

## The Upward Path to Structural Realism\*

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### Abstract

In a recent PSA paper (2001a) as well as some other papers ((1995), (2000), (2001b)) and a book chapter (1999, ch. 7), Stathis Psillos raised a number of objections against structural realism. The aim of this paper is threefold: 1) to evaluate part of Psillos' offence on the Russellian version of epistemic structural realism (ESR for short), 2) to elaborate more fully what Russellian ESR involves, and 3) to suggest improvements where it is indeed failing.

## 1. Introduction

Stathis Psillos has praised the revival of ESR, largely due to John Worrall, saying that it gives us an important insight into the scientific realism debate, namely that we need not believe to an equal degree all that a scientific theory ascribes to the world. In spite of this acknowledgement, he raises a number of objections against it. In what follows, I evaluate two of his objections that are directed against the Russellian version of ESR.

## 2. Epistemic Structural Realism

Although Russell's structuralist inclinations can be seen as early as *The Problems of Philosophy* (1912), a fully-fledged account first emerged in *The Analysis of Matter* (1927). There he argued that we only have direct epistemic access to percepts, i.e. the basic units of our perception.<sup>1</sup> These lie at the end of causal chains which originate in the external world. Indeed, because of this causal relationship, percepts are taken to encode information about the external world. Hence, the only way to attain knowledge of the external world, according to this view, is to draw inferences from perception. To underwrite such inferences Russell employed a number of principles, the following two of which are central:

(1) Helmholtz-Weyl Principle (H-W): Different effects (i.e. percepts) imply different causes (i.e. stimuli/physical objects) (1927, 255).<sup>2,3</sup>

(2) Mirroring Relations Principle (MR): Relations between percepts mirror (i.e. have the same logico-mathematical properties as) relations between their non-perceptual causes (1927, 252).<sup>4</sup>

Armed with these assumptions Russell argues that from the structure of our perceptions we can “infer a great deal as to the structure of the physical world” (1927, 400). More precisely, he argued that all that we can guarantee is that the structure of our perceptions is at most isomorphic to the structure of the physical world.

Redhead (2001) has called the notion of structure employed by Russell, ‘abstract structure’. To understand the notion of abstract structure we must first understand: a) what we mean by ‘structure’ and b) what it means for two structures to be isomorphic. A *structure*  $S = (U, R)$  is specified by two things: i) a non-empty set  $U$  of objects (the domain of  $S$ ) and ii) a non-empty set of relations  $R$  on  $U$ .<sup>5</sup> A structure  $S = (U, R)$  is *isomorphic* to a structure  $T = (U', R')$  just in case there is a bijection  $\phi: U \rightarrow U'$  such that for all  $x_1, \dots, x_n$  in  $U$ ,  $(x_1, \dots, x_n)$  satisfies the relation  $R_i$  in  $U$  iff  $(\phi(x_1), \dots, \phi(x_n))$  satisfies the corresponding relation  $R'_i$  in  $U'$ . If, like Russell, one wants to talk about a particular relation being isomorphic to some other relation, one need not go further than the definition of isomorphism between structures, for any particular relation specifies a structure, namely a structure whose set of relations contains one, and only one, member. We can now define the notion of abstract structure: An *abstract structure*  $\Sigma$  is an isomorphism class whose members are all, and only those, structures that are isomorphic

to some given structure. Qua isomorphism class, it can only identify what Russell calls the ‘logico-mathematical properties’ of its members (1927, 251-254).<sup>6</sup>

The notion of abstract structure is contrasted with what Redhead calls ‘concrete structure’. Whereas a concrete structure specifies one domain of objects that comes with a set of relations, an abstract structure just specifies a constraint as to which domains of objects and relations qualify, namely those domains equinumerous to some given number and those relations that share the same logico-mathematical properties.<sup>7</sup>

On the basis of these definitions we can now summarise Russell’s epistemic commitments as follows: (1) concrete observational/perceptual structures, (2) abstract structures (i.e. isomorphism classes) whose members are the concrete observational structures referred to in 1, and (3) the existence of concrete physical structures that (a) have as domain members the external world causes of the concrete observational structures’ domain members referred to in 1 and (b) are members of the isomorphism classes referred to in 2. It is extremely important to note here that Russell’s programme offers a rational *reconstruction* of scientific knowledge rather than a description of what goes on in science. Thus, it cannot, and should not, be criticised on account of its divergence from actual scientific practice.

Psillos calls the Russellian approach the ‘upward path’ to structural realism, in contrast to the Poincaréan/Worrallian approach or ‘downward path’ to structural realism. The difference primarily lies in the way the two views are motivated. The Poincaréan

approach takes the preservation of structure through theory change as indicative of its truth/approximate truth. The Russellian approach looks not in history but in perception to provide a reconstruction of our non-perceptual knowledge. Both accounts agree that the most that we can hope to know about the external world is its (abstract) structure. Other differences exist but I will refrain from dealing with these in this paper.<sup>8</sup>

An altogether different species of structural realism, which I will also be leaving out of this paper, has been proposed by James Ladyman (1998). Together with Steven French they argue that structural realism should be understood not just as an epistemological, but also as an ontological position, aptly calling it ‘ontic structural realism’ in contrast to Russell’s and Poincaré’s versions whose claims are merely epistemic. Put simply, the central claim of ontic structural realism is that all that exists in the world is structure. They thus argue that we should abandon individuals-based ontology and reconceptualise the role of individuals in terms of structures.<sup>9</sup>

### **3. The Objections**

One can single out two central objections from Psillos’ writings that are explicitly directed at the Russellian version of ESR. These are:

(1) ESR faces a dilemma: On the one hand, the H-W principle by itself can only establish a relation of embeddability between the external world and the ‘world’ of percepts, not a relation of isomorphism as required by ESR. Without a relation of isomorphism, the structural realists cannot establish inferential knowledge about the structure of the

external world. On the other hand, H-W and its converse, viz. different stimuli/physical objects imply different percepts, allow for the establishment of isomorphic relations but, in doing so, concede too much to idealism (2001a, S13-S16).

(2) ESR cannot justify the claim that the first-order properties and relations of unobservables are unknowable in principle (1999, 156); (2001a, S20-21).

I have omitted Psillos' centrepiece objection, namely the notorious Newman objection (see Newman (1928)). The reason for this omission is that it has been thoroughly discussed elsewhere (see, for example, Demopoulos and Friedman (1985), Ketland (2004) and Votsis (2003)).

#### **4. The First Objection**

##### *The First Horn of the Dilemma*

According to Psillos, the H-W principle can only establish a relation of embeddability between the external world and the world of percepts, falling short of a relation of isomorphism required by ESR. Without a relation of isomorphism, Psillos argues, structural realists cannot establish inferential knowledge about the external world. But in what way exactly is the H-W principle able to establish a relation of embeddability but not one of isomorphism?

Let us identify any set of percepts by the letter  $P$  and any set of external world causes, i.e. stimuli/physical objects, by the letter  $C$ . Psillos argues that the H-W principle cannot give us isomorphic mappings between  $P$  and  $C$ . To remind the reader, the H-W principle expresses the following conditional: If different percepts, then different stimuli/physical objects. This principle guarantees that a given  $C$  will have at least as many members as the corresponding  $P$ . More importantly, the principle is equivalent to saying that there is an injective mapping  $f: P \rightarrow C$ , such that if  $x$  and  $y$  are distinct members of  $P$ , then their images in  $C$ ,  $fx$  and  $fy$ , are also distinct. Now, for a mapping to be isomorphic it is not enough to be injective, it must also be surjective, i.e. for every  $c \in C$ , there is at least one  $p \in P$ , such that  $f(p)=c$ . To establish this, we need the converse of H-W, let us call it ‘W-H’, viz. if different stimuli/physical objects, then different percepts. H-W and W-H together guarantee a bijective mapping between the set of percepts and the set of stimuli/physical objects. A relation of isomorphism can be established when this bijection also maps all the relations found in the structured domains.<sup>10</sup>

Psillos correctly points out that the H-W principle cannot by itself give us the much-desired relation of isomorphism. He is wrong, however, in arguing that the H-W principle allows us to establish embeddability relations, unless he is using the term to mean injective mappings. The term ‘embedding’ is more often reserved for injections that also map relations, hence appeal to this notion implies one is already dealing with a structured domain. More formally, an embedding of structure  $S_1 (A, R)$  into structure  $S_2 (B, R')$  is a one-one mapping  $f$  of  $A$  into  $B$  such that: (1)  $f(a_j)=b_j$  for all  $a_j \in A$  and (2)  $\langle a_1, \dots, a_n \rangle \in R_i$  iff  $\langle f(a_1), \dots, f(a_n) \rangle \in R'_i$  for all  $i \in I$  and all  $a_1, \dots, a_n \in A$ . It is worth noting that

embeddings are isomorphic mappings of a particular kind. In general, we can say that a structure  $S_1$  is embedded in a structure  $S_2$  *if and only if*  $S_1$  is isomorphic to a substructure of  $S_2$ .

More crucially, Psillos is wrong in assuming that ESR requires a commitment to isomorphic relations only. Russell acknowledges that the W-H principle is unreliable because we often have different stimuli that lead to the same percepts. This can be easily illustrated in cases involving distance, as Russell's example shows: "If we are observing a man half a mile away, his appearance is not changed if he frowns, whereas it is changed for a man observing him from a distance of three feet" (1927, 255). Because of such examples, Russell suggests that "differences in the percept imply differences in the object, but not vice-versa" (1927, 339-340).<sup>11</sup> Moreover, he recognises the limitations of the inferential powers of the H-W principle, when it is not accompanied by W-H. Paradoxically, Psillos takes note of this when he says that "[p]recisely because Russell doesn't have the converse principle, he talks of 'roughly one-one relation' " (2001a, S15).<sup>12</sup>

The more potent objection that Psillos puts forward is that it is meaningless to speak of *roughly* one-one relations. Yet, even without the help of W-H, MR is a strong enough principle to guarantee inference at the isomorphic level. This can be done in the following way: Injective mappings can easily be given inverse mappings, i.e. for *any* injective mapping  $f: D \rightarrow E$  we can give an inverse mapping  $f^{-1}: E' \rightarrow D$  where  $E' = \text{ran } f$ . That is,  $E'$  contains as its members all and only those objects that are contained in the

range of  $f =_{\text{df}} \{fx: x \in \text{dom } f\}$ . Notice that by doing so we immediately satisfy the requirement of a surjective mapping, since for every object in  $E'$  there is at least one – in this case only one – corresponding object in  $D$ . In short, we get a bijective mapping between  $D$  and  $E'$  – where  $E'$  may or may not equal  $E$ .<sup>13</sup> The MR principle, i.e. that relations between percepts have the same logico-mathematical properties as relations between their external world causes, allows us to turn a bijective mapping into an isomorphic one, for it allows us to preserve any relations the set of external world causes may have. This is a crucial assumption without which very little can be established.

Psillos fails to register MR in his analysis. He thus misses a central part of Russell's programme and, more importantly, an opportunity to criticise this programme. After all, Russell is not clear on why we should accept MR. Unfortunately, I cannot pursue this issue here. In my opinion, it is hard to imagine how we can have knowledge of the external world without accepting something like MR. Epistemological realism requires belief in the correspondence between language and reality, i.e. belief in semantic realism. As many philosophers have argued over the years, the only type of correspondence that says anything coherent about the world is one that says something about the relations objects in the world stand in.

Psillos complains that “[f]rom a realist viewpoint, it should at least in principle be possible that the (unobservable) world has ‘extra structure’, i.e., structure not necessarily manifested in the structure of the phenomena” (2001a, S15).<sup>14</sup> If there is such extra structure, he continues, the required relation between the world of percepts and the

external world should be that of embeddability not isomorphism. Yet, Psillos argues, ESR cannot be upheld by appeal to embeddability since under this relation “the structure of the percepts doesn’t determine the *domain* of the stimuli” (2001a, S16) [original emphasis].<sup>15</sup>

Let us first reflect on the idea that the unobservable world could have some extra structure that is not manifested in the structure of the phenomena. There seems to be no reason why ESR should be inconsistent with this idea. ESR simply says that structures of phenomena mirror the structures of the unobservable world. It need only require that (at best) every phenomenal structure has a corresponding unobservable structure. It does not require the converse, i.e. that every unobservable structure has a corresponding phenomenal structure. In other words, ESR is compatible with the idea that the unobservable world may have extra structure.

What exactly does Psillos mean when he says that embeddings do not support ESR, since under embeddings the structure of percepts does not determine the domain of stimuli/physical objects? One way of understanding this claim is to take the absolute determination of the domain of the stimuli/physical objects as the complete description of the domain’s objects. If this is the case, his argument clearly falters on account of the fact that the relation of isomorphism does not require such a determination either. Indeed, one of the central points of ESR is that the stimuli/physical objects along with their properties and relations cannot be fixed absolutely, but only *up to* isomorphism. In short, this sort of underdetermination is not only compatible with ESR but constitutive of it. The only other

plausible reading of Psillos' claim is that a relation of isomorphism requires that the sets mapped have the same cardinality, whereas embeddability allows one to infer the minimum size of the set from which the range of the mapping is drawn. This difference does not amount to anything significant because there is no clause in ESR that requires the exact determination of the cardinality of a given set.

As Psillos admits, Russell's epistemic commitments are restricted to embeddings. These, as I have indicated, still offer isomorphic mappings, albeit of a special kind, namely that the structure of perceptions is isomorphic to a substructure of the external world. This still allows inferential knowledge from the structure of perceptions to the structure of the external world. Thus Psillos' first horn of the dilemma crumbles.

#### *The Second Horn of the Dilemma*

It is not entirely clear what Psillos means when he says that H-W and W-H allow inferences at the level of isomorphism but concede too much to idealism. In support of this claim he quotes certain passages from Hermann Weyl, where it seems that Weyl takes W-H to be "the central thought of idealism" and asserts that "science concedes to idealism that its objective reality is not given but to be constructed" (1963, 117). On the basis of this quote, Psillos complains that it should not be a priori false for a realist that there is a divergence between the structure of the physical world and the structure of the world of percepts. According to Psillos, "For all we know, the unobservable world may differ from the world of phenomena not just in its 'intrinsic nature', but in its structure too." (2001a, S16).

When Psillos argues that it should not be a priori false that there is a variance between the structure of the external world and the structure of perceptions, he mistakenly implies that this is the ESR-ist view. I do not see any good reasons why this should be the case. For example, nobody would deny that perceptual apparatus can *sometimes* malfunction. Just this point is sufficient to illustrate that the structure of the external world need not always be correctly reflected in the structure of our perceptions. A similar qualification should be made with regard to H-W, namely that *most* individuals would, when given the same stimulus, identify the same percept *most* of the time. Hence, ESR-ists can, and should, accept the view that some variance between the structure of the external world and the structure of perceptions exists. This qualification does not fundamentally undermine their programme, for the overall reliability of inferential knowledge about the structure of the external world is safeguarded.<sup>16</sup>

## 5. The Second Objection

The claim that we can know only the structure of the world, charges Psillos, is ambiguous. It may mean one of three things: (a) that everything is knowable apart from the individual objects, or (b) that everything is knowable apart from the individual objects and their first-order properties, or (c) that everything is knowable apart from the individual objects, their first-order properties and their relations. Each of these, Psillos claims, specifies a different version of epistemic structural realism. But which one do we choose? In other words, where exactly do we draw the line between what is knowable and what is not? Psillos thinks that option (c) “is the only characterisation of ESR which

can impose a principled limitation on what is knowable” (2001a, S21). But (c), according to him, is questionable since it commits us to the idea that some properties are unknowable *in principle*. He says:

...it isn't clear why the first-order properties of unobservable entities are unknowable. They are, after all, *part and parcel of their causal role*. So, if all these entities are individuated and become known via their causal role, *there is no reason to think that their first-order properties, though contributing to causal role, are unknowable* (2001a, S17) [my emphasis].<sup>17</sup>

It is thus implied that traditional varieties of scientific realism, of which Psillos is an advocate, are more reasonable than ESR because they do not preclude first-order properties from being knowable in principle.

Let us, first of all, make a clarification. Although (c) comes close to a faithful characterisation of ESR, it misrepresents the position in one important respect. ESR does not hold that we have absolutely no knowledge of the first-order properties of external world objects. Rather it holds that first-order properties of external world objects are knowable up to isomorphism. More precisely, ESR is best captured by (c'): Everything in the external world, i.e. objects, properties, and relations, is knowable up to isomorphism.

Since I presented isomorphism as a relation that holds between relations or structures, I must explain what I mean by the claim that *objects* and *properties* can be known

isomorphically. Structures specify objects, relations, and, potentially, one-place properties.<sup>18</sup> We take abstract structures to represent a certain isomorphism class of concrete structures, i.e. to represent the concrete structures isomorphically. Given this character of abstract structures, the contents of their domains of objects and any one-place properties (understood as sets) cannot be uniquely specified. Only their cardinalities and the (logico-mathematical properties of the) relations they stand in can be specified. For example, we can say that property  $P$  is instantiated by three objects and property  $Q$  is instantiated by two objects and that relation  $R$  holds between objects with  $P$  and objects with  $Q$ .<sup>19</sup> Thus, to say that we know objects or properties isomorphically just means that we know them to the extent that they are specified by abstract structures.

Despite Psillos' misconception of Russellian ESR, his objection can be reformulated thus: Why should properties of the external world be epistemically inaccessible beyond the level of isomorphism?

A satisfactory answer to this question can be given and finds some support in science. Optical science, for instance, tells me that when I see a coloured object it is the result of incident light waves of a given wavelength hitting my retina and producing nerve impulses that travel all the way to my brain where the relevant perception is formed. It thus tells me that the perception of colour gives us some information about the external world.<sup>20</sup> If I see two otherwise *perceptually identical* objects, one of which appears red and the other green, then, *ceteris paribus*, I postulate that there must be some difference in the two objects responsible for the difference I identify in perception.<sup>21</sup> In optics this

difference presumably arises from the different properties of the surface of the two objects, which determine the wavelength composition of the light reflected from them. This is just the H-W principle in use. I infer that there must be a non-empty set of properties that one object has while the other does not. (NB: If I am colour blind I may not be able to tell the difference but that just means that I will not pick up on this relation. The H-W principle holds that provided we identify a difference in perception, we should postulate that this corresponds to a difference in the world. It *does not* guarantee that we will identify a difference. It is the W-H principle that requires that there be a corresponding difference in perception provided there is a difference in the world).

On the basis of Russell's programme of rational reconstruction, although I can infer that there is a non-empty set of properties that one object has while the other does not, I cannot infer exactly what these properties are. To gain more information about these properties and their objects I must make more observations. In particular, I must find out whether any relations hold at the perceptual level. Placing the two objects under the microscope, for example, would presumably reveal such relations. Supposing MR to hold, I infer that the perceptual relations revealed under the microscope reflect relations between the constituent parts of the objects. So, at best, I know certain relations between these constituent parts of the objects, but I do not know the constituent parts themselves. But this (extensional) way of knowing a relation without knowing the relata just amounts to knowing the logico-mathematical properties of the relation. This is equivalent to saying that we know these relations, and the structure they specify, up to isomorphism.

More pertinently, it means that the properties of the relata can only be specified up to isomorphism.

Psillos insists that first-order properties are ‘part and parcel of their causal role’ and thus must be knowable. ESR does not deny that first-order properties are an essential or integral component of the causal chains that lead up to our perceptions. But it is one thing to argue for this, and quite another to claim it shows that we have epistemic access to the first-order properties, or indeed higher-order properties, of physical objects beyond the isomorphic level. Pending a more detailed argument explaining why this is the case, I do not see any force to this dimension of Psillos’ argument.

## **6. Conclusion**

Neither of Psillos’ objections stands up to scrutiny. The bottom line is that, provided one accepts Russell’s principles H-W and MR, some inferential knowledge about the structure of the external world can be safeguarded. The principles are questionable but not the target of Psillos’ critique. It remains to be seen whether H-W and MR can stand up to scrutiny. This, of course, is a topic for another paper.

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## FOOTNOTES

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<sup>1</sup> Percepts take on the role of the objects of direct acquaintance. In Russell's event-based ontological framework percepts are ultimately events that arise in one's head/mind. Though percepts are elusive entities, I will, just like Psillos, employ the term without commitment to any ontological scheme, but rather to convey that what is of importance is, loosely speaking, what we experience.

<sup>2</sup> Psillos (2001a) suggested this name for the principle on the basis of Helmholtz's and Weyl's appeal to it. Russell sometimes states the principle in its contrapositive (but equivalent) form, namely same causes imply same effects. Even Hume seems to endorse this principle as he advertises in the *Treatise* that "Like causes still produce like effects" (Book II, Part III, §1).

<sup>3</sup> Stimuli, according to Russell, are "the events just outside the sense-organ" (1927, 227). They are thus classed as physical events. Russell speaks about stimuli and physical objects interchangeably because he considers the former as lying in causal chains that can be traced to the latter. In other words, inferring something about the stimuli is seen as inferring something about the physical objects.

<sup>4</sup> In his own words: "My point is that the relations which physics assumes... are not identical with those which we perceive in the visual field, but merely correspond with them in a manner which preserves their logical (mathematical) properties" (1927, 252).

<sup>5</sup> The definition of structure sometimes includes a third condition, i.e. a set  $O$  of operations on  $U$  (which may be empty). This condition is optional because operations are functions and thus can be regarded as special kinds of relations capturable by condition two. A structure may also specify one-place relations, i.e. properties.

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<sup>6</sup> The view here is that whatever can be described in the language of mathematics and logic will be described only up to isomorphism.

<sup>7</sup> The equinumerosity requirement simply reflects the fact that for there to be a bijection between two sets, the sets must have the same number of objects.

<sup>8</sup> For more on the Poincaréan approach see Worrall (1989).

<sup>9</sup> For more on ontic structural realism see French and Ladyman (2003). Objections to this view can be found in a paper given by Anjan Chakravartty (2003) at the last PSA.

<sup>10</sup> This is an important detail that Psillos fails to mention in his discussion.

<sup>11</sup> As I indicate below, H-W can also be violated under some circumstances.

<sup>12</sup> Perhaps this illustrates why Russell refrains from saying that we can know the structure of the physical world and instead holds that we can ‘infer a great deal’ about it.

<sup>13</sup> E’ will be different from E only if the cardinality of E is greater than the cardinality of D.

<sup>14</sup> Notice that Psillos uses the terms ‘percept’ and ‘phenomena’ interchangeably. Though it is good practice to keep the two apart, I follow Psillos in using them interchangeably provided that the context allows it.

<sup>15</sup> This quote appears in the midst of Psillos’ discussion of the second horn of the dilemma but can be mustered here since it is an objection to the view that ESR sanctions embeddings.

<sup>16</sup> In discussing this first objection at the PSA meeting, Psillos conceded that if ESR asserts that isomorphic specification is the (perhaps ideal) limit of knowledge then his objection no longer holds. His only reservation was whether or not Russell endorsed this view.

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<sup>17</sup> See also (1999, 156).

<sup>18</sup> Recall that in the beginning of this paper, I indicated that a structure may also specify one-place properties not just relations, i.e. not just  $1+n$ -place properties, where  $n$  is a positive integer.

<sup>19</sup> An additional, crucial, component of ESR knowledge claims that is not contained in the isomorphism claim and is therefore sidelined here is the idea that the physical system exemplifying an abstract structure  $S^*$  can be indirectly identified as that system which is causally responsible for the concrete observational structure that led us to infer  $S^*$ .

<sup>20</sup> I am aware of the voluminous philosophical literature on colours. What I say here bodes well with eliminativist theories of colour, according to which physical objects have no colour.

<sup>21</sup> One potential worry here might be that colour, as well as other such properties, do not correspond to things or the structure of things in the world, but rather are products of our neurological apparatus and the external world. Such an objection would miss the point however since the external world, according to Russell, encompasses all that is non-perceptual which includes the neurological. After all, is not our nervous system composed of physical entities?