COUNTING 3D-SPACES
Classicality and probability in standard and many-worlds quantum mechanics from quantum-gravitational background-freedom

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I explain that background freedom in quantum gravity automatically leads to a dissociation of the quantum state into states having a classical 3d-space. That is, interference is not completely well-defined for states with different 3d-space geometries, even if their linear combination is.

The dissociation into 3d-space geometries still allows for interference at small scales, but precludes it at macro scales. It grants the possibility of classical-looking macroscopic objects, including measuring devices. Counting the 3d-space geometries automatically gives the Born rule.

But the wavefunction collapse turns out to be even more ad-hoc. Fortunately, the dissociation entails a kind of absolute decoherence, making the wavefunction collapse unnecessary. This naturally leads to a new version of the many-worlds interpretation, while solving its major problems:

1) the classical-3d-space states form an absolute preferred basis,
2) at any instant, the resulting branches look like classical worlds, with objects in the 3d-space,
3) the 3d-space geometries converge at the Big-Bang, favoring branching towards the future,
4) macro-branches stop interfering, even though micro-branches can interfere,
5) the coefficients $\Psi[\gamma, \phi]$ become real by absorbing the complex phases in the global U(1) gauge,
6) the ontology is a state vector uniquely dissociable into many gauged classical-3d-space states, each of them counting as a world by having local beables (the classical fields),
7) the density of the classical-3d-space states automatically obeys the Born rule.

Keywords: Everett's many-worlds interpretation; Born rule; quantum gravity; background-independence; many-spacetimes interpretation.

I. INTRODUCTION

I show that background free approaches to quantum gravity prevent most quantum state vectors from having physically meaningful superpositions. Interference effects require a way to relate the positions in space among different state vectors, but background freedom limits this possibility. Linear combinations exist mathematically, but interference effects are suppressed in most situations.

This leads to a new explanation of the emergence of classicality at the macro level, and to a natural derivation of the Born rule by counting states with definite classical 3d-space. The resulting approach to understand quantum mechanics works less naturally with the wavefunction collapse, but very well with the many-worlds interpretation, solving some of its main problems.

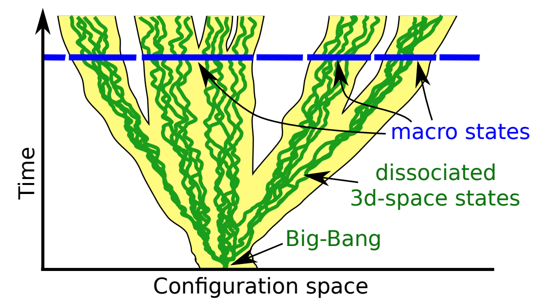

FIG. 1. **Wavefunctional dissociation due to quantum-gravitational background freedom.** The micro-states (in green) are 3d-space states. They are very similar at the Big-Bang, then background freedom makes them dissociate and form a branching structure like in the many-worlds interpretation. The dissociation is reversible at micro scales, allowing interference, but it becomes irreversible when it manifests at macro scales. The branching structure (in yellow) corresponds to the macro-states (in blue). Counting the 3d-space states for each macro-state or branch gives the Born rule.

In Sec. §II I sketch the generic features of wavefunctional formulations of background-free quantum gravity. This leads to the notion of classical 3d-space states, having a definite classical 3d-space (or other structure assumed to be more fundamental than the 3d manifold).

In Sec. §III I explain how background freedom makes the state vector dissociate into classical 3d-space states, by limiting their ability to interfere.

In Sec. §IV I show how counting the 3d-space states into which the state vector dissociates gives the Born rule. Each 3d-space state either is absent from the wavefunctional, or it appears in it with equal amplitude but varying density (see Fig. 2). The density is real, since the complex phases are absorbed into the gauge of the classical fields defining the 3d-space states.

In Sec. §V I argue that the 3d-space states approach works less well with the collapse postulate, but it works naturally with the many-worlds interpretation, resulting in a version of it named here the *many-spacetimes interpretation* of quantum mechanics.

In Sec. §VI I explain how the many-spacetimes interpretation solves some of the main problems of the many-worlds interpretation (Fig. 1). These include the existence of a preferred basis, the emergence of quasi-classical macro worlds, the existence of familiar, classical-looking objects in the 3d-space, the time-asymmetry of the branching structure, probabilities by counting states, the appearance of complex numbers in quantum mechanics, and the ontology, including the local beables, which justify counting each 3d-space state as a world.

Sec. §VII concludes the article with a discussion.

II. 3D-SPACE STATES IN QUANTUM GRAVITY

A. Classical 3d-space states

We do not have a yet final theory of quantum gravity, and even less so one that includes the other fields. But I will assume that such a theory is possible.

Many of the various currently known approaches to quantum gravity admit wavefunctional formulations.

The Wheeler-de Witt equation

$$\widehat{\mathsf{H}}\widetilde{\Psi} = 0 \tag{1}$$

involves a wavefunctional $\widetilde{\Psi} = \widetilde{\Psi}[\gamma_{ab}]$ on the space $\mathrm{Riem}(\Sigma)$ of all possible Riemannian geometries (Σ, γ_{ab}), where γ_{ab} is the intrinsic metric tensor on a three-dimensional manifold Σ, which is a time-dependent spacelike 3d-slice of the spacetime manifold $M = \Sigma \times \mathbb{R}$. Equation (1) was obtained [87] by quantizing the Hamiltonian formulation of classical general relativity by Arnowitt, Deser, and Misner (ADM) [71].

The quantization replaces the classical 3d metric γ_{ab} and its conjugate momentum π_γ^{cd} by operators,

$$\begin{cases} \widehat{\gamma}_{ab}(\boldsymbol{x})\Psi[\gamma_{ab}] = \gamma_{ab}(\boldsymbol{x})\Psi[\gamma_{ab}], \\ \widehat{\pi}_\gamma^{cd}(\boldsymbol{x})\Psi[\gamma_{ab}] = \dfrac{\hbar}{i}\dfrac{\delta\Psi[\gamma_{ab}]}{\delta\gamma_{cd}(\boldsymbol{x})}, \end{cases} \tag{2}$$

subject to the canonical commutation relations

$$\begin{cases} \left[\widehat{\gamma}_{ab}(\boldsymbol{x}), \widehat{\pi}_\gamma^{cd}(\boldsymbol{y})\right] = i\hbar\delta^c_{(a}\delta^d_{b)}(\boldsymbol{x}, \boldsymbol{y}), \\ \left[\widehat{\gamma}_{ab}(\boldsymbol{x}), \widehat{\gamma}_{cd}(\boldsymbol{y})\right] = \left[\widehat{\pi}_\gamma^{ab}(\boldsymbol{x}), \widehat{\pi}_\gamma^{cd}(\boldsymbol{y})\right] = 0, \end{cases} \tag{3}$$

where $\boldsymbol{x}, \boldsymbol{y} \in \Sigma$ and $\delta/\delta\gamma_{cd}(\boldsymbol{x})$ is the functional derivative.

The Wheeler-de Witt equation is a constraint equation, not an evolution equation, despite de Witt initially calling it the *Einstein-Schrödinger equation*. It is complemented by three other constraint equations that factor out the space diffeomorphisms. The wavefunctional $\widetilde{\Psi}$ is a timeless solution. A proposal to decode a dynamical solution, made by Page and Wootters [107], consists of interpreting it as a quantum system $|\psi(\tau)\rangle$ entangled with a clock $|\tau\rangle$, $\widetilde{\Psi} = \int_{\mathbb{R}} |\tau\rangle|\psi(\tau)\rangle d\tau$. This, and other proposals, were assessed critically in [94, 98]. According to Page and Wootters, we can consider that the state of the universe at the time t is represented by the vector $\Psi(t) := |t\rangle|\psi(t)\rangle$.

In the following we will assume the existence of a quantum theory of gravity based on time-dependent states.

Ashtekar's formalism [72] is similar, except that instead of γ and π_γ, its variables are an su(2) connection, whose conjugate variable is a densitized frame field on Σ. At the classical level the ADM formalism and the Ashtekar variables are equivalent. When quantized, the resulting operators satisfy commutation relations similar to (3) [96]. Its quantization was interpreted by Rovelli and Smolin in terms of *loop variables* [113].

We do not know with certainty that spacetime is continuous. Various approaches to quantum gravity are discrete, being based in general on structures that can be represented as graphs or hypergraphs that may have attached numbers at their vertices and (hyper-)edges. For example, in the *causal sets* approach [118], the vertices of the graph are events from spacetime, and oriented edges join pairs of events in causal relation, in the sense that the first event is in the past lightcone of the second one. The *Regge calculus* [111] is based on triangulations of spacetime into 4-simplices further approximated as flat. Distances are attached to the edges, and the spacetime curvature is concentrated at 2-faces, and expressed in terms of deficit angles *etc.* The *causal dynamical triangulation* approach is similar, but with fixed-length edges [99]. *Loop quantum gravity* can be formulated in terms of spin networks and spin foams. *Spin networks* are graphs with the edges labeled by half-integer numbers corresponding to irreducible representations of su(2) [73, 108, 114]. Two spin networks at different times are joined by a *spin foam*, a hypergraph used in the path integral formulation of loop quantum gravity.

All these graph or hypergraph structures are background-independent. They can also be seen as equivalence classes of (hyper)graphs embedded in the 3d-space Σ or in the spacetime M, where two such embedded structures are equivalent if they can be related by a diffeomorphism of the background manifold.

Many of these discrete approaches use Feynman's path integral quantization, but at the end a complex coefficient is associated to each classical 3d-space state, so it is likely that a wavefunctional representation always exists.

I will assume that quantum gravity can be described by a theory admitting a wavefunctional representation.

Let $\mathcal{C}_S$ be the set of classical 3d-space configurations. These may be the diffeomorphism equivalence classes of Riemannian geometries (Σ, γ), or more fundamental structures approximated by such geometries at low energies. For example, if quantum gravity is one of the discrete theories whose classical configurations are labeled (hyper)graphs, these will be the elements of $\mathcal{C}_S$.

While much of the following works well with both continuous and discrete spacetimes, we will see that continuous spacetimes have some advantages.

I will assume that there is a Schrödinger formulation of quantum gravity in terms of wavefunctionals over $\mathcal{C}_S$

endowed with a measure μ_S on $\mathcal{C}_S$. We assume that problems like the nonexistence of an infinite-dimensional Lebesgue measure are solved or avoided. The states of the universe are represented by unit vectors Ψ in the Hilbert space $\mathcal{H}_S$ spanned by states $|\gamma\rangle$, where $\gamma \in \mathcal{C}_S$ and $\Psi[\gamma] := \langle\gamma|\Psi\rangle$, with the Hermitian scalar product

$$\langle\Psi|\Psi'\rangle := \int_{\gamma\in\mathcal{C}_S} \Psi^*[\gamma]\Psi'[\gamma]\mathcal{D}\mu_S[\gamma]. \tag{4}$$

For matter quantum fields I will assume, like in the quantum field theory on the Minkowski spacetime, that there is a formulation in terms of wavefunctionals on the classical configuration space $\mathcal{C}_M$ of classical fields on Σ. The classical fields include bosonic fields, which commute, and fermionic fields, which are expressed using Grassmann numbers because they anticommute at equal times, see *e.g.* [95]. If additional variables are needed to specify how the 3d geometries integrate into 4d manifolds, for example the shift and lapse variables, I will assume that these are included as well in $\mathcal{C}_M$. If the 3d-space is a (hyper)graph $\gamma \in \mathcal{C}_S$, I will assume that matter can be described, in principle, by attaching various quantities or other structures to the elements of γ.

Let us summarize all of the above into the following

Assumption 1. The complete state of the universe is represented by a wavefunctional on a configuration space $\mathcal{C} = \mathcal{C}_S \times \mathcal{C}_M$, where $\mathcal{C}_S$ is the 3d-space configuration space and $\mathcal{C}_M$ is the matter configuration space. We assume a measure μ on $\mathcal{C}$, of the form $\mu[\gamma,\phi] = \mu_S[\gamma]\mu_M[\phi]$, where $(\gamma,\phi) \in \mathcal{C}$. Let the Hilbert space of such wavefunctionals be $\mathcal{H} \cong \mathcal{H}_S \otimes \mathcal{H}_M$, with a scalar product

$$\langle\Psi|\Psi'\rangle := \int_{(\gamma,\phi)\in\mathcal{C}} \Psi^*[\gamma,\phi]\Psi'[\gamma,\phi]\mathcal{D}\mu[\gamma,\phi]. \tag{5}$$

Definition 1. The states $|\gamma,\phi\rangle$ satisfying $\Psi[\gamma,\phi] = \langle\gamma,\phi|\Psi\rangle$, where γ represents the 3d-space and ϕ the matter fields, will be called 3*d-space states.*

B. 3d-space states are fundamental

Just because physicists first discovered classical physics, and later quantum theory, and formulated the latter by quantizing the former, it does not mean that quantum theory requires classical physics to exist. The universe is what it is, and it is fundamentally quantum.

However, the Hilbert space is too symmetric as it is, and without the existence of preferred structures that break its symmetry, there would be no relation between Hilbert space vectors and physical reality, or between Hermitian operators and physical observables. Physical properties cannot simply emerge from the abstract state vector, even if the Hamiltonian is known, because if they would, infinitely many entities with the very same properties, but able to represent completely different physical realities, would emerge as well [125, 128]. Therefore, the basis $(|\gamma,\phi\rangle)_{(\gamma,\phi)\in\mathcal{C}}$ of the Hilbert space $\mathcal{H}$ is special among the others, because of its physical meaning. This justifies

Assumption 2. The 3d-space states are fundamental, in the sense that, by their physical meaning, they are special among the other states represented by $\mathcal{H}$.

As explained earlier, these 3d-space states are not necessarily Riemannian geometries, they can be other structures approximated at low energies by such geometries. What is important is that they have a special physical meaning, in the same sense in which, in nonrelativistic quantum mechanics, the position operators and their eigenvectors have a special physical meaning compared to other operators or vectors in the Hilbert space.

C. Background freedom

To construct the configuration space $\mathcal{C}$, we eliminated the unphysical degrees of freedom due to diffeomorphisms and global gauge transformations. For example, two metric tensor fields on Σ may look different, but a coordinate transformation, which corresponds to a diffeomorphism of Σ, may be able to map them into one another, showing that they are isometric. For this reason, we took the equivalence class of metrics on Σ under diffeomorphisms.

Similarly if the 3d-space is a discrete structure like the ones that can be represented by graphs or hypergraphs from §II A, we took the configuration space consisting of such structures based on their internal relations, not as particular embeddings in a 3d manifold. But let us state this explicitly, since it will be central in the article:

Assumption 3. Our theory is background-free.

The case for background freedom was made for example by Smolin [117]. General relativity already shows that the structures have to be relational: we use coordinates, but they are not absolute, they are just ways to assign numbers to points in space or spacetime. The *hole argument* [106, 119] shows that taking the points of the underlying manifold as having an independent reality from the intrinsic relations introduced by the metric tensor leads to indeterminacy.

This is why many of the approaches to quantum gravity seem to require background freedom, or even have it built-in, including the formulation based on the Wheeler-de Witt equation (1), the discrete approaches based on (hyper)graphs discussed earlier, like causal sets, Regge calculus, causal dynamical triangulations, loop quantum gravity *etc.* For a discussion of background independence in *string theory* see Witten [137].

III. DISSOCIATION INTO CLASSICAL 3D-SPACE STATES

A. Background freedom and dissociation

In general, we make no difference between the concepts of linear combination and superposition, except maybe that a linear combination is understood as the mathematical expression of a superposition, which is a physical concept related to the position in the 3d-space and phenomena like interference. And they usually coincide.

In nonrelativistic quantum mechanics, any two wavefunctions can be superposed in the 3d-space, because the underlying geometry is the same, and the reference frames are the same. In the wavefunctional formulation of quantum field theory on Minkowski spacetime, the local information about the wavefunctional of a scalar field is obtained by using local operators at $\boldsymbol{x} \in \Sigma = \mathbb{R}^3$, definable in function of the operators $\widehat{\phi}(\boldsymbol{x})$ and $\widehat{\pi}_\phi(\boldsymbol{x})$ (to be rigorous, one uses operator-valued distributions, applied to a sequence of test functions that converge uniformly to the Dirac distribution $\delta_{\boldsymbol{x}}$).

In background-dependent theories of quantum gravity we can define local operators in a similar way, in function of the operators $\widehat{\gamma}(\boldsymbol{x})$ and $\widehat{\pi}_\gamma(\boldsymbol{x})$ from eq. (2), and $\widehat{\phi}_\alpha(\boldsymbol{x})$ and $\widehat{\pi}_{\phi_\alpha}(\boldsymbol{x})$ for each matter field ϕ_α, where α stands for the spin and the internal degrees of freedom.

But in background-free quantum gravity local operations on the 3d-space do not make sense for all states, and likewise superpositions, even if the linear combinations are always defined. If the theory is background-free, a difference appears when we apply local operators to linear combinations. Any local operator $\widehat{\mathsf{A}}(\boldsymbol{x})$ depends on $\boldsymbol{x}$, but background freedom prevents the matching of $\boldsymbol{x}$ for $|\gamma, \phi\rangle$ to $\boldsymbol{x}$ for $|\gamma', \phi'\rangle$, because in general $\gamma \neq \gamma'$. There is no definite correspondence between the points of Σ for $|\gamma, \phi\rangle$ and those of Σ for $|\gamma', \phi'\rangle$, because of background freedom. The situation is even more visible in background-free theories where (Σ, γ) is replaced by a labeled (hyper)graph.

If (Σ, γ) and (Σ, γ') are isometric, a correspondence between the points of Σ for $|\gamma, \phi\rangle$ and those of Σ for $|\gamma', \phi'\rangle$ exists, although it is not necessarily unique. Sometimes such a correspondence exists only between some open regions of Σ. So the dissociation is not always ensured, and we will see that this is important.

We arrived at the following:

Key observation 1. Background freedom implies the dissociation of the universal wavefunction into classical 3d-space states, because local operators and superpositions are not completely well-defined in the absence of a common background.

The dissociation is not necessarily complete, and various cases are captured in the following definition.

Definition 2. Two 3d-space states $|\gamma, \phi\rangle$ and $|\gamma', \phi'\rangle$ are *locally associable* if there exist two open subsets $U, U' \subseteq \Sigma$ and an isometry between (U, γ) and (U', γ'). In case that $U = U' = \Sigma$, they are *globally associable*.

Two 3d-space states are *dissociated* if they are not globally associable. They are *partially dissociated* if they are locally but not globally associable. They are *completely dissociated* if they are neither locally nor globally associable.

In the discrete case, in Definition 2, (local) isometries are replaced by (local) isomorphisms between the labeled (hyper)graphs γ and γ'.

As long as the dissociation is not complete, the 3d-space states can reassociate, at least partially. This allows quantum interference to exist at micro scales. This is the key to understanding why our quantum world looks quantum at small scales, and classical at macro scales.

B. Macro-states and classical micro-states

Macro-states correspond to equivalence classes of micro-states. There is a complete set of commuting projectors $(\widehat{\mathsf{P}}_\alpha)_{\alpha \in \mathcal{A}}$ on $\mathcal{H}$, so that $[\widehat{\mathsf{P}}_\alpha, \widehat{\mathsf{P}}_\beta] = 0$ for any $\alpha \neq \beta \in \mathcal{A}$, and $\bigoplus_{\alpha \in \mathcal{A}} \widehat{\mathsf{P}}_\alpha \mathcal{H} = \mathcal{H}$. Any *macro-state* is represented by a subspace of the form $\widehat{\mathsf{P}}_\alpha \mathcal{H}$. We will say that the states belonging to macro-states $\widehat{\mathsf{P}}_\alpha \mathcal{H}$ are *quasi-classical*.

Since the 3d-space states are classical, it makes sense to assume that they are also quasi-classical, *i.e.* every 3d-space state $|\gamma, \phi\rangle \in \widehat{\mathsf{P}}_\alpha \mathcal{H}$ for some α.

Assumption 4. All 3d-space states are quasi-classical.

If at a given time the state of the universe is a 3d-space state, it immediately evolves into a linear combination of 3d-space states. Dissociation and reassociation happen continuously. However, at the macro level, the state may remain quasi-classical for finite time intervals under unitary evolution. This accounts for the fact that macroscopic systems do not evolve all the time into linear combinations of macro-states like the Schrödinger cat, although it allows unitary evolution to lead to such linear combinations during quantum measurements.

IV. PROBABILITIES FROM COUNTING 3D-SPACE STATES

A. Taking dissociation seriously

Every vector $|\Psi\rangle$ from $\mathcal{H}$ has the form

$$|\Psi\rangle = \int_{(\gamma, \phi) \in \mathcal{C}} c_{\gamma, \phi} |\gamma, \phi\rangle \mathcal{D}\mu[\gamma, \phi], \tag{6}$$

where $c_{\gamma, \phi} = \Psi[\gamma, \phi] = \langle \gamma, \phi | \Psi \rangle$.

We may be tempted to simply proclaim the Born rule, that the probability density is

$$P[\gamma, \phi] = |c_{\gamma, \phi}|^2. \tag{7}$$

But let us resist this for a while, and explore the consequences of the dissociation. If we explore the consequences of a physical principle, we should do it in its own terms, and if the result contradicts the observations, we should drop the starting principle. So let us byte the bullet and see where the idea of dissociation leads. We will see that it leads to the Born rule, but in a natural way, not by fiat. The dissociation into classical 3d-space states suggests the following principle:

Principle 1. Each 3d-space state is either not present in $|\Psi(t)\rangle$, or it is present once (*i.e.* it cannot be "half-present", even if eq. (6) may suggest this possibility).

This may seem to contradict everything we know. However, we will get quantum theory back, with the familiar complex numbers, which will receive a geometric meaning in terms of a global gauge, and, since $\mathcal{C}$ is continuous, with the Born rule as we know it, but resulting from counting the 3d-space states.

B. Making the wavefunctional real

Background freedom implies that the quantum state dissociates automatically into 3d-space states, but since the coefficients $c_{\gamma,\phi}$ from eq. (6) are complex numbers, we need to understand their meaning.

First, while $|\gamma,\phi\rangle$ is a classical state, $c_{\gamma,\phi}|\gamma,\phi\rangle$ is not classical. Let us for the moment ignore γ, and consider that ϕ is a scalar field. In general, $c|\phi\rangle \neq |c\phi\rangle$. Even if $c \in \mathbb{R}$, if $c \neq 1$, $|c\phi\rangle$ represents a classical state $c\phi$ completely different from ϕ, so $|c\phi\rangle$ and $|\phi\rangle$ are orthogonal.

But if ϕ is an electrically charged field and $\varphi \in \mathbb{R}$, $e^{i\varphi}\phi$ represents a global gauge transformation of ϕ. The classical fields ϕ and $e^{i\varphi}\phi$ are physically the same. The state vectors $|\phi\rangle$ and $e^{i\varphi}|\phi\rangle$ are distinct, they differ by a phase factor, but they represent the same physical state. This suggests the following interpretation:

Key observation 2. If the matter fields admit an U(1) gauge symmetry, for any $\varphi \in \mathbb{R}$,

$$e^{i\varphi}|\gamma,\phi\rangle = |\gamma, e^{i\varphi}\phi\rangle. \tag{8}$$

This accounts for the fact that the physical equivalence of the classical fields ϕ and $e^{i\varphi}\phi$ corresponds to the physical equivalence of the state vectors $|\phi\rangle$ and $e^{i\varphi}|\phi\rangle$.

This approach works for fields admit an U(1) symmetry, like charged fields and spinor fields. Since the electromagnetic field can be put in a complex form, even the photon admits an U(1) symmetry [79].

Let us express the complex coefficients $c_{\gamma,\phi}$ from eq. (6) in the polar form

$$c_{\gamma,\phi} = r[\gamma,\phi]e^{i\varphi[\gamma,\phi]}, \tag{9}$$

with $r[\gamma,\phi] \geq 0$. Then, eq. (6) becomes

$$|\Psi\rangle = \int_{(\gamma,\phi)\in\mathcal{C}} r[\gamma,\phi]|\gamma, e^{i\varphi[\gamma,\phi]}\phi\rangle \mathcal{D}\mu[\gamma,\phi], \tag{10}$$

We see that, whenever a physical classical field contributes to $|\Psi\rangle$, it contributes only once, with a uniquely determined gauge $e^{i\varphi[\gamma,\phi]}$ and real coefficient $r[\gamma,\phi]$. As $|\Psi\rangle$ evolves in time, the gauge and $r[\gamma,\phi]$ can change.

It remains to explain the relation between $r[\gamma,\phi]$ and the probability density of the 3d-space states.

C. Emergence of the Born rule

Now that we have seen that gauge freedom allows the coefficients in the linear combination of 3d-space states to be real numbers, let us see what is their meaning and how it relates to probabilities.

I will assume that the configuration space $\mathcal{C}_S$ is continuous, so $\mathcal{C}$ is also continuous. This happens for example if Σ is a 3d manifold. I show that, under this assumption, the Born rule emerges by counting the 3d-space states. A more general derivation can be found in [127].

Let us choose all fields ϕ so that in eq. (9) $\varphi[\gamma,\phi] = 0$. We define $\xi := (\gamma,\phi)$.

First, we notice that a state vector of the form $|\Psi\rangle = \frac{1}{\sqrt{n}}\sum_{k=1}^{n}|\xi_k\rangle$, where $(|\xi_k\rangle)_{k\in\{1,\dots,n\}}$ are distinct basis vectors, leads to the Born rule. If $\widehat{\mathsf{P}}_\alpha$ is a macro projector and n_α basis vectors composing $|\Psi\rangle$ belong to $\widehat{\mathsf{P}}_\alpha\mathcal{H}$, then $\langle\Psi|\widehat{\mathsf{P}}_\alpha|\Psi\rangle = n_\alpha/n$. Therefore, the Born rule simply coincides with the usual counting rule "probability is the ratio of the number of favorable outcomes to the total number of possible outcomes". But only a small subset of the possible state vectors have this form, so this idea fails if the basis is discrete.

However, this idea works in the continuous case, since the basis vectors can be distributed with nonuniform density. More precisely, if $r[\xi] = r[\gamma,\phi]$ from eq. (10) is μ-measurable, we can define a new measure

$$\mathcal{D}\widetilde{\mu}[\xi] := r[\xi]\mathcal{D}\mu[\xi], \tag{11}$$

and obtain

$$|\Psi\rangle = \int_{\xi\in\mathcal{C}} |\xi\rangle \mathcal{D}\widetilde{\mu}[\xi]. \tag{12}$$

That's all.

At first sight, one may think eq. (12) cannot represent a normalized vector, so let us verify that it does:

$$\begin{aligned}
\langle\Psi|\Psi\rangle &= \int_{\xi\in\mathcal{C}} \langle\xi|\mathcal{D}\widetilde{\mu}[\xi] \int_{\xi'\in\mathcal{C}} |\xi'\rangle\mathcal{D}\widetilde{\mu}[\xi'] \\
&= \int_{\xi\in\mathcal{C}} \left(\int_{\xi'\in\mathcal{C}} \langle\xi|\xi'\rangle\mathcal{D}\widetilde{\mu}[\xi']\right)\mathcal{D}\widetilde{\mu}[\xi] \\
&= \int_{\xi\in\mathcal{C}} \left(\int_{\xi'\in\mathcal{C}} \langle\xi|\xi'\rangle r[\xi']\mathcal{D}\mu[\xi']\right)\mathcal{D}\widetilde{\mu}[\xi] \\
&= \int_{\xi\in\mathcal{C}} r[\xi]\mathcal{D}\widetilde{\mu}[\xi] = \int_{\xi\in\mathcal{C}} r^2[\xi]\mathcal{D}\mu[\xi] = 1.
\end{aligned} \tag{13}$$

Since $r[\xi]$ is μ-measurable, the measure $\widetilde{\mu}$ is absolutely continuous with respect to μ.

Now, consider a macro projector $\widehat{\mathsf{P}}_\alpha$ so that the macro-state $\widehat{\mathsf{P}}_\alpha\mathcal{H}$ is the closure of a subspace spanned by $(|\xi\rangle)_{\xi\in(C)_\alpha}$, where $\mathcal{C}_\alpha$ is μ-measurable. Then, from Assumption 4, we get

$$\langle\Psi|\widehat{\mathsf{P}}_\alpha|\Psi\rangle = \int_{\xi\in\mathcal{C}_\alpha} |\xi\rangle\mathcal{D}\widetilde{\mu}[\xi], \tag{14}$$

just like the Born rule says. Therefore, state counting gives the Born rule, in accord to Principle 1 (Fig. 2).

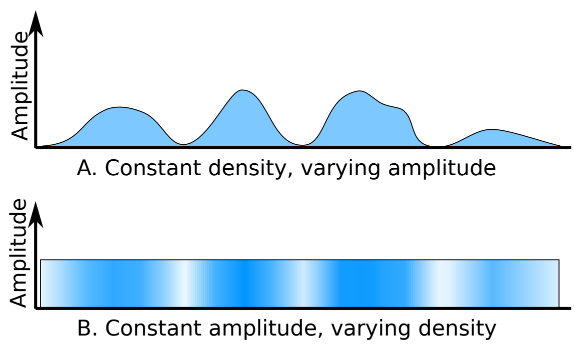

FIG. 2. **The Born rule from counting 3d-space states.** **A.** The usual interpretation of a wavefunction as a linear combination of basis state vectors of different lengths.
B. The interpretation of the wavefunction in terms of constant length basis state vectors, but with inhomogeneous density.

Key observation 3. If $\mathcal{C}_S$ is continuous, any state vector $|\Psi\rangle \in \mathcal{H}$ consists of mutually orthogonal 3d-space states whose density is $|\Psi[\gamma,\phi]|\mathcal{D}\mu[\gamma,\phi]$.

Therefore, the numbers from eq. (10) have a direct meaning: Principle 1 combined with the gauge freedom allows the interpretation of the states $|\Psi\rangle$ as consisting of 3d-space states that are either present or not. We obtained the Born rule from counting 3d-space states.

Remark 1. Note that the derivation of the Born rule from this Section is not limited to the case when the basis states are 3d-space states [127]. What is important is that the basis is continuous, and that the basis vectors belong to macro-states. In quantum field theory in the Schrödinger wavefunctional representation, one can use the classical field configurations to obtain the basis. In nonrelativistic quantum mechanics, one can use the classical positions of the $\mathbf{n}$ particles, which are represented by points in the configuration space $\mathbb{R}^{3\mathbf{n}}$, and this is consistent with the fact that ultimately every quantum measurement translates to a position measurement. But the 3d-space states have the advantage of dissociating in a natural way, and of including gravity. Moreover, the 3d-space states are the only ones consisting of local beables, which are γ and ϕ (see Sec. §VI G). This justifies counting these states to get the Born rule. □

V. COLLAPSE POSTULATE OR MANY-WORLDS?

Let us see how dissociation into 3d-space states works with quantum measurements, and whether it works better by assuming the collapse postulate or with the many-worlds interpretation.

A measuring device is a quasi-classical system. When interacting with the observed system, assumed to be microscopic in the sense that it is not directly observable, the combined system evolves into a linear combination of macroscopically distinct states. Each of these states contains the observed system in a different state, and the pointer of the measuring device indicating that state. So the Schrödinger equation predicts that two or more stories describing the measurement are simultaneously true. But we never observe such linear combinations: after the measurement, the pointer state is always in a definite macro-state.

QM Problem 1. Why can the state vector of the observed system be any linear combination at micro-scales, but not at macro-scale?

To resolve this problem, in standard quantum mechanics one invokes the *collapse postulate* [133], which simply states that quantum measurements suspend the Schrödinger evolution, so that from the linear combination we keep only the term that corresponds to only one of the possible pointer states, removing the others.

In doing this, standard quantum mechanics assumes, without explaining it, the pre-existence of measuring devices in quasi-classical states, but most quantum states are superpositions of quasi-classical states. So we have the following problem:

QM Problem 2. Why is the measuring device already in a quasi-classical state?

The collapse postulate purports to solve Problem 1 by assuming implicitly that Problem 2 is already solved. And, because of the collapse postulate, the Schrödinger equation is considered valid in some situations, but it is suspended in other situations.

There seems to be a double standard here. On one hand, linear combinations and entangled states appear and evolve in parallel as long as no observation is made, and the experiments are consistent with this. On the other hand, if we measure them, since we do not observe more parallel sets of outcomes simultaneously, we allow only one of the stories, and censor the other one, by appealing to the collapse postulate.

Let us see how measurements happen in the approach based on dissociation into 3d-space states proposed here.

Consider a measuring device assumed to be almost classical, having a locally well-defined 3d-space. Then, what enters in its range can be any state of the observed system, in any linear combination. Since the measuring device is localized, the instances in each 3d-space branch can be compared and collapse can be invoked. It may seem that the description of the measurement by using collapse became clearer.

But we still had to assume that Problem 2 is solved. And the collapse is still arbitrary, there is still no clear rule when it should be invoked. When no measurement is made, multiple 3d-space states are allowed to coexist, dissociate and associate in interference patterns in the wavefunctional. But when a measurement is made, only some of the 3d-space states seem to remain. Some linear combinations of 3d-space states seem to be "more equal" than others.

One may try to use the 3d-space states approach to solve Problems 1 and 2 at once, by reformulating the collapse postulate in the following way:

Tentative Postulate 1 (Alternative Collapse Postulate)**.** During the evolution of the system, the 3d-space states may become irreversibly dissociated into two or more sets of 3d-space states, determined by the macro projectors. Let us call these sets macro-branches. When this happens, only one of the macro-branches remains, and the others disappear.

This Tentative Postulate seems to provide a basis to explain macro systems, including measuring devices. If so, it can solve both Problems 1 and 2 at once.

But dissociation and reassociation happen all the time. Reassociation allows interference effects, but when dissociation is irreversible, these effects are suppressed automatically.

Remark 2. If we assume collapse and try to explain the Born rule by counting 3d-space states as in Sec. §IV, we will have to accept that the wavefunction consists of many micro-states that exist simultaneously, and part of them are eliminated by every collapse. But this would make quantum mechanics with the collapse postulate a strange version of the many-worlds interpretation, in which some of the micro-branches are removed with every collapse. On the other hand, the derivation of the Born rule from Sec. §IV works naturally with MWI. □

These remarks immediately prompt the following:

Key observation 4. Tentative Postulate 1 is unnecessary, because once the dissociation becomes irreversible, the macro-branches evolve independently and no longer interfere.

Therefore, since when dissociation becomes irreversible at macro scales the macro branches no longer interfere, the 3d-space states approach works more naturally with the many-worlds interpretation (MWI) rather than with the wavefunction collapse.

The key idea of MWI is to take the Schrödinger equation seriously, without introducing any ad-hoc rule that applies only to macro scales. This implies that all possible components of the total wavefunction continue to exist after the measurement, but thanks to decoherence, they no longer "see" each other. The linearity of the Schrödinger equation allows the macroscopically distinct states that result from a quantum measurement by unitary evolution to be independent, but in addition, they no longer interfere. The wavefunction branches so that the different branches occupy different regions in the configuration space. Interference is suppressed because the copy of any measuring device in one branch is unable to detect anything from another branch, so the branches no longer "know" about one another. And the branches become macroscopically distinct, in the sense that they correspond to projections of the state vector on different macro-states $\widehat{\mathsf{P}}_{\alpha_1}\mathcal{H},\ldots,\widehat{\mathsf{P}}_{\alpha_n}\mathcal{H}$.

Decoherence into macro-branches seems to explain the existence of measuring devices and solve the measurement problem without violating the Schrödinger equation by invoking an ad-hoc wavefunction collapse.

There are several problems that are not solved, at least not in a way that does not require a complete reinterpretation of well-established concepts like probabilities. They will be discussed in Sec. §VI, where I will propose that these problems are solved, or at least alleviated, by the dissociation into 3d-space states, which provides an absolute form of decoherence.

VI. THE MANY-SPACETIMES INTERPRETATION

We think that we are forced to suspend the Schrödinger equation as a result of measurements, because we observe only one of the stories that the Schrödinger equation describes as taking place in parallel. But could we observe more than one of these stories at once? The Schrödinger equation predicts that even the observers would be "multiplied", each of its instances participates in one of the stories and not in the others, of which they are oblivious. And the laws of physics are the same in all of these stories.

Everett noticed the perfect symmetry of the situation, and saw no reason to favor the story in which one gets an outcome against the competing stories. He proposed to trust the Schrödinger equation and accept that all stories continue to happen independently [88, 89]. Schrödinger himself proposed earlier something that he worried may "seem lunatic" along the same lines [75, 86, 116].

The result of Everett's realization is the *many-worlds interpretation* (MWI) of quantum mechanics. But there are still open questions in MWI. Various proposals were made to solve them, and some researchers think they are solved. Others think that they cannot be solved and MWI does not deserve to be taken seriously.

In this Section I argue that the 3d-space states approach solves some of these problems, or provides a more natural way to solve them. This leads to a variant of the many-worlds interpretation, which may be called "the many-3d-space states interpretation", but I will call it *the many-spacetimes interpretation* (MSTI).

A. Preferred basis: 3d-space states

Let us start with a problem whose solution is the key to solving other problems.

MWI Problem 1 (Preferred basis)**.** In what basis does the branching take place, so that the worlds appear classical at the macro level?

Presumably, this is solved by decoherence [100]. However, there has to be more to the preferred basis than that it simply "emerges". Otherwise, if a preferred basis emerges, either for the entire universe, or for a subsystem, infinitely many others emerge [125].

In nonrelativistic MWI, it is expected that the preferred basis is related to the positions in the configuration space. This would explain why branches no longer interfere – it is because they no longer overlap in the configuration space.

But the MSTI answer is different.

MSTI Answer 1 (Preferred basis)**.** The dissociation of the state vector automatically selects as the preferred basis the 3d-space states basis.

B. Macro world

Another important problem is the following

MWI Problem 2 (Macro world)**.** How does the classical-looking macroscopic world emerge from the wavefunction?

Often, Problem 2 is considered solved by decoherence [97, 100, 138], which appeared in the first place to solve it.

Without denying the importance of decoherence, the dissociation strengthens the idea, by introducing an absolute notion of decoherence.

MSTI Answer 2 (Macro world)**.** Each macro world corresponds to multiple classical 3d-space states that belong to the same macro-state, because they are not distinguishable at the macro level.

The 3d-space states gather together into macro states (Assumption4). Since each 3d-space state is also quasi-classical, and since they are not distinguished by the macro projectors, they can account for the macro world.

C. Classicality as classicality

Another problem is that the wavefunction is not defined on the 3d-space, but on the much larger configuration space. This disturbed Schrödinger [74], Lorentz ([110], p. 44), Einstein [90, 93], Heisenberg, Bohm [81] *etc*. This is true for the wavefunction of any state vector in the total Hilbert space. So even if MWI solves Problem 2, the following may remain:

MWI Problem 3 (Objects in space)**.** Given that the wavefunction is defined on the high-dimensional configuration space, how do familiar, classical-looking objects localized in space emerge from the wavefunction?

The wavefunction, being an element of a representation of the Galilei or the Poincaré group [136], is intrinsically associated to space or spacetime. Therefore, properly analyzed, it satisfies all expectations of standard geometric objects in space or spacetime [126]. Moreover, if one is not satisfied with this and wants the wavefunction to be expressed as classical-like fields in space or spacetime, this is also possible, albeit in an inaesthetic way that at least serves as a proof of concept [124]. But in the case of quantum gravity, the representation from [124] only works if the theory is background-dependent.

And even if the wavefunction is, in the sense of group theory or as fields, an object in space, it does not look like the familiar, classical-looking objects we see.

Maybe decoherence leads to branches that look like familiar, classical-looking objects localized in space. Wallace [135] thinks that the branches form patterns in the sense of Dennett [84], but are these patterns classical-looking enough?

Maudlin [101–103] and Norsen [105] think that Problem 3 is not solved, and that it is hard to solve it even if Problems 1 and 2 would be. They contrast this with the pilot-wave theory (PWT) [80], which includes, along with the wavefunction, point-particles at definite positions in space, and with the Ghirardi-Rimini-Weber (GRW) interpretation [91], where the wavefunction collapses around well-localized points in the configuration space, thereby appearing classical.

Their arguments can be seen as relying on the idea that the primitive ontologies of the PWT and GRW interpretation (especially in Bell's flash ontology [78]) are very similar to the classical ones. This similarity also seems to help solving the other problems of the PWT and GRW interpretations [1].

An important lesson that can be learned from their arguments is that classical physics is clearer, and so any interpretation of quantum mechanics that is closer to classical physics has an important advantage.

This suggests the following heuristic rule

Rule of Thumb 1. If a solution is considered to work without problems in classical physics, and if it can be applied to an interpretation of quantum mechanics, it should also be considered to work without problems in that interpretation of quantum mechanics.

We can see that the MSTI Answers 1 and 2 already align MWI to this Rule of Thumb, except for the multi-

[1] But for these interpretations to work, the wavefunction governing the motion of the particles in PWT and the probability of the spontaneous localization in the GRW interpretation has to be itself well localized around the points of the configuration space, so the MWI Problem 3 applies to these interpretations as well.

plicity of the worlds, which is not present in the classical theories.

It is therefore desirable to have a solution of Problem 3 along the Rule of Thumb 1 as in the PWT and GRW interpretations. Background freedom automatically makes this possible.

MSTI Answer 3 (Objects in space)**.** The 3d-space states consist of classical fields on the 3d-space.

What can be more classical than the classical itself?

D. Branching asymmetry from Big-Bang symmetry

Another problem is the following

MWI Problem 4 (Branching asymmetry)**.** Why is the branching happening only towards the future, and why do the branches remain separated?

This is also often claimed to be solved by decoherence, but since the Schrödinger equation is time-symmetric, without very fine-tuned initial conditions of the universe, decoherence would equally predict branching towards the past.

In the standard framework of the many-worlds interpretation, Wallace acknowledged this problem, analyzed it, and concluded that the branching asymmetry is correlated to the *thermodynamic arrow of time* [135]. But we do not have an explanation for the thermodynamic arrow of time either, although the second law of thermodynamics is a well-established fact.

The dissociation into 3d-space states allows us to make some progress, by relating branching asymmetry with the *cosmological arrow of time*. The cosmological arrow of time points from the Big-Bang to the direction of time in which the universe expands. The closer the state of the universe is to the Big-Bang, the more homogeneous and isotropic the universe is. Moreover, as the singularity is approached, the 3d-space contracts.

A way to interpret this is that it contracts to a point, which is the singularity. This would be problematic, since if Σ is a point at $t = 0$, we will need to explain how it evolves into a 3d manifold.

Another way to interpret it is that the 3d-space components of the metric tensor tend to 0 as $t \searrow 0$, but the topology of space does not contract to a point, it is still the 3d manifold $(\Sigma, \gamma_{ab}(\boldsymbol{x}) \equiv 0)$. By avoiding to make the assumption that the topology derives from distance, we can obtain equations for general relativity that continue to be valid under more general conditions. For this we need an alternative formulation of semi-Riemannian geometry and Einstein's general relativity, which is equivalent to these ones outside the singularity, but well-defined and free of infinities at the singularity. This was achieved and shown to work in many situations in which non-singular semi-Riemannian geometry is not defined [120, 122, 123]. Moreover, this approach works well together with Penrose's *Weyl curvature hypothesis*, whose motivation was to connect the cosmological and the thermodynamic arrows of time [109, 121].

Then, there is only one possible 3d-space state at the Big-Bang singularity. Of course, as $t \searrow 0$ the system may be chaotic, as in the Mixmaster model [104] or the Belinski–Khalatnikov–Lifshitz model [77]. Then, while at the singularity there is still only one possible 3d-space state, it can be approached in different ways as $t \searrow 0$. However, the limit $\gamma \to 0$ forces the solutions to depend on a small number of parameters as they converge to the unique 3d-space $(\Sigma, 0)$.

Therefore, the severe constraint of the initial conditions for (Σ, γ) implies that the branching structure of the wavefunctional is very asymmetric in time. This suggests a possible reason why, at macro scales, branching happens only towards the future.

MSTI Answer 4 (Branching asymmetry)**.** Branching happens only towards the future because at the Big-Bang the 3d-space states are characterized by a very small number of degrees of freedom and converge to the same initial 3d-space state.

This answer is of course incomplete. We do not know why the initial state had to be the Big-Bang, and it is not even sure that there was a singularity, many researchers think that quantum gravity will be able to remove it.

E. Probabilities from continuity

When a quantum measurement is made, the probability to obtain a certain outcome is given by the Born rule to be the square of the projection of the state vector. Different outcomes may therefore have different probabilities. However, in MWI, there is only one branch for each of these outcomes. A direct counting argument implies that all outcomes should be obtained with the same probability, contrary to the Born rule. Everett proposed that somehow the squared amplitude of the branch gives the probability that the observer ends out being the observer from that branch.

MWI Problem 5 (Probabilities)**.** Why are the probabilities proportional to the squared amplitudes of the branches?

There are various proposed solutions, based on many-minds [70], decision theory [85, 134], measure of existence [129] *etc.* For a review see [132]. Proposals that somehow the amplitude of a branch yields probability have merits and led to interesting insights into the nature of probability [135]. But if there is a way to obtain probabilities in the old-fashioned way, for example by branch counting (Saunders advocates this [115]) or as *the ratio of the number of favorable outcomes to the total number of possible outcomes*, the result would be more palatable, without necessarily contradicting other proposals.

The Rule of Thumb 1 suggests the desirability to solve the problem by micro-state or micro-branch counting. Fortunately, MSTI does just this:

MSTI Answer 5 (Probabilities)**.** "Counting" 3d-space states allowed to have an inhomogeneous density gives probabilities proportional to the squared amplitudes. Counting each 3d-space state is justified by the fact that only those states have local beables, see §VI G.

This derivation of the Born rule is consistent with, and provides a concrete realization of, Saunders' branch-counting [115], Vaidman's notion of measure of existence [129], and maybe, but I am not sure, the Deutsch-Wallace decision-theoretic argument [85, 134].

F. Real wavefunction

The Rule of Thumb 1 also suggests the following

MWI Problem 6 (Real-number-based probabilities)**.** It is true that the norm of the (complex) wavefunction is real. But is there a deeper reason why we get real probabilities?

MSTI suggests a solution according to the Rule of Thumb 1 for this too:

MSTI Answer 6 (Real-number-based probabilities)**.** The wavefunction is real, and the complex phases only represent a global U(1) gauge choice for the classical fields in the 3d-space states.

G. Ontology: a real wavefunction

Another problem is that of ontology:

MWI Problem 7 (Ontology)**.** What is the ontology of MWI? What are the local beables?

Some researchers consider that the abstract state vector and the Hamiltonian are sufficient to specify the ontology of MWI, and from it one can derive an essentially unique 3d-space, the tensor product structure, the preferred basis, and all there is to be known about the universe [83]. This is impossible, because if any of these structures can be derived from the state vector and the Hamiltonian, infinitely many other solutions exist [125].

Other researchers consider that the wavefunction is needed, in the sense that not only the state vector is required, but also the 3d-space, and this is sufficient to specify the complete ontology [130, 131].

Despite this, authors like Maudlin [101–103] and Norsen [105] consider that MWI does not have a primitive ontology in terms of local beables.

But in every micro-world in MSTI there are local beables, just like in classical physics.

MSTI Answer 7 (Ontology)**.** There is a wavefunctional, composed of 3d-space states and dissociable into them. Each 3d-space state consists of a 3d-space (Σ, γ), on which classical fields ϕ are defined, with a fixed gauge. Every 3d-space state appears at most once in the composition of the wavefunctional, but these states can be distributed with a nonuniform density. The distribution gives the real wavefunctional, and the gauge gives the complex phase of each term in the wavefunctional. The local beables are the classical fields ϕ and γ defined on the 3d manifold Σ, so they are defined only for 3d-space states. Because the 3d-space states are the ones having definite local beables, they correspond to (micro-)worlds. This justifies counting them to obtain the probabilities.

Therefore, local beables exist, and the Rule of Thumb 1 was followed.

VII. DISCUSSION

It is uncommon to use the wavefunctional formulation of quantum field theory in the interpretation of quantum mechanics. For some reason, it is considered more natural to take nonrelativistic quantum mechanics as a benchmark for these interpretations. But the wavefunctional formulation is natural too, if not even more natural.

Remark 3. When we perform a quantum measurement of a smaller system, we never observe directly its state, only the pointer state of the apparatus, which is macroscopic. A measuring device is dedicated to a particular location and type of quantum field (or subsystem in general), not to a particular particle (or subsystem). The result of any measurement translates into a change in the macro-state of the universe. All these are described adequately by the wavefunctional of the entire universe.

Wheeler and Everett considered MWI as the interpretation of quantum mechanics that is suitable for quantum gravity [76, 82]. According to DeWitt [87], p. 1141:

> Everett's view of the world is a very natural one to adopt in the quantum theory of gravity, where one is accustomed to speak without embarrassment of the 'wave function of the universe.' It is possible that Everett's view is not only natural but essential.

Here, we have seen that background free quantum gravity solves some foundational problems of quantum mechanics, and especially of MWI. It even suggests a version of MWI (which is MSTI) as the more natural interpretation of quantum mechanics. The relation between quantum gravity and MWI is therefore reciprocal.

Finally, I argued that MSTI solves some of the main problems of standard quantum mechanics and MWI.

The strategy to make this interpretation more palatable was to highlight similarities with classical physics, based on the Rule of Thumb 1. It turns out that, except for the existence of a multiplicity of worlds, MSTI is a more classical version of MWI, with respect to the appearance of classicality, the existence of local beables, the statistics, and even the understanding of the complex numbers inherent to the theory.

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