Existence of macroscopic spatial superpositions in collapse theories

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Abstract

It is shown that the collapse dynamics in collapse models such as the CSL model will entangle two independent systems under certain condition, and their state after collapse may be an entangled superposition of spatially separated states. However, the existence of such macroscopic spatial superposition is not inconsistent with experiments. One way to avoid this result is to assume that the noise responsible for the collapse of the wave function of a quantum system comes from the system itself. In this case, the states of two independent systems will always collapse independently.

It has been widely thought that the collapse theories of quantum mechanics do not permit the existence of macroscopic spatial superposition, and thus they can solve the measurement problem (Ghirardi, 2016). In this paper, we will present an interesting counterexample. We will show that the collapse dynamics in some collapse models such as the CSL model will entangle two independent systems under certain condition, and their state after collapse may be an entangled superposition of spatially separated states. This will permit the existence of macroscopic spatial superpositions. However, since the condition can hardly be satisfied in reality, the occurrence of such superpositions in Nature is very improbable, and thus these collapse models still provide a promising solution to the measurement problem.

Take the mass density version of the CSL model as an example (Pearle, 1989; Ghirardi, Pearle and Rimini, 1990; Pearle and Squires, 1994). In the model, there is a universal noise field in space, denoted by $w(x,t)$, and it couples with the smeared mass density operator of a quantum system to produce collapse toward its spatially localized eigenstates. The smeared mass number density operator for a particle with mass m is

$$M(x) = \frac{m}{(\sqrt{\pi}a)^3 m_0} \int dz N(z,t) e^{-\frac{1}{2a^2}(x-z)^2} \tag{1}$$

where $a \approx 10^{-7}m$ is a distance constant of the model, m_0 is a reference mass usually equal to the mass of a nucleon, $N(z,t) = \zeta^\dagger(z)\zeta(z)$ is the particle number density operator, $\zeta^\dagger(z)$ and $\zeta(z)$ are the particle creation and annihilation operators at position z. The eigenstates of $M(x)$ are the position eigenstates of the particle: $|x_i\rangle = \zeta^\dagger(x_i)|0\rangle$, and the eigenequation is:

$$M(x)|x_i\rangle = \frac{m}{(\sqrt{\pi}a)^3 m_0} e^{-\frac{1}{2a^2}(x-x_i)^2} |x_i\rangle . \tag{2}$$

For N such particles in the same position x_i, their joint position eigenstate $|N(x_i)\rangle = \zeta^\dagger(x_i)...\zeta^\dagger(x_i)|0\rangle$ is an eigenstate of $M(x)$, and the eigenequation is:

$$M(x)|N(x_i)\rangle = \frac{Nm}{(\sqrt{\pi}a)^3 m_0} e^{-\frac{1}{2a^2}(x-x_i)^2} |N(x_i)\rangle . \tag{3}$$

For a system containing n different types of particles, the smeared mass number density operator is

$$M(x) = \frac{1}{(\sqrt{\pi}a)^3 m_0} \int dz \sum_{j=1}^{n} m_j N_j(z,t) e^{-\frac{1}{2a^2}(x-z)^2} \tag{4}$$

where m_j and $N_j(z,t)$ are the mass and particle number density operators of type j particles.

Now consider two systems A and B which have no interactions and contain two different types of particles, N_A particles with mass m_A and N_B particles with mass m_B, respectively. Suppose the two systems are initially in a product state $|\psi(0)\rangle = \frac{1}{\sqrt{2}}[|N_A(x_1)\rangle + |N_A(x_2)\rangle]\frac{1}{\sqrt{2}}[|N_B(x_1)\rangle + |N_B(x_2)\rangle]$, where $|N_A(x_i)\rangle$ and $|N_B(x_i)\rangle$ are the eigenstates of $M(x)$ of A and B at position x_i, respectively.[1] According to the CSL model (Pearle, 1999), the state of the two systems at instant t for a given noise $w(x,t)$ is:

[1]Note that if the two systems contain the same type of particles, then this initial state is not permitted since it is not (anti-)symmetrical.

$$
\begin{aligned}
|\psi(t)\rangle_w &= e^{-\frac{1}{4\lambda}\int_0^t dt \int dx [w(x,t)-2\lambda M(x)]^2} |\psi(0)\rangle \qquad (5)\\
&= \frac{1}{2}[e^{-\frac{1}{4\lambda}\int_0^t dt \int dx [w(x,t)-2\lambda(n_A(x_1)+n_B(x_1))]^2} |N_A(x_1)\rangle |N_B(x_1)\rangle \\
&+e^{-\frac{1}{4\lambda}\int_0^t dt \int dx [w(x,t)-2\lambda(n_A(x_2)+n_B(x_2))]^2} |N_A(x_2)\rangle |N_B(x_2)\rangle \\
&+e^{-\frac{1}{4\lambda}\int_0^t dt \int dx [w(x,t)-2\lambda(n_A(x_1)+n_B(x_2))]^2} |N_A(x_1)\rangle |N_B(x_2)\rangle \\
&+e^{-\frac{1}{4\lambda}\int_0^t dt \int dx [w(x,t)-2\lambda(n_A(x_2)+n_B(x_1))]^2} |N_A(x_2)\rangle |N_B(x_1)\rangle]
\end{aligned}
$$

where λ is a parameter of the model that determines the collapse rate, $M(x) = M_A(x) + M_B(x)$, $M_A(x)$ and $M_B(x)$ are the smeared mass density operators of A and B, respectively, and $n_A(x_i) = \frac{N_A m_A}{(\sqrt{\pi}a)^3 m_0} e^{-\frac{1}{2a^2}(x-x_i)^2}$, $n_B(x_i) = \frac{N_B m_B}{(\sqrt{\pi}a)^3 m_0} e^{-\frac{1}{2a^2}(x-x_i)^2}$.[2]

The evolution equation (5) tells us what the initial wave function evolves into under a particular $w(x,t)$. The CSL model also requires the second equation, the probability rule (Pearle, 1999). It gives the probability density for $w(x,t)$ to be the actual noise that occurs in nature:

$$
\begin{aligned}
\mathcal{P}_t\{w\} &\equiv_w < \psi(t)|\psi(t) >_w \qquad (6)\\
&= \frac{1}{4}[e^{-\frac{1}{2\lambda}\int_0^t dt \int dx [w(x,t)-2\lambda(n_A(x_1)+n_B(x_1))]^2} \\
&+e^{-\frac{1}{2\lambda}\int_0^t dt \int dx [w(x,t)-2\lambda(n_A(x_2)+n_B(x_2))]^2} \\
&+e^{-\frac{1}{2\lambda}\int_0^t dt \int dx [w(x,t)-2\lambda(n_A(x_1)+n_B(x_2))]^2} \\
&+e^{-\frac{1}{4\lambda}\int_0^t dt \int dx [w(x,t)-2\lambda(n_A(x_2)+n_B(x_1))]^2}].
\end{aligned}
$$

Eq. (6) says that the wave functions with the largest norm are the most likely to occur.

Here is how these equations work. From Eq. (5) we can see that the most probable $w(x,t)$'s occur if $w(x,t) \approx 2\lambda[n_A(x_1) + n_B(x_1)]$ or $w(x,t) \approx 2\lambda[n_A(x_2)+n_B(x_2)]$ or $w(x,t) \approx 2\lambda[n_A(x_1)+n_B(x_2)]$ or $w(x,t) \approx 2\lambda[n_A(x_2)+n_B(x_1)]$. For example, suppose $w(x,t) \approx 2\lambda[n_A(x_1) + n_B(x_2)]$. Then, for large t, Eqs. (5), (6) become

[2] Here the Hamiltonian is set equal to 0 and the usual Schrödinger dynamics is ignored.

$$
\begin{aligned}
|\psi(t)\rangle_w \quad & \approx \frac{1}{2}[|N_A(x_1)\rangle\,|N_B(x_2)\rangle \\
& + e^{-\frac{t}{2}\int dx[n_B(x_2)-n_B(x_1)]^2}\,|N_A(x_1)\rangle\,|N_B(x_1)\rangle \\
& + e^{-\frac{t}{2}\int dx[n_A(x_1)-n_A(x_2)]^2}\,|N_A(x_2)\rangle\,|N_B(x_2)\rangle \\
& + e^{-\frac{t}{2}\int dx[n_A(x_1)-n_A(x_2)+n_B(x_2)-n_B(x_1)]^2}\,|N_A(x_2)\rangle\,|N_B(x_1)\rangle] \\
\mathcal{P}_t\{w\} \quad & \approx \frac{1}{4}[1 + e^{-t\int dx[n_B(x_2)-n_B(x_1)]^2} + e^{-t\int dx[n_A(x_1)-n_A(x_2)]^2} \\
& + e^{-t\int dx[n_A(x_1)-n_A(x_2)+n_B(x_2)-n_B(x_1)]^2}]
\end{aligned}
$$

Thus the probability associated with the $w(x,t)$ approaches $\frac{1}{4}$ and the state approaches $|N_A(x_1)\rangle\,|N_B(x_2)\rangle$. A similar argument holds for the other three branches. For other ranges of $w(x,t)$ the associated probability aproaches 0 for large t.

This result is within expectations; the states of two independent systems collapse independently. However, it is valid only when $N_A m_A \neq N_B m_B$. When $N_A m_A = N_B m_B$, we have $n_A(x_1) = n_B(x_1)$ and $n_A(x_2) = n_B(x_2)$. In this case, when $w(x,t) \approx 2\lambda[n_A(x_1) + n_B(x_2)]$, for large t, Eqs. (5), (6) become

$$
\begin{aligned}
|\psi(t)\rangle_w \quad & \approx \frac{1}{2}[|N_A(x_1)\rangle\,|N_B(x_2)\rangle + |N_A(x_2)\rangle\,|N_B(x_1)\rangle \\
& + e^{-\frac{t}{2}\int dx[n_B(x_2)-n_B(x_1)]^2}\,|N_A(x_1)\rangle\,|N_B(x_1)\rangle \\
& + e^{-\frac{t}{2}\int dx[n_A(x_1)-n_A(x_2)]^2}\,|N_A(x_2)\rangle\,|N_B(x_2)\rangle] \\
\mathcal{P}_t\{w\} \quad & \approx \frac{1}{4}[2 + e^{-t\int dx[n_B(x_2)-n_B(x_1)]^2} + e^{-t\int dx[n_A(x_1)-n_A(x_2)]^2}]
\end{aligned}
$$

This means that the state after collapse is not $|N_A(x_1)\rangle\,|N_B(x_2)\rangle$, but $|N_A(x_1)\rangle\,|N_B(x_2)\rangle + |N_A(x_2)\rangle\,|N_B(x_1)\rangle$ with probability $\frac{1}{2}$. In other words, after the collapse, each massive system is in an (entangled) superposition of two spatially separated states, and the separation distance may be arbitrarily large. Such spatially superposed states are usually avoided in collapse theories.

It can be further shown that this entanglement result holds true for other collapse models in which there is a universal noise field responsible for the collapse of the wave function of every quantum system (Bassi et al, 2013). The essential reason is as follows. The collapse dynamics will collapse the wave function to certain eigenstates of an operator (e.g. the smeared mass density operator in the CSL model), while these eigenstates are degenerate for a many-body system. Then, if the noise field is universal in the collapse dynamics, the final state after collapse may be a superposition of degenerate eigenstates, which is an entangled state of the whole system.

In addition, it can also be seen from the above analysis that if a collapse dynamics keeps the symmetry of the wave function of identical particles, it will permit entanglement for non-identical particles. On the other hand, if a collapse dynamics does not permit entanglement for non-identical particles, then it will not keep the symmetry of the wave function of identical particles. The reason is as follows. The entangled branch of two non-identical particles such as $|N_A(x_1)\rangle\,|N_B(x_2)\rangle + |N_A(x_2)\rangle\,|N_B(x_1)\rangle$ has the same form as a symmetric entangled state of two identical particles. Then, if a collapse dynamics keeps this entangled branch, it will also keep a symmetric entangled state of two identical particles, and vice versa. This is a more general argument independent of the noise field and collapse dynamics.

If the noise field in a collapse model is universal, responsible for the collapse of the wave function of every quantum system, then the above entanglement result seems inevitable. One way to avoid this result is to assume that the noise responsible for the collapse of the wave function of a quantum system comes from the system itself. In this case, two independent systems will have two independent noises, and thus their states will collapse independently. An example of such a collapse dynamics is Gao's collapse model (Gao, 2017).[3]

To sum up, we show that the collapse dynamics in some collapse models such as the CSL model will entangle two independent systems under certain condition, and their state after collapse may be an entangled superposition of spatially separated states. This will permit the existence of macroscopic spatial superpositions. However, since the condition can hardly be satisfied in reality, the occurrence of such superpositions in Nature is very improbable, and thus these collapse models still provide a promising solution to the measurement problem.

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