How Haag-tied is QFT, really?

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Abstract

Haag's theorem cries out for explanation and critical assessment: it sounds the alarm that something is (perhaps) not right in one of the standard way of constructing interacting fields to be used in generating predictions for scattering experiments. Viewpoints as to the precise nature of the problem, the appropriate solution, and subsequently-called-for developments in areas of physics, mathematics, and philosophy differ widely. In this paper, we develop and deploy a conceptual framework for critically assessing these disparate responses to Haag's theorem. Doing so reveals the driving force of more general questions as to the nature and purpose of foundational work in physics.

1 Introduction

> The edifice of science is not raised like a dwelling, in which the foundations are first firmly laid and only then one proceeds to construct and to enlarge the rooms. Science prefers to secure as soon as possible comfortable spaces to wander around and only subsequently, when signs appear here and there that the loose foundations are not able to sustain the expansion of the rooms, it sets about supporting and fortifying them. This is not a weakness, but rather the right and healthy path of development.

[Hilbert 1905, 102; translation by Corry 2004, 127]

Proven over six decades ago, Haag's theorem appears to present a problem for particle physics. The theorem seems to block a key technique—namely, the interaction picture and its attendant calculational methods—that has been widely used to generate successful

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predictions. It is clear that the theorem points to some sort of problem, driven by the empirical success of the calculations employing the interaction picture on the one hand and the logical force of the theorem on the other. Thus, while particle physics has secured for itself a "comfortable space" around which to wander, Haag's theorem appears as a sign that the foundation are too loose "to sustain the expansion of the rooms."

This paper aims to provide a framework for possible answers to a single, if double-faced, question: *What does Haag's theorem tell us about quantum field theory, present and future?* Several divergent answers have been given already. Indeed, these will shape the paper's framework substantially. Nevertheless, the shape the framework should take is less straightforward than it may at first seem. Even before getting to the nitty-gritty analysis of Haag's theorem, a framework must grapple with the problem of viewpoint, as the following exercise makes clear:

Regardless of your actual field, take whatever career stage you are at—early graduate student, doctoral candidate, early-, middle- or late-career researcher—and imagine yourself instead as a particle physicist. You may imagine yourself as an experimentalist or a theoretician, expert in QED or QCD—whatever comes to mind. Regardless, several things are true of you. First, you are committed to the development of particle physics (no matter what this means in practice). Second, you are steeped in, and reliant on, the interaction picture for your research in and teaching. And third, you have just learned of Haag's theorem and the trouble that it spells for the interaction picture. How might you respond?

Set the exercise up again, except now you are a mathematician committed to contributing to the development of QFT (no matter what this means in practice). Second, you are steeped in the implications of Haag's theorem, and you are intimately familiar with the axiomatic or algebraic approach to quantum field theory. And third, you believe that a full, conceptually coherent physically realistic replacement for the interaction picture is (presently) unavailable.

Set it up one last time, except now you are a philosopher of science. You may consider yourself a realist or an instrumentalist, interested in metaphysics or methodology—whatever comes to mind. Regardless, several things are true of you. First, you are committed to understanding the foundations of QFT (no matter what this means in practice). Second, you are familiar both with the major advances in conventional (Lagrangian) QFT that use the interaction picture as well as those based on algebraic QFT (AQFT).[1] But third, it is unclear to you if, or how, these advances can form a consistent whole.

It is not a given that your physicist, mathematician, and philosopher selves will share a single outlook on RQFT, even before considering Haag's theorem. Nor is it clear that they should. Inevitably, this will affect your reaction to our guiding question. Thus, a major contribution of our framework will be to highlight the influence these extra-Haagian outlooks have on understanding Haag's theorem and on assessing its implications for philosophy and for theoretical physics. As we show in section 4, many (but not all) of the disagreements about Haag's theorem derive ultimately from different extra-Haagian outlooks, such that the disagreement is far less about Haag's theorem itself than it is about how to do (foundations of) physics.

This work also aims to contribute to a larger discussion in philosophy of physics. As a quintessential and live example of work at the foundations of physics, the discussions

[1] See [Fraser, 2011] and [Wallace, 2011] for the classic debate over these two formulations of QFT.

of Haag's theorem draw our attention to important methodological questions: What role does (should) foundational work play in progress in physics? How is foundational work coordinated with non-foundational work, or how should it be? And, what does (should) foundational work even look like? These are undoubtedly heady questions, and we feign no complete answers. Nevertheless, our framework will reveal some of the answers that are being given by the authors under survey, and in so doing these answers open themselves to investigation. As we conclude the paper, we suggest several investigations we expect will sober up future discussions of these heady methodological questions.

The remainder of the paper is structured as follows. In section 2 we provide a synopsis of a standard proof of Haag's theorem and discuss the sense in which it raises an alarm that something is not right with the interaction picture. In section 3 we motivate the need for a framework for understanding the literature on Haag's theorem. The framework itself is given in section 4. The framework employs and extends Hilbert's construction analogy: the framework understands each author as something like a contractor giving their **assessment** of the problem in the foundations of physics heralded by Haag's theorem, their recommendation for **repair** work on the foundations, and their expectations of the needed long-term **maintenance or future renovations** to QFT's area of the edifice of science. This section applies the framework to seven leading contemporary viewpoints on Haag's theorem; the key results of this application are given in table 1. Section 5 demonstrates the framework's judicious balance of conceptual structure and flexibility in order to bring clarity to the space of responses to Haag's theorem; it further argues that, at the end of the day, the most important lesson for philosophers to take from Haag's theorem is that we need to put our own energies into clearly answering meta-level questions regarding the nature and purpose of foundational work in physics. Concluding remarks are given in section 6.

2 Haag's Theorem

2.1 History of Haag's Theorem

Haag's theorem is the culmination of two approaches in early quantum field theory. On the one hand, there had arisen a practical approach to calculating the results of scattering experiments in relativistic quantum field theory. This approach relied on the so-called interaction picture to model interactions [Schwinger, 1948b]. The approach was wildly successful, in particular in its use to calculate the anomalous magnetic moment of the electron [Schwinger, 1948a]. On the other hand, spurred by Wigner's precise characterization of special relativity's implications for quantum theory [Wigner, 1939], there had arisen a more mathematical approach to understanding the structure of relativistic quantum field theories. Naturally, relativistic QFT's mathematical structure was made transparent on this approach, characterized by a number of basic assumptions. However, this was not true of the interaction picture. Haag, in his lecture series given at CERN in 1953–4,[2] began by recounting the basic assumptions necessary for relativistic QFT (irreducible Hilbert space representation of the Poincaré group, etc.). Then, Haag used the mathematical tools at

[2]See [Haag, 2010] for his recollections of this aspect of the history. Note that Haag's placing of the lecture series in 1953–4 conflicts with [Lupher, 2005], who places it in 1952–3.

his disposal to characterize the interaction picture (Fock space, unitary intertwiner). The problem (Haag's theorem) was that the interaction picture, thus characterized, could not be used to represent non-trivial interactions.

Haag's theorem exists in several versions, notably [Haag, 1955] and [Hall and Wightman, 1957]. Opinions differ widely on its precise implications. However, loosely, the theorem states that any suitable representation for an interacting quantum field must be unitarily equivalent to a Fock representation for a free field. This means that any quantum field that is unitarily equivalent to a free field must also be a free field. Obviously, a field with a nontrivial interaction cannot be unitarily equivalent to a free field. Thus, Haag's theorem seems to show that if the interaction picture is mathematically consistent, it can only describe trivial interactions, in which the interacting field is in fact free.

2.2 The Interaction Picture

Haag's theorem is often construed as a no-go theorem for the use of what's called the interaction picture. This picture has been widely used for the calculation of many physical quantities that have matched experimental results to a high degree of accuracy, for example the celebrated computation of the anomalous magnetic dipole moment of the electron by [Schwinger, 1948a], and is a mainstay of undergraduate and graduate textbooks and has facilitated the calculation of many physical quantities that have matched experimental results to a high degree of accuracy. [3] As its name suggests, the interaction picture is one way to model interacting fields in conventional quantum field theory. Let us suppose we have a field, ϕ, with conjugate momentum π, generally taken (either explicitly or implicitly) to obey the equal time canonical commutation relations and the Wightman axioms.

The interaction picture is intermediate between the Schrödinger and Heisenberg pictures. In the Schrödinger picture of quantum mechanics, states evolve in time under the full Hamiltonian, whilst operators are stationary. In the Heisenberg picture of quantum mechanics, the operators evolve under the full Hamiltonian, whilst states are stationary. To form the interaction picture, we split the full Hamiltonian into a free and a (time-dependent) interaction part, $H = H_F + H_I$. The evolution of operators is governed by the free Hamiltonian, H_F, so the fields are free. The evolution of states is governed by the interaction part, H_I.

[3] As one crude measure of how salient the interaction picture is for actual physics, a search for the exact phrase "interaction picture" in Physics Reviews conducted on Nov 4, 2021 returned 4,377 hits. The interaction picture forms the basis of calculations using time-dependent perturbation theory. Crucially, the Gell-Mann and Low theorem makes use of the interaction picture. The original paper [Gell-Mann and Low, 1951] presenting this formula has (as of Nov 4, 2021) 1544 citations recorded on google scholar. This is likely a rather low estimate of how widely the formula is in fact used since the formula is now at the level of common knowledge among physicists, and thus in many cases it might be used without citation. For instance, the Gell-Mann and Low theorem was used to rigorously derive the Bethe–Salpeter equation, which describes the bound states of a system of two fermions in a relativistic formalism [Salpeter and Bethe, 1951]. This equation was used in subsequent quantum electrodynamics calculations of the fine structure of Hydrogen-like atoms [Salpeter, 1952], Helium [Douglas and Kroll, 1974] and heavy atoms [Mohr et al., 1998] and contemporary precision calculations of the electron magnetic moment [Aoyama et al., 2015]. Likewise, the equations have been utilized in the calculation of nucleon-nucleon potentials [Partovi and Lomon, 1969, Partovi and Lomon, 1972, Lomon, 1976, Partovi, 1978] and the derivation of scaling laws for interactions with large momentum transfer, confirmed by scattering experiments [Brodsky and Farrar, 1975], among many other examples.

We stipulate that the operator fields coincide with those of the Heisenberg picture at some time, t_0. Let $V(t_2, t_1)$ represent the unitary evolution of the interaction picture states from time t_1 to t_2, generated by the interacting part of the Hamiltonian, H_I,

$$V(t_2, t_1) = e^{-iH_I(t_2-t_1)} = e^{+iH_F(t_2-t_1)}e^{-iH(t_2-t_1)}. \tag{1}$$

We call this operator the intertwiner, or Dyson operator. Then, at all times, t, the Heisenberg (subscript F) and interaction (subscript I) operators are related by the intertwiner as follows,

$$\phi_I(x,t) = V^{-1}(t,t_0)\phi_H V(t,t_0), \tag{2}$$
$$\pi_I(x,t) = V^{-1}(t,t_0)\pi_H V(t,t_0). \tag{3}$$

As we will see, it is central to Haag's theorem that the relation of the interaction field, ϕ_I, to the free field, ϕ_F is characterized by a unitary map.

Generally, we seek to calculate physical amplitudes, taken in the limit in which the fields are free at times $t \to \pm\infty$. This is meant to capture the intuition that in an interaction, particles begin infinitely far apart (and hence not interacting) and then separate again infinitely far apart after an interaction. The interaction picture may be useful if we can treat the effects of H_I as a small, time-dependent perturbation on the evolution under H_F. In perturbation theory we perform approximate calculations by expanding the desired physical quantities in powers of the small interaction, H_I.

2.3 Proof of Haag's Theorem for Spin-free, Neutral, Scalar Fields

The proof of Haag's theorem assumes the Wightman Axioms. For convenience we number the axioms according to the convention of [Seidewitz, 2017].[4]

2.3.1 Wightman Axioms

Axiom 0. *States.* *We have a physical Hilbert Space, $\mathcal{H}$, for which the states, $|\phi\rangle$, are rays, such that,*

1. *The states transform according to a continuous unitary representation of the Poincare group, $U(\mathbf{\Delta x}, \Lambda)$, under Poincare transformations, $\{\mathbf{\Delta x}, \Lambda\}$.*
2. *There is a unique, invariant vaccum state, $|0\rangle$, in $\mathcal{H}$, invariant under U: $U(\mathbf{\Delta x}, \Lambda)|0\rangle = |0\rangle$.*
3. *Let $U(\mathbf{\Delta x}, \Lambda) = e^{iP^\mu \mathbf{\Delta x}_\mu}$. Then, $P^\mu P_\mu = -m^2$. We interpret P^μ as an energy-momentum operator and m as a mass. The eigenvalues of P^μ lie in the future lightcone.*

[4] This proof largely follows the proof of [Hall and Wightman, 1957], using some of the notation and conventions of [Seidewitz, 2017] and [Earman and Fraser, 2006] in order to make the role of each assumption transparent.

Axiom 1. ***Domain and continuity of fields.*** *The field $\phi(x)$ and its adjoint $\phi^\dagger(x)$ are defined on a domain D of states dense in $\mathcal{H}$ containing the vacuum state $|0\rangle$. The $U(\mathbf{\Delta x}, \Lambda)$, $\phi(x)$ and $\phi^\dagger(x)$ all transform vectors in D to vectors in D.*

Axiom 2. ***Field transformation law.*** *The fields transform under Poincare transformations as,*

$$U(\mathbf{\Delta x}, \Lambda)\phi(x)U^{-1}(\mathbf{\Delta x}, \Lambda) = \phi(\Lambda x + \mathbf{\Delta x}). \tag{4}$$

Axiom 3. ***Local commutativity.*** *If x and x' are two spacetime positions,*

$$[\phi(x), \phi(x')] = [\phi^\dagger(x), \phi^\dagger(x')] = 0 \tag{5}$$

Furthermore, if x and x' are space-like separated,

$$[\phi(x), \phi^\dagger(x')] = 0. \tag{6}$$

Axiom 4. ***Cyclicity of the vacuum.*** *The vacuum state $|0\rangle$ is cyclic for the fields, $\phi(x)$. That is, polynomials in the fields and their adjoints, when applied to the vacuum state, yield a set D_0 dense in $\mathcal{H}$.*

2.3.2 Proof of Haag's theorem

Haag's theorem follows from the results of two other theorems. For clarity of exposition, here we only prove the theorem for neutral, scalar fields; the generalization for other types of fields follows in a straightforward manner. From here on, it is convenient to decompose four-vectors as $x = (t, \mathbf{x})$, where the right hand side are the temporal and spatial components, respectively.

Theorem 1. ***Equality of equal-time vacuum expectation values.*** *Let ϕ_1 and ϕ_2 be two field operators, with associated conjugate momentum operators, π_1 and π_2, defined in respective Hilbert spaces $\mathcal{H}_1$ and $\mathcal{H}_2$, satisfying the Wightman axioms listed above, and satisfying the equal time commutation relations,*

$$[\phi_i(t, \mathbf{x}), \pi_i(t, \mathbf{x}')] = i\delta(\mathbf{x} - \mathbf{x}'), \tag{7}$$
$$[\phi_i(t, \mathbf{x}), \phi_i(t, \mathbf{x}')] = [\pi_i(t, \mathbf{x}), \pi_i(t, \mathbf{x}')] = 0. \tag{8}$$

Suppose that there exists a unitary operator G such that, at some specific time t_0,

$$\phi_2(t_0, \mathbf{x}) = G\phi_1(t_0, \mathbf{x})G^{-1}, \tag{9}$$
$$\pi_2(t_0, \mathbf{x}) = G\pi_1(t_0, \mathbf{x})G^{-1}. \tag{10}$$

We call G an intertwiner for the fields ϕ_1 and ϕ_2. Then the equal-time vacuum expectation values of the fields coincide. (Note that so far, this holds only at the particular time, t_0.)

Proof. Let $U_i(\mathbf{\Delta x}, \mathbf{R})$ be a continuous, unitary representation of the inhomogeneous Euclidean group of translations, $\mathbf{\Delta x}$, and three-dimensional rotations $\mathbf{R}$, defined on each $\mathcal{H}_i$, $i = 1, 2$. Let us further suppose that transformations $U_i(\mathbf{\Delta x}, \mathbf{R})$ induce Euclidean transformations of the field

$$U_i(\mathbf{\Delta x}, \mathbf{R})\phi_i(t_0, \mathbf{x})U_i^{-1}(\mathbf{\Delta x}, \mathbf{R}) = \phi(t_0, \mathbf{Rx} + \mathbf{\Delta x}), \tag{11}$$

$$U_i(\mathbf{\Delta x}, \mathbf{R})\pi_i(t_0, \mathbf{x})U_i^{-1}(\mathbf{\Delta x}, \mathbf{R}) = \pi(t_0, \mathbf{Rx} + \mathbf{\Delta x}), \tag{12}$$

as in axiom 0.1. From our supposition (equations 9 and 10), it follows that

$$U_2(\mathbf{\Delta x}, \mathbf{R}) = GU_1(\mathbf{\Delta x}, \mathbf{R})G^{-1}. \tag{13}$$

But since the representations possess unique invariant vacuum states $|0\rangle_i$ such that $U_i(\mathbf{\Delta x}, \Lambda) |0\rangle_i = |0\rangle_i$, as in axiom 0.2.,

$$c\,|0\rangle_2 = G\,|0\rangle_1\,, \tag{14}$$

where c is a complex number of absolute value 1, $|c| = 1$. In other words, up to a phase factor, $G\,|0\rangle_1$ is the vacuum state for field ϕ_2 at time t_0.

It further follows that the equal-time vacuum expectation values (Wightman functions, also known as correlation functions) of the two fields are the same,

$${}_1\langle 0|\phi_1(t_0, \mathbf{x}_1), \ldots \phi_1(t_0, \mathbf{x}_n)\,|0\rangle_1 = {}_2\langle 0|\phi_2(t_0, \mathbf{x}_1), \ldots \phi_2(t_0, \mathbf{x}_n)\,|0\rangle_2\,, \tag{15}$$

for $\mathbf{x}_1, \mathbf{x}_2$ up to at most $\mathbf{x}_4$, all at the same fixed time, t_0.[5] □

Theorem 2. *Jost-Schroer theorem.* *For any free scalar field ϕ, the two-point vacuum expectation values are given by*

$$\langle 0|\,\phi(x)\phi^\dagger(x_0)\,|0\rangle = \Delta^+(x - x_0), \tag{16}$$

where Δ^+ is the advanced Feynman propagator[6]*,*

$$\Delta^+(x - x_0) = (2\pi)^{-3} \int d^3p \frac{e^{i[-\omega_{\mathbf{p}}(x^0 - x_0^0) + p\cdot(\mathbf{x} - \mathbf{x}_0)]}}{2\omega_{\mathbf{p}}}, \tag{17}$$

and $\omega_p = \sqrt{\mathbf{p}^2 + m^2}$.

If, for any arbitrary field, the vacuum expectation values are given by equation 16, then that field is a free field.

[5] If at least one of ϕ_1 or ϕ_2 is a free field, then this can be proven to hold for all Wightman functions, i.e. for $\mathbf{x}_1, \mathbf{x}_2$ up to any $\mathbf{x}_n$.

[6] For example, see [Duncan, 2012, pages 209-210 and 256]

This result was first proved by Jost [Jost, 1961] for fields of positive mass, and extended to fields of zero mass by Pohlmeyer [Pohlmeyer, 1969].

Theorem 3. ***Haag's theorem for scalar fields.*** *Let ϕ_1 be a free, scalar field, which therefore satisfies equation 16. Let ϕ_2 be a second, locally Lorentz-covariant scalar field. Let us assume that ϕ_2 is unitarily related to ϕ_1 at time t_0, as in equations 9 and 10. Further let us assume that the conjugate momenta fields (written as adjoints $\phi_1^\dagger$ and $\phi_2^\dagger$) satisfy the hypotheses of Theorem 1. Then $\phi_2(x)$ is also a free field.*

Proof. Theorem 1 tells us that the equal time vacuum expectation values of the two fields, ϕ_1 and ϕ_2 must coincide at t_0, i.e.,

$$ {}_1\langle 0|\phi_1(t_0,\mathbf{x}_1)\phi_1(t_0,\mathbf{x}_n)\,|0\rangle_1 = {}_2\langle 0|\phi_2(t_0,\mathbf{x}_1)\phi_2(t_0,\mathbf{x}_n)\,|0\rangle_2\,. \tag{18}$$

Since the field ϕ_1 is free, it follows from Theorem 2 (equation 16) and equation 20 that the two-point vacuum expectation values for the two fields coincide at t_0, i.e.,

$$ {}_2\langle 0|\phi_2(t_0,\mathbf{x}_1)\phi_2^\dagger(t_0,\mathbf{x}_n)\,|0\rangle_2 = {}_1\langle 0|\phi_1(t_0,\mathbf{x}_1)\phi_1^\dagger(t_0,\mathbf{x}_n)\,|0\rangle_1\,. \tag{19}$$

So far, this holds *only* at t_0. However, any two spacelike separated position vectors, (t_1,x_1) and (t_2,x_2), can be brought into the equal time plane $t_1=t_2$ by a Lorentz transformation. Thus, the Lorentz-covariance of ϕ_2 allows us to extend the satisfaction of equation 19 to any two spacelike positions, and then, by analytic continuation, to any two positions:

$$ {}_2\langle 0|\phi_2(x_1)\phi_2^\dagger(x_n)\,|0\rangle_2 = \Delta^+(x-x_0). \tag{20}$$

Therefore, by Theorem 2, $\phi_2(x)$ must be a free field.

□

2.4 Implications of Haag's Theorem

By the criteria used by particle physicists, the interaction picture has undoubtedly produced numerous notable successes (see note 3). However, the interaction picture is generally taken to depend upon all of the assumptions needed for Haag's theorem, including Poincaré invariance and the existence of a unitary operator relating the two fields, that is, that the fields are unitarily equivalent. The apparent clash between the interaction picture and Haag's theorem arises as follows. If the fields, ϕ_F and ϕ_I, obey the Wightman axioms, and we have a unitary operator intertwining the two fields at even a single time (as in equations 9 and 10), and ϕ_F is free, then according to Haag's theorem ϕ_I must also be free. So the interaction must be trivial.

So Haag's Theorem appears to be a no-go theorem for calculations that use the interaction picture. At a glance, it would be mathematically inconsistent to use the interaction picture for calculations involving any of the non-trivial interactions that we care about in particle physics. However, a closer look reveals a whole labyrinth of philosophical, mathematical, and physical issues at stake in understanding the full significance of Haag's theorem. We take that closer look in the next section.

3 What the Haag Is Going on?

There are a few points on which all parties generally agree. First, judged by its own particular standards of success, particle physics –including the interaction picture– is highly successful.

Second, there is a consensus that Haag's theorem poses a *bona fide* problem for the standard presentation of the interaction picture, pre-renormalization. The theorem itself is mathematically correct: as Klaczynski puts it, "it is a mathematical theorem in the truest sense of the word; it brings with it the 'hardness of the logical must'." [Klaczynski, 2016]. Further, it is not disputed that the assumptions of Haag's theorem hold in the case of the standard textbook presentations of the interaction picture.[7] No one, to our knowledge, argues that the problem posed by Haag's theorem is illusory.[8] It is, rather, the severity and appropriate remedy of the problem that is subject to debate.

However, the severity of the problem raised by Haag's theorem clearly stops short of spelling the demise of that research program predicated on the application of quantum theories of fields to scattering experiments, commonly called particle physics.[9] Even the practitioners of AQFT, while operating far afield from the details of the Standard Model's phenomenology, still conceive of their work as contributing to the scientific enterprise whose primary goal is to develop the quantum theory of fields as the appropriate theoretical apparatus for understanding scattering experiments. So, then, what *is* going on with Haag's theorem such that these two stances —the interaction picture has been used successfully, and Haag's theorem poses a substantive problem for the interaction picture—can be held together? This central tension is widely recognized as a point of agreement. As Teller succinctly puts it,

> Everyone must agree that as a piece of mathematics Haag's theorem is a valid result that at least appears to call into question the mathematical foundation of interacting quantum field theory, and agree that at the same time the theory has proved astonishingly successful in application to experimental results. [Teller, 1995, 115]

And yet, despite this initial agreement, extant responses to Haag's theorem form a confusing lot. The literature on Haag's theorem reflects a number of different assessments of

[7] For example, standard presentations assume, either implicitly or explicitly, a Poincaré invariant theory, and that there are distinct free and interacting fields, related by a unitary operator.

[8] It may very well be that many practicing physicists hold such a view, and that, precisely because they see the problem posed by Haag's theorem as at most rooted in an unrealistic idealization of non-interacting states at temporal infinity, they therefore choose not to address Haag's theorem in their textbooks, lecture notes, or research articles. We can do little more than speculate that such an unspoken consensus fully explains the dearth of references to Haag's theorem in standard accounts of QFT—simple ignorance of the theorem may be just as strong of a causal factor. David Tong's lecture notes on QFT, for instance, do not explicitly mention Haag's theorem; but they do say that the assumption of non-interacting states in the interaction picture is wrong and should be replaced by the interactions-are-always-on interpretation of the LSZ reduction formula [Tong, 2012] p. 54-55, 79-80. Thus, while it is possible that this dismissive response to Haag's theorem is widespread, too little of it exists in print to be extensively covered in the remainder of this paper.

[9] Though see the discussion of Kastner (section 4.6) and Seidewitz (section 4.7) below for genuine proposals of new physical theories, each drawing some motivation from Haag's theorem.

the import of the theorem for both (mathematical) physics and philosophy. Moreover, the extent to which these different assessments make meaningful contact with each other is often unclear. In this section, we briefly illustrate the nature of the confusion in this literature. First, the confusion is *not* about the status of Haag's theorem as a mathematical result. All parties agree that the original theorem, and its several generalizations, have valid proofs. The confusion enters when trying to trace out the ramifications of Haag's theorem for the foundations of QFT. [Earman and Fraser, 2006] put it well, saying that "the theorem provides an entry point into a labyrinth of issues that must be confronted in any satisfactory account of the foundations of QFT" (p. 334). Once we have entered into this labyrinth via Haag's theorem, we encounter a host of conceptual and interpretive issues, enmeshed in technical issues of mathematics and physics, making even the range of options for a way forward through the labyrinth unclear, much less which one may be the best.

A reader interested in Haag's theorem and its implications for the use of the interaction picture in physics may first look for insight from Earman's and Fraser's seminal paper, "Haag's theorem and its implications for the foundations of quantum field theory." They conclude, "On any reading Haag's theorem undermines the interaction picture and the attendant approach to scattering theory" [Earman and Fraser, 2006, 333]. So, the reader naturally thinks, *the interaction picture is no good*. And yet for Duncan, "the proper response to Haag's theorem is simply a frank admission that the same regularizations needed to make proper mathematical sense of the dynamics of an interacting field theory at each stage of a perturbative calculation will do double duty in restoring the applicability of the interaction picture at intermediate stages of the calculation" [Duncan, 2012, 370]—the interaction picture survives! Miller concurs, adding that the success of calculations delivered from regularized and renormalized theories is explained by the conjecture that "perturbative expansions are asymptotic to exact solutions of a theory that generates them" [Miller, 2018, 818]. *So*, the reader concludes triumphantly, *the interaction picture works*!

Still more paths begin to emerge, however. According to Klaczynski, these renormalized theories evade Haag's theorem precisely by *denying* that the interaction picture exists [Klaczynski, 2016]. Maiezza and Vasquez agree, arguing that "due to *Haag's theorem*, it is *impossible to define QFT* starting from the interaction picture with free fields" [Maiezza and Vasquez, 2020, 10] (italics in original); indeed, they seem to argue that the interaction picture fails precisely because of the failure of the conjecture Miller relies on to save it. Yet confusingly, Maiezza and Vasquez *also* disagree with Klaczynski on what saves perturbative calculations from Haag's theorem.

The paths so far, while many, nevertheless seem to turn on what to say about the mathematical coherence of the interaction picture. Thus, *a labyrinth though it may be*, the reader thinks, *I can at least see its basic structure*. The reader has judged too soon, however, for it is not only the interaction picture *per se* that is at stake, but the metaphysics: "*either the assumptions of Haag's theorem do not hold, in which case there is no particle notion applicable to a scattering experiment at intermediate times, or they do, in which case the particle notion applicable at intermediate times is incommensurable with the ingoing/outgoing particle notions, if the interaction is non-trivial*" [Ruetsche, 2011, 252] (italics in original). *So*, the reader thinks, *the path out of the Haagian labyrinth requires the banishment of particles and the embracing of fields*! Not so, says Kastner: *particles* exist, and *fields* must be banished [Kastner, 2015].

The reader is thus confronted with paths out of the Haagian labyrinth diverging on both metaphysical and mathematical grounds and, worse still, she can't tell from her place in the labyrinth whether or where these paths coincide. As if this weren't bad enough, a fog sets in. *Are we even trapped at all?*, the reader asks, [Seidewitz, 2017, 356] in hand, for Haag's theorem arises in traditional QFT only because time is not "treated comparably to the three space coordinates, rather than as an evolution parameter." Thus, the Haagian labyrinth could have been entirely avoided had time been treated in a relativistically sensible manner at the outset.

How should the reader react to this? Is there a Haagian labyrinth? And if so, which path will lead us out? The framework given in the next section gives a fixed structure for organizing and assessing this confusing network of responses to Haag's theorem, thereby creating several distinct mappings of the Haagian labyrinth. More general lessons from studying these maps are given in section 5.

4 The Framework: Assessment, Repair, and Renovation and Maintenance

Before presenting the framework, we should carefully consider its goal. The purpose of this framework, as we said in the introduction, is to fruitfully structure and organize answers to the question "What does Haag's theorem tell us about quantum field theory, present and future?" Two desiderata for such a framework are fairly obvious. First, it should sensibly organize the answers that have already been given to this question. As such, we will apply the framework to prominent answers in the literature. Second, the framework should leave significant room for future developments. Given the interdisciplinary nature of the study of Haag's theorem, and given the recent increase of interest from physicists ([Klaczynski, 2016], [Maiezza and Vasquez, 2020], and [Seidewitz, 2017]), we expect there is much more to be said on this topic. A good framework for organization, therefore, must have space to accommodate these expected future developments.

There are additional desiderata, which are less obvious but of no lesser importance. These arise principally from the interdisciplinary nature of the research on Haag's theorem. Third, the framework should organize various extant and possible responses to Haag's theorem without itself providing a judgement as to which response is best. Thus, while a successful framework should assist the scholarly community in evaluating the success or failure of a given response to Haag's theorem *with respect to* or *along* a given criterial axis, it should in no way bias one axis as more or less important than any other axis. In particular, the criteria should not mark out one outlook as more important or relevant to a given criteria simply in virtue of that outlook being more interested or invested in that criteria. For example, "which assumption(s) of Haag's theorem must we give up?" is not unbiased insofar as it is of preeminent interest to axiomatic investigators; likewise, "what notion of particle, if any, is circumscribed by QFT?" is not agnostic insofar as it is of preeminent interest to the metaphysically-inclined philosopher. The importance of this desideratum is born of this framework's goal to *organize*, but not *finalize*, answers as to the significance of Haag's theorem. Nevertheless, for proper subsets of responses that coincide on at least one axis, sub-frameworks biased along that axis can provide further helpful organization (see section 5).

Fourth and finally, the framework should force the various extra-Haagian outlooks to be more explicitly acknowledged. In particular, it should illuminate what is at stake in discussion of Haag's theorem separately for mathematicians, physicists, and philosophers. This last desideratum is meant to draw attention away from products of miscommunication and toward more honest and fruitful dialogue.

Our framework consists of the following three criterial axes along which responses to Haag's theorem can be assessed, categorized, and critically compared. These axes have been chosen to expand upon Hilbert's leading construction analogy, namely, as the assessment and diagnosis, immediate repair, and longer-term renovation or maintenance of a building (the "edifice of science") that is continually under construction. Following the analogy, the authors surveyed here may be thought of as subcontractors brought in to assess and diagnose the building's status in light of Haag's theorem. Thus, in our capacity as organizers of the varied responses to Haag's theorem, we the authors may be thought of as something like the general contractor whose job is to arrange and make comparable the subcontractors' assessments and provide expert advice to the building's owner as to how to proceed with the construction. We accomplish this first aim in section 4 and the second aim in section 5.

- **Assessment:** What precisely is the problem posed by Haag's theorem, if any? For what objective(s) is this a problem?
- **Repair:** How should this problem be remediated?
- **Maintenance or Renovation:** Where should resources (time, attention, grant funding, conference and journal platforms, etc.) for the next (relevant) phase of research be allocated?

We now turn to applying this framework to organize seven major responses to Haag's theorem from several different perspectives. The results of applying our framework are summarized in table 1. Each result is discussed in detail in the following subsections. For clarity, each subsection begins with a discussion of the extra-Haagian outlook at play; however, note that the content of these discussions has been determined through application of our framework.

4.1 Earman and Fraser

Extra-Haagian outlooks. For Earman and Fraser, the most pressing question is, what is QFT? As they see it, answering this question requires an honest and complete marriage of SR with QM. As such, QFT cannot settle for the weakening of any fundamental postulates of either theory—dishonest or partial marriages of SR with QM are, by definition, *not QFT*.

This requirement of a complete and honest marriage is a significant departure from orthodoxy. For one, it immediately rules out the two most dominant approaches to constructing QFTs. On the one hand, renormalized canonical QFT is ruled out, according to Fraser, because the theory is so mathematically ill-defined that we cannot check if it has *honestly* married SR with QM. On the other hand, the introduction of cutoffs in canonical QFTs,

Name	**Assessment**	**Repair**	**Renovation or Maintenance**
Earman & Fraser	interaction picture (IP) rests on a set of inconsistent assumptions	Abandon the interaction picture	Interpret and assess AQFT and constructive QFT
Fundamental Interpreters	Standard particle notion inconsistent with axioms of QFT	Abandon standard particle interpretation	Develop univocal interpretation of axioms of QFT
Duncan & Miller	pre-renormalization IP is inconsistent	Renormalization (breaks Poincaré invariance)	Continue studying renormalization and EFTs, don't worry
Klaczynski	IP is inconsistent	Renormalization leads us to abandon the IP	replace the IP's unitary intertwiner
Maiezza & Vasquez	The full perturbative series has renormalon divergences	Resummation methods (but these are ambiguous)	New resummation methods and physical insights are needed
Kastner	Physics is non-local	Abandon notion of an independent field	Develop direct-action theories
Seidewitz	Use of time as the evolution parameter is at fault	Revise axioms to introduce new evolution parameter	Extend parameterized QFT to gauge theories and non-Abelian interactions

Table 1: Framework application summary

while mathematically acceptable, means that the marriage is *incomplete*—SR concerns infinite, continuous space, not finite lattices. But for another, such a requirement is born of an attitude toward theory development that is unorthodox in contemporary science (if not philosophy of physics). Rather than demand full unification up front, the average theoretical particle physicist contents themselves with combining SR and QM to the best of their current ability. At least for now, our current best practice gives up the strict Poincaré invariance demanded by SR. Arguably, the 'best of their ability' has improved over the decades such that, the average theoretical particle physicist could claim, we will inevitably *find* such a complete and honest marriage *if one is forthcoming*. In the meantime, we have inarguably successful partial (resp. dishonest) marriages of SR with QM with which we can keep experimental and theoretical particle physics going. While it is likewise a live option for Fraser (and presumably Earman) that an honest and complete marriage of QFT is not possible, this is a question of *logic*, not practical or empirical feasibility. This leaves only AQFT as a means for characterizing QFT.[10]

Assessment. As discussed above, the goal for Earman and Fraser is to determine what QFT is, i.e., to completely and honestly unify SR and QM. A successful unification, if it is possible, will at least be able to model interacting systems. Thus, according to Earman and Fraser, Haag's theorem is a demonstration of the logical impossibility of modeling interactions using the interaction picture in our would-be QFT (assumed to obey the Wightman axioms). All that the interaction picture is capable of modeling is, provably, free fields—a spectacular failure. As such, Haag's theorem is perhaps the first of many signs that SR and QM are, in the end, irreconcilable.

Repair. The repair work needed is in the task of modeling interactions in QFT. This might proceed either by abandoning the interaction picture altogether (e.g. with Haag-Ruelle scattering), or else by substantially revising it through giving up one or more of its assumptions. While assessing this second possible response, Earman and Fraser find the robustness of Haag's theorem, and its generalizations, to be significant: "Subsequent no-go results do not show that field theorists do not have to worry about Haag's theorem because some of its assumptions do not hold in all cases of interest; rather, what the subsequent results show [is] that even more assumptions have to be abandoned in order to obtain well-defined Hilbert space descriptions of interacting fields" (318). Thus, Earman's and Fraser's preferred remedy for Haag's theorem is to abandon the interaction picture altogether in favor of alternative ways of modeling interactions in QFT.

Note that this accords with Earman and Fraser's stated goal of unifying SR and QM. Regularization and renormalization techniques are not options available to them as they change the game or execute the unification dishonestly, respectively. Thus, the only option is to give up the interaction picture: Haag's theorem does not pose a problem for QFT per se, but it "does pose problems for some of the techniques used in textbook physics for extracting physical prediction from the theory" [Earman and Fraser, 2006](p. 306). QFT, therefore, is not to be identified with textbook physics. The textbook physics is one attempt at doing QFT, and Haag's theorem exposes serious problems in that particular attempt. The immediate technical moral of Haag's theorem is that (better attempts at) QFT must embrace the use of unitarily inequivalent representations of the CCR.[11] The philosophical insight from

[10] Fraser seems optimistic that, with more time, the as-of-yet unfinished project of constructing a fully rigorous (according to AQFT standards) interacting QFT in four spacetime dimensions could be completed. But she is clear that this has not happened yet, and that in principle it could be shown to be impossible. Fraser writes, "consistency is also a relevant criterion because quantum field theory is, by definition, the theory that integrates quantum theory and the special theory of relativity. Consistency is relevant to QFT for theoretical reasons—not for practical reasons (e.g., the derivation of predictions). As a result, it is necessary to either formulate a consistent theory or else show that this criterion cannot be satisfied (i.e., that there is no consistent theory with both quantum and special relativistic principles)\.. . . The formal variant is the only variant that satisfies the criterion; its set of theoretical principles are both consistent and well motivated. Neither the infinitely renormalized nor cutoff variant furnishes an argument that a consistent formulation of QFT is impossible; such an argument would require making the case that the axiomatic program cannot be completed." [Fraser, 2009] (p. 563).

[11] "[A] single, universal Hilbert space representation does not suffice for describing both free and interacting fields; instead, unitarily inequivalent representation of the CCR must be employed" (p. 333).

this technical result is that it designates the role of unitarily inequivalent representations as a distinctive feature of QFT in contrast to QM.[12]

Renovation. As a consequence of this logical diagnosis and logical remedy, Earman and Fraser advocate for future philosophical work and resources to be deployed in philosophical projects about AQFT and its attendant non-interaction-picture approaches to interactions. There are at least two such broad philosophical projects that benefit from the ways in which AQFT-practicing mathematicians have "digested the lesson of Haag's theorem" [Earman and Fraser, 2006] (p. 334). First there is the project of interpreting the mathematical structures used in AQFT and in constructive QFT. Second, and relatedly, there is the subtle matter of assessing the implications of theorems of AQFT for the philosophy of those areas of fundamental physics that make use of QFT.

4.2 The Particle Problem

There is a segment of the philosophical literature addressing Haag's theorem that is primarily motivated by the question, as put by Laura Ruetsche, 'Is particle physics particle physics?' [Ruetsche, 2011, 190]. In this context, Haag's theorem is most often taken to be a no-go theorem for a particle interpretation of QFT (most notably [Halvorson and Clifton, 2002] and [Fraser, 2008]). Others disagree with this assessment (e.g. [Wallace, 2011] and [Bain, 2011]).

Extra-Haagian outlooks. In this literature, the primary extra-Haagian divide is between those adhering to what [Ruetsche, 2011] calls the ideal of pristine interpretation and those that do not. As she noted when coining the phrase, this ideal is rarely explicitly stated. Nevertheless, sympathy for the ideal of pristine interpretation appears widespread, particularly in the philosophy of QFT.[13] [Halvorson and Clifton, 2002], [Fraser, 2008], and [Bain, 2011][14], for example, approach the particle problem from a methodological stance of pristine interpretation. What binds pristine interpreters in the philosophy of QFT together is a desire for a unimodal interpretation of the theory of quantum fields. These authors demand that an interpretation specify what metaphysical entities are *fundamental*, in the sense that they are present across all applications (models) of the theory of quantum fields.[15] Implicit in this is

[12] "Haag's theorem was instrumental in convincing physicists that inequivalent representation of the CCR are not mere mathematical playthings but are essential in the description of quantum fields" (p. 319).

[13] See [Arageorgis, 1995, 119–125] for an early interpretation-focused discussion of Haag's theorem. We do not discuss this work here because his status as pristine or not is unclear.

[14] Bain's precise view on the ideal of pristine interpretation is less clear, but we include him here because of the fundamentality of his notion of particle and its perspicuity in the LSZ formalism. See, e.g., [Bain, 2013] for work suggesting this classification is incorrect.

[15] Ruetsche characterizes the pristine interpreter as one who believes that "a theory's laws on their own—or even in concert with "a few general principles" in the form of duly abstract mathematical, metaphysical, methodological, etc., considerations—delimit what worlds are possible according to that theory"; in particular, "those worlds should [not] be characterized in different ways for different extranomic (factual or material or explanatory or maybe even practical) circumstances" [Ruetsche, 2011, 4]. For simplicity, here we prefer to talk of interpretations that are fundamental or not rather than of physical possibility being determined by a theory's laws on their own or not. A fundamental interpretation is one whose generalizations are determined solely by the theory's laws (with the same caveat given by Ruetsche).

the requirement that all representations be specifiable uniformly and precisely enough that a single, mathematically-precise interpretive schema is available throughout.

Despite widespread sympathy for pristine interpretation in the philosophy of QFT literature, there remain dissenters. In [Wallace, 2011, 124] and [Wallace, 2001], for example, scale-variant and emergent concepts, respectively, are given pride-of-place in interpretation. Likewise, Ruetsche herself argues at length that "[w]hat set of possible worlds we associate with a theory of QM_∞ can depend on what we'd like to do with that theory" [Ruetsche, 2011, 352]. Both authors champion non-pristine, non-fundamental interpretations of QFT, and they thereby approach the particle problem and Haag's theorem with a very different outlook. While this difference in outlook is often implicit, Fraser (in response to [Wallace, 2001]) is transparently skeptical of even the cogency of an ontology with non-fundamental entities or which is not built on an exact similarity relation with the mathematical model at hand [Fraser, 2008, 857–8]. This difference in outlook clearly informs reactions to the particle problem.

More generally, the pristine interpreters of QFT share an understanding of how no-go theorems should be addressed. On the one hand, they care almost exclusively about the fundamental-interpretive implications of Haag's theorem—its implications for scattering theory and calculations therein deemed irrelevant. Thus, this group fits uneasily alongside the other authors here discussed—with the possible exception of Earman and Fraser and Seidewitz—insofar as use or application of QFT plays no role in the discussion nor, indeed, motivation for discussion of no-go theorems. They furthermore expect a response to no-go results like Haag's theorem to apply for all theories of quantum fields. Whereas Duncan and Miller satisfy themselves with identifying violations of the assumptions of Haag's theorem in particular settings, thus potentially making available in those settings a classic (non-fundamental) particle interpretation, here a satisfactory response must apply across *all* settings. This feeds directly into the resource allocations proposed by pristine interpreters: since a response must work across all settings, axiomatic approaches to inquiry are naturally preferred. For this reason, satisfying responses are typically sought from AQFT; the effective field theory approach, in particular, cannot provide a satisfying response to the pristine interpreter because it makes no pretense of applying one set of concepts across all settings.

Assessment. For pristine interpreters, Haag's theorem is a diagnostic no-go result concerning interpretation: QFT cannot be interpreted as fundamentally about particles. For instance, in their [Halvorson and Clifton, 2002, 24], Halvorson and Clifton blame Haag's theorem, among other no-go results, when they conclude that the theory of quantum fields "does not permit an ontology of localizable particles; and so, strictly speaking, our talk about localizable particles is a fiction." Similarly, as Ruetsche summarizes the thinking of pristine interpreters, it is due to Haag's theorem that "within the confines of the interaction picture, no fundamental particle interpretation can frame explanations of particle physics phenomenology and its compliance with calculations mediated by Feynman diagrams" [Ruetsche, 2011, 253]. Fraser, too, comes to this conclusion on the basis of Haag's theorem [Fraser, 2008]. Thus, for these authors, Haag's theorem sits alongside results like the Reeh–Schlieder theorem [Reeh and Schlieder, 1961], the Unruh effect [Crispino et al., 2008] [Earman, 2011], or the Hegerfeldt [Hegerfeldt, 1998a] [Hegerfeldt, 1998b] and Malament [Malament, 1996] theorems (and extensions thereof [Halvorson and Clifton, 2002]) as evidence against the existence of a fundamental particle interpretation for RQFT. Indeed, canon seems to dictate—even for non-fundamental interpreters —that these results be discussed together when addressing the particle problem (e.g., [Bain, 2000, 380] [Halvorson and Clifton, 2002, 20] [Fraser, 2008, 842] [Ruetsche, 2011, Chs. 9–11] [Wallace, 2001, §2.4]

The most obvious path to a fundamental particle interpretation—relying on what we call the standard particle notion—is blocked by Haag's theorem in the following way. This path relies on a representation of the Weyl relations having a (global) number operator, which can be understood physically as counting the number of particles. Since number operators only exist for representations unitarily equivalent to a free-field Fock representation (theorem 3.3 of [Chaiken, 1968]), and since Haag's theorem implies that the latter cannot represent an interacting field, a particle interpretation for interacting fields cannot rely on the existence of (global) number operators.[16] But the situation is even worse, it would seem: violating any one of the assumptions of Haag's theorem would seem to undermine any notion of particle that could do the work we demand of a fundamental interpretation [Ruetsche, 2011, 253]. Thus, Haag's theorem seems to rule out fundamental particle interpretations of RQFT.

Crucially, the existence of a global number operator is necessary for the standard fundamental particle interpretation. That is, the existence of a global number operator is necessary for a particle interpretation to meet the fundamental interpreter's demand for a single, mathematically-precise interpretive schema applicable everywhere. This condition is meant to ensure both that would-be particles are like particles—i.e., are sufficiently localizable and countable—as well as that they are so *in the same way* throughout an interaction. Put so crudely, it is this latter aspect of the criterion that most directly rules out emergence- and coalescence-type particle notions like that found in [Wallace, 2001].[17]

Repair. Pristine interpreters have proposed two kinds of responses to the problems posed by Haag's theorem (and other no-go theorems) for a fundamental particle interpretation. Because of the strictures of the ideal of pristine interpretation, each is, in a sense, revisionary.

On the one hand, some bite the bullet and accept that there are no fundamental particles. Doing so entails providing an alternative fundamental ontology and, consequently, alternative explanations of erstwhile "particle" physics phenomena. This is the more typical repair proposal. For instance, Halvorson and Clifton conclude that "relativistic quantum field theory does permit *talk* about particles—albeit, if we understand this talk as really being about the properties of, and interactions among, quantized fields" [Halvorson and Clifton, 2002, 24]. Similarly, albeit with less patience for the talk of particles, Fraser concludes that special relativity and the non-linearity of the field equation for an interacting system—key ingredients for Haag's theorem—conspire against a quanta interpretation, i.e., "there is no quanta interpretation and there are no quanta" [Fraser, 2008, 858]. However, it should be noted that the most obvious way of constructing a field ontology appears to run into the same problems

[16] See references in [Fraser, 2008, 847] for further detail. See also [Heathcote, 1989] pp. 91-97 for a discussion of the consequences of Haag's theorem for Fock space representations and axiomatic approaches to modeling interactions (and see [Earman and Fraser, 2006] section 7 for their critical assessment thereof).

[17] See also [Feintzeig et al., 2021] for recent work on accounting for emergent particle phenomena using the classical limit of QFT.

as the particle ontology [Baker, 2009].[18] In each case, a revision of our ontology—or at least, ontological speech—is proposed.

On the other hand, some propose revision of the notion of particle required for a fundamental particle interpretation. Doing so entails revising the relevant definitions at play in the no-go on particles, e.g., the approach to scattering theory (Haag–Ruelle) [Reed and Simon, 1980], the condition for localizability and state equivalence (LSZ formalism) [Bain, 2000], or the definition of particle itself[19], in order to "save" the talk of fundamental particles, especially the explanatory work done by this talk. This repair proposal is less typical. Though each proposal may save (some of) the talk of fundamental particles, each nevertheless proposes revision of QFT or the standard definition of particle.

For non-pristine interpreters, Haag's theorem alone does not demand any repair. Instead of proposing repairs to either our ontology or QFT, it suffices for [Wallace, 2001] to show that a concept of particle can be recovered from a field-theoretic description. Given Wallace's background presumption that approximate concepts are legitimate in physics, it does not matter that the particle concept he deploys is vaguely defined and applies only to bosonic, massive QFTs; likewise, the satisfaction with approximate concepts removes the sting of Haag's theorem for a fundamental particle notion. For Ruetsche, Haag's theorem is only one chapter (namely, Ch. 9) in the story of particles [Ruetsche, 2011, Chs. 9–11], and that it rules out the standard particle interpretation *in general* is unremarkable in the larger story. Indeed, she concludes, "[s]ometimes particle physics is, adulteratedly, particle physics, and that's a good thing" [Ruetsche, 2011, 260].

Renovation. The long-term desire of the pristine interpreter group—including each of the above repairs—is to develop a pristine interpretation of QFT. Given this group's expectations for how one should be developed, a mathematically precise, general, and consistent formulation of QFT is a goal. Thus, expected renovations for this group are largely mathematical. We should note some reason for pessimism about the attainment of this goal, however. While the pristine interpreters have worked from the Wightman axioms for over half a century now, sufficiently realistic models of these axioms have not been forthcoming. In particular, these models must be able to represent interactions. This has proven especially challenging, and there is no guarantee that such models are even possible. One noteworthy recent development on what kinds of field theories can be rigorously constructed in four space-time dimensions is [Aizenman and Duminil-Copin, 2021]. Likewise, revisions of the relevant definitions (e.g., Bain's re-definition of 'particle') have led to difficulties [Fraser, 2008]. Long term renovations in this area will likely require deep conceptual work rather than mathematical advancement. Finally, the non-pristine interpreter does not see Haag's theorem itself as reason for any renovation.

[18] However this argument relies on the restriction that the field wave functionals are square-integrable. If we relax this restriction, then the space of wave functionals will be larger than the space of particle wave functions (see [Jackiw, 1990] and [Sebens, 2022], but see [Wallace, 2006] for a defense of the square-integrability restriction).

[19] After arguing that the Received View's notion of particle, which is inconsistent with relativistic quantum field theories, relies implicitly on the existence of an absolute temporal metric, [Bain, 2011] concludes by pointing to several potential modifications of the definition of particle.

4.3 Duncan and Miller

Tony Duncan [Duncan, 2012] and Michael Miller [Miller, 2018] argue that at least one of the assumptions needed to prove Haag's theorem is violated at some point in the actual process of calculating scattering theory results in perturbative QFT. Thus, in Duncan's words, we can "stop worrying" about Haag's theorem. These calculational processes include the methods of regularization and renormalization.

A similar view on Haag's theorem is given in [Fraser, 2017]. Here, James Fraser is primarily concerned with the much broader question of the status of perturbative QFT and diagnosing the 'real problem' with this area of physics. He argues that perturbative QFT is "a method for producing approximations without addressing the project of constructing interacting QFT models" (4). As a consequence of this view, the threat of inconsistency posed by Haag's theorem is defused for much the same reasons as given by Duncan and Miller. Fraser concludes section 4, "The perturbative method simply does not assert the set of claims shown to be inconsistent by Haag's theorem" (18). We set aside [Fraser, 2017] for the remainder of this section since Haag's theorem is not the primary target of that article; but readers interested in the broader question of how to assess perturbative QFT are encouraged to look there for an important contribution on that topic.

Extra-Haagian outlooks. For Duncan and Miller, it is a given that the interaction picture has been consistently applied in calculating various specific theoretical predictions. For Duncan, QFT as it is used in particle physics—including its use of the interaction picture—is "the most powerful, beautiful, and effective theoretical edifice ever constructed in the physical sciences" [Duncan, 2012, iv]. His goal, therefore, cannot coincide exactly with Earman and Fraser's of strictly unifying SR with QM. Rather, Duncan's broader goal for the book is to provide a "deep and satisfying comprehension" of QFT by addressing the important conceptual issues for which the traditional, "utilitarian" texts fail to provide careful explanations (pp. iii–iv).

Likewise, Miller is focused on understanding the QFT noted for its empirical successes [Miller, 2018, 802], hence not the QFT of Earman and Fraser. Ultimately, Miller's aim is to address "a general tension that exists in much of the literature engaged in the philosophical appraisal of the foundations of quantum field theory" (p. 803). This tension is essentially that between Earman and Fraser and the fundamental interpreters' approach to QFT on the one hand and the empirically-tractable QFT on the other: while the former is mathematically rigorous and hence (relatively) easy to interpret using standard philosophical tools, it has yet to produce a realistic model, so it is unclear how it could inform claims about the actual world; conversely, while the latter has generated wildly successful empirical predictions, it has done so through changes to the mathematical formalism whose interpretive significance is far from obvious (p. 803).

For both authors, then, the preferred approach seems to be to bring the philosopher's penchant for logical and conceptual clarity to bear on QFT as it is actually used. Given that both are sure (up to fallibility) that QFT is being applied consistently, questions about its foundations will naturally not concern *whether* they are consistent or correct but rather *how* they are so.

Assessment. Called in to assess the status of our QFT building in light of Haag's theorem, what Duncan and Miller are assessing is not *whether* QFT is consistently using the interaction picture, but *how*. Both Duncan and Miller recognize the logical nature of the problem posed by Haag's theorem—i.e., as the conflict between the interaction picture and free-field representation assumptions—and they take consistency to be a requirement for a viable theory. Indeed, they recognize that a response to Haag's theorem must come from negating at least one of its assumptions. However, unlike Earman and Fraser, their goal is to "explain why theoretical predictions for realistic experimental observables give empirically adequate results" [Miller, 2018, 803]. These predictions *in fact* use the interaction picture, so they cannot just jettison the interaction picture. Rather, they aim to identify where within the practical calculational techniques such violations of the assumptions must already take place for reasons entirely independent of Haag's theorem.

[Duncan, 2012, pages 359-370] and [Miller, 2018] each argue that we circumvent Haag's theorem in the messy calculational details of how the interaction picture is used in practice. This takes place in a two-step process. In the first step, regularization evades Haag's theorem at the price of also breaking the Poincaré invariance (and generally the unitarity) of the S-matrix, which we generally consider to be desirable. In the second step, renormalization allows us to remove the regularization, thus restoring the Poincaré invariance and unitarity of the theory. According to [Duncan, 2012, page 370], "the proper response to Haag's theorem is simply a frank admission that the same regularizations needed to make proper mathematical sense of the dynamics of an interacting field theory at each stage of a perturbative calculation will do double duty in restoring the applicability of the interaction picture at intermediate stages of the calculation." Thus, in their assessment, Haag's theorem poses no problem to our QFT building's integrity.

Repair. The interaction picture in practice makes use of renormalization and regularization techniques that, in pulling "double duty", both produce finite perturbative results and defuse the assumptions of Haag's theorem. The renormalization and regularization 'repairs' also evade Haag's theorem as a positive side effect. No additional repair is needed.

Maintenance. Maintenance is straightforward: maintain current practices of proper use of renormalization and regularization[20] techniques. For the conceptually and logically curious, following in Duncan and Miller's footsteps, this maintenance will likely also involve identifying precisely which assumptions have been violated, and where. While it remains to be shown that the perturbative power-series converge and thus correspond to exact non-perturbative objects post-regularization and -renormalization, [Miller, 2018, page 815] contends that this problem is unrelated to Haag's theorem.

[20]However, some methods of regularization do not break Poincaré invariance. This raises the question as to how this story should apply in such cases. [Miller, 2018, pages 814-815] suggests that the basic structure of the argument still stands. Any such regularization technique must provide a means for controlling infrared divergences. In so doing, one expects that one or more of the assumptions of Haag's theorem must have been violated through the regularization process.

4.4 Klaczynski

Extra-Haagian outlooks. Like the authors above, Klaczynski recognizes a bifurcation in QFT research. On the one hand there is canonical QFT. Canonical QFT has been spectacularly successful not only for making precise predictions but, as Klaczynski emphasizes, "*predict[ing] the existence of hitherto unknown particles* [Klaczynski, 2016, 2]. Nevertheless, canonical QFT is mathematically ill-defined: "canonical QFT presents itself as a stupendous and intricate jigsaw puzzle. While some massive chunks are for themselves coherent, we shall see that some connecting pieces are still only tenuously locked, though simply taken for granted by many practising physicists, both of phenomenological and of theoretical creed" [Klaczynski, 2016, 2]. One major contributor to this ill-definedness is owed to the use of renormalization.

On the other hand is constructive QFT, which use operator theory and stochastic analysis to attempt to construct models of quantum fields in a mathematically well-defined manner. A number of important results have been obtained by this research program, including many triviality results that can be seen as calling into question basic features expected of any rigorous QFT. Nevertheless, the construction of a renormalizable theory in 4 dimensions—i.e., realistic—has neither been achieved nor seems achievable using current methods [Klaczynski, 2016, 3]. That is, no rigorous and realistic model exists. Klaczynski's aim is to reconcile canonical and constructive QFT by elucidating the coherence brought about by renormalization.

Assessment. Given the above, Klaczynski approaches Haag's theorem intent on understanding what it says about canonical QFT. At first blush, the result appears to be negative. For instance, the Gell-Mann-Low formula, relating the ground states of the interacting and non-interacting fields, is built on the assumption that the time-evolution operator in the interaction picture, which relates the two fields, is unitary; this is exactly what Haag's theorem rules out (recall note 3 on the significance and wide-spread use of the Gell-Mann-Low formula.). However, on closer inspection, the contradiction is resolved, if nevertheless unfavorably: like Fraser and Duncan, Klaczynski blames Haag's theorem for the divergence of the perturbative expansion of the Gell-Mann-Low formula. This leads Klaczynski to conclude that the interaction picture is ill-defined and trivial [Klaczynski, 2016, 59].

While he points to similar symptoms as Duncan and Miller, Klaczynski's conclusion is more severe. In his final assessment, the interaction picture itself, relying as it does on a unitary intertwiner, is flawed—even renormalization does not save the interaction picture. While standard regularization methods may break Poincaré invariance, this is physically unacceptable; moreover, Poincaré invariance broken by regularization is restored when we take the adiabatic limit. This means that Haag's theorem again applies, albeit now to the renormalized theory [Klaczynski, 2016, 62–3].

Repair. Like Duncan and Miller, Klaczynski thinks that renormalization procedure still repairs the problem, but it works for a different reason. When we renormalize our theories, we replace the bare quantities with their renormalised counterparts. This process of renormalization does not merely cancel the divergences, but also fundamentally changes the theory, by bringing about a coupling-dependent mass shift. As a result, the renormalized free and interacting fields now have different masses. Two quantum fields of different mass are overwhelmingly likely to be unitarily inequivalent[21]. As such, the renormalized interacting and free theories are almost certainly unitarily inequivalent, and so the conditions for Haag's theorem do not apply in our renormalized theories.

Nevertheless, the un-renormalized and renormalized theories *are* still related—just not by a unitary transformation, as physicists still believe. Thus, we are working with something *like* the interaction picture insofar as the two theories are still related by an intertwiner of sorts, but *unlike* the interaction picture insofar as this intertwiner is manifestly non-unitary. So whilst renormalization is the correct treatment, physicists have not fully grasped how it allows us to evade Haag's theorem. By using renormalization to fix the divergences, physicists have "muddled our way through to successfully applying perturbation theory" [Klaczynski, personal communication, Sept. 8, 2021].

Renovation. Given that the interaction-picture presentation of canonical QFT still dominates, renovation is necessary. This has two parts. First, physicists have misunderstood, or at least mis-described, the tools they are using when performing scattering calculations with renormalized fields. While Haag's theorem says this tool cannot be the interaction picture, that does not mean renormalization is unconstrained or incoherent. Rather, renormalization "follows rules which have a neat underlying algebraic structure [the Hopf algebra] and are not those of a random whack-a-mole game" [Klaczynski, 2016, page 4].

Second, this program of identifying the structure of renormalization must continue. First and foremost, there are some mathematical lacunae in this process, as well as procedures that are not defined wholly rigorously; these should be addressed. However, at the end of the day, Klaczynski believes that renormalized quantum field theory "provides us with peepholes through which we are allowed to glimpse at least some parts of that 'true' theory" [Klaczynski, 2016, page 4]. As such, the ultimate goal here would be to glimpse what we can of the 'true' theory through the peepholes this work affords.

4.5 Maiezza and Vasquez

Extra-Haagian outlooks. Like Klaczynski, Maiezza and Vasquez are interested in determining the precise mathematical structure of canonical QFT. However, they seek a "consistent and generic (non-perturbative) formulation of QFT" [Maiezza and Vasquez, 2020, 2], in contrast to the perturbative formulation of canonical QFT. Given the centrality of the interaction picture to canonical QFT, the natural starting point for finding such a consistent and generic formulation of QFT would be to use the interaction picture, starting with free fields. But, Maiezza and Vasquez ask, is it possible to build such a non-perturbative formulation from the interaction picture and perturbative renormalization? Or, rather, is something new needed?

[21] The mathematical ill-definedness of the renormalized terms makes it hard to prove that renormalized free and interacting fields will be unitarily inequivalent in general. However, theorem X.46 of [Reed and Simon, 1980] is used to argue that it is "plausible beyond doubt" [Klaczynski, 2016, page 68].

Assessment. In Maiezza and Vasquez's assessment, Haag's theorem reveals a central problem when we try to improve upon the standard methods of perturbatively renormalized canonical QFT. We see this problem clearly when we consider the entire perturbative expansion series. The problem stems from vacuum polarization: because of vacuum polarization, the full power-series expansion diverges[22]. Maiezza and Vasquez prove that renormalon singularities arise in the total perturbative series, and trace these back to Haag's theorem. These renormalon singularities "are the concrete manifestation" of the impossibility of generating an unambiguous finite result in such cases [Maiezza and Vasquez, 2020, page 10]. [23] Attempts to generate a finite result for the whole series through analytic continuation methods (such as using a Borel-Laplace transformation) necessarily rely on a choice of arbitrary constants. Maiezza and Vasquez describe these dependencies on an arbitrary choice as "renormalization ambiguities." Thus, because of these ambiguities, ultimately arising from Haag's theorem, the interaction picture with perturbative renormalization cannot lead to their desired non-perturbative QFT.

Repair. According to Maiezza and Vasquez, there is no repair we can make at this time to address this problem. There remains a concerning flaw in perturbative renormalization, namely, the renormalized series' dependence on an arbitrary choice of constant. As Maiezza and Vasquez put it, perturbative renormalization "cannot be a self-consistent cure, because perturbative renormalization needs to be completed, or in practice resummed" [Maiezza and Vasquez, 2020, page 10].

In particular, they argue that the repair suggested by Klaczynski—essentially, to replace the interaction picture's unitary intertwiner with a non-unitary one—cannot work. The Dyson operator relating the free and interacting fields is unitary by construction, in such a way that simply undoing the time-ordered product present in its definition (as suggested by [Klaczynski, 2016]) does not therefore make it non-unitary [Maiezza and Vasquez, 2020, 5]. Something more is needed to repair the interaction picture, in particular to make it suitable for calculations using non-perturbative QFT.

That said, Maiezza and Vasquez are quick to note that the practical uses made of the interaction picture with perturbative renormalization need no repair: they evade Haag's theorem. Haag's theorem is, after all, a non-perturbative result, in that it applies to the *entire* power-series expansion, not just the perturbative expansion at some finite order. Because the power-series expansion is asymptotic, truncating at finite orders can give meaningful approximations, while also missing out on the non-perturbative effects that lead to the complete series' divergence. Thus, while regularization and renormalization do not 'fix' the problem posed by the divergences tracked by Haag's theorem, this is essentially because Haag's theorem does not apply in cases where the series is truncated. Perturbative renormalization fixes the divergences of individual terms within the series, but it does not address the non-perturbative divergences of Haag's theorem.

[22] This was first noted in [Dyson, 1952] concerning QED.

[23] Renormalons are singularities that arise as a function of the complex transform parameter when a formally divergent series is summed using Borel summation, see [Beneke, 1999] for a review.

Renovation. They suggest two forms of renovation. First, in the short run, research efforts should aim at improving our understanding of the non-perturbative singularities that arise, such as renormalons. Further research should be directed into mathematical techniques to better understand neglected non-perturbative effects (for example [Maiezza and Vasquez, 2021b, Maiezza and Vasquez, 2021a]). These include resurgence, a method for reading off certain non-perturbative effects from the perturbative expansions (for a review, see [Dunne and Ünsal, 2016]).

Second, further work is needed work *within* canonical QFT to account for non-perturbative effects. Ultimately, new physical insights are needed to complete quantum field theory. These must move beyond simply reformulating the perturbative quantum field theory framework [Maiezza and Vasquez, personal communication, Sept. 8, 2021].

4.6 Kastner

Extra-Haagian outlooks. [Kastner, 2015]'s outlook differs from the previous authors. Insofar as she seeks the correct formulation of our theory of quantum phenomena, she sits with the physicists above. However, insofar as she thinks this correct formulation requires a fundamental reinterpretation of quantum phenomena, she sits with the fundamental interpreters, who are overwhelmingly philosophers. Crucially, she has *already* given up on mediating interactions locally in favor of nonlocal, direct interactions between field sources. This is a profound departure from QFT.

Assessment. Naturally, given her prior commitment to abandoning QFT, Kastner's assessment of QFT in the light of Haag's theorem is grim: "QFT is not the correct model; a different, yet empirically equivalent, model is needed" (57). This goes beyond others' assessments locating the problem specifically with the interaction picture. Rather, Kastner locates the problem 'further back' conceptually. For her, QFT is wrong from outset: the foundational assumption of QFT—namely, that we use fields as mediators between particles (crudely, as "a 'bucket brigade' that is invoked in order to restrict causal influence to a local, continuous conveyance from spacetime point to spacetime point" (59))—is the problem. The revival of direct-action theories (DATs) is the natural solution for that problem. Thus, for DAT advocates such as Kastner, Haag's theorem "simply tells us what we already know: the interaction picture of quantized fields does not really exist" [Kastner, 2015, 58]. Haag's theorem is therefore an additional motivation for abandoning QFT in favor of DATs.

Repair. The treatment for such a deep problem is a major change in the modeling procedure for this area of physics. If we accept that Haag's theorem shows us that it simply does not work to model interactions by field operators creating and destroying Fock space states, then, Kastner urges, we are to instead model interactions through a direct, nonlocal interaction between sources of the field [Kastner, 2015, 57]. On this direct-action approach, one does away with fields as mediators of interactions, replacing them with non-local direct actions between charge sources. The key ingredient for avoiding Haag's theorem, according to Kastner, is that real and virtual particle propagation be treated distinctly. In particular, this means that virtual processes should not be assigned statehood, as they are in traditional QFT.

Kastner believes that this is a viable repair because she is convinced that DATs—which differ radically from QFTs—are empirically equivalent to QFTs. This places significant weight on (purported) demonstrations of the equivalence of direct-action theories and QFT (e.g., [Narlikar, 1968]). If the demonstrations are sound, *and* they extend such that all currently-used QFTs are approximations to some DAT, then they would go some way towards explaining how calculations in renormalized quantum field theories are able to generate successful predictions: QFT's "mathematical inconsistencies can be rendered inconsequential since they can be understood as arising from its 'makeshift,' nonfundamental character" [63].[24]

Kastner further invokes the abandonment of the idea of the ether due to the Michelson-Morley experiment to justify abandoning QFT, saying that abandoning interactions through local quantum fields in response to Haag's theorem would have an analogous "interpretive elegance" (58). She also quotes Wheeler (who co-developed with Feynman a direct-action theory of electromagnetism) and Wesley, who liken Haag's theorem to the EPR experiment insofar as it presents a serious challenge to the assumption that the laws of nature are entirely local. We repair all these problems, including those raised by Haag's theorem, by abandoning locality in favor of DATs.

Renovation. The prescribed renovation, on Kastner's view, is to develop a research program for DATs. This research program includes the rejection of alternative responses to Haag's theorem such as Haag-Ruelle scattering and constructive QFT; Kastner views these workarounds as "*ad hoc*, approximate, or partial measures" (63). Future work in the research program would need to assess the question of the calculational, pragmatic viability of DATs as well as the crucial need to check thoroughly for empirical equivalence with all of the accepted theoretical results of ordinary QFT. Kastner cites work from Narlikar and Davies developing a DAT empirically equivalent to QED; more work is needed to determine if other sectors of the Standard Model of particle physics can be recovered in the DAT approach. Furhter rennovation efforts should also explore the possibility that a DAT approach would lead to novel, empirically testable predictions, as much of theoretical particle physics is currently disappointed with the lack of direction from particle accelerator experiments for new lines of research.

Given the wide extent of Kastner's proposed work—not least, replacing the entire Standard Model with DATs—it would perhaps be more appropriate to describe the proposal as an entirely new foundation than as a renovation of the one that exists.

4.7 Seidewitz

Extra-Haagian outlook. Seidewitz's outlook is interesting in that it substantially coincides with several of the others already discussed, while at the same time departing from them. First, he seems to think of theory interpretation in a way similar to Earman and Fraser and the fundamental interpreters. Like the latter, he appears to take mathematical rigor as

[24] However, this is a non-trivial 'if', especially given the recent results suggesting that 'empirical equivalence' is a less straightforward concept than philosophers often presume. See [Weatherall, 2019] and references therein for more detail.

a precondition for providing a satisfactory interpretation [Seidewitz, 2017, 369]. However, unlike Earman and Fraser, Seidewitz is not satisfied taking the axioms of AQFT as gospel; indeed, his theory substantially revises these axioms in light of his conviction that the former axioms are an *incorrect* marriage of special relativity with quantum theory. This departs from Earman and Fraser insofar as Seidewitz is not merely marrying special relativity and quantum theory as they were traditionally understood. Rather, Seidewitz shares with the physicists a willingness to revise our theories in an effort to find the correct unification of them. Seidewitz also departs from the fundamental interpreters insofar as a univocal characterization of particle-ness is not forthcoming. This is because particles now come in two varieties—virtual and real—and a new definition of 'particle' will have to account for the significant mathematical distinction between the two.[25] While a univocal characterization of each *variety* of particle may still be forthcoming, nevertheless this is a departure from the expectations of fundamental interpreters.

Second, Seidewitz's outlook coincides with Kastner's. On the one hand, processes can evolve in a space-like way. That is, physics is non-local. Specifically, virtual processes need not evolve in a local fashion. On the other hand, particles come in two varieties—virtual and real. Indeed, the two agree on how these varieties are constituted with respect to locality. However, Seidewitz disagrees with Kastner that the non-locality of physics is a reason to abandon field theories. Rather, he suggests an alternative formulation of quantum field theory.

Assessment. Thus, [Seidewitz, 2017]'s assessment of Haag's theorem is that it is the symptom of a larger problem, namely the inconsistent treatment of time in traditional QFT. Seidewitz reads Haag's theorem as a corollary of two previous results, where these previous results are the more significant. First, let $\hat{\psi}_1(x, t)$ and $\hat{\psi}_2(x, t)$ be field operators defined in Hilbert spaces $\mathcal{H}_1$ and $\mathcal{H}_2$, respectively, and suppose there exists a unitary operator $\hat{G}$ such that, at a specific time t, $\hat{\psi}_2(x, t) = \hat{G}\hat{\psi}_1(x, t)\hat{G^{-1}}$. Then (Theorem 1) the equal-time vacuum expectation values of the fields at time t are the same. Second (Theorem 2), a given field's two-point expectation values satisfy a certain equation (Eq. (3) in Seidewitz) iff that field is free. Letting $\hat{\psi}_1$ be a free field and $\hat{\psi}_2$ a field related to the former as in Theorem 1, Haag's theorem merely observes that $\hat{\psi}_2$ also satisfies Theorem 2's equation for spacetime points at time t (by Theorem 1) and, by the Lorentz-covariance of $\hat{\psi}_2$ and analytic continuation, extends $\hat{\psi}_2$'s satisfaction of the equation to any two positions.

As Seidewitz sees it, Haag's theorem "essentially relies on a conflict between the presumption that the fields are Lorentz-covariant and the special identification of time in the assumptions of Theorem 1" [Seidewitz, 2017, 360]. This special identification is apparent in that the time-evolution is given by the (frame-dependent) Hamiltonian. This requires particles to evolve on time-like trajectories. Essentially, the (frame-dependent) Hamiltonian operator is playing double-duty as the generator of time translation, in addition to the generator of state evolution, with t as the evolution parameter for each. However, because of this, the translation group of t—the unitary operators $\hat{G}(t)$—coincides with the

[25] In particular, we should expect (an analog of) the (global) number operator characterization to hold only for real particles, given that the space-like evolution possible for virtual particles prevents their characterization as Fock space states to be created or destroyed (where this is done by operators used to define number operators).

group of Lorentz transformations, which guarantees that one can extend the coincidence $\hat{\psi}_2(x,t) = \hat{G}\hat{\psi}_1(x,t)\hat{G^{-1}}$ at a specific time t to *all* times t.

Repair. The immediate repair is to break the identification of the Hamiltonian with the generator of time translation and the energy observable. Seidewitz proposes we do so by dropping (i) the requirement that there is a unique Poincaré-invariant vacuum state and (ii) the spectral condition, i.e., the requirement that states are *on shell*. Essentially, this involves dropping the assumption that states transform according to an *irreducible* representation of the Poincaré group. This frees us up to define vacuum states as *relative* to the choice of a Hamiltonian operator, where the Hamiltonian is no longer equated with the time-translation generator. Instead, a Hamiltonian is required only to be Hermitian, commute with all spacetime translations, and have a unique null eigenstate. Then, one can consider the one-dimensional group generated by each Hamiltonian, where λ (instead of t) is the (now frame-independent!) evolution parameter. (Physically, λ can be thought of as the parameterization of a particle's path in spacetime—now unburdened from the constraint that the path be timelike.) Haag's theorem is evaded by limiting the equality of vacuum expectation values (Theorem 1) to equal values of λ, which is neither surprising—each Hamiltonian has a unique Poincaré invariant vacuum state—nor problematic—one can no longer use Lorentz transformations to extend the equality any farther. In particular, one cannot demonstrate that the fields coincide (Haag's theorem). Thus, interacting fields need not coincide with free fields, and yet a version of the interaction picture is reinstated since interacting fields can be related to free fields by a unitary intertwiner [Seidewitz, 2017, 369].

Renovation. Like Kastner, the renovation Seidewitz proposes is far-reaching. Most importantly, it will involve expanding the reach of parameterized QFT. As it stands, the theory does not cover gauge theories or non-Abelian interactions, nor does it resolve all of the problems with standard QFT. As such, significant work is required before this approach is of practical use. Nevertheless, insofar as it is conceptually closer to constructive QFT, one might reasonably suspect that the renovations required will be less thorough-going than those proposed by Kastner.

4.8 Framework Results Summary

Our framework accomplishes two main objectives. First, it maps out an otherwise wild jungle of scholarship on Haag's theorem. This organization is especially helpful as groundwork for making the interdisciplinary exchanges of ideas on Haag's theorem more efficient: it is not always easy for physicists, philosophers, and mathematicians to communicate. This is in part due to appropriately different aims, as well as differences in how these separate communities have developed their own forms of discourse, vocabulary, and standards of rigour.

Second, our framework goes beyond the organization of viewpoints on Haag's theorem by pulling each viewpoint's underlying disciplinary and methodological values to the foreground (see again table 2). How one diagnoses and treats Haag's theorem is profoundly influenced by what one expects of QFT, and of the interaction picture in particular, as well as one's expectations of theoretical physics in general. One's expectations for the mathematician's and the philosopher's appropriate relationship to theoretical physics also has an influence. Each response discussed here is shaped by the authors' background commitments, research purposes, and expectations of theoretical physics—what we have called Extra-Haagian outlooks. A breakdown summary of these Extra-Haagian outlooks is given in table 2.

Each viewpoint surveyed and assessed above has a distinctive purpose. Duncan and Miller are both concerned with executing a very specific explanatory task concerning the successful implementation of the interaction picture despite Haag's theorem, in the current practice of physics. Klaczynski is nearby doing a similar explanatory task of physical practice, but does not see the result as having saved the interaction picture. In contrast, Kastner and Seidewitz are interested in developing genuinely new physical theories. Maiezza and Vasquez are somewhere in between these previous two, seeking to explain how Haag's theorem eventually leads to renormalon divergences. They use this explanation to suggest further avenues for mathematical research that can make progress on the foundations of this area of physics. In this sense, their research aims at developing valuable technical groundwork for new physical theories, while stopping short of offering a new theory as Kastner and Seidewitz do. Finally, Earman and Fraser and the fundamental interpreters are doing the distinctly philosophical work of sorting out the downstream interpretive implications of Haag's theorem in conversation with the wider goals and conceptual foundations of QFT. Bringing these separate research aims to the surface helps to address the confusions described in section 3.

Finally, in bringing these background values and guiding aims to the forefront, our framework exposes, for general methodological consideration, several desiderata for any satisfactory theory of scattering experiments. First, we must be able to generate a consistent system for modeling interactions, that is, we must escape the inconsistency threat to the interaction picture demonstrated by Haag's theorem. Beyond that, we require the ability to consider interactions with either massive or massless particles (an issue raised in [Earman and Fraser, 2006], [Klaczynski, 2016], and [Kastner, 2015]), as well as a modeling procedure that is can be implemented in a realistic number of spacetime dimensions—the longstanding obstacle for constructive program within AQFT. Some in the philosophy community also require of QFT that it be formulated in a way amenable to traditional interpretive methods (e.g [Fraser, 2011] and [Halvorson and Clifton, 2002]). Finally, amongst those who take Haag's theorem as additional fuel on the fire of theory change, more far afield concerns over the desire to solve seemingly unrelated problems become relevant, such as doing away with gauge arbitrariness and the potential of a new formulation of QFT to lead to unification with gravity. [Kastner, 2015] and [Seidewitz, 2017] seem to have these considerations in the forefront of their thinking, and [Maiezza and Vasquez, 2020] perhaps have them in the background. And all of this is, of course, in addition to the basic requirement of empirical adequacy.

5 Cartography of the QFT Foundations Labyrinth

From what has been said so far, one may be drawn towards ordering the different viewpoints surveyed here, perhaps along a scale of *radical-ness of treatment* or *concern for mathematical rigor* or some other axis. We want to emphasize that it is a feature of our framework that it is sufficiently flexible to allow for multiple different orderings on this space of viewpoints. Our

Name	Extra-Haagian Outlooks
Earman & Fraser	QFT is, by definition, the unification of SR and QM. Therefore, giving up any postulate from either theory in order to defuse Haag's theorem is not an acceptable response.
Fundamental Interpreters	The larger project is one of fundamental theory interpretation. Hence, a satisfying interpretation must apply across all models of QFT. AQFT's axiomatic structure makes it the preferred formulation for interpretive work.
Duncan & Miller	The question of focus is: How can it be that actual calculations in theoretical physics, based on the interaction picture, have been successful despite Haag's theorem? This is a specific question for explaining the success of current practices, to be contrasted with more foundational questions in QFT.
Klaczynski	The question is how can we reconcile the success of scattering calculations with the consequences Haag's theorem? If we are committed to keeping the axioms of QFT, then all that is left to revise is the interaction picture itself.
Maiezza & Vasquez	We need to understand the implications of Haag's theorem for physically realistic quantum field theories. A perturbative procedure cannot give a complete answer to the problems posed by Haag's theorem. We must also attempt to trace the implications of Haag's theorem for non-perturbative methods.
Kastner	Kastner has a background commitment to the existence of non-local interactions. More generally, Kastner has a background expectation that much of mainstream formulations and interpretations of QFT are misguided. In this light, Haag's theorem becomes one more piece of evidence that the foundations of this area of physics need major revisions.
Seidewitz	QFT incorrectly treats time as an evolution parameter. This can be fixed by adding an evolution parameter, resulting in a new and improved field theory: parameterized QFT. The theory is mathematically rigorous, hence is suitable for interpretation. In particular, the theory is non-local.

Table 2: Extra-Haagian outlooks summary.

framework gives structure and clarity to the literature on Haag's theorem, but it intentionally does not fix any one line of debate as the most important issue, nor does it settle any one of the many lines of debate to be drawn. The framework thus makes it possible to draw these lines of debate clearly, as if it were producing competing maps of the labyrinth of conceptual issues at the foundations of QFT. In this section we give three such maps.

In table 3, we map viewpoints in terms of their response to the bare logical problem posed by the contradiction between Haag's theorem and the interaction picture: Which assumptions should be abandoned in order to escape the contradiction? On this mapping, some authors locate the problem in the QFT axioms while others point to key assumptions in the interaction picture. Within these two branches, viewpoints further diverge. Note, too, that some respondents' viewpoints are left out, in virtue of not responding to Haag's theorem in this way. In particular, neither Maiezza and Vasquez nor the fundamental interpreters have tended to respond to Haag's theorem by considering the rejection of (some of) its assumptions.

While the bare logical problem raised by Haag's theorem is widely agreed upon, it is not the only illuminating way to survey the terrain of the literature on Haag's theorem. In figure 1 we give an alternative mapping according to the driving motivation in each response: some authors are primarily motivated by questions about conceptual foundations/interpretation, whereas others are motivated by a desire to explain and assess physics practice. Authors further diverge according to *which* issue at the foundations or within physics practice is most pressing. Extra-Haagian outlooks play a key role in this mapping.

Figure 2 shows a third option for mapping out the different positions in terms of which disciplinary speciality is best suited for the kind of repair and maintenance work needed. Loosely, we can conceive of three general areas where energy could be directed in response to Haag's theorem. First, energy could be put into developing new mathematical techniques. This could include new mathematical models, renormalization methods, and methods such as regularization and resurgence to better deal with the asymptotic series. Second, energy could be put into theoretical physics, such as studying cutoff invariance, beta functions, or more generally seeking out new physical insights. Third is philosophical work readdressing how we interpret quantum field theory (or whatever theory should replace it). However, none of these areas are clear cut and all overlap somewhat.

These three mappings are likely not exhaustive. They are meant to illustrate the value of our framework in simultaneously giving a fixed structure for understanding each response (the structure of assessment/repair/maintenance & renovation), while also leaving enough conceptual room to accommodate these various further cartographic options.

The mapping in figure 1 is particularly helpful in that it reveals where there are points of genuine disagreement. Moving upwards through the diagram, the first disagreement is about whether, and how, regularization and perturbative renormalization allow us to circumvent Haag's theorem. Here, Duncan and Miller, Klaczynski, and Maiezza and Vasquez each come to contradicting conclusions. According to Duncan and Miller, the regularization process allows us to circumvent Haag's theorem by temporarily breaking the Poincare invariance of the theory. However, Klaczynski argues that Haag's theorem cannot be circumvented in this way. Instead, he argues that the fault lies with the interaction picture: once we have fully renormalized our theory, there is no longer a unitary intertwiner. Maiezza and Vasquez assert that the intertwiner is unitary by definition, thus Haag's theorem cannot be fully evaded by a perturbative procedure like renormalization. The problems of Haag's theorem still appear as divergences when we consider the entire perturbative series of our renormalized theory. These three views cannot all be correct.

A second clear disagreement emerges with regards to the status of conventional quantum field theory. Both Kastner and Seidewitz propose alternatives to conventional QFT. Seidewitz proposes modifying the axioms of QFT, in which we drop the requirement that states transform according to an irreducible representation of the Poincaré group. In Kastner's proposal, we replace quantum field theory with a direct action theory, in which interactions take place non-locally between charge-sources. Even if both approaches can recapture our empirical data, they are fundamentally different theories, built on different assumptions. They cannot both be right.

Having now thoroughly and multiply mapped out the Haagian labyrinth, the question remains: how do we get out? The foregoing suggests that this is not so simple. Each proposed path is sensible enough on its own. However, as the last two paragraphs emphasize, the paths cannot all overlap—a non-trivial choice must still be made. To make this choice, we require further information. We have highlighted the extra-Haagian outlooks throughout this section because they together provide a glimpse of where that further information should come from.

The central lesson of our framework is this: charting a way out of the Haagian labyrinth will require us to answer meta-level questions of what foundational work is, and about what it does for physics. We highlight the following three as central:

1. **What role does (should) foundational work play in progress in physics?**
2. **How does (should) foundational work coordinate with non-foundational work?**
3. **And, what does (should) foundational work even look like?**

All those who are lost in the Haagian labyrinth would agree that they are doing work at the foundations of physics. Doing this work inevitably involves taking a stand on these meta-level questions. Nevertheless, these questions are rarely addressed head-on. However, doing so is necessary in order to adjudicate between different escape routes from the Haagian labyrinth.

We can phrase this in terms of our general contractor metaphor, too. You have received all of your subcontractors' assessments, proposed repairs, and any renovation plans they recommend for your QFT building in response to Haag's theorem. Each has even toured the building with you to evidence their assessment and lobby for their proposal. Nevertheless, you remain unsure of how to proceed: you want to ensure the building's integrity as its rooms continue to expand, but precisely *how* the building will evolve is beyond you. Our advice as to how to make an informed decision is to reconsider the fundamental design principles that inform the relationship of the building and its rooms to its foundation. Indeed, you may also investigate that relationship in *similar* buildings, past and present.

[26] There are several different ways of doing this. For instance, Haag–Ruelle scattering gives up the idea of building out from free-field representation. Alternatively, LSZ formalism. [Earman and Fraser, 2006], [Duncan, 2012, pages 268-281]

Name	Problem Source	We should give up on...
Duncan & Miller	QFT Axioms	exact Poincaré invariance.
Kastner	QFT Axioms	independent fields.
Seidewitz	QFT Axioms	time as evolution parameter.
Klaczynski	Interaction Picture	a unitary intertwiner.
Earman & Fraser	Interaction Picture	the entire IP. [26]

Table 3: Diagnoses of those who take the import of Haag's theorem to be the identification of a contradiction in the foundations of physics, specifically between the axioms of QFT and the assumptions of the interaction picture.

For our own part, we plan to carry out the following investigations. First, how should we think about the relationship between (foundational) no-go theorems and non-foundational physics? We believe there is a more constructive way of construing no-go theorems that would more closely and more fruitfully connect foundational and non-foundational work. Second, what distinguishes fruitful uses of axiomatization from unfruitful uses in physics? Here we plan to investigate the origins of, and expectations for, the Wightman axioms and Haag's theorem. Finally, how *precisely* are the assumptions of Haag's theorem undermined using the various regularization, renormalization, and resummation methods? Duncan and Miller offer good reason to believe that at least one assumption is undermined *at some point* during many calculations, but they neither verify this for all methods nor characterize precisely where in a given calculation the assumptions are undermined. Both have implications for the significance of Haag's theorem.

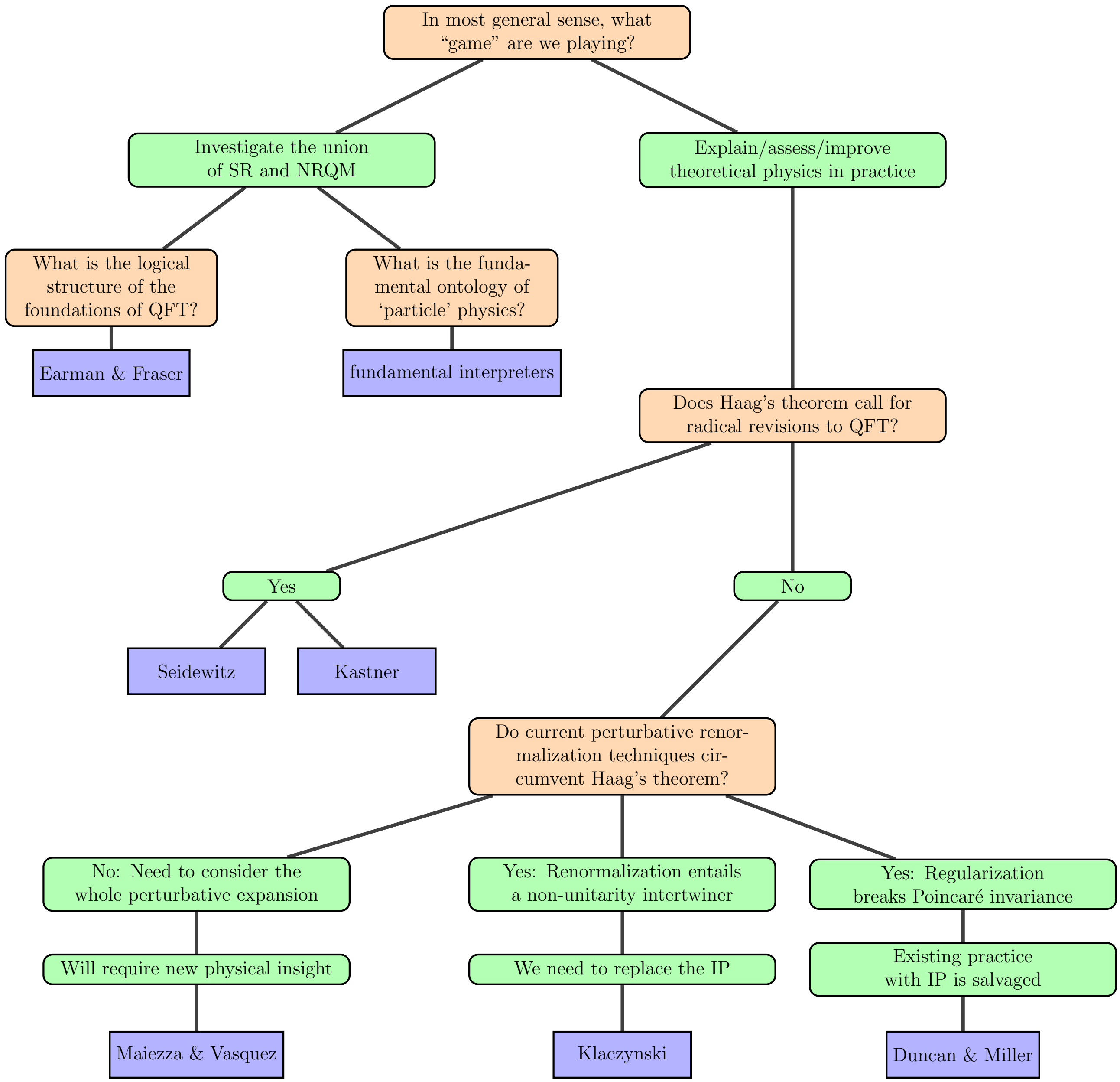

Figure 1: A mapping of the responses to Haag’s theorem according to the driving motivations.

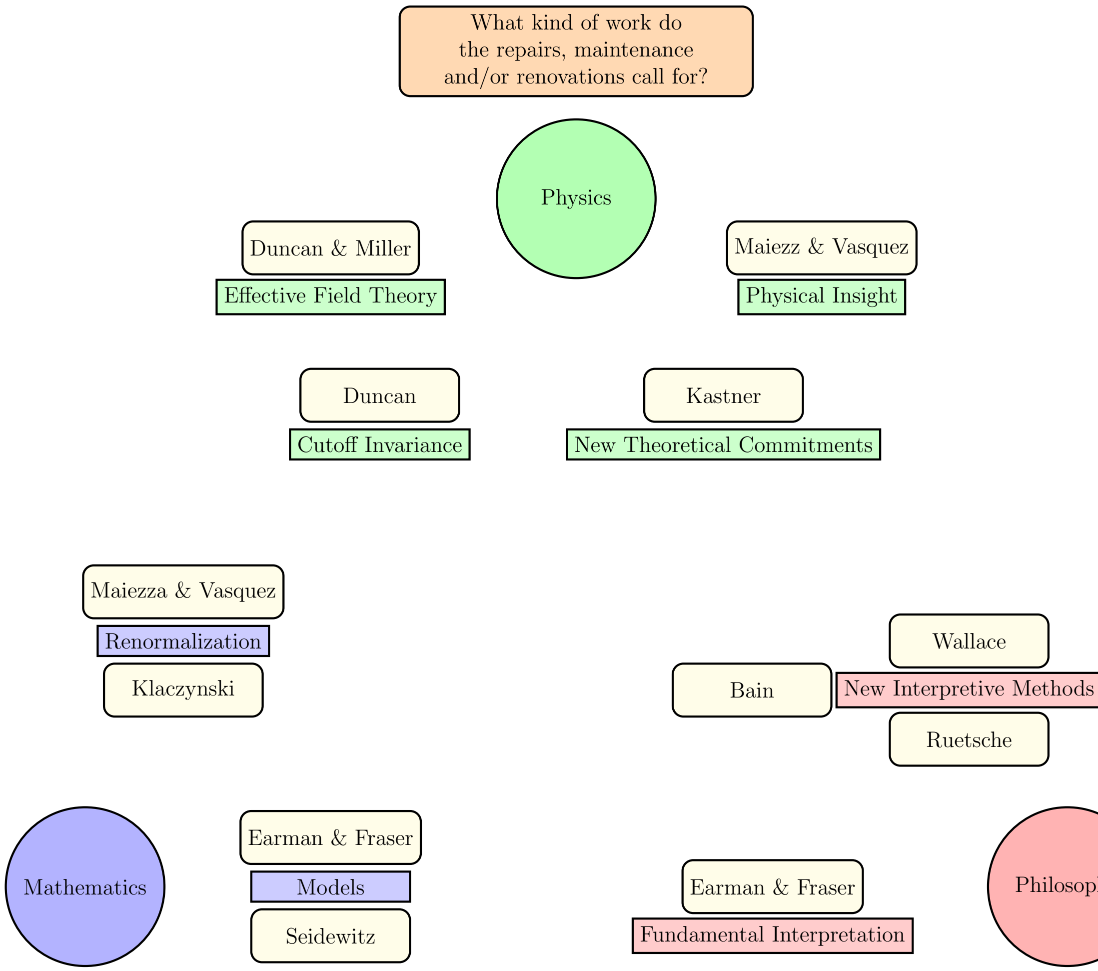

Figure 2: What kind of work do the repairs, maintenance and/or renovations call for?

6 Conclusion

Haag's theorem cries out for explanation and critical assessment: it sounds the alarm that something is (perhaps) not right in one of the standard way of constructing interacting fields to be used in generating predictions for scattering experiments. Viewpoints as to the precise nature of the problem (assessment), the appropriate solution (repair), and subsequently called-for developments in areas of physics, mathematics, and philosophy (maintenance or renovation) differ widely. Moreover, the extant literature presenting these differing views constitutes a complex mix of arguments at cross-purposes, generating substantive confusion as to the precise issues to be addressed. In this paper, we have worked to address this confusion by cataloging and comparing a number of these viewpoints, and we have developed and applied a framework for understanding these distinct viewpoints. The application of our framework reveals each authors' background disciplinary and methodological commitments and expectations of QFT—what we have termed 'extra-Haagian' outlooks. While regimenting and structuring the literature, our framework does not fix any one line of debate regarding viable or preferable responses to Haag's theorem. Rather, within our framework we have mapped out three distinct ways of drawing the possible lines of debate, and we hope that these maps will prove useful for future work on many issues within the labyrinth of the foundations of QFT.

Finally, in light of these maps of the Haaging labyrinth, charting a way out requires that we turn our attention to meta-level questions regarding the nature of foundational work in physics. What role does (should) foundation work play in the progress in physics? How does (should) foundational work coordinate with non-foundational work? What does (should) foundational work even look like? Each practitioner, be they physicist, mathematician, or philosopher, must have at least some implicit answer to these questions in order to find a path out. It is perhaps more seriously incumbent upon the philosopher to develop and debate explicit answers to these questions.

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