Does quantum cognition imply quantum minds?

Shan Gao
Research Center for Philosophy of Science and Technology,
Shanxi University, Taiyuan 030006, P. R. China
E-mail: gaoshan2017@sxu.edu.cn.

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Abstract

Based on the PBR theorem about the reality of the wave function, we show that the wave function assigned to a cognitive system, which is used to calculate probabilities of thoughts/judgment outcomes in quantum cognition, is a real representation of the cognitive state of the system. In short, quantum cognition implies quantum minds. However, this result does not mean that we have quantum minds and our brain is a quantum computer, since quantum cognition by its standard formulation has not been fully confirmed by experiments. We hope that more crucial experiments can be done in the near future to determine whether or not quantum cognition is real.

Key words: quantum cognition; wave function; PBR theorem

1 Introduction

Quantum cognition is a theoretical framework for constructing cognitive models based on the mathematical principles of quantum theory. Due to its success in explaining paradoxical empirical findings in cognitive science, quantum cognition has been the subject of much interest in recent years (see Wang et al, 2013; Busemeyer and Bruza, 2014; Yearsley and Busemeyer, 2016 for helpful reviews). However, it is still unknown what quantum cognitive models tell us about the underlying process of cognition. It is widely thought that quantum cognition is only an effective theory of cognition, where the actual brain processing may take place in an essentially classical way. In this paper, we will address this issue. Based on the recent advances in the research of quantum foundations, especially the PBR theorem about the reality of the wave function, we will show that the wave function assigned to a cognitive system such as our brain, which is used to calculate probabilities of thoughts/judgment outcomes in quantum cognition, is a real representation of the cognitive state of the system. We will also discuss possible implications of this interesting result.

2 The PBR theorem

Quantum theory, in its minimum formulation, is an algorithm for calculating probabilities of measurement results. The theory assigns a mathematical object, the so-called wave function or quantum state, to a physical system prepared at a given instant, and specifies how the wave function evolves with time. The time evolution of the wave function is governed by the Schrödinger equation, whose concrete form is determined by the properties of the system and its interactions with environment. The connection of the wave function with the results of measurements on the system is specified by the Born rule, which roughly says that the probability of obtaining a particular result is given by the modulus squared of the wave function corresponding to the result.

At first sight, quantum theory as an algorithm says nothing about the actual state of a physical system. However, it has been known that this is not true due to the recent advances in the research of the foundations of quantum mechanics (see, e.g. Leifer, 2014; Gao, 2017). First, a general and rigorous approach called ontological models framework has been proposed to determine the relation between the wave function and the actual state of a physical system (Spekkens 2005; Harrigan and Spekkens 2010). The framework has two fundamental assumptions. The first assumption is about the existence of the underlying state of reality. It says that if a physical system is prepared such that the quantum algorithm assigns a wave function to it, then after preparation the system has a well-defined set of physical properties or an underlying ontic state, which is usually represented by a mathematical object, λ. In general, for an ensemble of identically prepared systems to which the same wave function ψ is assigned, the ontic states of different systems in the ensemble may be different, and the wave function ψ corresponds to a probability distribution $p(\lambda|\psi)$ over all possible ontic states, where $\int d\lambda p(\lambda|\psi) = 1$.

There are two possible types of models in the ontological models framework, namely ψ-ontic models and ψ-epistemic models. In a ψ-ontic model, the ontic state of a physical system uniquely determines its wave function. In this case, the wave function directly represents the ontic state of the system, or it is a mathematical representation of the physical state of the system.[1] While in a ψ-epistemic model, there are at least two wave functions which are compatible with the same ontic state of a physical system. In this case,

[1]Note that the wave function is not necessarily complete, i.e. it does not necessarily represent the complete physical state of a system, such as in Bohm's theory.

the wave function merely represents a state of incomplete knowledge - an epistemic state - about the actual ontic state of the system.

In order to investigate whether an ontological model is consistent with the quantum algorithm, we also need a rule of connecting the underlying ontic states with measurement results. This is the second assumption of the ontological models framework, which says that when a measurement is performed, the behaviour of the measuring device is determined by the ontic state of the system, along with the physical properties of the measuring device. Concretely speaking, for a projective measurement M, the ontic state λ of a physical system determines the probability $p(k|\lambda, M)$ of different results k for the measurement M on the system. The consistency with the quantum algorithm then requires the following relation: $\int d\lambda p(k|\lambda, M)p(\lambda|\psi) = p(k|M, \psi)$, where $p(k|M, \psi)$ is the Born probability of k given M and the wave function ψ.

Second, several important ψ-ontology theorems have been proved in the ontological models framework (Pusey, Barrett and Rudolph, 2012; Colbeck and Renner, 2012, 2017; Hardy, 2013), the strongest one of which is the Pusey-Barrett-Rudolph theorem or the PBR theorem (Pusey, Barrett and Rudolph, 2012). The PBR theorem shows that in the ontological models framework, when assuming independently prepared systems have independent ontic states, the ontic state of a physical system uniquely determines its wave function, or the wave function of a physical system directly represents the ontic state of the system. This auxiliary assumption is called preparation independence assumption.

The basic proof strategy of the PBR theorem is as follows. Assume there are N nonorthogonal quantum states ψ_i (i=1, ... , N), which are compatible with the same ontic state λ.[2] The ontic state λ determines the probability $p(k|\lambda, M)$ of different results k for the measurement M. Moreover, there is a normalization relation for any N result measurement: $\sum_{i=1}^{N} p(k_i|\lambda, M) = 1$. Now if an N result measurement satisfies the condition that the first state gives zero Born probability to the first result and the second state gives zero Born probability to the second result and so on, then there will be a relation $p(k_i|\lambda, M) = 0$ for any i, which leads to a contradiction.

The task is then to find whether there are such nonorthogonal states and the corresponding measurement. Obviously there is no such a measurement for two nonorthogonal states of a physical system, since this will permit them to be perfectly distinguished, which is prohibited by quantum theory. However, such a measurement does exist for four nonorthogonal states of two copies of a physical system. The four nonorthogonal states are the following product states: $|0\rangle \otimes |0\rangle$, $|0\rangle \otimes |+\rangle$,$|+\rangle \otimes |0\rangle$ and $|+\rangle \otimes |+\rangle$, where $|+\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$. The corresponding measurement is a joint measurement

[2] It can be readily shown that different orthogonal states correspond to different ontic states. Thus the proof given here concerns only nonorthogonal states.

of the two systems, which projects onto the following four orthogonal states:

$$\begin{aligned}
\phi_1 &= \tfrac{1}{\sqrt{2}}(|0\rangle \otimes |1\rangle + |1\rangle \otimes |0\rangle), \\
\phi_2 &= \tfrac{1}{\sqrt{2}}(|0\rangle \otimes |-\rangle + |1\rangle \otimes |+\rangle), \\
\phi_3 &= \tfrac{1}{\sqrt{2}}(|+\rangle \otimes |1\rangle + |-\rangle \otimes |0\rangle), \\
\phi_4 &= \tfrac{1}{\sqrt{2}}(|+\rangle \otimes |-\rangle + |-\rangle \otimes |+\rangle),
\end{aligned} \tag{1}$$

where $|-\rangle = \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle)$. This proves that the four nonorthogonal states are ontologically distinct. In order to further prove the two nonorthogonal states $|0\rangle$ and $|+\rangle$ for one system are ontologically distinct, the preparation independence assumption is needed. Under this assumption, a similar proof for every pair of nonorthogonal states can also be found, which requires more than two copies of a physical system (see Pusey, Barrett and Rudolph, 2012 for the complete proof).

To sum up, the PBR theorem shows that quantum theory as an algorithm also says something about the actual state of a physical system. It is that under the preparation independence assumption, the wave function assigned to a physical system, which is used for calculating probabilities of results of measurements on the system, is a mathematical representation of the actual physical state of the system in the ontological models framework.

There are two possible ways to avoid the result of the PBR theorem. One is to deny the preparation independence assumption. Although this assumption seems very natural, it may be rejected in some ontological models (Lewis et al, 2012). The other is to deny that an isolated system has a real physical state, which is objective and independent of other systems including observers, i.e. dening the first assumption of the ontological models framework. Indeed, this assumption is rejected by Quantum Bayesianism or QBism (Fuchs et al, 2014) and other pragmatist approaches to quantum theory (Healey, 2017), where the wave function represents information about possible measurement results or it is only a calculational tool for making predictions concerning measurement results.

3 Implications for quantum cognition

Let's now analyze the possible implications of the PBR theorem for understanding quantum cognition.

In its most conservative form, quantum cognition is only an algorithm for calculating probabilities of thoughts/judgment outcomes.[3] In order to understand the underlying process of cognition in the brain, we need a similar

[3]There may exist different formulations of quantum cognition. Our following analysis is based on the standard formulation given by Busemeyer and Bruza (2014) and Yearsley and Busemeyer (2016).

ontological models framework for cognitive systems. The first assumption of the framework is about the existence of cognitive states. It says that when quantum cognition assigns a wave function to a cognitive system, the system has a well-defined cognitive state, which can be represented by a mathematical object, λ_c. This assumption is accepted by quantum cognitive models explicitly or implicitly. If one denies this assumption, then it will be impossible to understand the underlying process of quantum cognition.[4]

The second assumption is that when a measurement/judgment is made, the behaviour of a cognitive system is determined by the cognitive state of the system, along with the concrete measurement setting such as asked questions or evidence presentation. Concretely speaking, for a measurement M, the underlying cognitive state λ_c of a cognitive system determines the probability $p(k|\lambda_c, M)$ of different results k for the measurement M on the system. The consistency with the quantum algorithm in quantum cognitive models then requires the following relation: $\int d\lambda_c p(k|\lambda_c, M)p(\lambda_c|\psi) = p(k|M,\psi)$, where $p(k|M,\psi)$ is the Born probability of k given M and the wave function ψ. This assumption is necessary for connecting the underlying cognitive states with the results of measurements. If denying this assumption, then we cannot investigate whether an ontological cognitive model is consistent with the quantum algorithm, and as a result, we cannot understand why the quantum algorithm works and what it tells us about the underlying process of cognition.

In addition, there is also an auxiliary assumption besides the above ontological models framework for cognitive systems, namely the preparation independence assumption. It says that independently "prepared" cognitive systems such as independent persons have independent cognitive states. This assumption holds true in quantum cognitive models.

Then, quantum cognition satisfies the three preconditions of the PBR theorem, namely (1) the quantum algorithm; (2) the ontological models framework; and (3) the preparation independence assumption. Thus, like the proof of the PBR theorem, we can prove that the wave function assigned to a cognitive system such as our brain, which is used to calculate probabilities of thoughts/judgment outcomes in quantum cognition, is a real representation of the cognitive state of the system. This means that the cognitive state of our brain and its dynamics are not classical but quantum in quantum cognition. In short, quantum cognition implies quantum minds.

[4]Note that denying this realistic assumption will pose more serious difficulties for understanding macroscopic cognitive systems than for understanding microscopic physical systems such as atoms. We cannot directly perceive the microscopic objects after all. But we can directly perceive macroscopic objects, and we also have self-awareness. Thus it is arguable that QBism and other pragmatist approaches to quantum theory are not applicable here.

4 Further discussion

Does this result mean that we have quantum minds and our brain is a quantum computer? Not really. Here is the reasons.

As we have shown above, this result is derived based on three assumptions about a cognitive system. Although the ontological models framework and the preparation independence assumption seem to be uncontroversial, what quantum cognition really is is a debated issue. Our analysis is based on the standard formulation of quantum cognition (Busemeyer and Bruza, 2014; Yearsley and Busemeyer, 2016), which is the standard quantum theory applied to cognitive systems. Thus, strictly speaking, the above result is valid only in this formulation of quantum cognition.

The question is: is the standard formulation of quantum cognition true? Admittedly, it has not been fully confirmed by experiments, although there is strong evidence (see Wang et al, 2014). On the one hand, the existing quantum cognitive models use only a part of the complete quantum formalism. On the other hand, it seems that the models based on the standard formulation cannot explain some cognitive experiments (see, e.g. de Barros and Suppes, 2009). This also motivates some researchers to propose some quantum-like models (Aerts, 2009; Khrennikov, 2010) or generalized quantum models (Atmanspacher et al, 2002).

We think it is fair to say that existing cognitive experiments have not conclusively determined whether the standard formulation of quantum cognition is true or false. And thus the above result, which is derived based on the formulation, does not show that we have quantum minds and our brain is a quantum computer (Hameroff and Penrose, 1996; Hagan et al., 2002; Fisher, 2015; Wendt, 2015). We need more crucial cognitive experiments to support this conclusion. For example, if a violation of the Bell inequality is found for separated human systems, then it will provide a strong support.[5]

5 Conclusions

Based on the ontological models framework and the PBR theorem, we have shown that the wave function assigned to a cognitive system in the standard formulation of quantum cognition is a real representation of the cognitive state of the system. This result means that the cognitive state of our brain and its dynamics are not classical but quantum in quantum cognition. In short, quantum cognition implies quantum minds. However, it does not imply that we have quantum minds and our brain is a quantum computer, since the theory has not been fully confirmed by experiments. We hope

[5]Note that even if quantum cognition is true, it does not necessarily assume that two people can get entangled since there may not exist unitary interactions between them to form the entangled state. I thank ... for this insightful comment.

that more crucial experiments in cognition can be done in the near future to determine whether or not quantum cognition is real.

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