On The Galilean Invariance of the Pilot-Wave Theory

Valia Allori

Philosophy Department
Northern Illinois University
915 Zulauf Hall
60115 DeKalb IL USA

Abstract

Many agree that the pilot-wave theory is to be understood as a first-order theory, in which the law constrains the velocity of the particles. However, while Dürr, Goldstein and Zanghì maintain that the pilot-wave theory is Galilei invariant, Valentini argues that such a symmetry is mathematical but it has no physical significance. Moreover, some wavefunction realists insist that the pilot-wave theory is not Galilei invariant in any sense. It has been maintained by some that this disagreement originates in the disagreement about ontology, as Valentini, contrary to Dürr, Goldstein and Zanghì, has been taken to endorse wavefunction realism. In this paper I argue that Valentini's argument is independent of the choice of the ontology of matter: it is based on the notion of natural kinematics for a theory, and the idea that the kinematics should match the dynamics. If so, I also argue that there are several reasons to dispute Valentini's claim that the kinematical symmetries should constrain dynamical ones.

Keywords: Galilei invariance; the pilot-wave theory; wave function realism; kinematics; dynamics; symmetries

1. Introduction

There are several controversies surrounding the pilot-wave theory. First, there is discussion about its *dynamical law*: is it fundamentally a first order theory, or a second order theory? Then, there is disagreement about the *ontology* of matter according to the theory: is the theory about particles, or about particles and waves? If both, then how are we to understand a wave in configuration space? I do not focus on these debates in this paper. Rather, I wish to discuss an issue which has been barely addressed in the literature, namely whether the theory is Galilei invariant. One may be surprised that there is such a disagreement, as one may reasonably think that specifying a theory, in particular its material ontology and its laws, would also fix its symmetry properties: isn't the fact that, e.g., classical mechanics is Galilei invariant part of the definition of the theory? Nonetheless, putatively all possible positions about the matter have been advocated for: some, most notably Albert (1996), have maintained that the theory is not Galilei invariant;[1] Dürr, Goldstein and Zanghì (1992) have argued that it is; while Valentini (1997) has insisted that the theory is mathematically Galilean invariant, but this symmetry is not physical. One may think that those who take the pilot-wave theory as Galilei invariant have in mind that it has to be fundamentally understood as a second order theory, as it would be more similar to classical mechanics. Instead, this is not the case: all those who disagree about the Galilei invariance of the pilot-wave theory believe that the theory is fundamentally of the first order.

Some[2] then have argued that the heart of the matter is the ontology of matter according to the theory: wavefunction realists think of the wavefunction as a physical, material, field, while others have a different view of the wavefunction. I have similarly argued in another context that material ontology is important to determine the symmetry property of a theory: if the wavefunction is a material field in configuration space then it would not transform as needed to preserve invariance, while a nomological view of the wavefunction can account for the transformation required to allow invariance.[3] However, I do not think this is at the heart of the disagreement in this case. In fact, this would correctly account for Dürr, Goldstein and Zanghì's claim that the theory Galilei invariant, and that in general for the wavefunction realist's case against Galilei invariance. Nonetheless, it seems unable to make sense of Valentini's position: *contra* wavefunction realists, he accepts that the theory is mathematically invariant while *contra* Dürr, Goldstein and Zanghì he denies that this transformation has any physical significance. Indeed, his reasoning is independent on the choice of the ontology of matter. In fact, it is based on the idea that one can identify two sources for the motion of a body: one coming from spacetime, which identifies the natural kinematics of the theory and thus its natural motion, and one coming for the influence of other objects, which determines the body's dynamics. Valentini first argues that the natural motion for the pilot-wave theory is rest, thereby making the theory not Galilei invariant. Then he argues that the kinematical symmetries should constrain the dynamical symmetry, which is why Galilei invariance has no physical significance.

After reconstructing Valentini's position, I discuss an argument that Valentini's conclusion relies on an unreasonable assumption about the nature of the natural free motion in the pilot-wave theory. Moreover, I show how there are additional good reasons not to prioritize the kinematics, as Valentini wishes to do. In addition, I discuss

[1] See also Skow (2010).
[2] Belousek (2003), Solè (2013).
[3] Allori (2015, 2019).

how Valentini's argument is heavily influenced by general relativity but the analogy with that theory soon breaks down: while in general relativity the kinematics is relevant for the dynamics, this is not the case for the pilot-wave theory. Also, I present reasons that the relation between kinematics and dynamics is not necessarily what Valentini suggests.

Before continuing, let me add a remark on the relevance of this debate. One may ask why it is of any relevance to inquire whether some theory is Galilei invariant or not, when we know that Galilei invariance is false already, given relativity theory. I think it is important to understand where the disagreement about this symmetry lies because it may shed some light on what determines the symmetry properties of a theory: the ontology of matter, the laws, or something else? Determining this is important to better understand what it means to require for a theory to be Lorentz invariant. Indeed, Valentini wishes to claim that the theory being non-Galilei invariant shows that imposing Lorentz invariance is misguided. Instead, I am going to argue that the very opposite is true: if the pilot-wave theory were not Galilei invariant it would be further away, rather than close by, an important aspect of relativity, albeit not Lorentz invariance.

Here is the roadmap of the paper. I start with section 2, in which I discuss the controversies over the material ontology and the type of dynamics of the pilot-wave theory. In section 3, I consider symmetries in general and Galilei invariance, in particular within the pilot-wave theory. In section 4, I present Albert's argument that the pilot-wave theory is not Galilei invariant, and in section 5 the argument by Dürr, Goldstein and Zanghì that it is. Then, in section 6 I continue discussing Valentini's kinematical approach. I make the point of the situation in section 7, where I further elaborate Valentini's argument, focusing on his motivations. I provide several objections to Valentini's approach in section 8, and I argue that he is ultimately mistaken. I end with the summary of what has been done in the paper and with some concluding remarks.

2. On the Different Ways of Being a Pilot-Wave Theory

The pilot-wave theory has a long story of controversies. Two prominent ones have to do with which formalism should be considered fundamental, and with which is the ontology of the theory. In this section I briefly review some positions which are relevant for the discussion in this paper.

2.1 The Wavefunction as Material or as Nomological

While almost everyone considers the theory as describing the behavior of particles in three-dimensional space, it is controversial how to interpret the wavefunction. Leaving aside those who think of the wavefunction as epistemic, even among those who considers it to be ontic one can distinguish a variety of distinct positions.[4] As a first approximation, among ontic approaches one can distinguish between those who think that the wavefunction is a material physical field (let's dub them, simplistically, the 'materialists'), and those who think it is not (the 'non-materialists').[5] According to the materialists, in the pilot-wave theory the wavefunction is part of the ontology of matter, together with the particles. There are several ways of being a materialist within the pilot-wave theory; for instance, one can think of the wavefunction as a multi-valued field in three-dimensional space.[6] A particularly radical view among materialists has been proposed by Albert (1996) who argues that the fundamental ontology of matter according to the theory is given by a single particle and a single wave in configuration space, so that the objects and the there-dimensionality of our experience are derivative (sometimes this view is called the 'marvelous point'). We will come back to this theory in section 4.

Otherwise, there are several ways of being non-materialists with respect to the wave function. They would all agree that the wavefunction does not represent material objects but nonetheless it would still be part of the (more general) ontology of the theory, namely it exists objectively, one way or another: one can think that the wavefunction is in its own category,[7] or broadly think that the wavefunction is

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[4] A common distinction is between epistemic and ontic views of the wavefunction. According to the former, the wavefunction is not objective, it is not part of the ontology of the theory but rather it represents the observer's state of knowledge of a physical system. Instead, ontic approaches have in common the idea that the wavefunction represents some objective feature of reality, even if they disagree about what this feature is.

[5] Notice that these distinctions are closely connected with the debates concerning the notion of primitive ontology (see Allori 2013). The idea is that a theory is more explanatory if matter in it is represented by some low-dimensional (most likely three-dimensional) mathematical object. So, for what is relevant for this paper, the wavefunction, understood as a field in a high dimensional or abstract space (configuration space or the like), is not a suitable material ontology. This is the case even if it is, broadly speaking, part of the ontology of the theory in the sense that it represents an objective feature of reality. I do not think that introducing the notion of primitive ontology is necessary here, and I am afraid it will only generate more confusion. Thus, I will write about ontology of matter (instead of primitive ontology) to refer to the variables in the theory which represent material objects, and ontology in general to include also the variables representing other objective feature, such as the forces, the potentials, the Hamiltonians, the velocities, and so on. In any case, materialists with respect to the wavefunction, as defined above, are those who go against the primitive ontology approach, while non-materialists are more likely to be sympathetic to it.

[6] Forrest (1988), Belot (2012), Hubert and Romano (2018).

[7] Maudlin (2019).

nomological. That is, one can maintain that the wavefunction should be understood as influencing the motion of the particle in terms of laws,[8] properties,[9] forces,[10] or more generally having suitable functions in the theory,[11] among others.[12]

2.2 The Guidance View or The Causal Formulation

There are two ways of presenting the theory: the guidance view, and the causal formulation. The former was first introduced by de Broglie (1924) and it is explicitly endorsed by two main groups of supporters of the pilot-wave theory, namely Dürr, Goldstein and Zanghì (1992) and Valentini (1997). In this approach, the pilot-wave theory is a theory of (three-dimensional) particles, like classical mechanics, but moving according to a new, non-classical dynamical law, proportional to the velocity rather than the acceleration. The wave ψ evolves according to the Schrödinger equation and one can write it in polar form as $\psi(r_1, .., r_N, t) = R(r_1, .., r_N, t)e^{\frac{i}{\hbar}S((r_1,..,r_N),t)}$. The first-order guidance equation then looks as follows: $m_i \frac{dr_i}{dt} = \nabla_i S(r_1, .., r_N)$ (the index i runs from 1 to N, the number of particles). It can also be written as $\frac{dr_i}{dt} = \frac{\hbar}{m_i} Im \frac{\nabla_i \psi}{\psi}$ (where $'Im'$ is an operator telling us to take the imaginary part of what follows).

The causal formulation, first advocated by Bohm (1952), and then endorsed also by Hiley (Bohm and Hiley 1993) and Holland (1993), in contrast proposes that the theory is to be understood as fundamentally about particles evolving according to a second order equation involving the particles' acceleration. The (three-dimensional) particles are thought as moving under the influence of forces, among which there is a new one, derived by the so-called quantum potential $Q = \sum_{i=1}^{N} \frac{\hbar^2}{2m_i} \frac{\nabla_i R}{R}$. Therefore, the dynamics of the causal formulation is given by: $m_i \frac{d^2 r_i}{dt^2} = -\nabla_i (V + Q)$. In this formulation $m_i \frac{dr_i}{dt} = \nabla_i S(r_1, .., r_N)$ is not regarded as an equation of motion (as in the guidance approach), but as a restriction imposed on the initial momenta.

There has been some discussion on which formulation should be taken to be fundamental,[13] and much more about the status of the wavefunction.[14] I will not focus of

[8] Dürr, Goldstein and Zanghì (1997), Goldstein and Zanghì (2013).
[9] Monton (2006), Suàrez (2007), Deckert and Esfeld (2018).
[10] Valentini (1992, 1997), Belousek (2003).
[11] Allori (2021).
[12] See Chen (2019) for more ways of understanding the wavefunction in quantum theory, including the pilot-wave theory.
[13] See Goldstein (2021), Valentini (1992, 1997), Belousek (2003), Solé (2013).
[14] See, for starters, the contributions in Albert and Ney (2013).

these issues here: I will merely take on what is needed to examine the question at hand, namely the Galilean invariance of the theory. In any case, the driving motivation for the causal view seems to be its alleged superior explanatory power: Belousek (2003) has argued that the nomological view, which he takes to be wedded to the guidance formulation, does not truly explain the phenomena. Thus, he argues, one needs to reformulate the theory in terms of particles being subject to forces. However, even granting Belousek that forces are needed for proper explanation, one does not have to choose the causal view. In fact, Valentini (1992) has maintained that the pilot-wave theory is fundamentally a first order theory, in which the particles should be seen as moving under the influence of 'Aristotelian' forces, namely forces generating velocities rather than acceleration. That is, Valentini both endorses the guidance view and explains the particles' motions in terms of these forces, thereby satisfying Belousek's criterion for satisfactory explanation.

For what is worth, I agree with those who claim that the causal view is misleading when taken as the fundamental formulation of the pilot-wave theory: mathematically the particle velocity, rather than its acceleration, is affected by the wavefunction, and this means, by definition, that the theory is of the first order. The causal formulation, by deriving again the velocity field, seems to make the same sense as deriving again the Newtonian equation in classical mechanics: one can certainly do that, but it is unclear what one could gain by doing it, as in classical mechanics forces affect acceleration, rather than the rate of change of acceleration. One could protest that writing down the theory as a second order dynamics may be useful for a variety of reasons, for instance to better understand its classical limit. In fact, one can show that this happens under conditions in which the quantum potential goes to zero.[15] Given that, one can easily see how velocities become independent of the wavefunction, recovering the Newtonian schema. Nonetheless, that does not mean that the second order formulation is to be preferred as the fundamental formulation of the theory. This seems like saying that the Lagrangian formulation, say, of classical mechanics is to be preferred to the Newtonian one because it makes it easier to do calculations.

Be that as it may, one does not need to show that one formulation is better than the other in order to discuss the Galilei invariance of the theory. In fact, the pilot-wave theory understood as a second order theory looks very similar to classical mechanics, and so it seems reasonable to assume that it preserves its symmetries, including Galilei

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[15] See Allori & Zanghi (2009).

invariance. In addition, all those who disagree about Galilei invariance endorse the first order approach, and this is what I will take for granted.

3. Galilei Invariance in the Pilot-Wave Theory

Before presenting the various positions, let me briefly discuss symmetries in general. A theory is said to have a dynamical symmetry under a given transformation S, or invariant under S, when S 'transforms solutions into solutions.' To clarify, let's start from the fact that the dynamics of a theory is the dynamics of something in the theory. That is, the theory has a dynamical equation which describes the temporal evolution, namely the dynamics, of something, namely the material ontology as specified by the theiry. In other words, in addition to the specification of the dynamics, one needs to be clear about what matter is represented by in the theory in order to talk about what the dynamics is a *dynamics of*. Then the solutions of the dynamical equation describe possible ways the world can be according to the theory. To say that a theory is invariant under a given transformation S then means that the dynamical equation of motion is transformed under S as to have the same solutions of the original, untransformed, equation. One often says in this case that that the law is unchanged in form by S.

To be more explicit, consider for instance classical mechanics, and ask about its time reversal invariance. Time reversal is the transformation T which inverts the direction of time: $t' = -t$. According to classical mechanics, matter is made of particles, which are therefore the material ontology of the theory, evolving according to Newton's second law, which therefore specifies the dynamics of the particles. Newton's second law remains the same in form under T: the velocity, because of its definition as rate of change of position, flips sign, while the acceleration changes sign again, being the rate of change of velocity, thereby cancelling out the first change. That means that classical mechanics is time reversal invariant: the solutions of the 'forward' equation are also solutions of the 'backward' equation.

The case of Galilean invariance in classical mechanics is similar: Newton's theory is Galilei invariant, that is, Newton's law holds in all inertial (i.e. non-accelerated) frames. A pure Galilean transformation, or Galilean boost, G is a transformation which translates a system into one moving at uniform velocity (uniform translations, so to speak): $r'(t) = r(t) - vt$, where v is the constant velocity. Since the velocity is constant, the acceleration (its derivative with respect to time) is zero, so that Newton's equation remains the same in form after the transformation G. That means, again, that Newtonian mechanics is Galilean invariant.

Summarizing, the basic algorithm to figure out whether a theory is invariant under a symmetry S is therefore the following:

1- Identify the material ontology O of the theory and its dynamical law L.
Example: classical mechanics has a particle ontology and L is $F = ma$.
2- Apply S to the material ontology to obtain the transformed ontology of matter $S(O)$.
Example: in classical mechanics, the particle position does not change under T and under G;
3- Apply S to all the other variables in the dynamical law L.
Example: in classical mechanics, under both T and G the mass, the force and the acceleration (being the second derivative of position) are unchanged;
4- If the dynamical law L is unchanged in form, then the transformed material ontology $S(O)$ is still a possible way the world can be according to the theory, and the theory is invariant under S.
Example: given what we just established (all ingredients of L are unchanged by both transformations), classical mechanics is invariant under T and G.
5- If instead L changes, then the theory is not invariant.

4. Albert's Wavefunction Realist, non-Galilean Invariant Theory

Moving to the pilot-wave theory, we need to be clear about what the ontology of matter is. First consider Albert's marvelous point approach to the pilot-wave theory, with one $3N$-dimensional particle, represented by its configuration X, and one $3N$-dimensional field, ψ, are both representing the material constitution of the pilot-wave world. The material ontology of this theory is thus given by $O = (X, \psi)$, and the laws are given, respectively, by the guidance equation and the Schrödinger equation. It has been argued that in virtue of such a material ontology, quantum theory, and in particular the pilot-wave theory, is not invariant under T and G.[16] Here is the argument. Suppose we apply T to the ontology of matter of the theory in this view, namely $O = (X, \psi)$: the position remains the same, but also the wavefunction. In fact, the transformation should act on an object accordingly with the object's nature. As anticipated, and so the argument goes, in classical mechanics velocities, being defined as rate of change of position, flip direction under T. Instead, there is no reason why the wavefunction, which in this view is part of the ontology and it is seen as field in a high dimensional space, would change in any way: assuming that a history of the world can be imagined as a sequence of snapshots depicting the ontology of the world at various times, and assuming also that applying T means to reverse the order of these snapshots, then the

[16] Albert (2000). For time reversal invariance, see also Callender (2000).

content of the picture should stay the same.[17] That is, not only $X' = T(X) = X$, but also $\psi' = T(\psi) = \psi$, so that $T(O) = (X, \psi) = O$. However, if we now plug in this transformed material ontology in the Schrödinger equation, a minus sign comes up. In order to restore time reversal invariance, the wavefunction would have to transform into its complex conjugate. But there is no reason why it would do that, so it won't do that. Because of this, it is concluded then that the theory is not time reversal invariant.[18]

The same reasoning can be done for G: a Galilean boost transforms a given system into a system in uniform motion, and the ontology (X, ψ) should stay the same, as there is no reason why it should change in any way. If that is the case, the theory is not Galilean invariant: to be invariant the wavefunction would have to change in this inexplicable way: $\psi' = e^{\frac{i}{\hbar}\sum_j m_j v \cdot x_j}\psi$. Thus, to come back to our question regarding the Galilean invariance of the pilot-wave theory: if it is interpreted along the lines of wavefunction realism, then it follows that the theory is not Galilean invariant, as solutions are not transformed into solutions by the transformation G.

5. DGZ Particle Only, Galilean Invariant Theory

A different conclusion is reached by Dürr, Goldstein and Zanghì (1992) and by Valentini (1997). Let's discuss Dürr, Goldstein and Zanghì's view in this section, and Valentini's position in the next. In their first paper on the theory, Dürr, Goldstein and Zanghì (DGZ) start assuming that matter is made of particles only, so that $O = X$, and that the particles motion is governed by some equation. To determine which evolution equation the particles follow they use two principles: simplicity and symmetries. The simplest equation for the motion of the particles is a first order differential equation in terms of the velocities: $\frac{dx_i}{dt} = f_i(x_1, \dots, x_N)$, for some unknown function f_i. By imposing rotation invariance, time reversal symmetry, and Galilean invariance, they obtain the guiding equation for the particles as $\frac{dx_i}{dt} = \frac{\hbar}{m_i} Im\ \frac{\nabla_i \psi}{\psi}$, assuming that there is a function ψ which transforms under time reversal into its complex conjugate, under a Galilean transformation as $\psi' = e^{\frac{i}{\hbar}\sum_j m_j v \cdot x_j}\psi$, and it is such that $\psi = z\psi$, for some complex number z. The simplest evolution equation for such a function, compatible with the

[17] Allori (2019).

[18] For a proposal on how to make the theory time reversal invariant even in this context, see Struyve (2021). The proposal, however, requires a different understanding of the field the wavefunction is supposed to be represent. For the way in which the nomological approach deals with this, see Allori (2019).

above-mentioned symmetries, is the Schrödinger equation.[19] Thus, in the formulation of DGZ, the pilot-wave theory is Galilean invariant by construction.

Skow (2010) has criticized this proposal as question begging. Following Albert, he argued that a symmetry transforms an object as the nature of the object prescribes, as we saw above.[20] Thus, the Galilean invariance of the theory can be assessed only discussing first what the wavefunction is: only in this way one can establish its natural ways of transforming under the various symmetries. This is a '(material) ontology first' approach, where first one specifies the ontology of matter according to the theory, and then one sees whether the theory is invariant under a certain symmetry, given that material ontology. As we have seen in the previous section, if the wavefunction is a material field understood as Albert does, then the theory is not time reversal and Galilean invariant. In the account of DGZ, the wavefunction has to transform as needed to get the invariances, so for Skow the question becomes: what kind of object is a wavefunction which transforms naturally under these transformations as to maintain the corresponding invariances?

As anticipated, DGZ have argued that the wavefunction does not represent material objects but rather it is a nomological entity. Symmetries instead seems to leave only the ontology of matter 'alone': a theory is invariant under a given transformation S if a possible physical evolution of matter is transformed by S into another possible physical evolution in the sense previously specified. This allows for the wavefunction to transform as needed for invariance: the wavefunction is flexible in this respect.[21]

Regardless of the possible criticisms to the nomological approach, it does seem to be the case that the material ontology of the theory determines its symmetries.[22] Nonetheless, the situation has to be more subtle than that, as Valentini's position will show.

6. Valentini's Aristotelian, Unphysically Galilean Invariant Theory

A completely different approach is the one put forward by Valentini (1992, 1997). He argues that the pilot-wave theory is not Galilei invariant in virtue of being

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[19] As discussed in other papers, DGZ show that one can obtain the guidance law also through other considerations. Nonetheless, it is always clear that DGZ regard the theory as Galilei invariant, regardless of how the guidance equation is obtained (see also Dürr and Teufel 2009).

[20] This type of argument has been put forward in the case of time reversal by other authors as well: Earman (2002), Malament (2004), Arntzenius (2004), Arntzenius and Greaves (2009), Roberts (2017, 2021).

[21] See also Allori (2018, 2019, 2021).

[22] See also Solè (2013).

fundamentally a first order theory. More precisely, he thinks that the theory is mathematically Galilean invariant, but this symmetry has no physical significance.[23] Here is my reconstruction of Valentini's argument.

6.1 A Newtonian Analogy to Determine the Natural Motion

Valentini distinguishes between kinematics and dynamics, the former describing the structure of spacetime, and the latter describing the motion of matter in such background. The natural kinematic of a theory arises from the definition of free system, namely a system subject to no influence from other bodies. According to Valentini, any effect on the motion of a body which is independent of the particular properties and the composition of the bodies involved should be regarded as originating from spacetime. This in turns defines the natural kinematics and the natural geometry of the spacetime in which the motion happens.

In classical mechanics the natural, or free, motion is one in which there are no forces. This is the uniform motion because the dynamical law involves the acceleration: since $F = ma$, when there are no forces, we also has to be no acceleration, meaning a constant velocity. The theory is then Galilean invariant as it does not distinguish uniform motion from rest. The natural motion for a given particle i is then $r_i(t) = vt + r_{0,i}$, where v and $r_{0,i}$ are constants. These free particle trajectories are independent of the particle masses, so they can be considered as properties of spacetime itself: "that is, as geodesics associated with an appropriate affine structure" (Valentini 1997). Similarly, in general relativity, a body subject only to gravitational forces is free, and since the trajectories are independent of the mass and the composition of the body, they are regarded as geodesic in curved spacetime.

Valentini then argues that the first order character of the pilot-wave theory selects a kinematic which one could label Aristotelian, as it generates velocities, and which is incompatible with physical Galilean invariance. He claims that in the pilot-wave theory a body is free if the guidance law $m_i \frac{dr_i}{dt} = \nabla_i S$, which now constrains velocities, becomes $m_i \frac{dr_i}{dt} = 0$. That is, when there are no 'Aristotelian forces' $f_i = \nabla_i S$. This means that the natural motion is $r_i(t) = r_{0,i}$. Since this motion does not depend on any feature of the body involved, Valentini thinks it should be attributed to spacetime itself. Thus, the natural spacetime for the pilot-wave theory is Aristotelian, in which there is a standard of absolute rest, and Galilean symmetry does not hold.

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[23] See also Brown *et al.* (1996).

6.2 The Wavefunction Transforms Exponentially so the Theory is Mathematically Galilean Invariant

Valentini also notices that even if the equations of motion are Galilean invariant (in the dynamical sense noted above: they 'transform solutions into solutions'), this mathematical symmetry has no physical significance. Mathematically, under G, a Galilean boost, we have $x'_i = x_i - vt$, and $\psi' = e^{\frac{i}{\hbar}\sum_j m_j v \cdot x_j}\psi$. This leads to Galilei invariant laws where $S' = S - \sum_i m_i v \cdot x_i$, so that $\nabla'_i S' = \nabla_i S - m_i v$. However, Valentini notices, the term $m_i v$ does not represent a physical, 'Aristotelian', force, or casual agent, responsible for the particles motion. Rather, it is a fictitious 'force' which is introduced in the boosted (i.e. non-inertial, according to the pilot-wave dynamics) frame only to be able to use the guidance equation in the same form as in the rest frame. Valentini maintains that this is analogous for apparent forces in classical mechanics. A non-inertial frame (in the classical case, an accelerated frame) will not obey the second law unless one adds some fictitious forces, such as for instance the Coriolis force and the centrifugal force in the case of a rotating frame. These forces do not originate in any physical body, and thus, are to be taken to be part of the geometry of spacetime. Similarly in the pilot-wave theory, the 'forces' appearing in a non-inertial (in this case, a boosted) frame are to be attributed to the spacetime geometry. Thus, the supposed Galilean invariance of the pilot-wave theory is, in Valentini's view, a first-order analogue of the above fictitious invariance of second-order classical mechanics. Just as the true, physical invariance group of classical mechanics leaves acceleration and (Newtonian) force invariant, so the true, physical invariance group of pilot-wave dynamics leaves velocity and (Aristotelian) 'force' invariant. In the pilot-wave theory, a Galilean transformation does not respect the true geometry of spacetime even if leaves the dynamics invariant. This is what Valentini means, I take it, when he says that Galilean symmetry is unphysical.

7. The Puzzle

As anticipated in the introduction, Valentini's result that the pilot-wave is Aristotelian has been attributed to him endorsing wavefunction realism.[24] Indeed, his conclusion seems the same as Albert's: the theory is not (physically) Galilei invariant. However, I think that the similarity is only superficial. In fact, wavefunction realists such as Albert will not concede that the wavefunction transforms as to be multiplied by a suitable exponential factor. Rather, they flat out deny that Galilei invariance is even a mathematical symmetry. Instead, that is exactly what Valentini grants, even if he denies its physical significance.

[24] Belousek (2003), Solè (2013).

Moreover, by 'mathematical symmetry' Valentini means that one could artificially impose it to the theory, out of convenience, even if physically it means nothing. In doing that one introduces fictitious forces like in classical mechanics, to make it work even in non-inertial frames. If so, one may think of the wavefunction as transforming in strange ways, but also acknowledging that this is so because of convenience. However, this is not the standard brand of wavefunction realism, according to which the wavefunction is what it is, and transforms as its nature dictates, regardless of our convenience of description. Rather, this approach seems more compatible with a weak nomological approach: the wavefunction is to be understood not as a physical field, but as some sort of force, something that makes stuff move. This is, in my opinion, a better interpretation of Valentini's position, also because he explicitly writes things like the following: "the pilot wave ψ should be interpreted as a new causal agent, more abstract than forces or ordinary fields" (Valentini 1997).

In any case, even understanding Valentini in this weak nomological sense, it remains unclear, from considerations of ontology alone, why Valentini thinks Galilei invariance is only a mathematical, fictitious symmetry. Indeed, understanding Valentini this way exacerbates the problem of figuring out why he disagrees with DGZ. In fact, they do not disagree about the dynamical law, as they both think the pilot-wave theory is fundamentally of the first order, and they both agree that the material ontology is the one of particles. Still, they disagree about the Galilei invariance of the theory.

How can they possibly disagree if symmetries are determined by the material ontology and its laws? Either it is not true that symmetries are determined by the ontology of matter and its dynamics alone, or there is a mistake somewhere. If there is something more to determine symmetries, what is it? If there is a mistake, what is it?
I am going to argue in the next section, that the disagreement lies in the role and the importance of kinematics: contrary to DGZ, Valentini gives more importance to the kinematics than to the dynamics, and believes that the symmetries of the former should constrain the latter.

8. Valentini's 'Kinematics First' Approach

Let me discuss Valentini's argument in more detail. I think that the crux of the disagreement is what makes a symmetry having physical significance: for DGZ dynamical symmetries are all physical, while for Valentini a dynamical symmetry is physical only if it 'respects' the geometry of spacetime. For Valentini, ultimately, a dynamical symmetry which does not respect the spacetime geometry is unphysical. In other words, there is a sense in which, roughly put, for Valentini kinematics comes first, as it constrains the symmetries of the dynamics: given that in the pilot-wave theory the

particles have a natural kinematics in terms of Aristotelian forces, rather than Newtonian forces, then Galilean invariance is not a physical symmetry, he concludes. The algorithm discussed above to identify the symmetries of a theory, according to Valentini, determines the dynamical symmetries, but among them only the symmetries which are compatible with the theory's kinematics are physical. This is what he means that kinematical and dynamical symmetries need to match.
But why does he think that? I believe that this boils down to the idea that there are two ways the motion of a material body can be affected: it can be affected by the geometry of spacetime, and by the presence of other bodies. Imagine there is only one body in the universe: the way in which it moves solely depends on the features of spacetime: if it is flat, the body goes straight; if it is curved, it moves along the geodesics, the shortest lines between two points. This motion is what we refer to as free motion, or natural motion, because it is not due to the interaction with other material bodies, but it depends only on the features of spacetime itself. According to Valentini, the natural motion is captured by the kinematics: we recognize effects to be kinematical if they are the same for all bodies, namely they are independent of the features of a given body. If something affects all bodies equally, he wants to say that it is due to the geometry, rather than to the interaction with other bodies. Once we have determined the true kinematics of a theory, namely the geometry of spacetime suggested by the theory's kinematics, then we have our symmetries. If the dynamics has additional symmetries, for Valentini they are merely mathematical: they are convenient to postulate to make the theory applicable to all frames, like classical mechanics in accelerated frames.
Even if Valentini does not stress this, I think that this view is heavily influenced by Einstein's equivalence principle between the gravitational mass and the inertial mass. From the fact that these two masses were the experimentally the same, Einstein inferred that they were actually the same, and since this implies that all bodies are affected by gravity in the same way, he came to believe that gravity was a feature of spacetime rather than an effect of the interaction between bodies. Valentini is going along these lines in the sense of considering everything affecting all bodies the same way as originating in spacetime.

The bottom line is therefore that we need to first identify the kinematics, and then have the dynamics match whatever symmetries the kinematics suggests. This can be done as follows:

1- *From the dynamics to the natural motion*. Extract the natural motion for the theory defined as the motion a body would have without any external influence. This is obtained by switching off all external influences on a body in the theory's dynamics. An influence is called external if it is independent of the features of the body itself. In classical mechanics it means switching off all forces, so that the natural motion is uniform motion. In the pilot-wave theory, for Valentini it means switching off the Aristotelian forces, so the natural motion is rest.

2- *From the natural motion to the natural kinematics*. The natural kinematics is the one which puts constraints on the natural motion. For classical mechanics the kinematics is Galilean, as the law for the natural motion constrains the acceleration, so that there is no distinction between uniform motion and rest. In the case of the pilot-wave theory, it is Aristotelian, as the natural motion constraints the velocity.
3- *From the natural kinematics to the natural geometry*. The natural kinematics determines the natural geometry of spacetime: if an influence is external to the body, it needs to be attributed to spacetime. In classical mechanics the spacetime is Galilean because Newton's law does not require absolute velocity, given that it constrains the acceleration. In the pilot-wave theory the guidance equation constrains the velocity, so it does need absolute velocity.
4- *From the natural geometry to kinematical symmetries*. This determines the kinematical symmetry properties of a theory: if the theory does not distinguish between two possible ways the world can be, then there is a symmetry associated with it. Classical mechanics does not distinguish between uniform motion and rest so there is a symmetry transforming them into one another. This symmetry corresponds to Galilean boosts. In the pilot-wave theory there is no such symmetry because the theory does distinguish between these two possible motions. The natural invariance group is translations and rotations, without Galilean boosts, according to Valentini.
5- *From kinematical symmetries back to dynamical symmetries*. The dynamical symmetries should match the natural kinematics. In the case of classical mechanics, kinematics and mechanics do match, and if we want them to match also in the pilot-wave theory, we should say that the theory is not dynamically Galilean invariant.

9. Objections to Valentini's Approach

Somewhat inexplicably, this argument has not been discussed in the literature at all. Nonetheless, some objections were anticipated by Valentini in his paper, while others have been informally put forward by various people in discussion with me, as indicated in the text. Others instead come from my considerations and reflections. These objections range from attributing the lack of Galilei invariance of the pilot-wave theory to wavefunction realism, as already discussed, to questioning the validity of the Newtonian analogy, to other considerations about the role of kinematics and dynamics. I discuss the problems with the Newtonian analogy first, then I move to some other considerations concerning the relationship between kinematics and dynamics. In this regard, I wish to argue first that while Valentini's argument that kinematics is important may be persuasive, it still does not imply that the dynamics necessarily needs to match its symmetries.

9.1 The Newtonian Analogy Fails

One set of objections to Valentini's argument, part of which Valentini already addresses in his paper, has to do with the Newtonian analogy. One could insist that the idea of natural motion does not make sense even in classical mechanics because we cannot actually turn off the forces. Additionally, one could argue that the pilot-wave analogy of free motion would not be the gradient of the phase being zero, as Valentini maintains, but rather the wavefunction being zero everywhere, and this is impossible (Tumulka, Maudlin, p.c.).
Valentini (1992) replies by saying that even if it may not be possible to actually have such a state in practice that does not mean it is not possible to think about it in principle, and this is enough to make his point.

Similarly, one may argue that the analogy with Newtonian mechanics does not hold because while physical forces always have their origin in bodies and decrease with distance, in the pilot-wave theory neither of these is true.
Valentini (1992) replies that this is of no importance: each theory has its own natural kinematics, which can be determined by following the steps above, regardless of how appropriate the analogy is between the nature of the forces in the two theories: what is important is how we can find the free motion in both.

I think that Valentini is correct in these replies. In my opinion, a little more problematical is Bricmont's remark (p.c.) that the free motion in the pilot-wave theory is not what Valentini suggests. That is, the free motion is not obtained putting the gradient of the phase of the wavefunction to zero. Since free motion corresponds to no interaction between physical bodies, Bricmont says that the potential has to be zero instead. Nonetheless, he continues, it does not mean that the wavefunction is zero (or constant) as well. Indeed, absence of interaction could also be described by a wavefunction which is the product of the wavefunction for each particle. In this way, since the potential is zero, the free particle wavefunction is typically a plane wave $\psi(x,t) = Ae^{i\hbar(kx-\omega t)}$, so that we have $\nabla S = \hbar k$, as one finds in physics textbooks. So, the free motion equation is $mv = p$, which is compatible with Galilean invariance, because $p = \hbar k$ is constant. Also, one can show[25] that a free Gaussian wave packet (i.e. a wave packet evolving according to an Hamiltonian with no potential) initially $\psi(x,0) =$

[25] Holland (1993) chapter 4, Bricmont (2016) chapter 5.

$\frac{1}{\pi^{1/4}}e^{-x^2/2}$ evolves into $\psi(x,t)=\frac{1}{\left(1+i\frac{t}{m}\right)^{1/2}\pi^{1/4}}e^{-\frac{x^2}{2\left(1+i\frac{t}{m}\right)}}$. This means that $S(x,t)=\frac{tx^2}{2m\left(1+i\frac{t}{m}\right)}$ so that the equation of motion is $\frac{dX(t)}{dt}=\frac{t}{m^2+t^2}X(t)$, whose solution is $X(t)=X(0)\left(1+\left(\frac{t}{m}\right)^2\right)^{\frac{1}{2}}$, showing that the natural motion is not rest.

Valentini replies (p.c.) that it is a classical prejudice to think that free motion amounts to have no potential: the relevant object one needs to look at when there is no interaction is the gradient of the phase, not the potential, as this is what appears in the guidance equation for the motion of the particles. It is just a mistake that people consider the motions described above as 'free:' the above-mentioned Gaussian 'free' particles are not free at all.

9.2 Fictitious Forces are Not Part of the Geometry of Spacetime

I believe Valentini's approach is coherent, thereby successfully defeats the 'Newtonian' objections above. Regardless, I believe that there are other considerations to be made about Valentini's understanding of the role of kinematics which I think will make his account less plausible. Let's go back to Valentini's claim that kinematics combines the effects of spacetime structure: any effect on a body's motion that is independent of the body itself (namely an effect that affects each body in the same way) is to be thought as due to the geometry of spacetime. As anticipated, the motivation for entertaining this view presumably comes from general relativity, according to which the effects of the gravitational force being independent of the object allows gravity to be geometrized. This is the sense in which one can think of gravity as a fictitious force, and this is also the sense in which the most fitting comparison with the pilot-wave theory seems to be with general relativity rather than with classical mechanics. In fact, Valentini's understanding of kinematics in this way has the counterintuitive consequence that fictitious forces in classical physics are part of spacetime structure, while this does not seem to be the way they are normally considered, in contrast with what happens in general relativity.

Nonetheless, one may insist that what Valentini means here is simply that these forces point at the existence of a preferred frame, and this is similarly the case for the pilot-wave theory, where the fictitious forces pop up in non-inertial (i.e. boosted) frames. Whether that counts as 'geometrizing' the forces is unclear, as we will discuss later.

9.3 We Do Not Feel the Fictitious Forces in the Pilot-Wave Theory

Let's assume now that this is the right way to think about fictitious forces. Then one has the problem that, while fictitious forces in classical mechanics and general relativity are felt in the appropriate frames (accelerated ones), this is not true for the pilot-wave theory (for uniformly moving frames): we do not feel any (Aristotelian) force when moving in a uniform motion. Indeed, in classical mechanics we can tell we are in an accelerated frame because we experience forces which we identify as fictitious because they have no origin in other physical bodies. Also in general relativity, we realize that we are in a curved spacetime because we experience gravity, and we categorize it as fictitious because it is the same for all bodies. If Valentini were correct, in the pilot-wave theory we should experience similar fictitious forces in boosted frames. The problem. I think, is that we do not feel anything of the sort.

Valentini recognizes this, and disputes that in classical mechanics we can reliably identify the forces in the accelerated frame as fictitious. He argues that we can identify these forces as fictitious only because we assume they are not generated by other bodies, while they could well be: "They might be generated, for instance, by acceleration with respect to distant matter" (Valentini 1997). He could say something similar for gravity: we cannot prove it is a fictitious force, namely we cannot prove that spacetime is curved, as we could be in an accelerated frame.

Again, I am not sure this is a convincing reply: if Valentini wishes to insist that fictitious forces in classical mechanics might not be fictitious after all, then there is no reason to conclude that the transformation to accelerate frames in which the fictitious forces come out is unphysical, and conversely there is no reason to conclude that Galilean invariance in the pilot-wave theory is unphysical as well.

9.4 The Dynamics Does Not Have to Match the Kinematics

Even granting Valentini that the fact that we do not detect the fictitious forces in the uniformly moving frame in the pilot-wave theory is not a problem, I still think that his position fails to convince that the kinematics should constrain the dynamics. I think that Valentini's view about the importance of kinematics and its connection with the dynamics is very plausible in the sense that the free motion is what reveals to us the geometry of spacetime: the body will freely move as it does in virtue of the properties of spacetime, as there is nothing else to constrain it. Once we know what the arena of the physical phenomena looks like, then we can put physical objects into it, and have them interact with one another *via* the dynamical law.

However, this at most shows that kinematics is informative, without showing that it necessarily has to have the final word about symmetries. In fact, there seems to be at least one good reason to think that the dynamics should be prioritized over the kinematics, as opposed to the other way around as Valentini proposes: *the kinematics of a theory is a special case of the dynamics of that theory*. The theory's kinematics, by Valentini's own admission, is obtained by switching off the forces in the dynamics, so we start off with the dynamics in his approach as well. By insisting that the kinematics constrains the dynamics, Valentini is looking at the features of a special case to infer the properties of a more general case. It seems to me that, in virtues of its generality, whatever the dynamics tells you is a physical symmetry, then it should be a symmetry also for the kinematics. If instead we insist on the contrary, we end up in a situation in which the symmetries of the special case are attributed to the more general case, and this does not seem to be sensible. It seems like inferring that the graph of a generic cubic function $f(x) = ax^3 + bx^2 + cx + d$ is symmetric with respect to a line parallel to the y-axis because that is what we find when looking at a parabola, which is the special case in which $a = 0$. Or, more mundanely, like inferring that all American boys wear a blue uniform when they go to school by looking solely at the children going at the school behind my house. Therefore, since Galilei invariance is a symmetry for the pilot-wave dynamics (in the sense that 'solutions are transformed into solutions'), then it also has to be a symmetry for the kinematics, namely for spacetime, even if the kinematics does not 'see' this symmetry.

If that is the case, we cannot really know what the true geometry is simply by looking at a body's free motion. While it may be true that switching off the interaction in the dynamical law will tell us what the free motion is, it seems to be unreasonable to read too much from it about the properties of the 'true' geometry of spacetime: the free motion might identify *some of* the effects which are independent of the presence of other bodies, and which thus could be attributed to spacetime. However, there is no guarantee it will identify all of them, as the presence of other bodies could modify the symmetry properties of spacetime itself, or the geometry of spacetime itself, as general relativity teaches us. Indeed, in general relativity in absence of any other material object a body moves along straight lines. If Valentini's argument were correct, then it seems that we should conclude that the natural geometry of spacetime is flat. However, when we put other bodies in the world, so to speak, they change the geometry of spacetime, and also its dynamics. So, we cannot gain much information on the geometry of spacetime by looking at the free motion alone.

9.5 The Kinematic Does Not Affect the Dynamics

Indeed, the disanalogy with general relativity continues in the following sense. In general relativity gravity is a fictitious force in the sense that its effect is accounted for by the curved spacetime structure. In the pilot-wave theory instead the forces which appear according to Valentini in the uniformly moving frames are fictitious in the sense that they pick up a preferred frame. One can think of the free trajectories as the equivalent of the geodesics, and they do not change in time. Instead, in general relativity the fictitious force (gravity) changes the geodesic by curving spacetime, so the kinematics changes in time. In this way the kinematics is relevant for the dynamics: if the geometry of spacetime changes, the dynamics changes as well. Instead, this is not the case for the pilot-wave theory: having a preferred frame or not does not change the body's motion. In other words, whether the uniformly moving frame is preferred or not does not change the dynamics, while in general relativity the motion changes depending on whether or not spacetime is curved. That is, while in general relativity the natural kinematics, as Valentini would put it, influences the dynamical symmetries, this is not the case for the pilot-wave theory, where the kinematical symmetries are entirely irrelevant for the dynamics.

In classical mechanics the situation is similar to the pilot-wave theory: the kinematics does not affect the dynamics, because the geometry of spacetime is fixed. However, since there is no mismatch between kinematics and dynamics (they are both Galilean invariant), we do not have a problem to start with.

9.6 A Step Further Away from Relativity

As a final remark, let me notice the following, which is not an argument against Valentini's view, but rather only an unwelcomed consequence, in my opinion. Assume that Valentini is correct that the theory is not Galilei invariant. Valentini takes this to be an advantage: since the theory is not Galilei invariant, it does not make sense to look for Lorentz invariant extensions of the pilot-wave theory, so it is no surprise one cannot find any.

I find this problematical for a variety of reasons. First, it is not true that there are no Lorentz invariant extensions of the theory: several such proposals have been put forward.[26] In addition, it may well be that case that even if a theory is not Galilei invariant, it would be Lorentz invariant: after all, Galilean symmetry is false, as we know that relativity is true, so it is unclear why we should expect Galilean invariance to hold.

Regardless, it seems to me that going from Galilean spacetime back to Aristotelian spacetime, as Valentini is proposing, is exactly the opposite of what Einstein would have liked to have obtained with his principle of relativity. In fact, I take it, the principle of relativity embodies the belief that the form of the physical laws should be as independent as possible form the reference frame in which we describe them, so that

[26] Even if it is controversial that they are truly relativistic invariant. In any case, see Dürr *et al.* (2014) and references therein.

they are as independent as possible from our description of them. In classical mechanics all inertial (non-accelerated) frames are equivalent, as reflected in Galilean relativity, while non-inertial frames can be identified by the presence of fictitious forces. As it is known, Einstein wanted a theory in which all inertial frames are equivalent (like the principle of Galilean relativity) but he also wanted to reconcile this with classical electrodynamics, so in special relativity he introduced the Lorentz transformations for the velocity, instead of the Galilean ones. General relativity expanded the principle of relativity to non-inertial frames by asserting that gravity is embodied in the distortion of spacetime.

Now, with respect of the relativity principle, which amounts of making the description of the phenomena as independent from our point of view as possible, general relativity was a step closer than classical mechanics toward that goal: in classical mechanics non-accelerated frames are privileged while in general relativity they are not. Instead, Aristotelian spacetime is a step backwards: it distinguishes between uniform motion and rest, while classical mechanics does not. In other words, with general relativity we go towards a more frame-independent description than classical mechanics, while if the pilot-wave theory is Aristotelian in the sense of Valentini then we go towards a more frame-dependent description. So, if one believes the principle of relativity to be correct then this result seems undesirable.

10.Conclusions

In this paper I have tried to make sense of the various conflicting positions concerning the Galilei invariance of the pilot-wave theory. I have attributed the claims of those who think the theory is not Galilei invariant to their commitment to wavefunction realism, as opposed to those endorsing the nomological view who take the theory to be invariant. Then, I have focused on Valentini's claim that the theory has an Aristotelian geometry which constrains the dynamics of the theory.

I have argued that this is independent on his view about the status of the wavefunction, as I have shown that his argument would go through even for someone endorsing the nomological view. I have then discussed how the free motion that Valentini attributes to the pilot-wave theory is questionable. Moreover, I argued that kinematics should not impose its symmetries on the dynamics because kinematics is a special case of the dynamics. Even if Valentini's proposal is inspired by general relativity, the comparison with this theory leads to disanalogies that suggest that at best kinematical symmetries might constrain the dynamics when they influence it, and this is not the case for the pilot-wave theory. Because of these reasons, while I found Valentini's argument to be very persuasive in certain respects, I ultimately do not find it convincing that the pilot-wave theory is not Galilei invariant.

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