# Can the ontology of Bohmian mechanics consists only in particles? The PBR theorem says no

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**Abstract**

The meaning of the wave function is an important unresolved issue in Bohmian mechanics. On the one hand, according to the nomological view, the wave function of the universe or the universal wave function is not ontic but nomological, like a law of nature. On the other hand, the PBR theorem proves that the wave function in quantum mechanics or the effective wave function in Bohmian mechanics is ontic, representing the ontic state of a physical system in the universe. It is usually thought that the nomological view of the universal wave function is compatible with the ontic view of the effective wave function, and thus the PBR theorem has no implications for the nomological view. In this paper, I argue that this is not the case, and these two views are in fact incompatible. This means that if the effective wave function is ontic as the PBR theorem proves, then the universal wave function cannot be nomological, and the ontology of Bohmian mechanics cannot consist only in particles. Moreover, I argue that although the nomological view can be held by rejecting one key assumption of the PBR theorem, the rejection will lead to serious problems, such as that the results of measurements and their probabilities cannot be explained in ontology in Bohmian mechanics.

## 1 Introduction

Bohmian mechanics or the pilot-wave theory of de Broglie and Bohm provides an ontology of quantum mechanics in terms of particles and their trajectories in space and time (de Broglie, 1928; Bohm, 1952). One important unresolved issue in this theory is the meaning of the wave function. Is the wave function of the universe ontic, representing a concrete physical entity, or nomological, like a law of nature? In recent years, the nomological view of the wave function becomes more and more popular (Dürr, Goldstein and Zanghì, 1997; Allori et al, 2008; Esfeld et al, 2014; Goldstein, 2021). On this view, there are only particles in three-dimensional space in Bohmian mechanics. At the same time, a general and rigorous approach called ontological models framework has been proposed to determine the relation between the wave function and the ontic state of a physical system (Harrigan and Spekkens, 2010), and several $\psi$-ontology theorems have been proved in the framework (Pusey, Barrett and Rudolph, 2012; Colbeck and Renner, 2012, 2017; Hardy, 2013). In particular, the Pusey-Barrett-Rudolph theorem or the PBR theorem proves that the wave function in quantum mechanics or the effective wave function in Bohmian mechanics is ontic, representing the ontic state of a physical system in the universe (Pusey, Barrett and Rudolph, 2012). An interesting question then arises: is the nomological view of the universal wave function is compatible with the ontic view of the effective wave function?

This issue has not received much attention in the literature. For example, Goldstein's (2021) comprehensive review of Bohmian mechanics does not mention the PBR theorem, and Esfeld et al's (2014) insightful paper about the ontology of Bohmian mechanics refers to the theorem once but without discussion. Presumably it is thought that the two views of the wave function are obviously compatible. They both say that the wave function is real for single physical systems after all. Moreover, according to Esfeld et al (2014), although Bohmian mechanics says that the universal wave function is nomological, it regards the effective wave function of a subsystem in the universe as ontic, representing "an objective, physical degree of freedom belonging to the subsystem", and thus the nomological view is compatible with the ontic view. In this paper, I will argue that this received view is debatable, and a careful analysis of the (in)compatibility between the nomological view and the ontic view will deepen our understandings of the meaning of the wave function in Bohmian mechanics.

The rest of this paper is organized as follows. In Section 2, I first introduce Bohmian mechanics and the nomological view of the wave function. According to the nomological view, the wave function of the universe or the universal wave function is not ontic but nomological, like a law of nature, and the ontology of Bohmian mechanics consists only in particles. In Section 3, I then introduce the ontological models framework and the PBR theorem based on the framework. The PBR theorem proves that the wave function in quantum mechanics or the effective wave function in Bohmian mechanics is ontic, representing the ontic state of a physical system in the universe. In Section 4, I argue that the nomological view of the universal wave function is incompatible with the ontic view of the effective wave functions. This means that if the effective wave function is ontic as the PBR theorem proves, then the universal wave function cannot be nomological, and the ontology of Bohmian mechanics cannot consist only in particles. In Section 5, I point out that the nomological view can be held by rejecting one key assumption of the PBR theorem. But the rejection will arguably lead to serious problems, such as that the results of measurements and their probabilities cannot be explained in ontology in Bohmian mechanics. Conclusions are given in the last section.

## 2 Bohmian mechanics and the nomological view

Bohmian mechanics or the pilot-wave theory of de Broglie and Bohm provides an ontology of quantum mechanics in terms of particles and their trajectories in space and time (de Broglie, 1928; Bohm, 1952). In Bohmian mechanics, a complete realistic description of a quantum system is provided by the configuration defined by the positions of its particles together with its wave function. The law of motion is expressed by two equations: a guiding equation for the configuration of particles and the Schrödinger equation, describing the time evolution of the wave function which enters the guiding equation. The law of motion can be formulated as follows:

$$\frac{dQ(t)}{dt} = v^{\Psi(t)}(Q(t)), \tag{1}$$

$$i\hbar\frac{\partial\Psi(t)}{\partial t} = H\Psi(t), \tag{2}$$

where $Q(t)$ denotes the spatial configuration of particles, $\Psi(t)$ is the wave function of the particle configuration at time $t$, and $v$ equals to the velocity of probability density in standard quantum mechanics. Moreover, it is postulated that at some initial instant $t_0$, the epistemic probability of the configuration, $\rho(t_0)$, is given by the Born rule: $\rho(t_0) = |\Psi(t_0)|^2$. This is called quantum equilibrium hypothesis, which, together with the law of motion, ensures the empirical equivalence between Bohmian mechanics and standard quantum mechanics.

The status of the above equations is different, depending on whether one considers the physical description of the universe as a whole or of a subsystem thereof. Bohmian mechanics starts from the concept of a universal wave function (i.e. the wave function of the universe), figuring in the fundamental law of motion for all the particles in the universe. That is, $Q(t)$ describes the configuration of all the particles in the universe at time $t$, and $\Psi(t)$ is the wave function of the universe at time $t$, guiding the motion of all particles taken together. To describe subsystems of the universe, the appropriate concept is the effective wave function in Bohmian mechanics.

The effective wave function is the Bohmian analogue of the usual wave function in standard quantum mechanics. It is not primitive, but derived from the universal wave function and the actual spatial configuration of all the particles ignored in the description of the respective subsystem (Dürr, Goldstein and Zanghì, 1992). The effective wave function of a subsystem can be defined as follows. Let $A$ be a subsystem of the universe including $N$ particles with position variables $x = (x_1, x_2, ..., x_N)$. Let $y = (y_1, y_2, ..., y_M)$ be the position variables of all other particles not belonging to $A$. Then the subsystem $A$'s conditional wave function at time $t$ is defined as the universal wave function $\Psi_t(x, y)$ evaluated at $y = Y(t)$:

$$\psi_t^A(x) = \Psi_t(x, y)|_{y=Y(t)}. \tag{3}$$

If the universal wave function can be decomposed in the following form:

$$\Psi_t(x, y) = \varphi_t(x)\phi_t(y) + \Theta_t(x, y), \tag{4}$$

where $\phi_t(y)$ and $\Theta_t(x, y)$ are functions with macroscopically disjoint supports, and $Y(t)$ lies within the support of $\phi_t(y)$, then $\psi_t^A(x) = \varphi_t(x)$ (up to a multiplicative constant) is $A$'s effective wave function at $t$. It can be seen that the temporal evolution of $A$'s particles is given in terms of $A$'s conditional wave function in the usual Bohmian way, and when the conditional wave function is $A$'s effective wave function, it also obeys a Schrödinger dynamics of its own. This means that the effective descriptions of subsystems are of the same form of the law of motion as given above.

Bohmian mechanics raises the question of the status of the wave function that figures in the law. According to the nomological view of the wave function, the relationship between the universal wave function and the motion of the particles should be conceived as a nomic one, instead of a causal one in terms of one physical entity acting on the other (Dürr, Goldstein and Zanghì, 1997; Goldstein and Teufel, 2001; Goldstein and Zanghì, 2013; Esfeld et al, 2014). In the words of Dürr, Goldstein and Zanghì (1997),

> The wave function of the universe is not an element of physical reality. We propose that the wave function belongs to an altogether different category of existence than that of substantive physical entities, and that its existence is nomological rather than material. We propose, in other words, that the wave function is a component of a physical law rather than of the reality described by the law. (p. 10)

The reasons to adopt this nomological view of the wave function come from the unusual kind of way in which Bohmian mechanics is formulated, and the unusual kind of behavior that the wave function undergoes in the theory. First of all, although the wave function affects the behavior of the configuration of the particles, which is expressed by the guiding equation (1), there is no back action of the configuration upon the wave function. The evolution of the wave function is governed by the Schrödinger equation

(2), in which the actual configuration $Q(t)$ does not appear. Since a physical entity is supposed to satisfy the action-reaction principle, the wave function cannot describe a physical entity in Bohmian mechanics.

Next, the wave function of a many-particle system, $\psi(q_1, ..., q_N)$, is defined not in our ordinary three-dimensional space, but in the $3N$-dimensional configuration space, the set of all hypothetical configurations of the system. Thus it seems untenable to view the wave function as directly describing a real physical field. In fact, the sort of physical field the wave function is supposed to describe is even more abstract. Since two wave functions such that one is a (nonzero) scalar multiple of the other are physically equivalent, what the wave function describes is not even a physical field at all, but an equivalence class of physical fields. Moreover, Bohmian mechanics regards identical particles such as electrons as unlabelled, so that the configuration space of $N$ such particles is not the familiar high dimensional space, like $R^{3N}$, but is the unfamiliar high-dimensional space ${}^N R^3$ of $N$-point subsets of $R^3$. This space has a nontrivial topology, which may naturally lead to the possibilities of bosons and fermions. But it seems odd as a fundamental space in which a physical field exists.

Thirdly, the wave function in Bohmian mechanics plays a role that is analogous to that of the Hamiltonian in classical Hamiltonian mechanics (Goldstein and Zanghì, 2013). To begin with, both the classical Hamiltonian and the wave function live on a high dimensional space. The wave function is defined in configuration space, while the classical Hamiltonian is defined in phase space: a space that has twice as many dimensions as configuration space. Next, there is a striking analogy between the guiding equation in Bohmian mechanics and the Hamiltonian equations in classical mechanics. The guiding equation can be written as:

$$\frac{dQ}{dt} = der(log\psi), \tag{5}$$

where the symbol *der* denotes some sort of derivative. Similarly, the Hamiltonian equations can be written is a compact way as:

$$\frac{dX}{dt} = der(H), \tag{6}$$

where $der(H)$ is a suitable derivative of the Hamiltonian. Moreover, it is also true that both $log\psi$ and $H$ are normally regarded as defined only up to an additive constant. Adding a constant to $H$ doesn't change the equations of motion. Similarly, when multiplying the wave function by a scalar, which amounts to adding a constant to its *log*, the new wave function is physically equivalent to the original one, and they define the same velocity for the configuration in the equations of motion in Bohmian mechanics. Since the classical Hamiltonian is regarded not as a description of some physical entity, but as the generator of time evolution in classical mechanics, by the above analogy it seems natural to assume that the wave function is not a description of some physical entity either, but a similar generator of the equations of motion in Bohmian mechanics.

However, it seems that there is a serious problem with the nomological view of the wave function. The wave function of a quantum system typically changes with time, but laws are supposed not to change with time. Moreover, we can prepare the wave function of a quantum system and control its evolution, but laws are not supposed to be things that we can prepare and control. This problem indeed exists for the effective wave function of a subsystem of the universe, but it may not exist for the wave function of the universe, only which deserves to be interpreted nomologically (Goldstein and Zanghì, 2013). The wave function of the universe is certainly not controllable. And it may not be dynamical either. This can be illustrated by the Wheeler-DeWitt equation, which is the fundamental equation for the wave function of the universe in canonical quantum cosmology:

$$H\Psi(q) = 0, \tag{7}$$

where $\Psi(q)$ is the wave function of the universe, $q$ refers to 3-geometries, and $H$ is the Hamiltonian constraint which involves no explicit time-dependence. Unlike the Schrödinger equation, the Wheeler-DeWitt equation has on one side, instead of a time derivative of $\Psi$, simply 0, and thus its natural solutions are time-independent. Moreover, the wave function of the universe may be unique. Although the Wheeler-DeWitt equation presumably has a great many solutions, when supplemented with additional natural conditions such as the Hartle-Hawking boundary condition, the solution may become unique. Such uniqueness also fits nicely with the conception of the wave function as law.

The above analyses suggest that the wave function is nomological, describing a law and not describing some sort of concrete physical entity in Bohmian mechanics. A law of motion tells us what happens in space and time given the specification of initial conditions, but it is not itself a physical entity existing in space and time. The exact meaning of the wave function then depends on what exactly a law is. There are two main views about laws of nature in the literature, namely Humeanism and dispositionalism, and both of them can be drawn upon for developing the nomological interpretation of the wave function in Bohmian mechanics (Esfeld et al, 2014). By Humeanism about laws, there are only particles' positions in the ontology, while dispositionalism admits more in the ontology than particles' positions, namely the holistic disposition of all the particles in the universe.[1]

[1]Note that Bohmian mechanics is also compatible with a primitivism about laws as suggested by Maudlin (2007). It has been argued that primitivism about laws faces a dilemma: "either it has to bite the bullet of conceiving the law as developing itself in time and as including differences that correspond to different initial wave-functions, or it has

My following analysis of Bohmian mechanics and the nomological view of the wave function is independent of how to understand laws of nature.

## 3 The PBR theorem

Although there are various reasons to adopt the nomological view of the universal wave function in Bohmian mechanics, we in fact know little about the universal wave function itself, since the final theory of quantum gravity is not yet available. A more feasible approach is to analyze the meaning of the effective wave function of a subsystem in the universe, such as whether the effective wave function has a tenable physical explanation under the nomological view of the universal wave function. In recent years, there appear several rigorous arguments supporting the ontic view of the wave function in quantum mechanics or the effective wave function in Bohmian mechanics, which has been called $\psi$-ontology theorems. In this section, I will introduce the ontological models framework and an important $\psi$-ontology theorem proved in the framework, the PBR theorem.

Quantum mechanics, in its minimum formulation, is an algorithm for calculating probabilities of measurement results. The theory assigns a mathematical object, the wave function, to a physical system appropriately prepared at a given instant, and specifies how the wave function evolves with time. The time evolution of the wave function is governed by the Schrödinger equation, and the connection of the wave function with the results of measurements on the system is specified by the Born rule. At first sight, quantum mechanics as an algorithm says nothing about the actual ontic state of a physical system. However, it has been known that this is not true due to the recent advances in the research of the foundations of quantum mechanics (see Leifer, 2014 for a helpful review).

First of all, a general and rigorous approach called ontological models framework has been proposed to determine the relation between the wave function and the ontic state of a physical system (Harrigan and Spekkens 2010). The framework has two fundamental assumptions. The first assumption is about the existence of the underlying state of reality. It says that if a physical system is prepared such that the quantum algorithm assigns a wave function to it, then after preparation the system has a well-defined set of physical properties or an underlying ontic state, which is usually represented by a mathematical object, $\lambda$. In general, for an ensemble of identically prepared systems to which the same wave function $\psi$ is assigned, the ontic states of different systems in the ensemble may be different, and the wave function $\psi$ corresponds to a probability distribution $p(\lambda|\psi)$ over all possible ontic states, where $\int d\lambda p(\lambda|\psi) = 1$.

to conceive the universal wave-function as a physical entity." (Dorato and Esfeld, 2015)

There are two possible types of models in the ontological models framework, namely $\psi$-ontic models and $\psi$-epistemic models. In a $\psi$-ontic model, the ontic state of a physical system uniquely determines its wave function, and the probability distributions corresponding to two different wave functions do not overlap. In this case, the wave function directly represents the ontic state of the system.[2] While in a $\psi$-epistemic model, the probability distributions corresponding to two different wave functions may overlap, and there are at least two wave functions which are compatible with the same ontic state of a physical system. In this case, the wave function merely represents a state of incomplete knowledge - an epistemic state - about the actual ontic state of the system.

In order to investigate whether an ontological model is consistent with the quantum algorithm, we also need a rule of connecting the underlying ontic states with measurement results. This is the second assumption of the ontological models framework, which says that when a measurement is performed, the behaviour of the measuring device is determined by the ontic state of the system, along with the physical properties of the measuring device. Concretely speaking, for a projective measurement $M$, the ontic state $\lambda$ of a physical system determines the probability $p(k|\lambda, M)$ of different results $k$ for the measurement $M$ on the system. The consistency with the quantum algorithm then requires the following relation:

$$\int d\lambda p(k|\lambda, M)p(\lambda|\psi) = p(k|M, \psi), \tag{8}$$

where $p(k|M, \psi) = |\langle k|\psi\rangle|^2$ is the Born probability of $k$ given $M$ and the wave function $\psi$.

Second, several important $\psi$-ontology theorems have been proved in the ontological models framework (Pusey, Barrett and Rudolph, 2012; Colbeck and Renner, 2012, 2017; Hardy, 2013), one of which is the PBR theorem (Pusey, Barrett and Rudolph, 2012). The PBR theorem shows that in the ontological models framework, when assuming independently prepared systems have independent ontic states, the ontic state of a physical system uniquely determines its wave function, or the wave function of a physical system directly represents the ontic state of the system. This auxiliary assumption is called preparation independence assumption.

The basic proof strategy of the PBR theorem is as follows. Assume there are $N$ nonorthogonal quantum states $\psi_i$ ($i$=1, ... , $N$), which are compatible with the same ontic state $\lambda$.[3] The ontic state $\lambda$ determines the probability $p(k|\lambda, M)$ of different results $k$ for the measurement $M$. Moreover, there is a normalization relation for any $N$ result measurement: $\sum_{i=1}^{N} p(k_i|\lambda, M) = 1$.

[2]Note that the wave function is not necessarily complete, i.e. it does not necessarily represent the complete ontic state of a system.

[3]It can be readily shown that different orthogonal states correspond to different ontic states. Thus the proof given here concerns only nonorthogonal states.

Now if an $N$ result measurement satisfies the condition that the first state gives zero Born probability to the first result and the second state gives zero Born probability to the second result and so on, then there will be a relation $p(k_i|\lambda, M) = 0$ for any $i$, which leads to a contradiction.

The task is then to find whether there are such nonorthogonal states and the corresponding measurement. Obviously there is no such a measurement for two nonorthogonal states of a physical system, since this will permit them to be perfectly distinguished, which is prohibited by quantum mechanics. However, such a measurement does exist for four nonorthogonal states of two copies of a physical system. The four nonorthogonal states are the following product states: $|0\rangle \otimes |0\rangle$, $|0\rangle \otimes |+\rangle$,$|+\rangle \otimes |0\rangle$ and $|+\rangle \otimes |+\rangle$, where $|+\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$. The corresponding measurement is a joint measurement of the two systems, which projects onto the following four orthogonal states:

$$\begin{aligned}
\phi_1 &= \frac{1}{\sqrt{2}}(|0\rangle \otimes |1\rangle + |1\rangle \otimes |0\rangle), \\
\phi_2 &= \frac{1}{\sqrt{2}}(|0\rangle \otimes |-\rangle + |1\rangle \otimes |+\rangle), \\
\phi_3 &= \frac{1}{\sqrt{2}}(|+\rangle \otimes |1\rangle + |-\rangle \otimes |0\rangle), \\
\phi_4 &= \frac{1}{\sqrt{2}}(|+\rangle \otimes |-\rangle + |-\rangle \otimes |+\rangle),
\end{aligned} \tag{9}$$

where $|-\rangle = \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle)$. This proves that the four nonorthogonal states are ontologically distinct. In order to further prove the two nonorthogonal states $|0\rangle$ and $|+\rangle$ for one system are ontologically distinct, the preparation independence assumption is needed. Under this assumption, a similar proof for every pair of nonorthogonal states can also be found, which requires more than two copies of a physical system (see Pusey, Barrett and Rudolph, 2012 for the complete proof).

To sum up, the PBR theorem shows that quantum mechanics as an algorithm may also say something about the ontic state of a physical system. It is that under the preparation independence assumption, the wave function assigned to a physical system, which is used for calculating probabilities of results of measurements on the system, is a mathematical representation of the ontic state of the system in the ontological models framework.

## 4 Is the nomological view compatible with the ontic view?

The proof of the PBR theorem raises an intriguing question: if the effective wave function is ontic as the PBR theorem proves, can the universal wave function be nomological? or is the nomological view of the universal wave function compatible with the ontic view of the effective wave function? The received view seems to be that these two views are obviously compatible, and thus the PBR theorem has no implications for the nomological view of the universal wave function. In the following, I will argue that this is not the case.

Before my analysis, it is worth pointing out that if the PBR theorem applies not only to the subsystems of the universe, but also to the universe as a whole, then it is obvious that the ontic view and the nomological view are two different views of the universal wave function, and thus they are incompatible. According to the ontic view, the universal wave function is ontic, representing the ontic state of the universe or a concrete physical entity, while according to the nomological view, the universal wave function is not ontic but nomological, and it does not represent a concrete physical entity. However, it is arguable that the PBR theorem may not apply to the universe as a whole, since the proof of the theorem concerns measurements and copies of a single physical system, while there is only one universe and no measurements can be made on the universe as a whole either.

Now consider the effective wave function of a subsystem in the universe. On the one hand, the PBR theorem proves that the effective wave function of a subsystem is ontic, representing the ontic state or a physical property of the subsystem. On the other hand, according to the nomological view, the universal wave function is not ontic but nomological, and the ontology of Bohmian mechanics consists only in particles. This means that in order that the nomological view of the universal wave function is compatible with the ontic view of the effective wave function, the effective wave function of a subsystem must represent a physical property of these particles. If the effective wave function of a subsystem represents a physical property of another physical entity different from these particles, then the nomological view of the universal wave function, which requires that the ontology of Bohmian mechanics consists only in particles, cannot be true.

Let's see if the effective wave function of a subsystem represents a physical property of the Bohmian particles. First of all, as Esfeld et al (2014) have already pointed out, it is impossible to interpret the effective wave function of a subsystem as an intrinsic property of the particles of the subsystem. The reasons are that (1) one can assign a wave function only to the subsystem as a whole but not to each particle individually in general, and (2) the effective wave function depends on the universal wave function and the configuration of all the other particles in the universe (Esfeld et al, 2014).

Then, the only possibility is that the effective wave function of a subsystem represents a physical property of the particles in the environment of the subsystem. Since the effective wave function of a subsystem influences the motion of the particles of the subsystem by the guiding equation, this physical property of the particles in the environment must have the efficiency of influencing the particles of the subsystem. Moreover, since the particles in the environment and the particles of the subsystem are spacelike separated at each instant, the influence, if it exists, must be nonlocal.

This is the view supported by Esfeld et al (2014). According to these authors, the effective wave function of a subsystem encodes the nonlocal influence of other particles on the subsystem via the nonlocal law of Bohmian mechanics. For example, in the double-slit experiment with one particle at a time, the particle goes through exactly one of the two slits, and that is all there is in the physical world. There is no real physical field that guides the motion of the particle and propagates through both slits and undergoes interference. The development of the position of the particle (its velocity and thus its trajectory) is determined by the positions of other particles in the universe, including the particles composing the experimental setup, and the nonlocal law of Bohmian mechanics can account for the observed particle position on the screen (Esfeld et al, 2014). If this view is true, then one can say that the nomological view of the universal wave function regards the effective wave function as ontic, and thus it is consistent with the ontic view of the effective wave function.

However, it can be argued that the effective wave function of a subsystem does not represent a physical property of the particles in the environment, such as encoding the nonlocal influence of these particles on the subsystem (see also Gao, 2017). Let's first consider the simplest case in which the universal wave function factorizes so that

$$\Psi_t(x, y) = \varphi_t(x)\phi_t(y), \tag{10}$$

where $x = (x_1, x_2, ..., x_N)$ is the position variables of $N$ particles of a subsystem $A$ of the universe, and $y = (y_1, y_2, ..., y_M)$ is the position variables of all other particles not belonging to $A$. Then $\psi_t^A(x) = \varphi_t(x)$ is subsystem $A$'s effective wave function at $t$. In this case, subsystem $A$ and its environment, which are in a product state, are independent of each other. Thus, the effective wave function of subsystem $A$ is independent of the particles in the environment, and it does not represent a physical property of these particles. Moreover, since subsystem $A$ and its environment are in a product state, the particles in the environment do not have nonlocal influence on the particles of subsystem $A$, and thus the effective wave function of subsystem $A$ cannot encode such a non-existent nonlocal influence either.

Next, consider the general case in which there is an extra term in the factorization of the universal wave function:

$$\Psi_t(x, y) = \varphi_t(x)\phi_t(y) + \Theta_t(x, y), \tag{11}$$

In this case, the effective wave function of subsystem A is determined by both the universal wave function $\Psi_t(x, y)$ and the positions of the particles in its environment $Y(t)$. If $Y(t)$ lies within the support of $\phi_t(y)$, $A$'s effective wave function at $t$ will be $\varphi_t(x)$. If $Y(t)$ does not lie within the support of $\phi_t(y)$, $A$'s effective wave function at $t$ will be not $\varphi_t(x)$. For example, suppose $\Theta_t(x,y) = \sum_n f_n(x) g_n(y)$, where $g_i(y)$ and $g_j(y)$ are functions with macroscopically disjoint supports for any $i \neq j$, then if $Y(t)$ lies within the support of $g_i(y)$, $A$'s effective wave function at $t$ will be $f_i(x)$.

First of all, it can be seen that the role played by the particles in the environment is only selecting which function the effective wave function of subsystem A is, while each selected function is completely determined by the universal wave function. Thus the effective wave function of subsystem $A$, as part of the universal wave function, will only represent a physical property of the part of something represented by the universal wave function (if there is any), and it does not represent a physical property of the particles in the environment. This is like the parable of blind men touching an elephant. If a blind man touches the tail of an elephant, he would say that the elephant is like a rope. The property of being like a rope is a property of one part of the elephant, and it is not the property of the blind man.

Next, even if the effective wave function of subsystem $A$ represents a physical property of the particles in the environment, this property has no efficiency of influencing the particles of the subsystem nonlocally. According to the Bohmian law of motion, when the effective wave function of subsystem $A$ has been selected (via a measurement-like process), the particles in the environment have no nonlocal influence on the particles of subsystem $A$; these particles reside in an effective product state such as $\varphi_t(x)\phi_t(y)$. For example, in the double-slit experiment with one particle at a time, the development of the position of the particle will not depend on the positions of other particles in the universe (if only the positions of these particles select the same effective wave function of the particle during the experiment, e.g. $Y(t)$ has been within the support of $\phi_t(y)$ during the experiment).

Finally, it is worth noting that even if the effective wave function of subsystem $A$ encodes the nonlocal influence of the particles in the environment on the subsystem, it does not imply that the whole effective wave function is a property of these particles. The reason is that the nonlocal influence, which determines the velocities of the particles of the subsystem, is determined only by the phase of the effective wave function, and not by the amplitude of the effective wave function. Thus, if there exists such nonlocal influence, it only indicates that the phase of the effective wave function of subsystem $A$ represents a property of the particles in the environment, and it does not imply that the amplitude of the effective wave function of subsystem $A$ also represents a property of these particles.

To sum up, I have argued that the effective wave function of a subsystem represents neither a physical property of the particles of the subsystem nor a physical property of the particles in the environment, such as encoding their nonlocal influence on the subsystem. In short, the effective wave function of a subsystem does not represent a physical property of the Bohmian particles. This means that the nomological view of the universal wave function is not compatible with the ontic view of the effective wave function. If the effective wave function is ontic as the PBR theorem proves, then it must represent a property of another physical entity different from the Bohmian particles, and thus the universal wave function cannot be nomological, and the ontology of Bohmian mechanics cannot consist only in particles.

## 5 A possible way out

Based on the above analysis, it can be seen that the only way to hold the nomological view of the universal wave function in Bohmian mechanics is to avoid the result of the PBR theorem by rejecting one or more assumptions of the theorem. Let's see if this is possible.

As we have seen, the PBR theorem is proved based on three preconditions: (1) the quantum algorithm; (2) the ontological models framework; and (3) the preparation independence assumption. Bohmian mechanics is consistent with the quantum algorithm. Moreover, it also accepts the preparation independence assumption, since two unentangled systems (whose wave function is a product state) have independent ontic states in the theory.[4] The crux is whether Bohmian mechanics also accepts the ontological models framework when assuming the nomological view of the universal wave function.[5]

The ontological models framework has two fundamental assumptions. The first assumption is a realist assumption, which says that if a physical system is prepared such that quantum mechanics assigns a wave function to it, then after preparation the system has a well-defined set of physical properties or an underlying ontic state. This assumption is accepted by Bohmian mechanics. According to the nomological view, a subsystem of the universe which has an effective wave function is composed of particles, and the positions of these particles are the ontic state of this subsystem. Note that the ontic state of a subsystem also includes the disposition of its particles which determines their velocities via the guiding equation according to the dispositionalism about laws of nature (Esfeld et al, 2014).

The second assumption of the ontological models framework says that when a measurement is performed, the behaviour of the measuring device is determined by the ontic state of the measured system, along with the physical properties of the measuring device. For a projective measurement

[4]Note that the result that different orthogonal states correspond to different ontic states can be derived in the ontological models framework without resorting to the preparation independence assumption or other auxiliary assumptions. Thus, even if Bohmian mechanics rejects the auxiliary assumptions such as the preparation independence assumption, there is still the question of whether it is consistent with the ontological models framework.

[5]The question of whether Bohmian mechanics is consistent with the ontological models framework has been discussed by several authors (Feintzeig, 2014; Leifer, 2014; Drezet, 2015). Here I will focus on the issue of whether the nomological view of the wave function is consistent with the ontological models framework.

$M$, this means that the ontic state $\lambda$ of a physical system determines the probability $p(k|\lambda, M)$ of different results $k$ for the measurement $M$ on the system. Then, in order that the nomological view of the universal wave function is valid in Bohmian mechanics, the theory must reject this assumption. Concretely speaking, the revised assumption will be that when a measurement is performed, the behaviour of the measuring device is determined not only by the ontic state of the measured system and the measuring device, but also by something else not in the ontology, a nomological component represented by the effective wave function of the system. In particular, for a projective measurement $M$, the complete ontic state $\lambda$ of a physical system and its effective wave function $\psi$ both determine the probability of different results $k$ for the measurement $M$ on the system, which may be denoted by $p(k|\lambda, \psi, M)$.[6]

It can be seen that the proof of the PBR theorem cannot go through based on this revised assumption. Let's remind the basic proof strategy of the PBR theorem. Assume there are $N$ nonorthogonal quantum states $\psi_i$ ($i$=1, ... , $N$), which are compatible with the same ontic state $\lambda$. According to the second assumption of the ontological models framework, the ontic state $\lambda$ determines the probability $p(k|\lambda, M)$ of different results $k$ for a measurement $M$. Moreover, there is a normalization relation for any $N$ result measurement: $\sum_{i=1}^{N} p(k_i|\lambda, M) = 1$. Since there is an $N$ result measurement that satisfies the condition that the first state gives zero Born probability to the first result and the second state gives zero Born probability to the second result and so on, there will be a relation $p(k_i|\lambda, M) = 0$ for any $i$, which contradicts the normalization relation.

Now if the second assumption of the ontological models framework is replaced by the revised assumption, namely that the probability of different results $k$ for a measurement $M$ on a physical system is determined not only by the ontic state $\lambda$ of the system, but also by its wave function $\psi$, i.e. $p(k|\lambda, M)$ is replaced by $p(k|\lambda, \psi, M)$, then the above contradiction cannot be derived. The reason is as follows. Under the revised assumption, the original normalization relation for an $N$ result measurement $\sum_{i=1}^{N} p(k_i|\lambda, M) = 1$ holds true only for systems with the same wave function, and for systems with different wave functions $\psi_j$ ($j = 1, ..., N$), the normalization relation should be $\sum_{j=1}^{N}\sum_{i=1}^{N} p(k_i|\lambda, \psi_j, M) = 1$. Then, even if there is an $N$ result measurement that satisfies the condition that the first state gives zero Born probability to the first result and the second state gives zero Born probability to the second result and so on, it will only lead to the relation $p(k_i|\lambda, \psi_i, M) = 0$ for any $i$. But this relation does not contradict the new normalization relation.

[6]Note that when the effective wave function is not nomological but related to the state of reality, the second assumption of the ontological models framework should not be revised this way but keep unchanged, since the complete ontic state $\lambda$ already includes all parts of the state of reality (see also Leifer, 2014; Drezet, 2015).

Although the above revised assumption can help the nomological view of the universal wave function avoid the result of the PBR theorem, it has several issues. First of all, the revised assumption already admits that the wave function is real for a single system. According to the revised assumption, the probability of different results $k$ for a measurement $M$ on a physical system is determined not only by the ontic state $\lambda$ of the system, but also by its effective wave function $\psi$, i.e. it replaces the response function $p(k|\lambda, M)$ with $p(k|\lambda, \psi, M)$. If the wave function is not real for a single system, then the response function for a single system should not explicitly depend on the effective wave function of the system. However, it is arguable that the wave function being real for a single system should not be assumed before our analysis; rather, it should be a possible result obtained at the end of our analysis. For it directly excludes the possibility that the wave function is not real for a single system (e.g. the $\psi$-epistemic view). This is unsatisfactory.

Next, and more seriously for a realist view, the results of measurements and their probabilities cannot be explained in ontology under the revised assumption. According to the original assumption of the ontological models framework, when a measurement is performed, the behaviour of the measuring device is fully determined by the complete ontic state of the measured system and the measuring device. But according to the revised assumption, the behaviour of the measuring device is not fully determined by the complete ontic state of the measured system and the measuring device. Then, the results of measurements and their probabilities will be unexplainable in ontology.

Let me give a simple example. Consider a measurement of an observable (other than position) on a system being in an eigenstate of the observable such as a measurement of the energy of a particle in one of its energy eigenstates in a box. The measurement will always yield a definite result, the corresponding energy eigenvalue. According to the ontic view of the effective wave function, this measurement result is determined by the ontic state of the measured system represented by its wave function, and thus it is explainable in ontology. This is consistent with the original assumption of the ontological models framework. But according to the nomological view of the universal wave function, the ontic state of the measured system is only the position of its particle which is at rest in the box, and it does not include the energy eigenstate of the system. Since the particle may be in the same position in the box for different energy eigenstates, the ontic state of the measured system cannot determine the measurement result, which are different for different energy eigenstates. This is just what the revised assumption says. Thus, when assuming the nomological view of the universal wave function or according to the revised assumption, the definite results of certain measurements will be unexplainable in ontology.

Finally, it seems that one can argue that the revised assumption cannot be true in some cases. Consider two measurement situations in which the initial ontic states of the measured system and the measuring device are the same but the effective wave functions of the measured system are different.[7] Since two identical ontic states cannot be distinguished, the law of motion must be the same for them. Then, the behaviours of the measuring device will be the same for the two measurements, and thus they cannot depend on the effective wave functions of the measured system, which are different for the two measurements. This means that the revised assumption cannot be true. Conversely, in order to make the revised assumption true, the behaviour of the measuring device must depend on the effective wave function of the measured system, while this requires that two identical ontic states can be distinguished. This seems to be an impossible task.

## 6 Conclusions

It is widely thought that the nomological view of the wave function in Bohmian mechanics, which says that the universal wave function is not ontic but nomological, is consistent with the PBR theorem, which proves that the effective wave function in Bohmian mechanics is ontic. In this paper, I argue that this is not the case, and the nomological view of the universal wave function is not compatible with the ontic view of the effective wave function. This means that if the effective wave function is ontic as the PBR theorem proves, then the universal wave function cannot be nomological. Moreover, I argue that although the nomological view can be held by rejecting one key assumption of the PBR theorem, the rejection will lead to serious problems, such as that the results of measurements and their probabilities cannot be explained in ontology in Bohmian mechanics.

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[7]This is possible as can be seen from the previous example. In the example, the effective wave function of the measured system is an energy eigenstate in a box. The ontic state of the measured system, which is represented by the position of its particle in the box, may be the same for different energy eigenstates. Moreover, the ontic states of the measuring device are supposed to be the same before different measurements.

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