Functionalism, Reductionism, and Levels of Reality

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Abstract

I consider a problem for functional reductionism, based on the following tension. Say that b is functionally reduced to a. On the one hand, a and b turn out to be identical, and identity is a symmetric relation. On the other hand, functional reductionism implies that a and b are asymmetrically related: if b is functionally reduced to a, then a is not functionally reduced to b. Thus, we ask: how can a and b be asymmetrically related if they are the same thing? I propose a solution to this tension, by distinguishing between ontological levels and levels of description.

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1 Introduction

Functional reductionism brings functionalism and reductionism together, making reduction a matter of recovering the right behaviour. The goal of this paper is to consider a problem for functional reductionism, and then propose a solution.

To begin with, let's introduce the central notions of the debate. First, functionalism is the view that 'to be x is to play the role of x'. In this sense, x can be deemed as *functionally defined*. This view has been the main position in the philosophy of mind for a long time.[1] For instance, a functionalist account of phenomenal states would define 'pain' in terms of its causal roles, i.e. as 'that state that is caused by bodily injury, that cause the belief that there is something wrong with the body' and so on. Functionalism is now becoming increasingly prominent in different areas within the philosophy of science as well, in particular in the philosophy of physics.[2] For example, according to Knox (2019) and Lam and Wüthrich (2018), we should define spacetime in terms of its functional role, i.e. as that thing that plays the theoretical role of spacetime, and not in terms of some intrinsic features. In a slogan, 'spacetime is as spacetime does'.

Second, reductionism about scientific theories is roughly the view that worse or less detailed theories can be derived from better or more detailed theories. According to the mainstream view, i.e. Nagel's (1962) model of reduction, a theory can be reduced to another theory iff the laws of the reduced theory can be deduced from the laws of the reducing one, with the addition of auxiliary assumptions and bridge laws connecting the vocabularies of the two theories in case they do not share their theoretical terms. More precisely, according to the refined version of the view, since in most cases we cannot derive the exact laws of the reduced theory, we should rather frame reduction as the deduction of a corrected version of the to-be-reduced theory from the reducing theory, plus auxiliary assumptions and bridge-laws.[3]

Then, there is functional reductionism. Versions of this view have been defended in the philosophy of science by several authors, especially in recent years.[4]

[1] See e.g. Kim (1998, 2005), Levin (2021).

[2] See e.g. Wallace (2012), Albert (2013, 2015), Ney (2012), Knox (2019), Lam and Wüthrich (2018).

[3] This is the essence of the refined version of Nagelian reduction developed by Schaffner (1967). See also Butterfield (2011a, 2011b) and Dizadji-Bahmani et al. (2010).

[4] See Lewis (1970), Kim (2005), Morris (2020), Esfeld and Sachse (2007), Lam and Wüthrich (2018, 2020), Butterfield and Gomes (2020b, 2020a), Robertson (2020), Lorenzetti (2022). Moreover, the functionalist account by Albert (2013, 2015) and Ney (2012) can be considered as a form

The target of this paper is, in particular, the Lewisian formulation of functional reductionism, which is the most developed version of the account to date. This framework has been originally proposed by Lewis (1970, 1972), and recently defended and improved by Butterfield and Gomes (2020b, 2020a). Summing up the proposal, say that we have a top theory T expressed in a certain vocabulary and a bottom theory T^* which is expressed in a new vocabulary and which can reduce the former. Let's say we adopt a functionalist account of theoretical terms, according to which theoretical terms are defined in terms of the roles they play within a given theory. Then we can draw bridge laws between terms of the two theories – in the forms of identities – when they share the same functional profile. That is, in this account bridge laws are obtained via functionalism. Furthermore, on the assumption that theoretical terms designate actual entities or properties, we can use this functionalist approach to define entities and properties as the occupants of certain causal roles within a theory. Thus, if we find something in the bottom theory that, in the right regime, plays the role that we attributed to a distinct entity or property within the top theory, we can draw a theoretical identification of the entities/properties across the two theories. In this case we can say that the functionalised entity b is *functionally reduced* to its realiser a introduced by the bottom theory. It is in this sense that one could say, e.g., that the spacetime of general relativity can be functionally reduced to non-spatiotemporal structures of quantum gravity theories, as Lam and Wüthrich (2018) suggest. Section 2 presents the view in more details. For now, the crucial point to highlight is that, within functional reductionism, when a bottom entity a behaves as an upper entity b, b is functionally reduced to a, and a and b turn out to be identical.

This essay considers a problem for functional reductionism – in the Lewis-Butterfield-Gomes model – that I call 'the puzzle of identity', which is based on the following tension. Say that b is functionally reduced to a. On the one hand, as mentioned above, a and b are identical, and identity is a symmetric relation. On the other hand, functional reductionism is expressed in terms of a distinction between a top and a bottom level, and arguably implies that a and b are asymmetrically related: if b is functionally reduced to a, then a is not functionally reduced to b. Thus, one may ask, how can a and b be asymmetrically related given that they are the same thing?

This paper argues for a way to dissolve the tension, based on the following idea: while the identity (and thus the symmetry) is ontological, the asymmetry

of functional reductionism too, as they closely follow Kim's functionalist account which is a form of functional reduction. Indeed, Allori (2021) classifies Albert's view as functional reductionist.

is descriptive. That is, the functional reductionist can maintain that *a* and *b* are ontologically identical – i.e. they refer to the same thing, they are coextensive – but, at the same time, say that the *a*-description is more fundamental than the *b*-description. This paper shows how we can give a formal account of this claim and of the idea that one description can be more fundamental than another, by appealing to the formal notion of *levels of description*, as introduced by List (2019). According to this proposal, while *a* and *b* belong to the same ontological level, in virtue of their identity, they also belong to different levels of description, that are asymmetrically ordered. In other words, the fact that within functional reductionism the world is not divided into higher-level entities and lower-level entities does not entail that functional reductionism cannot make any room for asymmetry at all – in this case, in terms of higher-level and lower-level descriptions.

This solution is important for two main reasons. First, without a strategy of this kind, we would have to bite the bullet and accept that functional reductionism cannot make room for any sense of asymmetry between the relata of the reduction. This would clash with our intuitions about reduction, according to which asymmetry is a constitutive feature of such relation, as the reduced is in some sense 'dependent on' or 'less fundamental than' the reducing element. This attitude can be clearly found in the literature on functional reduction, where e.g. philosophers broadly talk about spacetime being recovered from non-spatiotemporal structures, and not vice-versa, and thus some kind of asymmetry is presupposed. This proposal finds a way to satisfy this desideratum and accommodate the intuitions about asymmetry within functional reductionism, expressed in the Lewisian way. Second, there is also a reason why this specific solution to the puzzle is particularly helpful. That is, the specific use of List's framework in this context is motivated by the fact that it provides a formal way of introducing a *hierarchy* of descriptions. In fact, we are not merely saying that *a* and *b* are identical entities represented in different ways, but rather we are employing List's formal machinery to show that the two descriptions are hierarchically ordered and thus that we can re-introduce the asymmetry in that context. This is the novelty bestowed by List's account of levels.[5]

[5] I elaborate on this second point at the end of Section 4.

2 Functional Reductionism

This section introduces functional reductionism as presented by Lewis (1970, 1972) and Butterfield and Gomes (2020a). In Section 2.1, I introduce the view and highlight the features of the account that will be crucial for the topic of the paper. In Section 2.2, I present an example of functional reduction in the Lewisian model, in order to situate our discussion within a realistic case of reduction in science.

2.1 Lewisian Functional Reductionism

Take a theory (this will be our 'top' theory) and call T-terms the theoretical terms $\tau_1, \ldots \tau_n$ introduced by the theory, and call the rest of the terms in which the theory is couched O-terms.[6] Let's then form the *postulate* of the theory T:

$$T(\tau_1, \ldots \tau_n)$$

This is a sentence that contains all the theoretical postulates of the theory (e.g. $\vec{F} = m\vec{a}$), expressed as a long conjunction. If we replace the T-terms with open variables, we obtain the *realization formula* of T:

$$T(x_1, \ldots x_n)$$

Any n-tuple of entities that satisfies this formula may be said to realize the theory T. We can now introduce the *Ramsey sentence*, which says that T is realized:

$$\exists x_1, \ldots x_n T(x_1, \ldots x_n)$$

Accordingly, we can also define a *modified Ramsey sentence*, which states that T is uniquely realized, i.e. that there is just one set of entities that realize the theory:

$$\exists y_1, \ldots y_n \forall x_1, \ldots x_n (T[x_1, \ldots x_n] \equiv .y_1 = x_1 \wedge \ldots \wedge y_n = x_n)$$

Then, let's introduce the *Carnap sentence*, whose role is to interpret the T-terms:

$$\exists x_1, \ldots x_n T(x_1, \ldots x_n) \rightarrow T(\tau_1, \ldots \tau_n)$$

[6] The theory can – but does not need to – be a physical theory. E.g. it can also be a theory of mental states which describes how mental states relate with each other and with beliefs and physical states.

It says that *if* T is realized, then the n-tuple of entities named by $\tau_1,...\tau_n$ is a *realization* of T. Thus, assuming that T is uniquely realized,[7] the Carnap sentence is logically equivalent to a series of sentences which explicitly defines the T-terms, purely by means of O-terms:

$$\tau_1 \stackrel{def}{=} \iota y_1 \exists y_2,...y_n \forall x_1,...x_n (T[x_1,...x_n] \equiv .y_1 = x_1 \wedge ... \wedge y_n = x_n)$$
$$\tau_n \stackrel{def}{=} \iota y_n \exists y_1,...y_{n-1} \forall x_1,...x_n (T[x_1,...x_n] \equiv .y_1 = x_1 \wedge ... \wedge y_n = x_n)$$

That is, assuming that our theory is uniquely realized, once we write down the postulate of the theory and we derive the realization formula, we can derive an explicit definition for each of the theoretical terms in the theory. As Lewis claims:

> This is what I have called functional definition. The T-terms have been defined as the occupants of the causal roles specified by the theory T; as *the* entities, whatever those may be, that bear certain causal relations to one another and to the referents of the O-terms. (Lewis, 1972, p. 255)

Thus, this is a formal way to functionally characterize the theoretical entities postulated by a certain theory. But suppose now that a second theory T^* is introduced. This will be our 'bottom' theory. T^* introduces a new set of theoretical terms, which we can call O^*-terms. O^*-terms are either T^*-terms or O-terms. Suppose further that:

$$T^* \vdash T[\rho_1...\rho_n]$$

where $\rho_1...\rho_n$ are O^*-terms, introduced independently from the terms $\tau_1,...\tau_n$. $T[\rho_1...\rho_n]$ is called the *weak reduction premise* for T, and it does not contain T-terms. It says that T is realized by a n-tuple of entities $\rho_1...\rho_n$. Thus, T is realized by a n-tuple of entities expressed in the vocabulary of the new theory. Now, Lewis points out that the postulate $T(\tau_1,...\tau_n)$ can be derived from the weak reduction premise together with some bridge laws of the following form, which are usually taken as separate empirical hypotheses:

[7] Unique realisability seems *prima facie* unattainable in the face of multiple realisability, which is allegedly very widespread. For instance, pain seems to be multiply realised by human brains, dog brains, and so on. To avoid the challenge and ensure unique realisability, Lewis (1970, 1972) argues that we should focus on domain-relative functional roles. E.g. we should functionally reduce human-pain to human-brain states, etc. Thus we secure unique realisability by making theory T and its terms specific enough.

$$\rho_1 = \tau_1, \ldots, \rho_n = \tau_n$$

Alternatively, the bridge laws can be derived from T^* alone. In the case in which T is uniquely realized by a n-tuple of entities named by $\rho_1 ... \rho_n$, we can accept the following sentence, which we can call the *strong reduction premise* for T:

$$\forall x_1, ... x_n (T[x_1, ... x_n] \equiv . \rho_1 = x_1 \wedge ... \wedge \rho_n = x_n)$$

This sentence logically implies the following definitions, which are O^*-sentences and can be therefore theorems of T^*:

$$\rho_1 = \iota y_1 \exists y_2, ... y_n \forall x_1, ... x_n (T[x_1, ... x_n] \equiv . y_1 = x_1 \wedge ... \wedge y_n = x_n)$$
$$\rho_n = \iota y_n \exists y_1, ... y_{n-1} \forall x_1, ... x_n (T[x_1, ... x_n] \equiv . y_1 = x_1 \wedge ... \wedge y_n = x_n)$$

Which entails the theoretical identifications $\rho_1 = \tau_1, \ldots, \rho_n = \tau_n$ by transitivity of identity. That is, the strong reduction premise entails the theoretical identifications by itself. We thus have bridge laws in form of identities between the two theories. Thanks to functionalism and functional identifications, these bridge laws are directly deduced from the reducing theory.

Having introduced the core of the Lewisian framework, let's draw some further considerations. To begin with, we can see how this account exemplifies functional reductionism. First, it is a form of functionalism, since the 'Ramseyfication' of the two theories is explicitly used to formulate functional definitions of the entities described by the theories. Second, it is a reductionist account. The Lewisian framework sets out inter-theoretic reduction in the Nagelian sense, since the upper theory is taken to be derivable from the bottom theory, with the advantage of having bridge laws as deduced and not postulated, as stressed by Butterfield and Gomes (2020a). These bridge laws have the special form of identity statements, as they follow from the identifications of the functional profiles given by functionalism.[8] Moreover, the account can accommodate the revised version of Nagelian reduction, as we can take as T the corrected version of the original theory from the outset.

Furthermore, as you can notice, this form of functional reductionism is spelled out in terms of theories. However, in Lewis' account, this functional reductionism about theories, which leads to identity relations between theoretical terms, is meant to be a way to ensure functional reduction about ontology as well. Lewis makes this clear in several places, for instance in the passage quoted above, where

[8] However, even though they are bridge laws formulated as identities, the account does not run into multiple realisability objections, due to the reason discussed in ft. 7.

he stated that the T-terms refer to "the entities, whatever those may be, that bear certain causal relations to one another and to the referents of the *O*-terms." (Lewis, 1972, p. 255).[9] The passage from theory to ontology is indeed straightforward. On the assumption that the theoretical terms refer to actual entities, the theoretical functionalisation is just a means to codify in a scientifically accurate way the causal roles played by the worldly entities referred to by the theoretical terms. Thus, functional reduction of theoretical terms is a guide to functional reduction of entities. That is, once we functionally define a theoretical term in the upper theory and we find some other theoretical term in the bottom theory with the same role, we can infer that there is a bottom entity (referred by the term ρ_i) to which the upper entity (denoted by τ_i) is reduced to.

This scientific realist and ontologically-laden reading of functional reductionism is not peculiar to Lewis, but rather explicitly underlies most of the cases in which the view is applied. As such, we can take it as an integral part of the view. In the philosophy of mind (cf. Kim (1998)) the reduction of folk-psychology to physiology is used to functionally reduce mental states to brain states; in Lam and Wüthrich (2018, 2020) theoretical reduction of general relativity to quantum gravity backs the functional reduction of spacetime to non-spatiotemporal structures; in Albert (2015) and Lorenzetti (2022) theoretical reduction between laws of quantum mechanics and classical laws is used to argue for the functional reduction of three-dimensional entities to quantum wavefunctions; finally a realist stance is supported by Butterfield and Gomes (2020b) in the recovery of time from geometrodynamics.[10]

We can thus notice that, given that the Lewisian model delivers (deduced) bridge laws in form of identities between the theoretical terms, then if we give an ontological interpretation of those terms it follows that functional reduction entails (type) identity relations between the elements of the two ontologies.[11] That is, once we functionalise a theory and then find out another theory whose entities can

[9]Lewis (1970, 1972) is explicit about this: for him, theoretical terms like 'electron' are meant to refer to actual entities, as he clearly wants to maintain a scientific realist stance.

[10]This widespread ontological reading of functional reduction is not surprising. In fact, one crucial reason why functional reductionism is defended is that this is an approach to reduction which allows for a non-eliminativist position about the higher-level reduced entities: we can be realists about the reduced entity as far as we have another entity that realises the functional role of the former. In this sense, it is an approach to theoretical reduction which bears a clear and strong link with ontological reduction.

[11]It might naturally be the case that some theoretical terms are not ontologically interpreted and thus this implication does not hold. However, we are interested here in those cases in which this happens, like in the examples mentioned above.

realize the former theory, we are committed to drawing theoretical identifications across the two theories. If an entity belonging to the bottom theory plays the role that we associated with another entity within the top theory, those two entities are identical. This is the feature the puzzle of identity revolves around and that will be crucial for the coming sections. Before that, however, I introduce a more concrete example of functional reduction to see how the identification works.

2.2 An Example of Functional Reduction

I present here an example of functional reduction in physics, concerning classical and quantum systems, drawing on Lorenzetti (2022). This is a simple instance of functional reduction, but it is nevertheless a realistic case study and it allows us to see more closely how functional reductionism works in the Lewisian model, and how it leads to identity relations. The scheme followed here would be the same for more complex cases, although the details would be less tractable. For example, the model could be similarly applied to cases such as the functional reduction of spacetime structures to spin networks in loop quantum gravity (cf. Lam and Wüthrich (2018), sect. 5) and the functional reduction of thermodynamic entropy to Gibbsian entropy in the context of thermodynamics and statistical mechanics (cf. Robertson (2020)).

Our example concerns the functional reduction of a single-particle classical system to a single-particle quantum system.[12] Take first a quantum system associated with a wavefunction ψ, subject to a potential $V(x)$. The Schrödinger equation for the system is:

$$i\hbar \frac{\partial \psi}{\partial t} = \hat{H}\psi, \tag{1}$$

where $\hat{H}$ is the self-adjoint Hamiltonian: $\hat{H} = \dfrac{\hat{p}^2}{2m} + V(\hat{x})$. Now say that we have an isolated and localised wavepacket defined over configuration space. Its position, according to Ehrenfest's theorem, can be said to evolve in this way:[13]

$$\frac{d}{dt}\langle \hat{x} \rangle = \frac{\langle \hat{p} \rangle}{m} \tag{2}$$

[12]For instance, you can take the latter to be the quantum system denoted by the quantum wavefunction of the Hydrogen atom.

[13]Ehrenfest's theorem says that, for a generic operator $\hat{Q}$, with associated expected value $\langle \hat{Q} \rangle$, the time evolution of $\langle \hat{Q} \rangle$ can be stated as: $\dfrac{d}{dt}\langle \hat{Q} \rangle = \dfrac{i}{\hbar}\langle [\hat{H}, \hat{Q}] \rangle + \langle \dfrac{\partial \hat{Q}}{\partial t} \rangle$.

Similarly, for the momentum operator:

$$\frac{d}{dt}\langle\hat{p}\rangle = -\langle\frac{\partial V(\hat{x})}{\partial x}\rangle \tag{3}$$

If we assume that $\langle\frac{\partial V(\hat{x})}{\partial x}\rangle$ is equal to $\frac{\partial V(\langle\hat{x}\rangle)}{\partial x}$ – an assumption that is justified here by the fact that this is a localised wavepacket – then the expectation values of the position and the momentum evolve like the classical position and momentum, and thus (3) is equivalent to Newton's second law. In fact, we can write:

$$m\frac{d^2\langle\hat{x}\rangle}{dt^2} = \frac{d\langle\hat{p}\rangle}{dt} = -\frac{\partial V(\langle\hat{x}\rangle)}{\partial x}, \tag{4}$$

which is, for narrowly localised wavepackets, equivalent to a high approximation to:

$$F = m\frac{d^2x}{dt^2} = \frac{dp}{dt} = -\frac{dV(x)}{dx} \tag{5}$$

This means that, within the quantum mechanical picture, the centre of the localised wavepacket has a trajectory that is identical, up to a very high approximation, to the trajectory of a point particle of mass *m* within classical mechanics (in the Hamiltonian formulation). Thus, the trajectory of the wavepacket can be practically considered as a solution to the classical dynamic equation for a classical particle.

This leads to the conclusion that, up to a high approximation, a localised wavepacket can behave as a classical point particle. For the functional reductionist this is all we need. If all it takes to be a classical point-particle is to behave according to Newton's law for the evolution of a point-particle, then we have just recovered a classical particle from the evolution of a wavepacket.[14] Following the Lewisian approach, functional reduction would proceed by functionally defining the concept of being a 'classical point-particle' in terms of its role in classical mechanics – as it is expressed by the laws of the theory properly restated in the form of a Ramsey sentence – and then by showing that such behaviour is realised by a 'highly localised one-particle quantum system'. In such a case, the classical system can be functionally reduced to the quantum system, provided the appropriate conditions. If we adopt a scientific realist stance to functional reduction, as discussed above, this entails that the localised quantum system turns out to be

[14] Within classical mechanics every quantity is fixed by the position and the momentum, so here we have really obtained a full-fledged classical particle.

identical to the single classical point-particle, in the sense that we talk about the *same* entity.

3 The Puzzle of Identity

This section introduces a challenge to functional reductionism, which I call 'the puzzle of identity'. The issue stems from the fact that functional reductionism embeds a 'levelled' and asymmetrical picture of reality, which clashes *prima facie* with the identity relations described in Section 2.

Start by considering the functionalist aspect of the functional reductionist framework. The first step of the functionalist account is the functionalisation of a certain property or entity. The second step is to find a realiser for that functional role. The reductionist aspect of functional reductionism then follows from how this functionalisation procedure formally works. The functional definition is first picked out from a top theory and then the functional realiser is found within the ontology of the bottom theory. In this way, the ontology of the upper theory is reduced to the ontology of the bottom theory, in the right context. Thus, not only the account makes a clear distinction between a bottom and an upper level, but it seems to imply an asymmetrical relation between the two.[15] This is a feature that can be found in the general context of reductionism as well, as stressed by van Riel and van Gulick (2019, p. 1), given that, when an entity x is said to be reduced to an entity y, "then y is in a sense *prior to* x, is *more basic than* x, is such that x *fully depends* upon it or is *constituted by* it".

This levelled and asymmetrical conception of reality can indeed be found in those situations in which functional reductionism is applied in practice. For instance, if we functionally reduce classical systems to quantum systems (as we did in our simple example above and as it is done by Albert (2013, 2015) and Lorenzetti (2022)), or spacetime to non-spatiotemporal structures (cf. Lam and Wüthrich (2018)), the assumption would be that the former kind of entities is functionally reduced to the latter *and not the other way round*. The same would hold for the relation between thermodynamic quantities and statistical mechanical ones, as in the functional reductionist account proposed by Robertson (2020), and within the cases considered by Butterfield and Gomes (2020b), and also in the

[15] It might be the case that this picture does not hold in the way for some possible cases of reduction where functional reduction has not been discussed, e.g. intra-level examples reductions different from the cases discussed here (cf. Nickles (1973)). I leave this open for future discussion and focus here on the extant functional reductionist accounts.

philosophy of mind, where mental states are functionally reduced to brain states (cf. Lewis (1972) and Kim (1998)).

However, here comes the challenge. In fact, as we have seen, the functional reductionist account presented entails identity. And identity relations are, of course, *symmetrical*. We can call this issue 'the puzzle of identity': how can reduced and reducing entities be asymmetrically related if they are identical? For instance, how can we combine the asymmetry implicit in the intuition that it is the classical system that is functionally reduced to the quantum one with the fact that the two are the same system?

This puzzle has been firstly raised by van Riel (2013) as related to reductionism, but it has received little attention, and it is unexplored in the context of functional reductionism – where bridge laws, and thus identity relations, follow deductively from the functionalisation process. Notice that, due to this fact, the tension is more pressing within functional reductionism than it is for general reductionism. In fact, whereas a reductionist can simply avoid the challenge by appealing to a reductionist account that is not formulated in terms of identity, the functional reductionist is necessarily committed to the identity relations, since those follows from the way in which reduction is obtained within the account, i.e. via functionalism, as shown before.

One way for the functional reductionist to dissolve the tension would be to reject the claim that reductionism requires any form of asymmetry. In contrast with this counter-intuitive move, the aim of the next section is to dissolve the tension while also vindicating the asymmetrical nature of functional reduction, by moving the asymmetry from the ontological to the descriptive level. In other words, there is no tension because symmetry and asymmetry operate on different domains.

4 Reconciling Identity and Asymmetry

This section presents a strategy to deal with the the puzzle of identity. To do so, it appeals to List's (2019) systematic framework of *systems of levels*.[16] Recall that, since functional reduction entails identity, the entities belonging to the top theory (the realised entities) and those entities belonging to the bottom one (the realisers) do not occupy distinct levels within a hierarchy of levels, ontologically speaking. However, the ontological notion of levels at play here is not the only available

[16]Cf. also Dewar et al. (2019).

conception of levels. Most importantly, one can distinguish between *ontological* and (more fine-grained) *descriptive* levels. List introduces a formal account of both notions, showing how systems of descriptive levels can be systematized. This section presents List's framework for levels of description, and argues that this notion can help us to solve the riddle of identity and to make room for (non-ontological) asymmetry within functional reductionism.

Before discussing levels of description, let's see how List (2019, p. 854) defines the generic notion of systems of levels:

> A *system of levels* is a pair $\langle \mathscr{L}, \mathscr{S} \rangle$ defined as follows:
>
> - $\mathscr{L}$ is a class of objects called *levels*, and
> - $\mathscr{S}$ is a class of mappings between levels, called *supervenience mappings*, where each such mapping σ has a *source level* L and a target level L' and is denoted by $\sigma : L \to L'$

Such that the following conditions hold:

> (S1) If $\mathscr{S}$ contains $\sigma : L \to L'$ and $\sigma' : L' \to L''$, then it also contains the composite mapping $\sigma \bullet \sigma' : L \to L''$;
>
> (S2) For each level L, there is an *identity mapping* $\mathbf{1}_L : L \to L$ in $\mathscr{S}$, such that, for every mapping $\sigma : L \to L'$, we have $\mathbf{1}_{L'} \bullet \sigma = \sigma = \sigma \bullet \mathbf{1}_L$;
>
> (S3) For any pair of levels L and L', there is at most one mapping from L to L' in $\mathscr{S}$.

When $\mathscr{S}$ contains the mapping $\sigma : L \to L'$, the level L' can be said to be *supervenient* (or dependent on, determined by, necessitated by) on the level L, and thus L' is the *higher* level, while L is the *lower* level. Also, supervenience is taken here to have its usual meaning, i.e. a change in L' is impossible without any change in L.

Now that we have introduced the generic structure for a system of levels – which was needed to characterize the class $\mathscr{S}$ of supervenience mappings – we can move to the more particular framework of levels of description. Its purpose is to give a model of levels which can account for the fact that different sciences describe the world in different ways, ranging over different levels of description. Following List (2019, p. 862), we introduce the notion of a *language*. We define a language $\mathbf{L}$ as a set of *sentences*, plus (i) a negation operator $\neg$ such that for every sentence $\phi \in \mathbf{L}$, there is $\neg\phi \in \mathbf{L}$; and (ii) a consistency criterion, according to which, once we have fixed some sets of sentences as consistent, the remaining sets[17] are classified as inconsistent. According to List, any language $\mathbf{L}$ introduces a corresponding ontology, i.e. "a minimally rich set of worlds Ω_L such that each world in Ω_L 'settles' everything that can be expressed in $\mathbf{L}$" (ibid.), where settling a sentence means to assign it a truth-value. By linking truth-conditions to the sentences, we are indeed committed to positing the ontology induced by $\mathbf{L}$. In sum, the set Ω_L represents all the possible ways the world could be according to $\mathbf{L}$.

At this point, we can finally introduce the notion we need. Call a *level of description* any pair of a language $\mathbf{L}$ and its corresponding set of worlds Ω_L. Then, a *system of levels of descriptions* $\langle \mathscr{L}, \mathscr{S} \rangle$ is defined as follows (p. 863):

- $\mathscr{L}$ is some non-empty class of levels of description, each of which is a pair $\langle \mathbf{L}, \Omega_L \rangle$;
- $\mathscr{S}$ is some class of surjective functions of the form $\sigma : \Omega_L \rightarrow \Omega_{L'}$, where $\langle \mathbf{L}, \Omega_L \rangle$ and $\langle \mathbf{L'}, \Omega_{L'} \rangle$ are levels of description in $\mathscr{L}$, such that $\mathscr{S}$ satisfies (S1), (S2) and (S3).

Now, one can take $\mathscr{L}$ to contain levels of description corresponding to any science, from physics to chemistry. Any such level is going to embed a pair of a level-specific language and the corresponding set of induced (level-specific) worlds. For example, using the model presented here one can argue that the chemistry level of description is determined by the physical level of description. More precisely, notice that the determination relation between the levels does not hold directly, but holds in virtue of the supervenience mapping instantiated between the induced ontologies that respectively constitute the two levels of description. But, there is also something more. In fact, levels of description are more fine grained than ontological levels, i.e. they encode more information. Different languages, and thus different levels of description, can entail the same system of ontological levels. As List crucially remarks, given the framework we have just introduced, "it should be no surprise that different languages can in principle be used to describe the same sets of level-specific worlds, while describing them differently" (ibid.). Therefore, it could be the case that the same ontology, the same ontological level, turns out to be described by different languages, and so by distinct (perhaps supervenient) levels of description.

[17] With the exclusion of the subsets of the consistent sets.

It is now time to take stock of what we have seen so far, and make clear how List's framework can help us with functional reductionism. To recap the problem at stake, identities between realisers and realised entities are at odds with the asymmetry between the two ontologies. However, we can break this impasse by appealing to levels of description. That is, even if functional reduction entails that there is only one ontological level (and ontological symmetry), there is still a sense in which we can say that there are *two* distinct levels of description in place, that can accommodate a kind of asymmetry.

Consider the following quote. Butterfield and Gomes (2020a, p. 4), in their introduction to functional reduction, highlight the fact that, within functionalism:

> A single entity (extension) is picked out in two independent ways:
>
> (a) as the unique occupant of a functional role extracted from the first [top] theory, and
>
> (b) as specified by the second [bottom] theory.

This is the functional model that we have seen in Section 2 and which entails identity in the form of co-extensionality. The same extension is picked out in two different ways, i.e. by the top and by the bottom theory. However, let's now pay attention to *how* that extension is picked out by those distinct theories. Recall that – in the second section – the top theory was first introduced using theoretical terms (T-terms) and other/observational terms (O-terms). Then, after the "functionalization" – i.e. after the application of the Ramsey sentence – the same theory was expressed only via O-terms. At that point, we constructed explicit (functional) definitions for the T-terms $\tau_1, ... \tau_n$. At the same time, we introduced also a bottom theory, which was supposed to reduce the top theory. The bottom theory was embedded with a new vocabulary containing new theoretical terms (T^*-terms). The terms in which the theory was expressed ($\rho_1 ... \rho_n$) were called O^*-terms, i.e. either T^*-terms or O-terms. Then, we showed that we can build bridge laws $\rho_1 = \tau_1, \ldots, \rho_n = \tau_n$ between the terms of the two theories. In this sense, the O^*-term ρ_i introduced by the bottom theory was shown to pick out the same entity which was picked out by the top-term τ_i which we previously functionally defined. This is the way in which, as Butterfield and Gomes remark, the same entity can be specified both by the bottom theory (using the theoretical terms of that theory) and by the top theory (via functional definitions).

At this point, notice that the two theories were couched in different languages, one using sentences containing T-terms and O-terms and the other using O^*-sentences (containing T^*-terms and O-terms). In the end, the entities postulated by the top theory are co-extensional with entities postulated by the bottom theory, but what it is important here is that those entities are independently introduced by two distinct theories expressed in two different languages. If we now recall List's notion of levels of description, we can say that each of those theories corresponds to a different pair $\langle \mathbf{L}, \Omega_L \rangle$, which denotes a level of description. The language $\mathbf{L}_T$ of the top theory induces a corresponding ontology, and the language $\mathbf{L}_B$ induces a different ontology. Then, it turns out that some of the entities in those ontologies are co-extensional. Yet, through the notion of system of levels of description, we can say that one level (the pair $\langle \mathbf{L}_T, \Omega_{L_T} \rangle$) supervenes on the other level (the pair $\langle \mathbf{L}_B, \Omega_{L_B} \rangle$).

Before concluding, I stress here two important points, also to anticipate some possible questions. The first point I want to highlight concerns the supervenience relation. That is, to secure the ordering of the two levels, we just need the supervenience relation to hold between the induced ontologies Ω_L and $\Omega_{L'}$. This means that the supervenience mappings hold *between the possible worlds* and not within the actual world.

Second, a potential question that can be asked is the following. A central goal of the present approach is to account for the fact that, even though the functionally reduced entities turn out to be identical with their realisers, the two entities are significantly different in certain respects. For example, they may be different epistemologically-wise. Thus, one could wonder why cannot we appeal to the old extension-intension distinction, instead of the complex formal machinery presented here. That is, we could maintain that the reduced and reducing entities are co-extensive and identical, but have a different intension. In other words, reduction generates intensional contexts, in which expressions like 'a reduces to b' do not allow for substitutions *salva veritate* of co-referential terms to 'a' or 'b'. This would resemble the classic Kripkean account of the Hesperus/Phosphorus case, and is roughly the strategy endorsed by van Riel (2013), in his response to a challenge analogous to the puzzle of identity. However, the problem with this strategy is that – while the intension/extension distinction accounts for the fact that the reduced and the reducing are somehow different – this approach is unable to account for the asymmetry and the hierarchical ordering between the two. In contrast, a central advantage of the proposal defended here is to make room for asymmetry. This is why List's specific framework is particularly suitable for our task, as suggested in the introduction.

To conclude, let's thus sum up the intuition behind the strategy proposed here to avoid the puzzle of identity. When the ontology of the bottom theory (e.g. quantum systems) plays the right roles, then the entities described by the upper theory (e.g. classical systems) come into play. However, these are not new ontological posits. It's just the fundamental/bottom ontology behaving in a certain way. This ontology, in the right context, can thus be expressed in terms of the upper theory. This is just a redescription of the same entities. However, since levels of description give us a way to hierarchically order different descriptions, we can maintain that the two levels of description are not on a par. In yet other words; ontologically speaking, the entities which are functionally reduced are straightforwardly identical, since they are co-extensive. In this respect, they belong to the same ontological level. On the other hand, those entities are originally picked out by different theories, which introduce them via different languages, at different levels of description – which are hierarchically ordered via supervenience mappings. Thus, even within the Lewisian functional reductionist model, there is still room for saying that reduction embeds asymmetrical relations. It is not ontological asymmetry, but asymmetry of description.

5 Conclusion

The Lewisian account of functional reduction, i.e. the main model of the view available in the literature, leads to an evident tension, which becomes even more apparent when we apply the view to those cases in which functional reductionism is employed, like the functional reduction of classical systems to quantum ones. That is, how can we make sense of the asymmetry underlying functional reduction given that the account entails that the functionally reduced entities are identical to their realisers?

The paper proposes to distinguish between ontological levels and levels of description. While we acknowledge that the functional reductionist account gives us ontological identity, we move the asymmetry to the levels of description. We do not simply claim that the same entity can be described in different ways, but rather we employ List's formal account to show that we can build a hierarchy between the different descriptions, and it is this hierarchy which satisfies the asymmetry desideratum.

This original solution will be of interest for all the specific debates in which functional reduction is employed, as it dissolves a potential lingering tension. We have been mainly focused here on the reduction between physical theories and on the applications of functional reduction in science, but our strategy can be carried over to the philosophy of mind since the Lewisian approach can be applied to that context as well, as stressed along the way in several places. Finally, I suggest that the present discussion can be of interest also beyond the debate on functional reduction. In fact, this strategy can be plausibly applied to any Nagelian account of reduction in which the bridge laws have the form of identity statements since nothing within the discussion of Section 4 essentially relied on details about functional reduction.

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