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Nested modalities in astrophysical modeling

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Abstract

In astrophysics, where typically phenomena cannot be reproduced in laboratory, the adoption of models and simulations becomes mandatory. Here, we investigate the modalities implied in astrophysical modeling practices by taking into account different levels of abstraction or generality at which phenomena can be framed. To this aim, we analyze two case studies involving modeling gravity at different regimes: namely, gravity at the solar system level and gravity at the cosmological level. At first glance, modeling practices and implied modalities could appear significantly different in these cases. On the contrary, by framing the analysis in terms of the how-possibly/how-actually distinction (as in Bokulich (2014)) we show that, in both cases, we can identify a web of nested modalities arranged across different levels. On this basis, we argue that, independently from the gravitational regime, there is not necessarily a direct correspondence between the degree of possibility and the level of detail at which the phenomena are considered.

Keywords: Astrophysical modeling practices · Modal modeling · How-possibly/how-actually distinction · Post-Newtonian models · Cosmological models

1 Introduction

In the current literature on modeling practices in science, there has been a renewal of interest in the modal aspects implied. In particular, some recent papers (Verrault-Julien (2019); Sjölin Wirling and Grüne-Yanoff (2021, 2022); Grüne-Yanoff and Verrault-Julien (2021)) are devoted to tackle the epistemology of modal modeling, with a special focus on the how-possibly/how-actually distinction in regard to model-based explanations. Without entering into the details of this debate, owing its origin to the literature on mechanism and mechanistic models (e.g., Machamer et al (2000); Craver (2006)), here we will use the how-possibly/how-actually distinction as a conceptual framework for examining the modalities involved in representative cases of astrophysical modeling.

Astrophysics, intended in a broad sense as including both the physics of the solar system and the physics at large and very large scales (i.e., cosmology),[1] notoriously deals with phenomena and processes taking place in extreme conditions, hardly reproducible in a laboratory. Given also the broad range of physical scales and the complexity of the systems considered, the adoption of models and computer simulations becomes indispensable in this field, thus offering exemplary case studies for the role of models in scientific practice.

In fact, the functions and meaning of modeling and simulations in astrophysics have been objects of extensive philosophical study, especially in the last decade (e.g., Anderl (2018); Massimi (2018); Smeenk and Gallagher (2020); Gueguen (2020); Jacquart (2020)). Here, we take a slightly different direction with respect to this literature by focusing – in the framework of the debate on modal modeling in science – on a more specific issue: that is, in the background of the how-possibly/how-actually distinction, on the question as to how the degree of possibility is related to the level of generality at which the astrophysical phenomena are considered in the modeling practice. On this aim, we consider two case studies of astrophysical modeling at very different scales, namely, at the solar system scale and at the cosmological scale, respectively. More precisely, since astrophysical phenomena are mostly driven by gravitational interactions, the cases considered involve, in fact, modeling gravity at the two different regimes. In more detail: in the framework of modeling gravity at the solar system level, we focus on the case of the "relativity experiment" of the ESA space mission BepiColombo; concerning modeling gravity at cosmological scales, we discuss the case of the rotational curves of spiral galaxies.

As we will see, these modeling practices draw a web of nested modalities, which can be framed at different levels of detail. In particular, we will show how setting the analysis in terms of the how-possibly/how-actually distinction allows to identify the different kind of modalities into play, depending on the level. On this basis, we will argue that, independently from the gravitational

[1] This is a common use of the term though not the only one. See for example Anderl (2016), where astrophysics is characterised as the physics operating at "intermediate" scales, that is between the physics of the solar system and the cosmology of the entire universe

regime, there is no direct link between the degree of possibility and the level of detail at which the phenomena are considered.

The paper is organized as follows. In Section 2, we precise the conceptual framework adopted, that is, how we will use such notions as "model", "simulation", "data set" and the how-possibly/how-actually distinction. Section 3 provides an outlook on modeling practice at different astrophysical regimes, as a background for the discussion of our case studies, examined in Sections 4 and 5, respectively. Finally, in Section 6 we discuss the nature of the modality implied in the modeling practices at stake in the two case studies.

2 Conceptual framework

Over time, and especially in the last decades, a huge amount of literature on models has been proposed, debating questions regarding their nature, functions and epistemic import.[2] As clearly shown by this literature, "models" are meant in different senses, depending on the context in which they are applied and on their intended use. Here, following Weisberg (2013), we will adopt the notion of a model in the sense of an *interpreted structure* used for studying physical phenomena, properties or evolution in a given domain.

More precisely, we will focus on astrophysical *theoretical models*, and consider their relations with *data sets* by means of *simulations*. For the aim of this paper, "theoretical models" are intended in the sense of non-concrete interpreted structures, used and tested to explore a space of physical possibilities characterized in terms of parameters which may take different values according to given purposes (e.g., Datteri and Schiaffonati (2019)). "Data sets" are the results of the procedures (synthesising, filtering, correcting or smoothing) by means of which are processed and elaborated the "raw" astronomical data, collected by using a telescope or a space mission or whatever other observing methodology.[3] In this sense, data sets function as the "observational" basis to be taken into account, providing the so-called "observed observables".

"Simulations", in the cases we consider, come into play by mediating between theoretical models and data sets. To be more concrete, the models examined are related to data sets through the computer simulations which are used to obtain numerical results from the model equations.[4] In substance, the

[2] See the useful overview provided by Frigg and Hartmann (2020) and references therein.

[3] Here we follow the common scientific usage of the term (see Kelleher and Tierney (2018), Chap. 2). In fact, there is some ambiguity in the use of the term in the literature, especially in the philosophical one, where they are often identified with data models (see, for example, the discussion in Bokulich (2014); Bokulich and Parker (2021); Antoniou (2021)).

[4] Here, we take the term "simulation" in the narrow sense of running a computer process and, following Datteri and Schiaffonati (2019), we adopt their working definition according to which a (computer) system is said to simulate a theoretical model if it can be characterised in terms of parameters whose values depend on one another according to the regularities mentioned in the theoretical model. Of course, how to define a "simulation system" and under which conditions it can be said to effectively simulate a target system (or a theoretical model of the target system) is not such a simple issue and the different approaches debated in the literature on scientific modeling sensibly depend on the context considered (physics, climate science, economics, social science, ..). See, e.g., the detailed investigation on the relation between models and simulations proposed by Winsberg (2018). A recent philosophical discussion of the role of computer simulations in astrophysics is provided by Jacquart (2020).

procedure is as follows: once a theoretical model is chosen or properly built, by simulating the corresponding dynamics it is possible to generate the so-called "simulated observables", which are the data that would be recorded if the model was, in fact, the actual description of the phenomena under study. In general terms, these modeling practices are articulated in building a suitable theoretical model, running a computer simulation on its basis and then comparing the output of the simulation with the available data sets. At this point, simulated and "observed" observables can be directly compared by different sort of fitting algorithms, in order to check the validity of the theoretical model (e.g., Lari et al (2021)).

By definition, theoretical models are possible models, that is, models of possible state of affairs. Is there a way of of being more precise about the degree of possibility these models represent? In this respect, a helpful conceptual tool turns out to be the distinction between "how-actually" *vs* "how-possibly" models, widely discussed in the modeling literature. Note that this distinction, as introduced in Craver (2006), is usually discussed in regards to the explanatory role of models, accordingly taking the form of a distinction between how-actually and how-possibly model explanations.

In Craver's terms, how-possibly models are "only loosely constrained conjectures", heuristically useful in "constructing a space of possible mechanisms", while how-actually models offer an adequate explanation of what in fact produces the phenomenon (Craver (2006), p. 361). Between the two typologies of models, there is "a range of how-plausibly models" that are more or less consistent with the known constraints on the details of the system studied. Thus, moving from how-possibly to how-actually models is just a matter of progressively restricting the space of possibilities by increasing the constraints.

In fact, this is not the only way to intend the difference between how-possibly and how-actually modalities. In particular, here we will refer to the how-possibly/how-actually distinction as discussed in Bokulich (2014). In that paper, a special attention is paid to the different contexts in which an explanation can be given and the different levels of abstraction at which the explanandum phenomenon can be framed. By taking as a case study the geological phenomenon known as "tiger bush" (a characteristic striking periodic banding of vegetation appearing in semi-arid region) and its various possible model explanations, Bokulich shows how alternative models can compete both at the how-actually level and at the how-possibly level, forming a kind of hierarchical branching tree. On the one side, how-actually explanations, i.e., explanations referring in some way to the observable effects of the phenomenon, are shown to be deployed also at a very abstract level. On the other side, within the corresponding class of how-actually models, Bokulich shows how it is possible to identify a split at a second most abstract level between different how-possibly models, each providing a possible further specification of the explanatory mechanism (p. 331–332). In this case, the how-actually/how-possibly distinction does not merely refer to a more or less detailed description of the phenomenon (in the sense of fine-grained explanation). In particular, it does not follow that, the more fine-grained a model is, the closer it is to a how-actually explanation (p. 334).

Here, we will set in similar terms the analysis of the modalities involved in the gravitational modeling practices taken as our case studies. As we will see, also in these astrophysical cases, however different from the geological case studied by Bokulich, it will be possible to draw the same kind of conclusion.

3 Modeling practices at different astrophysical regimes

As said, astrophysical modeling practices are mostly concerned with gravity modeling. Let us start, then, with discussing how modeling gravity can be approached at different astrophysical regimes. The issue of modeling gravity frames in the context of the current efforts in the experimental testing of GR, a central topic in the actual debate in astrophysics. Indeed, although GR has passed a large number of experimental tests, observations over the last decades pointed out some shortcomings of the theory both at the infrared (i.e., galactic) and ultraviolet (i.e., quantum) scales, highlighting the fact that GR could not be the final theory for gravitational interaction (see, for example, Capozziello and de Laurentis (2011) for a review on this issue). In this paper, we will consider only "classical" gravitation (i.e., issues at quantum scales will not be taken into account).

In addressing the problem of the observational limitations of classical GR, two main approaches can be distinguished. One approach is to give up relativity altogether and look for a new theory of gravitation, based on different physical principles. Up to now, the most successful attempt in this direction has been the case of MOdified Newtonian Dynamics (MOND) theories, first proposed by Milgrom (1983). The other approach is to preserve the basic relativistic framework, but modify or extend the classical GR model in such a way that different or additional physical mechanisms are involved, in order to obtain a better fit with the observational data. In this direction, several modifications of GR have been developed over time. Here below, we shortly give an idea of the most known types of modifications:

- *Scalar-tensor (S-T) modifications*: while in Newton's theory gravity is determined by means of a scalar field and in GR by the metric tensor $g_{\mu\nu}$, in this case gravity is determined both by a metric tensor and a scalar field ϕ, so that the metric can be put in the form $\bar{g}_{\mu\nu} \equiv A^2(\phi)g_{\mu\nu}$ (for a comprehensive introduction see, for example, Fujii and Maeda (2003)). Different formulations can be given depending on the behaviour of the scalar field: the best known is the Brans–Dicke theory (Brans and Dicke (1961)).
- $f(R)$ *theories*: in this case, the idea is to substitute the Ricci scalar curvature R with a suitable function, $f(R)$, chosen in such a way that at cosmological scales the universe would experience an accelerated expansion, without the need to resort to a cosmological constant or dark energy. This family of

modifications of GR was first proposed in Buchdahl (1970). It can be shown that eventually $f(R)$ theories are equivalent to S-T modifications (see, for example, Jain and Khoury (2010)).

- *Vector-tensor (V-T) modifications*: here gravity is determined both by the metric tensor and by a dynamical four-vector field u^{μ}, which can be unconstrained or, as in Einstein-Aether theories, constrained to be timelike with unit norm. This type of modification is motivated by the idea of exploring possibilities for a violation of Lorentz invariance in gravity, thus allowing for preferred frame effects (a detailed description is provided in Jacobson and Mattingly (2001)).

On the grounds of the above distinctions, we can identify three different levels of *abstraction* or *generality* at which a gravitational phenomenon can be framed:[5]

1. *First level*: at a first, most general level, we can distinguish between theories of gravity based on different kind of physical principles. The two main alternatives in this frame are General Relativity, on the one side, and MOND theories, on the other side. In fact, a third proposal has been advanced, on the aim of framing the empirical approach of MOND in a relativistic framework,[6] in order to exploit the best of both GR and MOND (and their related success at different gravitational regimes). At this first, most abstract level the distinction is among *theories*, where different theories correspond to different sets of physical principles.
2. *Second level*: once the basic physical framework is chosen (GR, MOND, or relativistic MOND), different classes of dynamical processes can be invoked, leading to alternative types of theoretical models. Thus, at this second level, a bit less abstract than the first level, we mainly distinguish between *classes of theoretical models*: within the same theoretical framework (i.e., the same set of principles), different classes are based on different physical mechanisms.
3. *Third level*: at this level, less abstract than the preceding one, within the same class of theoretical models one can further distinguish between *specific theoretical models*, which, though based on the same general physical assumptions and mechanisms, invoke different specific sub-mechanism. These are the models that are, then, compared with the data sets at disposal.

A general sketch of these three levels of abstraction (corresponding, respectively, to distinguishing theories, classes of theoretical models or specific theoretical models) is provided in Fig. 1: for each of the theories (A; B; C; etc.) based on different physical principles, classes of theoretical models (A.1, A.2, A.3; B.1, B.2, B.3; C.1; C.2, C.3; etc.) can be distinguished, based on different physical mechanisms; finally, for each class of theoretical models, i.e., for

[5] We adopt the expression "levels of abstraction" (or generality) with a similar meaning as adopted in Bokulich (2014), p. 325.

[6] Referred as "relativistic MOND" in the following.

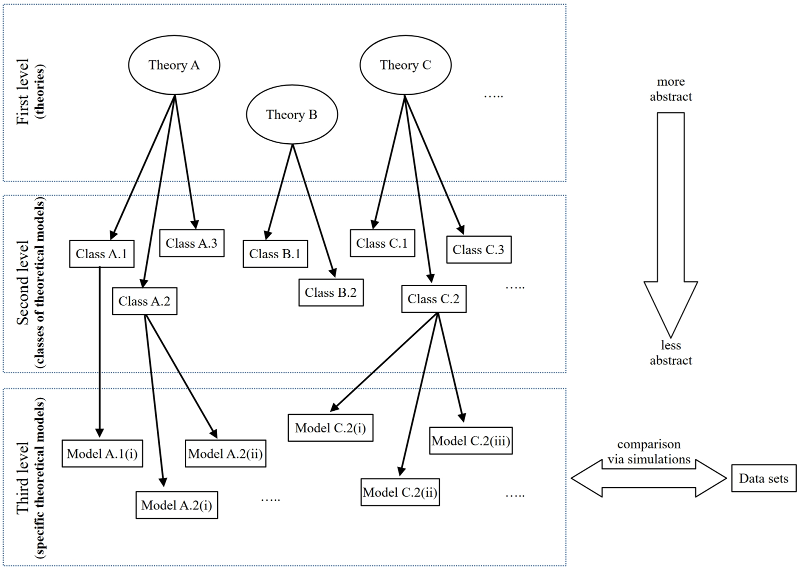

Fig. 1 Sketch of how a phenomenon can be framed within three levels of abstraction or generality (corresponding, respectively, to a distinction between theories, classes of theoretical models and specific theoretical models).

a given set of physical mechanisms and processes, specific theoretical models (A.1(i); A.2(i), A.2(ii); C.2(i), C.2(ii), C.2(iii); etc.) can be distinguished, each one based on different sub-mechanisms. We thus obtain a resulting hierarchical tree structure, corresponding to the levels of abstraction: going from the first level down to the third one, there is a shift through a progressively less abstract description of the phenomenon under study.

At the third level, the ideal case would be that each model could be assessed or discarded when checked against the available data sets. The typical situation that arises in practice is that two or more competing models of gravity can be mutually exclusive but still both equally conceivable if compared with the available data. In this sense, the issue of the unavoidably limited accuracy of the astronomical data at disposal turns out to be a key feature in discriminating between models.[7]

Gravitational phenomena can take place at very different spatial, temporal and energy scales. Accordingly, different gravitational regimes can be distinguished, and the related gravitational phenomena are studied by adopting different appropriate scientific methodologies and tools.

[7]With respect to the limited accuracy of the available data, a natural strategy is to plan and devise new space missions and surveys, in order to provide increasingly more accurate observations.

In what follows, we focus on modeling practices taking place at two very different astrophysical regimes, namely, at the solar system scale and at cosmological scales. A natural assumption is that the scale difference implies corresponding differences in the modeling practices. The modeling activities at different gravitational regimes employ different tools and different kinds of observational data, due to the remarkably different features that need to be addressed. Moreover, it is a fact that the physicists studying solar physics and cosmology, respectively, form two well distinct scientific communities.

However, as we will see, this conclusion is too hasty. This will be clear in the light of the two cases we will examine in the next two sections, where, for each regime, we will present a specific case study:

- Case study I: modeling gravity in the solar system by means of the the relativity experiment of the ESA BepiColombo mission (Section 4.1);
- case study II: modeling gravity at galactic scales by investigating the debate on the interpretation of the rotational curves of spiral galaxies (Section 5.1).

4 Modeling gravity in the solar system

The standard practice in modeling gravity at the solar system scale is to frame the gravitational phenomena within a relativistic framework.[8] This means that at the first "level of abstraction" (in the terminology adopted here), i.e., at the level of theories, GR is the best candidate option.

At the solar system level the dynamics can be described by an approximate solution of the Einstein's field equations, known as the *post-Newtonian* (PN) *approximation*, corresponding to the limit of slow moving particles under the effect of weak gravitational fields (*slow-motion weak-field* limit). In rough terms, the PN approximation allows to expand the spacetime metric, $g_{\mu\nu}$, about the Minkowski metric, $\eta_{\mu\nu}$, as a sum of dimensionless gravitational potentials of varying degrees of smallness. In particular, in the parameterized PN (PPN) formalism, dimensionless parameters are put in the place of the coefficients of the potentials, where each of these PN parameter describes a specific property of the spacetime metric. Classical GR provides for a total 10 PN parameters, which have well defined values.[9]

The most remarkable advantage of adopting the PPN formalism is that the only way in which one class of theoretical models can differ from another is in the values of the PN parameters. This allows for an easy way to compare different models and fit models with data sets. Thus, following the hierarchy shown in Fig. 1, at the second (less abstract) level we can distinguish three main classes of theoretical models within the PN approximation:

[8] Indeed, the relativistic framework fits most of the observational data with very good accuracy (see, for example, the discussion in Will (2014)).

[9] The list of the 10 classical PN parameters, with their meaning and the assumed value in classical GR and in generalized relativistic theories can be found in Will (2014), p. 31. Note that in classical GR, the only two not null PN parameters are the Eddington parameters γ and β, whose value is expected to be unity.

1. Classical GR model: in this case, the validity of GR in its classical formulation is assumed and the value of the PN parameters is the one provided by GR; no further PN parameters are allowed.
2. *Modified* models of gravity: here we intend the classes of models such that the values of the 10 standard PN parameters are allowed to be different from the value predicted by GR. Examples are the S-T modifications of GR and the $f(R)$ theories, which can be re-written in PN approximation.
3. *Alternative* models of gravity: in this case we mean the classes of models which allow for additional dynamical effects not expected in classical GR; the 10 PN parameters have the same values as predicted in GR, but further PN parameters are introduced by representing the additional dynamical effects by means of small additional terms in the metric, each one characterized by a new PN parameter.

A this point, a further distinction can be made within the classes of modified and alternative models. We thus obtain a third (even less abstract) level, depending on the specific physical sub-mechanisms invoked: different theoretical models can be specified within each of the classes defined at the second level (an example will be discussed in 4.1).

On the other side, despite the general consensus in adopting a relativistic framework to approach the gravitational phenomena at the solar system level, there are some attempts to frame the available observations within the MOND approach and to check for detectable effects at the solar system level. In particular, a possible prediction could be the violation of the strong equivalence principle.[10] In this way, the MOND paradigm could be tested phenomenologically and constrained by fitting the available planetary data (e.g., Magueijo and Bekenstein (2007); Iorio (2008); Blanchet and Novak (2011)).

Summing up, in the case of modeling gravity in the solar system we have two (branching) trees, starting one with GR and the other with MOND at the higher level (level 1). An illustration of how these levels of abstraction arrange is provided in Fig. 2.

Let us now turn to discuss a specific case study concerning testing gravity at the solar system level.

4.1 Case study I: Modeling gravity with BepiColombo

BepiColombo is an ESA/JAXA space mission for the exploration of the planet Mercury and the inner solar system (e.g., Benkhoff et al (2013)).[11] The spacecraft was launched at the end of 2018 and it is planned for orbit insertion

[10] In a MOND framework, the solar system would be affected by the external gravitational field in which it is embedded, that is, the gravitational field of the Milky Way. This would produce a detectable effect consisting in a precession of the perihelia of the Sun and the planets due to the galactic field and, consequently, in a violation of the strong equivalence principle (according to which the laws of gravitation are independent of the velocity and location of the observer and they can be described, locally, within the framework of special relativity).

[11] The mission is named after Giuseppe "Bepi" Colombo (1920–1984), an Italian mathematician and astronomer who first discover the 3:2 spin-orbit resonance of the planet Mercury and contributed to develop the 'gravity assist' technique to reach the planet.

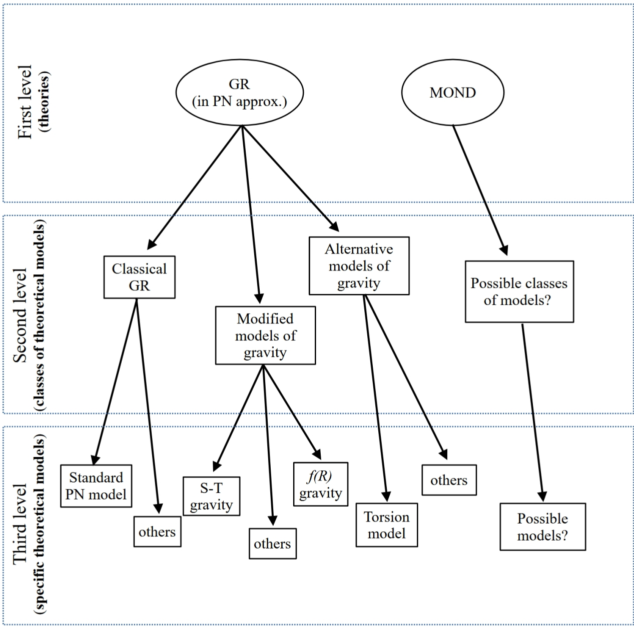

Fig. 2 The three levels of abstraction in the case of modeling gravity at the solar system level.

around Mercury at the end of 2025. It is equipped with a competitive suit of instruments to perform different scientific experiments. One of the mission goals is to perform a test of GR. Indeed, Mercury is the best placed planet in the solar system in order to test for gravitational theories, as it is the nearest planet to the Sun and, therefore, the most subject to its gravitational force. Thanks to the possibility of achieving a very accurate determination of both the orbits of Mercury and the Earth, the Mercury Orbiter Radio science Experiment (MORE) (e.g., Iess et al (2021)) on board BepiColombo will allow to perform a "relativity experiment" consisting in the very precise determination of the value of the main PN parameters by means of a non-linear least squares fit (e.g., Milani et al (2002)).

Let us see how this example of astrophysical modeling practice at the solar system level fits in the hierarchical structure illustrated in Section 3. Assuming at the first (more general) level of abstraction that a relativistic framework is the "actual" one for describing the solar system (as it is widely supported by observations), we can distinguish different classes of theoretical models (second level), each one providing a set of possible specific theoretical models (third level) – as illustrated in Fig. 2. These specific theoretical models can differ one from each other in two ways: by predicting a different value with respect to classical GR for one or more of the 10 standard PN parameters (modified models) or by predicting additional PN parameters to include further physical effects, inhibited in classical GR (alternative models). Hence, depending on the physical assumptions, defining each class of models and each specific model, we can have at our disposal a number of competing theoretical PN models of gravity. From a computational point of view, each of these models can give rise to a set of simulations, representing how the dynamics of the solar system would be if the given model were its actual description. At this point, by comparing each set of simulations with the available data sets, it would be possible, in principle, to constrain the value of the PN parameters and, thus, to discriminate between theoretical models.

In the case of the relativity experiment of BepiColombo, the comparison between data sets and different PN models will be conducted by means of a non-linear least squares fit, which will provide the values of PN parameters together with the state vectors of Mercury and other fundamental physical parameters, such as the gravitational mass of the Sun (e.g., Milani and Gronchi (2010) for details on this fitting method). Once obtained the best estimate of the PN parameters by such fit, it will become possible to check which of the available competing models provides the most accurate description of the dynamics of the solar system.

At the third level of abstraction, an example of an alternative model of gravity which can be tested with BepiColombo implies the generalization of GR which allows for non-vanishing spacetime torsion (see Schettino et al (2020) and references therein). Such a model belongs to a general class of alternative models of gravity known as "teleparallel gravity" (e.g., Bahamonde et al (2021)). While in Riemann spacetime of classical GR the anti-symmetric part of the affine connection vanishes, generalizing to the Riemann-Cartan spacetime an anti-symmetric term can be added to the Levi-Civita connection, leading to a possible non-vanishing torsion tensor (mathematical details can be found, for example, in Hehl et al (1976)). The resulting theoretical model, called "torsion model" in what follows, can be written in PN approximation by parameterizing the dynamical contribution due to spacetime torsion in terms of 3 additional PN parameters (called *torsion parameters* t_1, t_2, t_3: see March et al (2011) for details). The values of the torsion parameters is fixed as zero in the case of classical GR, while they can be equal to some non-null value in the case of the extended torsion model. Hence, the BepiColombo data set will be fitted with two competing specific theoretical models: the classical GR model, which predicts $t_1 = t_2 = t_3 \equiv 0$, and the torsion model, which allow for $t_1 \neq 0$, $t_2 \neq 0$, $t_3 \neq 0$. Depending on the estimated values for the three torsion parameters, it will be possible to discard one of the two specific PN models, up to the accuracy threshold of the BepiColombo experiment.

Following the same modeling practice, other competing PN models of gravity (at the third level of abstraction) could be tested and the estimated values of the PN parameters will allow to discriminate between the corresponding specific models. As already pointed out, the capability of assessing or rejecting a given model is intrinsically linked to the accuracy threshold provided by the experiment. In fact, two competing models can be both admissible up to a given accuracy. Conversely, once the available data allow to discard a given model – i.e., it turns out that the model is not an "actual" one – no more accurate observation is expected to overturn this conclusion.

5 Modeling gravity at cosmological scales

Constraining models of gravity at cosmological scales by means of observational data is obviously more problematic than in the previous case at the solar system scale. Current data mainly support the "standard model of cosmology", which is based on the so-called Λ Cold Dark Matter (ΛCDM) model, first proposed in 1995 by Ostricker and Steinhardt (1995). The standard model is framed within GR and depicts the universe as dominated by vacuum energy (∼70% of the total energy budget) plus some cold matter (i.e., moving at non-relativistic speed) (e.g., Ferreira (2019); Perivolaropoulos and Skara (2021)). In some more detail, ΛCDM is a specific theoretical model within the general class of Friedman-Lemaître-Robertson-Walker (FLRW) models, charaterized by assuming that the equations of GR hold for a isotropic and homogeneous universe (see, for example, Hamilton (2014), Bambi and Dolgov (2015)). Hence, in terms of our three-level distinction, what is usually referred to as the standard model of cosmology in the literature (see, e.g., Ellis (2014)) can be specified as follows: at the first level, GR is the accepted theory of gravitation; at the second level, the FLRW class of models is adopted (i.e., the Einstein's field equations hold for a isotropic and homogeneous universe); at the third level, ΛCDM represents the simplest parameterization within all the possible FLRW models. There is a rich, growing literature, both from the scientific and the philosophical sides, on the standard cosmological model, its implications and open questions.[12] In fact, notwithstanding its remarkable success (both theoretical and observational),[13] the availability of increasingly accurate cosmological observations have highlighted two main drawbacks:

- A *theoretical* drawback: a new kind of matter (dark matter) and a new kind of energy (dark energy) are postulated, making up about the 95% of the universe but with no direct observational confirmation for the time being. Moreover, some parameters of the model appear to be fine-tuned.[14]
- A *phenomenological* drawback: on the grounds of high precision cosmological measurements, some observations at galactic scales turned out to be problematic within the ΛCDM framework. In fact, while ΛCDM is mostly successful at very large scales (from Gpc to Mpc scales), when dealing

[12] Ferreira (2019) is an extensive review on the ΛCDM model, its possible extensions and the current cosmological tests of gravity. On the philosophical side, recent analyses of modeling practices in the cosmological context are, for example, Massimi (2018), Gueguen (2020) and Smeenk and Gallagher (2020).

[13] Examples are: the consistency of the Hubble parameter with the ages of the oldest stars (see Ostricker and Steinhardt (1995)), the power spectrum of density perturbations (see Cole et al (2005)), the accelerated expansion rate of the universe (see Perlmutter et al (1999)), the scale of the baryonic acoustic oscillations (see Eisenstein et al (2005)).

[14] One of the most discussed topics in cosmology is the issue of the cosmological constant, whose theoretical value differs from the observed one up to 120 orders of magnitude (e.g., Copeland et al (2006); Sahni and Starobinsky (2006)).

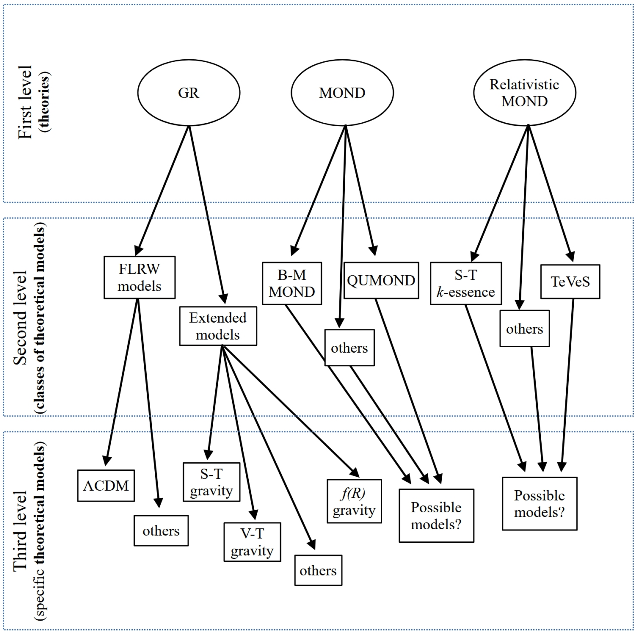

Fig. 3 Levels of abstraction in the case modeling gravity at cosmological scales.

with less than ~ 1 Mpc length scales and less than $\sim 10^{11} M_{\odot}$ mass scales (i.e., galactic scales) many predictions of the model do not match with observations (e.g., Freedman (2017); Bullock and Boylan-Kolchin (2017)).

The issue of how to describe gravity at large scales has animated, and still animates, an intense debate concerning theories as well as models – that is, in the terminology used here, all the three levels of abstraction distinguished above. Let us summarize the main approaches to address the open problems of the standard model, framed in terms of our three-level distinction, as follows (cfr. Fig 3).

1. **First approach: GR-based models.**
   This approach, while maintaining a relativistic framework at the first level, proposes at the second level, besides the FLRW class of models, different classes of *extended* models of gravity, i.e., models which extend or modify some of the physical assumptions of the standard model. Most known attempts to extend GR, as mentioned in Section 3, are: S-T modifications of GR, $f(R)$ gravity and V-T modifications of GR. [15] In general, an extended model can follow two possible (not necessarily alternative) strategies: (1) assume a modification of Einstein's field equations, (2) assume

[15] Of course, many other extended models have been proposed over time: reviews can be found, e.g., in Ferreira (2019); Ishak (2019).

a modification of the energy-momentum tensor, as proposed, for example, in teleparallel gravity. Finally, for each of these classes of models (at the second level), a further specification can be achieved at the third level by choosing more specific models (as illustrated in Fig. 3).

2. **Second approach: alternative theories of gravity**
In this kind of approach, the idea is to search for alternative theories of gravity with respect to GR, i.e., based on different physical principles, in order to obtain a better fit with the cosmological observations. In particular, the new theory should bypass the need for dark matter and/or dark energy. At the same time, it should fit also the observations in favor of GR, at least with the same observational accuracy. Up to now, the most promising attempt is MOND. Milgrom (1983) proposed the idea that the open challenges of the ΛCDM model, in particular the problem of the "missing mass" (see Section 5.1), could reflect a breakdown of Newtonian dynamics in galaxies. On this aim, he devised a formula ("Milgrom's law") to link the Newtonian gravitational acceleration to the "true" gravitational acceleration in galaxies. Introducing a new universal constant $a_0 \simeq 10^{-10}$ m s^{-2}, Milgrom's law predicts that Newtonian mechanics is achieved for large accelerations, while for small accelerations ($a \ll a_0$) a modification to Newton's law is applied. On the one side, Milgrom's law allows for general predictions on galactic systems; on the other side, many observations, unpredicted by ΛCDM at galaxy scales, naturally ensue from this simple law. [16] Since the original formulation by Milgrom, many attempts have been made in order to derive his heuristic law from a universal force law, reflecting a modification of the dynamics. These attempts have given rise to a number of classes of models (in our terms, at the second level): examples are Bekenstein-Milgrom (B-M) MOND (see Bekenstein and Milgrom (1984)), or QUMOND (see Milgrom (2010)).
3. **Third approach: relativistic MOND**
Actually, MOND can be framed within a relativistic framework by intending it as the weak-field limit of a relativistic theory. This is possible since ΛCDM achieves its maximum success at extra-galactic scales, while MOND at the galactic level. In this way, the best of the two approaches is kept. At present, these mixed models have many limitations; accordingly, the search for a suitable relativistic formulation of MOND is an active field of research. In particular, a great challenge for relativistic MOND has to do with the strong equivalence principle, one of the cornerstones of GR, which has to be broken in a MOND framework. A possible solution to this problem is to resort to a Scalar-Tensor (S-T) strategy (at the second level of abstraction, in our terminology) and to account for the violation of the strong equivalence principle as an effect due to the external field.[17] This solution has been developed in the so-called Scalar-Tensor k-essence theories (e.g.,

[16] An extensive discussion on the empirical evidences of Milgrom's law can be found, for example, in Famaey and McGaugh (2012).

[17] See the attempts to detect the effects of MOND at the solar system scale mentioned in Section 4.

Armendariz-Picon et al (2001)). Moreover, to overcome some limitations of this class of models, the Tensor-Vector-Scalar (TeVeS) class of models proposes the addition of both a scalar field and a dynamical vector field (see Bekenstein (2004)).[18]

5.1 Case study II: the issue of the rotation curve of spiral galaxies

As an example of a modeling practice at cosmological scales, let us consider here the well-known case of modeling the mass discrepancy observed in the rotation curves of spiral galaxies.[19]

For almost one century significant discrepancies between the luminous and the dynamical masses of cosmological objects have been pointed out.[20] Indeed, the rotational velocity of these galaxies "should" decrease with the distance from the galaxy center, as expected in Newtonian gravity; on the contrary, it is observed to remain constant. Such discrepancy has been accounted in two different, though not mutually exclusive, ways: 1) in terms of the presence of a significant quantity of non-luminous matter, which clusters on galactic scales (i.e., dark matter); 2) by modifying the gravitational law with respect to the classical inverse square law.

Moreover, the are some peculiar observations of the kinematics of galaxies that need to be properly framed and interpreted in a theoretical framework. First, accurate observational tests (e.g., Lelli et al (2016)) have shown the existence of a simple power-law relating the rotational velocity of galaxies (not only spiral) with the galaxy's baryonic mass, M_b, known as the baryonic Tully-Fisher (TF) relation, $v_{rot}^4 \propto M_b$ (see Tully and Fisher (1977)). In particular, the relevance of such TF relation relies in the fact that it links the rotational velocity (which is an indicator of the total matter content, baryonic and not) to the baryonic content alone. Furthermore, another observational relation to be accounted for is the so-called "mass discrepancy - radial acceleration relation" (MDAR) (e.g., Sanders (1990)). More precisely, this relation describes the observed anti-correlation of the galactic mass discrepancy with the radial acceleration of baryonic matter: the discrepancy is higher for slow radial accelerations, where 'slow' in this case means accelerations such that $a \ll a_0$, where a_0 is a critical acceleration ($a \simeq 10^{-8} \mathrm{cm\,s^{-2}}$).

At this point, let us illustrate how the mass discrepancy issue has been addressed, by referring to the three approaches (GR-based models, MOND, relativistic MOND) described above within the framework of the three-level distinction adopted here.

1. According to the first approach (i.e., assuming GR at the first level of abstraction), the detected discrepancy can be regarded as a confirmation of

[18] An extensive review of relativistic MOND can be found, for example, in Famaey and McGaugh (2012).

[19] This is a widely discussed and central phenomenon in the debate concerning ΛCDM and MOND. In particular, for a philosophical point of view, see, for example, Massimi (2018).

[20] The most detailed quantitative evidence of such a discrepancy is found in the spectroscopic observations (21 cm line of emission of neutral hydrogen) of the rotation curves of spiral galaxies.

dark matter (e.g., Van Albada and Sancisi (1986)). Indeed, a constant velocity implies the proportionality between the mass content and the distance from the galactic center: spiral galaxies should be surrounded by a dark matter halo, extending far from the visible observed galaxy, with a mass density behaving as $\rho \propto r^{-2}$. At the second level of abstraction, different classes of theoretical models account for dark matter in different ways. For example, in the case of the FLRW models dark matter, can be spatially distributed in different ways, giving rise to different specific models at the third level of abstraction. The standard picture, corresponding to the ΛCDM model, predicts that dark matter is distributed in a separated extended spheroidal component (called "dark halo") around the visible galaxy. Conversely, other models can consist in a disc which gets progressively darker in its outer regions.[21] In general, this kind of models shows some inconsistencies between simulations and observations, regarding in particular the TF relation and MDAR (e.g., McGaugh (1983)).[22] Note that these inconsistencies may be bypassed, for example, by imposing some constraint or modification to the acceleration profile of the dark matter halos in order to fit (in some sense, *ad hoc*) the observed rotation curves (e.g., Navarro et al (2017)).

2. Following the second approach (i.e., adopting MOND at the first level), the flat behaviour of the rotation curves of spiral galaxies can be explained by assuming that, in the MOND regime ($a \ll a_0$), the gravitational law is modified with respect to the inverse square law. Moreover, the MOND framework offers a natural explanation for the TF relation and MDAR. In particular, McGaugh (1983) describes different classes of models (at the second level of abstraction, in our terminology) which account for the mass distribution in disk galaxies without resorting to dark matter but assuming a modification of the gravitational law in the regime below a_0. Despite the success of MOND in explaining mass discrepancy features, this remains a minority view in the scientific community.[23]
3. There attempts to frame the rotation curve issue according to the third approach (i.e., assuming relativistic MOND at the first level of abstraction). At the second level, S-T k-essence and TeVeS classes of models have not provided, as of now, a totally satisfactory picture of galactic mass discrepancy. Some alternatives have been considered, giving rise, at the third level, to hybrid models, built *ad hoc* to fit at the same time dark matter and modified gravity.[24] One attempt has been proposed by Berezhiani and Khoury (2015), invoking dark matter superfluidity (i.e., dark matter is

[21] From a physical point of view, there is no preference for a dark halo with respect to a different geometrical distribution of dark matter (see the discussion in Sanders (1990)).

[22] Taking extended dark matter halos to account for the galaxy mass discrepancy yields to the so-called "too big to fail" problem, taking place mainly in dwarf galaxies: that is, the extent of halos capable to explain the observed rotation curves should be so massive that it seems very unlikely that they do not comprise any visible star (this problem was first identified by Boylan-Kolchin et al (2011)).

[23] Indeed, its phenomenological nature and the challenges at extra-galactic scales have made hard so far to consider MOND as an exhaustive theoretical alternative to GR.

[24] See Berezhiani and Khoury (2015) and references therein.

assumed to behave as a superfluid at low temperatures). In this way, dark matter and MOND components are assumed to have a common origin, representing different phases of a single underlying substance, thus saving both the success of MOND at galactic scales and the success of ΛCDM at much larger scales. A different model, still at the third level, has been proposed in Khoury (2015) by invoking two scalar fields, one mediating dark matter and one mediating a MOND-like law force (at the price of having two *a priori* distinct components).

6 Conclusion: Nested modalities

As seen, the two cases illustrated in Sections 4.1 and 5.1, respectively, naturally fit in the three-level distinction proposed in this paper on analogy with the hierarchical structure discussed in Bokulich (2014). For what regards the two astrophysical cases, it is worth underlining that this structure applies independently from the gravitational regime at stake. This prompts the question as to whether there is, in fact, a correspondence between the levels of abstraction and the degree of modality implied, from "possible" to "actual". Moreover, this common approach in modeling gravity allows us to wonder how much the modeling practices acting at these two gravitational regimes substantially differ, if they do.

In order to address these questions, let us go back to our starting point, that is, the how-possibly/how-actually distinction. As mentioned in Section 2, Bokulich (2014) argues that the different models proposed as an explanation of the geological phenomenon she uses as a case study (i.e., the tiger bush phenomenon) fall, in turn, into a branched hierarchy of possible model explanations, occurring at different levels of abstraction. On this basis, Bokulich aims to "show how our understanding of the distinction between how-possibly and how-actually model explanations needs to be revised" (p. 321). More precisely, Bokulich claims that the how-possibly/how-actually distinction "does not track how detailed the explanation is, in the sense that the more fine-grained the explanatory mechanism is specified, the closer it is to a how-actually explanation" (p. 334). This allows her to conclude that "it is not the amount of detail that is relevant, but rather whether the mechanism represented in the model is the mechanism operating in nature" (ibid.).

Notwithstanding the great difference between Bokulich's geological case and the gravity modeling practices examined here, it can be shown that there is a common scheme underlying the modality structures of all these cases. This is illustrated in Fig. 4, where Bokulich's geological case and our two astrophysical cases are framed in terms of our three-level distinction. The typical hierarchical branching characterizes all the three cases, highlighting the affinity within the corresponding modeling practices.

Accordingly, for what regards the comparison between the level of detail (as opposed to the level of abstraction, in the sense used here) and the how-possibly/how-actually distinction, a similar conclusion as Bokulich's one can

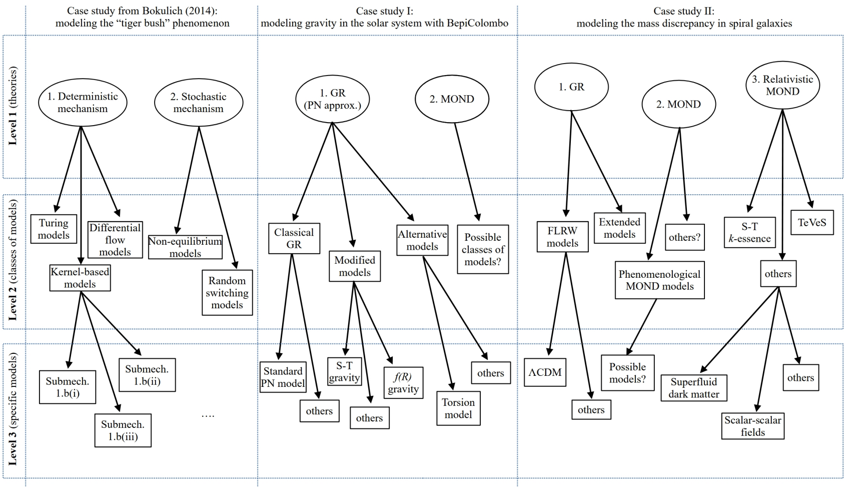

Fig. 4 Comparison of the three levels of abstraction in three cases (from the left)): Bokulich's tiger-bush case, case study I and case study II.

be drawn also for the astrophysical cases we have considered. However, before entering into more detail, let us note that, in the context of astrophysical modeling, it turns out to be more appropriate to categorize modality in terms of gradation of possibilities (from *less-possibly* to *more-possibly*) instead of resorting to a strict distinction between *possibly* and *actually*.[25] Hence, in what follows, we will intend the how-possibly/how-actually distinction in the sense of a *less-possibly/more-possibly* distinction.[26]

At this point, let us consider in some more detail how modalities are involved in the modeling practices acting in the astrophysical cases, I and II, previously described. By referring to the hierarchical structures illustrated above, we will show that, at each level of abstraction, the descriptions adopted can figure both as how-possibly and how-actually, depending on the perspective and purpose of the analysis performed. In other words, at a given level a description can play the role of one of the how-possibly scenarios framed within a how-actually scenario provided at the higher, more abstract level; or it can represent the how-actually scenario for further fine-tuned how-possibly descriptions, to be provided at the subsequent, less abstract level.

a) Case I

[25] As it is known, data sets are limited to a given accuracy by their very nature. Therefore, a given description can be regarded as how-actually in the sense that is how-actually description "subject to a given level of accuracy". In other words, the description cannot be how-actually in a strict sense; rather, it is a more-possibly description with respect to others, according to the accuracy of the available data sets.

[26] Thanks to Mauricio Suarez for suggesting this phrasing.

According to the scientific community consensus and on the grounds of the available data sets, when modeling gravity in the solar system the common option is to assume GR as the how-actually description for framing gravitational phenomena. Thus, we can fix GR as the how-actually scenario at the first level of abstraction. At the second, less abstract level, as seen in Section 4, we have at disposal a number of how-possibly classes of models (the "modified models" and the "alternative models"). Then, for each of these classes of models, we have seen (cfr. Fig. 4) that a number of competing PN models can be provided at the third, least abstract level.

In the case of testing relativity with BepiColombo (see Section 4.1), one of the experiment's intents is to test the possibility of a spacetime torsion.[27] If the aim is to test the goodness of the torsion model, which represents one of the how-possibly scenarios considered at the third level, this implies that, at the second level, the class of alternative models has been chosen as the how-actually description of the physical mechanisms at stake. Analogously, if a different PN model needs to be tested as a how-possibly scenario at the third level, it will be its corresponding class of models at the second level to be chosen as the how-actually description of the dynamics. And so on.

Conversely, a researcher with different purposes in mind could assume that, at the most abstract level (first level), MOND is a more-possibly description than GR, thus playing the role of the how-actually scenario at that level. Accordingly, the classes of models framed within MOND (i.e., along the hierarchical branching provided by considering MOND at the first level) will represent the how-possibly scenarios available at the second, less abstract level. Then, if the aim is to study specific MOND models (i.e., models at the third level), this means that the corresponding second-level class is chosen as the how-actually description of the dynamics.

b) Case II

In the case of modeling the rotational curve of galaxies, at the first, most abstract level, three different approaches (GR-based models, MOND, relativistic MOND) are available as how-possibly scenarios, as shown in Section 5. Depending on the analysis's purpose, one of the three scenarios can be chosen as the how-actually description of gravitational phenomena. For example, if GR is chosen as the how-actually scenario at the first level, then, at the second, less abstract level two how-possibly scenarios have been distinguished, i.e., the FLRW models and the "extended models" (cfr. Fig. 4). Furthermore, if the aim is to test the goodness of ΛCDM, which is one of the how-possibly scenarios at the third, more specific level, this means that the class of FLRW models plays the role of the how-actually scenario at the second level.

To sum up: in both the astrophysical cases considered as well as in the geological case discussed by Bokulich (2014), the hierarchical branching of possibilities represents a web of nested modalities. At each level, a given description can

[27] As said, spacetime torsion is inhibited in classical GR, while it is allowed in the case of the torsion model.

be interpreted as one of the how-possibly options along one arm of the branching generated at the higher level of abstraction. At the same time, this very description, in turn, can be interpreted as a how-actually scenario giving rise to a further branching at the subsequent, less abstract level.

In other words, the same hierarchical branching tree of possibilities, triggering a web of nested modalities, can be recognized in the three case studies. This appears to be a common modal feature in the modeling practices considered here, however different are the scientific domains at stake (astrophysics *vs* geology) when comparing the tiger-bush case with the astrophysical cases, and however different are the gravitational regimes (solar system scale *vs* galactic scale), when comparing BepiColombo relativity experiment (case I) with the rotational-curve case (case II). Therefore, *a fortiori* we can conclude that there is not a necessary correspondence between the how-possibly/how-actually distinction and the level of detail at which the phenomena are taken into account.

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References

Anderl S (2016) Astronomy and Astrophysics. In: Humphreys P (ed) The Oxford Handbook of Philosophy of Science. Oxford University Press, Oxford, p 652–670

Anderl S (2018) Simplicity and simplification in astrophysical modeling. Philosophy of Science 85(5):819–831

Antoniou A (2021) What is a data model? An anatomy of data analysis in high energy physics. European Journal of Philosophy of Science 11(101):1–33

Armendariz-Picon C, Mukhanov V, Steinhardt PJ (2001) Essentials of k-essence. Physical Review D 63(10):103 510

Bahamonde S, Dialektopoulos KF, Escamilla-Rivera C, et al (2021) Teleparallel Gravity: From Theory to Cosmology. Preprint at https://arxiv.org/abs/arXiv:2106.13793

Bambi C, Dolgov AD (2015) Introduction to Particle Cosmology: the Standard Model of Cosmology and its Open Problems. Springer-Verlag, Berlin

Bekenstein JD (2004) Relativistic gravitation theory for the modified Newtonian dynamics paradigm. Physical Review D 70:083 509

Bekenstein JD, Milgrom M (1984) Does the missing mass problem signal the breakdown of Newtonian gravity? The Astrophysical Journal 286:7–14

Benkhoff J, Casterena JV, Hayakawa H, et al (2013) BepiColombo-comprehensive exploration of Mercury: mission overview and science goals. Planetary and Space Science 58:2–20

Berezhiani L, Khoury J (2015) Theory of Dark Matter superfluidity. Physical Review D 92:103 510

Blanchet L, Novak J (2011) Testing MOND in the solar system. Preprint at https://arxiv.org/abs/arXiv:1105.5815v1

Bokulich A (2014) How the tiger bush got its stripes: "how possibily" vs. "how actually" model explanation. The Monist 97(3):321–338

Bokulich A, Parker W (2021) Data models, representation, and adequacy-for-purpose. European Journal of Philosophy of Science 11(1):1–26

Boylan-Kolchin M, Bullock JS, Kaplinghat M (2011) Too big to fail? the puzzling darkness of massive milky ways subhaloes. Monthly Notices of the Royal Astronomical Society 415(L40):1–6

Brans C, Dicke RH (1961) Mach's principle and a relativistic theory of gravitation. Physical Review 124(3):925–935

Buchdahl HA (1970) Non-linear lagrangians and cosmological theory. Monthly Notices of the Royal Astronomical Society 150:1–8

Bullock JS, Boylan-Kolchin M (2017) Small-scale challenges to the ΛCDM paradigm. Annual Review of Astronomy and Astrophysics 55:343–387

Capozziello S, de Laurentis M (2011) Extended theories of gravity. Physics Reports 509(4–5):167–321

Cole S, Percival WJ, Peacock JA, et al (2005) The 2df Galaxy Redshift Survey: power-spectrum analysis of the final data set and cosmological implications. Monthly Notices of the Royal Astronomical Society 362(2):505–534

Copeland EJ, Sami M, Tsujikawa S (2006) Dynamics of Dark Energy. International Journal of Modern Physics D 15(11):1753–1935

Craver CF (2006) When mechanistic models explain. Synthese 153:355–376

Datteri E, Schiaffonati V (2019) Robotic simulations, simulations of robotics. Minds and Machines 29:109–125

Eisenstein DJ, Zehavi I, Hogg DW, et al (2005) Detection of the baryon acoustic peak in the large-scale correlation function of SDSS luminous red galaxies. The Astrophysical Journal 633(2):560–574

Ellis GFR (2014) On the philosophy of cosmology. Studies in History and Philosophy of Science, Part B: Studies in History and Philosophy of Modern Physics 46:5–23

Famaey B, McGaugh SS (2012) Modified Newtonian Dynamics (MOND): Observational phenomenology and relativistic extensions. Living Reviews on Relativity 15:10

Ferreira PG (2019) Cosmological tests of gravity. Annual Review of Astronomy and Astrophysics 57:335–374

Freedman WL (2017) Cosmology at a crossroad. Nature Astronomy 1:0121

Frigg R, Hartmann S (2020) Models in sciences. The Stanford Encyclopedia of Philosophy at https://plato.stanford.edu/entries/models-science/

Fujii Y, Maeda K (2003) The Scalar-Tensor Theory of Gravitation. Cambridge University Press, Cambridge

Grüne-Yanoff T, Verrault-Julien P (2021) How-possibly explanations in economics: anything goes? Journal of Economic Methodology 28(1):114–123

Gueguen M (2020) On robustness in cosmological simulations. Philosophy of Science 87(5):1197–1208

Hamilton JC (2014) What have we learned from observational cosmology? Studies in History and Philosophy of Modern Physics 46:70–85

Hehl F, von der Heyde P, Kerlick G, et al (1976) General relativity with spin and torsion: Foundations and prospects. Reviews of Modern Physics 48:1393–416

Iess L, Asmar SW, Cappuccio P, et al (2021) Gravity, geodesy and fundamental physics with BepiColombo's MORE investigation. Space Science Reviews 127(1):id.21

Iorio L (2008) Constraining MOND with solar system dynamics. Journal of Gravitational Physics 2(1):26–32

Ishak M (2019) Testing General Relativity in cosmology. Living Reviews on Relativity 22:1

Jacobson T, Mattingly D (2001) Gravity with a dynamical preferred frame. Physical Review D 70:024 028

Jacquart M (2020) Observations, simulations, and reasoning in astrophysics. Philosophy of Science 87(5):1209–1220

Jain B, Khoury J (2010) Cosmological tests of gravity. Annals of Physics 325:1479–1516

Kelleher JD, Tierney B (2018) Data Science. The MIT Press, Cambridge (MA)

Khoury J (2015) An alternative to particle Dark Matter. Physical Review D 91:024 022

Lari G, Schettino G, Serra D, et al (2021) Orbit determination methods for interplanetary missions: development and use of the Orbit14 software. Experimental Astronomy, forthcoming

Lelli F, McGaugh SS, Schombert JM (2016) The small scatter of the baryonic Tully-Fisher relation. The Astrophysical Journal Letters 816(1):L14

Machamer P, Lindley D, Craver CF (2000) Thinking about mechanisms. Philosophy of Science 67(1):1–25

Magueijo J, Bekenstein J (2007) Testing strong MOND behaviour in the solar system. International Journal of Modern Physics 16(12a):2035–2053

March R, Bellettini G, Tauraso R, et al (2011) Constraining spacetime torsion with the Moon and Mercury. Physical Review D 83:104,008

Massimi M (2018) Three problems about multi-scale modelling in cosmology. Studies in History and Philosophy of Modern Physics 64:26–38

McGaugh SS (1983) The surface density profile of the galactic disk from the terminal velocity curve. The Astrophysical Journal 270:365–370

Milani A, Gronchi GF (2010) Theory of Orbit Determination. Cambridge University Press, Cambridge

Milani A, Vokrouhlicky D, Villani D, et al (2002) Testing General Relativity with the BepiColombo radio science experiment. Physical Review D 66:082,001

Milgrom M (1983) A modification of the newtonian dynamics as a possible alternative to the hidden mass hypothesis. The Astrophysical Journal 270:365–370

Milgrom M (2010) Quasi-linear formulation of MOND. Monthly Notices of the Royal Astronomical Society 403:886–895

Navarro JF, Benitez-Llambay A, Fattahi Ae (2017) The origin of the mass discrepancy-acceleration relation in ΛCDM. Monthly Notices of the Royal Astronomical Society 471:1841–1848

Ostricker JP, Steinhardt PJ (1995) The observational case for a low-density universe with a non-zero cosmological constant. Nature 377:600–602

Perivolaropoulos L, Skara F (2021) Challenges for ΛCDM: an update. Preprint at https://arxiv.org/abs/arXiv:2105.05208v2

Perlmutter S, Aldering G, Goldhaber G, et al (1999) Measurements of Ω and Λ from 42 high-redshift supernovae. The Astrophysical Journal 517(2):565–586

Sahni V, Starobinsky A (2006) Reconstructing Dark Energy. International Journal of Modern Physics A 15(12):2105–2132

Sanders RH (1990) Mass discrepancies in galaxies - Dark Matter and alternatives. Astronomy and Astrophysics Review 2:1–28

Schettino G, Serra D, Tommei G, et al (2020) A test of gravitational theories including torsion with the BepiColombo radio science experiment. Universe 6(10):175

Sjölin Wirling Y, Grüne-Yanoff T (2021) The epistemology of modal modeling. Philosophy Compass 16(10):e12,775

Sjölin Wirling Y, Grüne-Yanoff T (2022) Epistemic and objective possibility in science. British Journal for the Philosophy of Science, forthcoming

Smeenk C, Gallagher SC (2020) Validating the universe in a box. Philosophy of Science 87(5):1221–1233

Tully RB, Fisher JR (1977) A new method for determining the distances to galaxies. Astronomy and Astrophysics 54(3):661–673

Van Albada TS, Sancisi R (1986) Dark Matter in spiral galaxies. Philosophical Transactions of the Royal Society of London Series A 320:447–464

Verrault-Julien P (2019) How could models possibly provide how-possibly explanations? Studies In History and Philosophy of Science Part A 73:22–33

Weisberg M (2013) Simulation and Similarity: Using Models to Understand the World. Oxford University Press, Oxford

Will CM (2014) The confrontation between General Relativity and experiment. Living Reviews on Relativity 17:4

Winsberg E (2018) Philosophy and Climate Science. Cambridge University Press, Cambridge