A Note on Density Matrix Realism

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Abstract

Quantum mechanics with a fundamental density matrix (W-QM) has been proposed and discussed recently. It motivates a new view called density matrix realism, according to which the ontic state of the universe is represented not by a wave function in quantum mechanics (QM), but by a density matrix in this theory, which may be a mixed state. In this paper, I argue that this view is inconsistent with the empirical equivalence between W-QM and QM.

Quantum mechanics with a fundamental density matrix (W-QM) has been proposed and discussed recently (Dürr et al, 2005; Maroney, 2005; Chen, 2018, 2019, 2020). It replaces the wave function in quantum mechanics (QM) with the density matrix and correspondingly the Schrödinger equation with the von Neumann equation. Since quantum dynamics can be formulated directly in terms of the density matrix, it seems reasonable to assume that the ontic state of the universe is represented not by a wave function in QM but by a density matrix in W-QM, which may be a mixed state. This view has been called density matrix realism (Chen, 2018). In this paper, I will present a new analysis of density matrix realism.

According to Dürr et al (2005) and Chen (2019), W-QM and QM are empirically equivalent when assuming that in QM a random wave function is assigned to the universe such that the associated statistical density matrix equals the fundamental density matrix assigned to the universe by W-QM. Suppose the fundamental density matrix of the universe at a given instant t_0 is

$$W_0 = \sum_{i=1}^{N} p_i \left|\psi_i\right\rangle \left\langle\psi_i\right|, \tag{1}$$

where N is the dimension of the Hilbert space, $p_i \in (0,1)$ satisfies the nomalization relation $\sum_i p_i = 1$, and $|\psi_i\rangle$ is a set of orthogonal states in the Hilbert space. The equivalence between W-QM and QM then means that one can assign a random wave function $|\psi_i\rangle$ or a mixed state W_0 to the universe and use either QM or W-QM for the same empirical predictions.

According to density matrix realism, each density matrix in W-QM, whether it is a pure state or a mixed state, is ontic, representing an ontic state of the universe. Then, not only each pure state $|\psi_i\rangle\langle\psi_i|$ corresponds to an ontic state λ_i, but also the mixed state W_0 corresponds to an ontic state λ_0. Moreover, we have $\lambda_0 \neq \lambda_i$ for any i, and $\lambda_i \neq \lambda_j$ when $i \neq j$. This means that W_0 and $|\psi_i\rangle\langle\psi_i|$ (or the wave function $|\psi_i\rangle$) correspond to different ontic states of the universe, and thus they cannot be assigned to the same universe at an instant which has a unique ontic state. Therefore, density matrix realism is inconsistent with the equivalence between W-QM and QM; the equivalence requires that pure states and mixed states cannot be both ontic, but density matrix realism says that all density matrices, including both pure states and mixed states, are ontic.

In order to avoid the above inconsistency, one may assume a revised version of density matrix realism, which says that in W-QM pure states are not ontic and only mixed states are ontic.[1] This view may be called impure density matrix realism.[2] However, this view is not a unified view about a fundamental theory. The whole space of density matrices in W-QM is composed of both pure states and mixed states. If W-QM is a fundamental theory that directly describes the physical world, then a unified view is that each state in the state space of W-QM is ontic, no matter it is a pure state or a mixed state. By contrast, if some states such as pure states in the state space of W-QM are not ontic but compatible with different ontic states, then the theory will be neither fundamental nor complete.

Furthermore, it can be argued that the reality of mixed state and the unreality of pure states in W-QM are incompatible. In the above example, W_0 is ontic and it corresponds to one unique ontic state for all $p_i \in (0,1)$. But when $p_i = 1$, the pure state $|\psi_i\rangle\langle\psi_i|$ is not ontic and it corresponds to at least two different ontic states, such as λ_{i1} and λ_{i2}. Then, when p_i changes from 1 to $1-\epsilon$, where ϵ is arbitrarily small, the ontic state, λ_{i1} or λ_{i2}, will also undergo an arbitrarily small change. Since λ_{i1} and λ_{i2} have a finite difference, the two new ontic states will be also different. This means that the mixed state W_0 for which $p_i = 1-\epsilon$ also corresponds to at least

[1]Note that every mixed state being ontic requires that no pure states are ontic, since each pure state is compatible with at least two mixed states due to the equivalence between W-QM and QM. This argument is similiar to the above inconsistency argument.

[2]The claim that the actual ontic state of the universe is a mixed state does not mean that all possible ontic states of the universe must be mixed states, not pure states. Thus I think Chen's (2018) view is still density matrix realism, not impure density matrix realism (see also Chen, 2019).

two different ontic states.[3] Therefore, if some pure states are not ontic, then some mixed states are not ontic either. As a result, the reality of mixed state and the unreality of pure states in W-QM are incompatible.

Finally, it is worth noting that impure density matrix realism can hardly be consistent with the ample evidence for the reality of pure states such as the PBR theorem (Pusey, Barrett and Rudolph, 2012). In particular, impure density matrix realism says that no pure states are ontic, while this claim seems too strong to be true. Take energy as an example. This claim means that all energy eigenstates are not ontic, and two energy eigenstates do not correspond to different ontic states but are compatible with the same ontic state. Then an electron being in the gound state and an electron being in the first excited state may be in the same ontic state, which means that after emitting a photon the ontic state of the electron, including all its properties such as energy, does not change. This seems impossible. In general, it is arguable that two (nondegenerate) eigenstates of an observable cannot be compatible with the same ontic state, since the same measurement of the observable on these two states will yield two definite (not random) results, namely two different eigenvalues of the observable.

To sum up, I have argued that density matrix realism is inconsistent with the empirical equivalence between W-QM and QM. If this result is valid, then not only density matrix realism is not true, but also W-QM is not a fundamental and complete theory.

References

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[3] Due to the appearance of more terms relating to probabilities p_j (where $j \neq i$), the mixed state W_0 may correspond to more ontic states.

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