Review of "On the Ostrogradski Instability; or, Why Physics Really Uses Second Derivatives", by Noel Swanson (*British Journal for the Philosophy of Science* 73(1), pp. 23–46, 2022)

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Why are candidate fundamental physical laws so rarely—if ever—higher than second order in derivatives? This question was taken up in 2014 by Easwaran [2], who argued that a particular package of metaphysical commitments (regarding, *inter alia*, causation, grounding, physical change) afford an answer to this question. In the present article, after criticising the account offered by Easwaran (essentially on the grounds that it boils down to a judgement that higher-order dynamical laws are less 'simple': see §2), Swanson argues that a more naturalistic answer is both available and superior: "generic higher-order Lagrangian theories are energetically unstable" (p. 24).

To be more precise, the theorem upon which Swanson's argument is based is this:

> **Theorem (Ostrogradski):** If a non-degenerate Lagrangian, $\mathcal{L}(q, \ldots, q^{(n)})$, depends on the nth derivative of a single configuration variable q, with $n > 1$, then the energy function in the corresponding Hamiltonian is unbounded from below. (p. 29)

Swanson outlines an argument for this result which assumes the non-degeneracy of the Lagrangian—this being the condition that

$$\det \left[\frac{\partial^2 \mathcal{L}}{\partial \dot{q}^2} \right] \neq 0. \tag{1}$$

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Although the theorem can be extended to the case of degenerate Lagrangians (see [4]), additional constraints must be assumed, which, as Swanson points out, "opens up a possible avenue for evading the no-go result" (p. 32). I'll return to this below.

But first: what, exactly, is problematic about an energy function which is unbounded from below? After dismissing some bad answers to this question (§4), Swanson builds on an argument originating in Pais and Uhlenbeck [3] and Woodard [6, 7]: interacting Lagrangian field theories that satisfy the Ostrogradski theorem have an unstable vacuum state, so, ultimately, "no field theory which an Ostrogradski Hamiltonian could possibly describe a stable world like ours" (p. 34). Thus,

> if nature is described by an interacting Lagrangian field theory with a stable vacuum, then higher than second-order equations of motions [*sic*] are either impossible or very special, requiring just the right interplay between constraints to eliminate the Ostrogradski instability without reducing the dynamics to second-order laws. (p. 38)

Note that latter case here arises when the Lagrangian is degenerate.

Throughout the article, Swanson's reasoning is clear and sharp, and—to those of a naturalistic disposition—plainly superior to the account offered by Easwaran. It does, however (as Swanson himself registers), invite further discussion, along a number of axes:

1. As Swanson recognises, the above reasoning has a "weak anthropic character" (p. 35). One can thereby ask: does it qualify as an acceptable/uncontroversial explanation? (Tegmark argues in [5] for the universe having four spacetime dimensions on similar grounds of stability—a direct comparison might well prove illuminating.)

2. In what sense is it legitimate to appeal to the Lagrangian framework in accounting for the salient properties of physical laws (namely, their being at most second-order)? Brown, for example, maintains that "the real meat in the physics ultimately resides in the equations of motion" [1].

   Again, Swanson anticipates this concern: even if one regards Lagrangians as (in some sense or other) subordinate to equations of motion, the former may nevertheless have a bearing on physical goings-on in the actual world. In particular, inferences such as the following are warranted: ‘*If* my equations of motion are such that they can be derived by varying a Lagrangian satisfying the Ostrogradski theorem, *then* the vacuum will be unstable.’

Of course, though, this does not rule out equations of motion with higher derivatives which are *not* derivable from a variational principle (p. 35), so questions remain. In particular: why are the physical laws of the actual world second-order and derivable from an action principle, rather than higher-order and not so derivable? It’s not obvious that Swanson’s appeal to the ‘fruitfulness’ of Lagrangians (an epistemological fact) fully resolves this puzzle (which, of course, is metaphysical in nature), so arguably there is more work here to be done.

3. Swanson begins by asking ‘Why does $F = ma$?’ (p. 23), but ends up discussing Lagrangian field theories. It would be helpful if more could be said to bridge the gap between the latter and the former.

In any case, Swanson’s argument is profound and significant: any world in which the laws satisfy the conditions of the Ostrogradski theorem will not be one in which agents such as ourselves can exist. Though there is more work to be done in analysing this result (especially along the lines of (2) above), Swanson’s paper certainly represents a substantial step forward in addressing the question of why the laws of our universe are (seemingly) never higher than second-order.

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References

[1] Harvey R. Brown, “Do Symmetries “Explain” Conservation Laws? The Modern Converse Noether Theorem *vs* Pragmatism”, in J. Read and N. J. Teh, *The Philosophy and Physics of Noether’s Theorems*, Cambridge: Cambridge University Press, 2022.

[2] Kenny Easwaran, “Why Physics Uses Second Derivatives”, British Journal for the Philosophy of Science 65, pp. 845–62, 2014.

[3] A. Pais and G. Uhlenbeck, “On Field Theories with Nonlocalized Action”, Physical Review D 79, pp. 145–65, 1950.

[4] J. M. Pons, "Ostrogradski's Theorem for Higher Order Singular Lagrangians", Letters in Mathematical Physics 17, pp. 181–9, 1989.

[5] Max Tegmark, "On the Dimensionality of Spacetime", Classical and Quantum Gravity 14, pp. L69–L75, 1997.

[6] R. P. Woodard, "Avoiding Dark Energy with $1/R$ Modifications of Gravity", in L. Papantonopoulos (ed.), *The Invisible Universe: Dark Matter and Dark Energy*, Berlin: Springer, 2007.

[7] R. P. Woodard, "Ostrogradski's Theorem on Hamiltonian Instability", Scholarpedia 10, 32243, 2015.