systems that measure the geometry of space. In the case of the classical solution to the PoS, the physical interpretation of the notion of congruence is given by the notion of free mobility of rigid bodies, which is then associated with the group of transformations that are permissible according to the mathematical notion of congruence. In other words, we have a mathematical notion, congruence, that provides a sense of correspondence for mathematical objects, and its physical counterpart given by the idea of rigid body. This allows us to determine a group of transformations as those respecting certain internal relations, and from then to determine, a least partially, the geometry of space.

In any case, I want to stress that these approaches provide a framework within which to formulate the connection between geometry and dynamics. Note that the general strategy can be taken, independently of the specific results that it renders, to consist of providing the mathematical characterization of a certain notion of congruence through a group of transformations, which will be interpreted as defining a geometry. If one starts with a prior notion of congruence (equality of lengths for vectors, for instance), this determines the group of transformations. But we could also think, inversely, of the group as defining congruence. This perspective might be especially relevant when we leave the context of pure mathematics. In this case, we might think that a notion of mathematical congruence will be justified insofar as it represents a certain concept of equivalence for physical systems that is relevant in some specific way. Whatever that notion of equivalence may be, by generalizing the lessons from the PoS we can see that it is its eventual association with a group of transformations (if they can be interpreted—using Weyl's terminology—as allowing congruence transfers) that will provide the connection with the geometry of spacetime. This points to the desired link between spacetime symmetries and dynamical symmetries, as I will next elaborate, but coming from the opposite direction.

Let us now consider the treatment of measuring devices as subsystems interacting with other subsystems. Generally, we can take them to be measuring empirical quantities that can be used to confirm/refute a given dynamical theory. The quantities themselves need not be spatio-temporal, but the assumption is that, in order for them to have empirical relevance (some authors call this empirical salience), the measuring must provide some kind of parametric marking of the events in such a way that the measured relations can be taken as data to test the theory. It seems difficult to see how this assumption could be avoided (which does not mean that it is being assumed that the relations are determined). Perhaps a less loaded assumption about the functioning of certain subsystems as devices would just be that the measurement contents must be coordinated in such a way that relations between the events can be expressed, and some of them sanctioned, as being derived from the dynamics.

In any case, initially bracketing the question about the degree of commitment that one is ready to make to the minimal structure of events needed to formulate a dynamical theory, measuring devices can be taken to be physical systems that can be characterized as subsystems that interact with other subsystems. Since we will be interested in the description of dynamical symmetries as they apply to different subsystems, we can use the framework discussed in the previous section. The dynamics of subsystems acting as measuring devices can be represented in a configuration space with coordinates capable of encoding the dynamics of target systems (whether this is the same subsystem device or other subsystems). Borrowing Wallace' notation that I previously introduced, we can represent the combined target system–device state as $(O, g; O', g')$. The state of the measuring device after the measurement will be a function of the state of the target device before measuring, meaning that it covaries with the state of the target system. As discussed before,[19] the Unobservability Thesis implies, assuming that the device is a physical system capable of encoding the state of the target, that internally the recording is not capable of distinguishing whether a transformation which is a symmetry of the dynamics of the device has taken place. Now, the device must interact with external target systems in order for us to be able to interpret the quantities measured as relevant for the testing of the dynamics of those systems. Some of these quantities, even if observable, might be variant under some of the symmetries of the dynamics of the device when only applied to the target or device systems: quantities that can be interpreted as relational (encoding information about the target–device relation). Using the previous notation, the quantities represented by $g'^{-1}g$ would be invariant under symmetry transformations (if they are symmetries of the combined system) but could be interpreted, assuming that the change in the device is undetectable (therefore taking g' as fixed), as detecting transformations of the target system. From a perspective that is internal to the device some quantity that encodes the relation between the device and an external target system does change. Thus, the record of the states before and after the transformation will be interpreted as two states of the target permitted by its dynamics. However, these same transformations could, in principle, be interpreted as (relative) 'motions' of the device subsystem for which nothing changes internally while its relation to other subsystems varies.[20]

This is the conceptual basis that connects certain dynamical symmetries with the notion of congruence: they share some formal characteristics (being defined by a group of transformations that are not observable via the change

[19] Wallace (2020, p.10).

[20] One might wonder to what extent being able to detect these type of quantities is necessary for the characterization of measuring devices. Behind this is the idea of measuring devices being able to capture empirical contents that can be used as evidence to test the theory, together with the idea of dynamics establishing relations between contents that observers like us can experience. This is clearly not sufficient to prove the necessity of this characterization of devices and much more work needs to be done to stablish this kind of connection, but my view is that such a characterization is part of the way in which we define empirical content. Provisionally, we can say the following: insofar as part of the characterization of measuring devices is the possibility of capturing such quantities, as argued below, certain dynamical symmetries with the specified features when applied to subsystems will be, as argued below, spacetime symmetries.

in quantities measured internally, but nonetheless making sense of the claim that the transformation has taken place because some quantities that encode external relations have changed). From establishing this connection, the next step is to say something about the relation between the symmetries of the dynamics of the device and the dynamics of target systems. For this, we just need to recover the following result: in order for these quantities to be observable externally, the symmetries of the subsystem device must be extendible and, using Wallace' terminology, subsystem-global: the same group must be a symmetry group of the different interacting subsystems. This means that the dynamics of target systems must have the same symmetries as the dynamics of the subsystem device.

Formally, the rationale for such a connection is given by the equivalence of some structure in both cases: the existence of transformations with the structure of a group (which is given as part of the definition of congruence, and eventually of geometry) and the identification of relevant invariances of the dynamics. This is one way of expressing what has been done here: a mathematical notion of congruence (which can be taken as defined through a group of transformations) has been coordinated with a dynamical notion of congruence. This latter is based on the idea of subsystems that can detect some symmetry transformations that are internally unobservable but observable by measuring quantities whose variation with the transformation is interpreted as detecting change in some relation between the subsystems. The motivation for the coordination between these notions comes from the idea that such transformations for measuring devices share essential features with mathematical congruence transformations; they provide a criterion of equivalence which is somehow internal, together with a distinction between initial and final state which is external. There is a class of dynamical symmetries that can accomplish this: those that can be interpreted as dynamical congruences and define a subclass of dynamical symmetries.

We can define *D-congruent symmetries* as those dynamical symmetries of the subsystems such that for a measuring device operating in them are internally unobservable but observable through changes of invariant relational quantities between the device and any other subsystem.

The general motivation behind this definition is that D-congruent symmetries can be interpreted as providing a dynamical counterpart of the notion of congruence. Let us reflect on this. Congruence, originally, is a geometrical notion referring to the equivalence of figures in space. The idea is that congruent figures can be perfectly superposed when one is moved to the other's position. Helmholtz conceptualizes this through the notions of rigid body and free mobility. In a simplified manner[21] one can say that the geometrical notion of congruence is defined by some procedure for determining the equivalence between bodies at the same place and some rules for comparing distant bodies, all of which determine certain transformations. Mathematically such transfor-

[21] See for instance ([10], p. 123) for a recent presentation of Helmholtzian congruence conditions.

mations form a group. Alternatively, one can think of this characterization as providing a procedure to determine certain intrinsic properties of the figures (length, angles...) and a group of transformations that keeps the intrinsic properties invariant while changing the extrinsic relations to other figures. Naturally, depending on what the procedure to determine the intrinsic property is, the group of transformations found is going to be different and, one might think, the fact that a certain group of transformations define a geometry is dependent on having originally chosen properties that are, let us say, spatial or geometrical.

The bold step taken here, inspired by Helmholtz' treatment, consists on abstractly focusing on the properties of the dynamics that the physical systems that implement the notion of congruence must meet and, together with this, generalizing by abstracting the initial geometrical features. The main leading question can be posed in the following terms: What general conditions must the symmetries of the dynamics meet in order to be at the base of a definition of congruence? The leap is taken by thinking that any dynamical symmetry that meets such conditions could be considered as able to support a definition of congruence. To put it differently, if we blindly started by looking at the dynamics without a previous geometrical background, we could use those properties to define a subset of dynamical symmetries that, eventually, might be taken to define a congruence. The answer to the question, I claim, is found in the features that certain dynamical symmetries when applied to subsystem have. They define a group of transformations that are not detectable by measuring devices detecting intrinsic quantities but can be detected by variations in some relational quantities between subsystems. Formally, they will be congruences.[22]

It must be noted that this general scheme necessarily involves what can be seen as conventional elements. These are linked to the fact that, by declaring certain structures as spatio-temporal, an interpretation of a physical theory incorporates a tacit decision as to the notion of dynamical symmetry that is taken as relevant for a definition of congruence, from which spacetime geometry follows. The minimal requirement, according to the previous discussion, is linked to the existence of symmetry transformations which, when applied to subsystems, are internally unobservable and externally detectable; but this leaves some degree of freedom.

At this point, it is important to stress a couple of things. The first is that there might be further requirements that are needed to coordinate the dynamical notion of congruence with a geometrical one, but it seems unlikely that we could formulate them in general without using concepts that are already chronogeometrical (more on this at the end, when I discuss why this linking

[22] I have not provided a full proof of this claim in this paper. To do so, one should start by deciding which kind of quantities our devices should detect (vectorial, for instance) and show that in order to observe relational quantities of that type, the group must be one that defines a congruence. I leave the full discussion for another paper.

of spacetime to formal properties of receptivity[23] is a (the best) functionalist perspective on spacetime). Here I have specified the minimal structure that is behind the connection between spacetime and dynamics in a theory, intentionally eluding any mention of plainly phenomenological notions.

The second remark has to do explicitly with the nature of the convention involved here. This can be seen from two complementary perspectives. From the geometrical point of view, it involves taking some physical systems as suitable for the implementation of a notion of geometrical congruence; from the perspective of the dynamics, it means assuming, in cases in which a full dynamical description of the measuring devices is inviable, that the laws governing the dynamics of the devices have certain symmetries. These two aspects are derived from the original conventional dimension involved in the coordination of a notion of dynamical symmetry and a geometrical congruence. This could also be expressed in a slightly different way: through its coordination to a notion of mathematical congruence, we are using a notion of dynamical congruence to define spacetime symmetries. In this sense, this approach is at the base of an eventual dynamical definition of spacetime symmetry, which is the principal motivation of the DA.

5.1 Limitations of the applicability of the principles

Earman's principles can be, and usually are, taken as heuristic principles concerning the formulation of spacetime theories that recommend the equality of spacetime symmetries and dynamical symmetries. One of the main problems of Earman's proposal has to do with the allegedly insufficient justification for the principles, which can lead to a lack of clarity as to whether the principles are materially adequate or whether some counterexamples to them can be found. The core of the previous discussion allows us to complete the justification for the principles and, consequently, to delimit their range of validity. According to the view I defend here, the relation between spacetime symmetries and dynamical symmetries is arrived at by referring both notions to certain transformations that are symmetries of subsystems which can act as measuring devices. This is achieved by coordinating a notion of geometrical congruence with certain dynamical symmetries applied to subsystems. So, stated in a simplistic way, this proposal amounts to a reformulation of Earman's principles in which the terms 'spacetime symmetries' and 'dynamical symmetries' are precisely interpreted in the following way: spacetime symmetries and D-congruent symmetries should coincide. A slightly different way of putting this involves understanding the principles as recommending an (empirical) interpretation of the theory in which there is coincidence of the dynamics responsible for the behaviour of devices taken to determine the spacetime structure (rods and clocks) and the dynamics of measuring devices that provide empirical tests of the theory. Conversely, according to this proposal, the discrepancy between

[23] I borrow this term from Kant's work to allude to a generalized faculty of ideal observers as represented in physical theories.

spacetime and dynamical symmetries—once the notion of dynamical symmetry is taken as fixed and interpreted as applied to subsystems—can be traced back to a posit of spacetime symmetries which, even if only tacitly, must be linked to the behaviour of certain systems that obey a dynamics that is different from that attributed to measuring devices (many times, the discrepancy is due to the postulation of spacetime structures whose determinations have lost all trace of their connection to any dynamics whatsoever). This can be made explicit through a very well-known example.

Special Relativity (SR) can be characterized as a theory stating either that the geometry of spacetime is Minkowskian or that (locally) all the physical laws are Poincaré invariant. From the perspective defended in this paper, both characterizations arise from a more basic implicit assumption: the empirical procedures (physical systems) idealized and assumed by the theory as standard for the determination of spacetime chronogeometry (this is somehow, implicitly, encoded in the formalism of the theory), are the same (or at least are governed by analogous dynamics) as the measuring devices that provide us with the empirical basis of the physical theory. Historically, this was encoded in the behaviour of rods and clocks as derived from the relativity principle and the light principle. Now, this does not mean that it is inconsistent to formulate a theory empirically equivalent to SR but set in a different spacetime structure, or that it is impossible to have dynamical laws with symmetries differing from Poincaré invariance in Minkowski spacetime. But in such cases, the coordination presumed by the principles fails.

An example of this might be present in Lorentz' theory.[24] In this case, the main question can be posed in terms of how to accommodate a dynamical law that is Lorentz invariant in Newtonian or Galilean spacetime. If one is willing to keep one of these spatio-temporal structures (ultimately due to the assumed dynamical functioning of some material systems governed by Newtonian dynamics) and faces a theory encoded by Maxwell's equations, two possibilities exist. One is to hope eventually to formulate the theory in equations with the same symmetries as the assumed spatio-temporal structure; doing so has some empirical consequences that should be sanctioned by experiment. The other option is to allow the difference in symmetries. But this means that, in principle, these laws should imply the possibility of detecting certain inertial motions in spacetime. If this was not the case—as the null result of the Michelson–Morley experiment eventually showed—then there is the possibility of trying to explain the null result by postulating some dynamical effects on measuring devices due to their motion in absolute space, as Lorentz did. But in such a case, the question is then: What justification is there for maintaining that the spatio-temporal structure is Newtonian? Simplifying, this might be sustained by a priori, geometrical or dynamical reasons. Considered from a bluntly a priori standpoint, it would involve assuming that spacetime must be Newtonian. A more sophisticated version of this position, based on geometri-

[24] Philosophical discussions of Lorentz' theory and its relation to SR abound. See, for example [26], [17], [1], [4].

cal considerations, involves assuming that rods and clocks measure a Galilean spacetime (note that from this vantage point it is difficult to justify Newtonian absolute space). Finally, one might justify the, again, Galilean structure of spacetime as being derived from the symmetries of Newtonian mechanics. At the same time, as noted above, one would need to use some machinery (similar to Lorentz' *theorem of corresponding states* and *generalized contraction hypothesis*) to explain why we do not detect motion in absolute space.[25]

The problem with all these defences of the Newtonian or Galilean nature of spacetime, from the perspective of the principles, is that they have lost all contact with the dynamical notion of congruence that is the basis of the geometric determination of spacetime. Postulating, as Lorentz did, some dynamical effects to explain the null result of the Michelson–Morley experiment, flies in the face of taking rods and clocks—if governed by Newtonian dynamics—as a basis for the dynamical definition of congruence. The recommendation extracted from the principles (under my interpretation) would be something like the following: prefer the theory in which the measuring systems used to determine the geometry of spacetime are governed by the same dynamics as that governing the measuring devices that operate in electrodynamic experiments, which would be used to detect the alleged motion in absolute space. The rationale behind this, in this particular case, is that after realizing that some fundamental theory is incompatible with the dynamics of systems that probe spacetime, there are reasons to doubt that those systems are adequate for measuring the metric of spacetime. The notion of congruence defined for such systems does not seem to hold any more if we take into account electromagnetic phenomena. The principles recommend coordinating the mathematical notion of congruence with a dynamics of measuring systems that cannot detect inertial motion internally. This has the consequence, following the logic of the previous section, of establishing that spacetime is Minkowskian.

This way of formulating Earman's principles allows us to derive different conclusions when they are applied to different pairs of theories: absolute space is surplus in Newtonian gravitation but ether rest is not in Lorentz' theory because the principles recommend eliminating the former (by congruence reasoning) but not the latter (the same reasoning in this case recommends shifting to Minkowski spacetime). This addresses the problems, fairly pointed out by Bradley [4], of a merely formalistic interpretation of Earman's principles (in line with Norton's [26]) that recommends eliminating surplus structure. The principles defended here can be taken as indicating the origin of changes in interpretation that result in assigning different roles to the same structures: they have to do with the difference in how the connection between the procedures for the determination of spacetime geometry and the behaviour of measuring systems is interpreted in both theories. It is then an interpretative, not a merely formal, question. What they recommend is not to eliminate alleged surplus structure but to make these two procedures convergent; in other words, they recommend a strategy by which to relate the notion of congruence

[25] See [19].

behind the definition of spacetime structure and a notion of dynamical symmetries, interpreted as being applied to subsystems that can measure/detect certain quantities that relate them to other subsystems.

6 Conclusions

The relation between spacetime and dynamical symmetries can be traced back to the interpretation of some features of physical theories as potentially codifying the notion of ideal observers. This interpretive framework provides a justification for the connection between a notion of congruence, from geometry and essential for the determination of physical space(time), and certain features that characterize the measuring devices that can be used to empirically test the theory. So, such a connection can be understood as a way of giving content to an epistemological framework that assumes that the same procedures that are used to measure the geometry of spacetime are also part of the means through which we arrive at the empirical content that supports our physical theories. In both characterizations (congruence and measuring devices) certain transformations play an essential role; the present proposal attempts to state under what conditions those transformations can be equated. This is what provides the qualified relation between spacetime and dynamical symmetries expressed by the symmetry principles.

These are the terms involved in this formulation of the principles. Spacetime symmetries, nominally invariances of spacetime structures, must be understood as being determined by the congruences associated with certain (idealized) systems that are taken to probe spacetime. Dynamical symmetries are transformations that take solutions to solutions; and are such that when applied to subsystems that act as measuring devices, they are internally unobservable but detectable as changes in quantities that are relational between subsystems. With these definitions, and the discussion in this paper, we have a justification for Earman's type of symmetry principles.

Such principles are restricted in two senses. First, this formulation assumes that every dynamical symmetry so defined is going to be a spacetime symmetry but, as I have suggested, this assumes that all such dynamical symmetries can be interpreted as congruences. This might not always be the case (think of the controversial case of global "internal/phase" symmetries). Second, it depends on assuming a certain degree of idealization for the physical systems that determine spacetime structures. Such an idealization might be based on physical considerations or, as in the case of Weyl, on phenomenological ones.

The principles so explicated can be understood as heuristic principles for the interpretation of spacetime theories. They recommend an interpretation of the theory in which certain dynamical symmetries (those meeting the conditions referred to above) are interpreted as a group of congruence transformations that is at the base of the definition of spacetime structures. Such an interpretation, as we have seen in the case of Lorentz' theory, might have to be accompanied by different kinds of formal modifications; sometimes, but not always, these might recommend considering some structures as surplus.

We can distinguish different interpretive levels in this proposal. At the base, we have the claim that the relation between spacetime symmetries and dynamical symmetries arises from the coordination between some dynamical symmetries and a mathematical notion of congruence. This might be inserted into an interpretation of the formalism of a theory in which certain features are taken as codifying the work of measuring devices. Finally, the connection can be justified in a general framework in which this is related to the representation of ideal observers. The full package is what I believe motivates us to regard the proposal as a version of a functionalist approach to spacetime.

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