Is decoherence necessary for the emergence of many worlds?

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March 15, 2023

Abstract

It is usually thought that decoherence is necessary for the emergence of many worlds. In this paper, I propose a thought experiment and argue that the decoherence requirement leads to a contradiction.

The many-worlds interpretation of quantum mechanics (MWI) assumes that the wave function of a physical system is a complete description of the system, and it always evolves in accord with the linear Schrödinger equation. In order to solve the measurement problem, MWI further assumes that after a measurement with many possible results there appear many equally real worlds, in each of which a definite result occurs (Everett, 1957; Barrett, 2018; Vaidman, 2021). This many-worlds assumption is supported by an extensive analysis of decoherence and emergence in the modern formulations of MWI (Wallace, 2012). In this paper, I will propose a thought experiment and argue that if the decoherence condition is required for the emergence of many worlds, then there will be a contradiction.

Suppose there is a closed system containing two experimenters Alice and Bob. They are initially in an entangled state:

$$|0\rangle_{Alice} |1\rangle_{Bob} + |1\rangle_{Alice} |0\rangle_{Bob}, \tag{1}$$

where $|0\rangle_{Alice}$ and $|1\rangle_{Alice}$ are two result states of Alice in which she obtains the results 0 and 1, respectively, and $|0\rangle_{Bob}$ and $|1\rangle_{Bob}$ are two result states of Bob in which he obtains the results 0 and 1, respectively.

Consider a unitary time evolution operator U_N which changes $|0\rangle_{Alice}$ to

$|1\rangle_{Alice}$ and $|1\rangle_{Alice}$ to $|0\rangle_{Alice}$ after a time interval T.[1] Then by the linearity of the dynamics, the time evolution of the initial state under U_N is

$$|0\rangle_{Alice}|1\rangle_{Bob} + |1\rangle_{Alice}|0\rangle_{Bob} \rightarrow |1\rangle_{Alice}|1\rangle_{Bob} + |0\rangle_{Alice}|0\rangle_{Bob} \tag{2}$$

At each instant $t \in [0, T]$ during the evolution, $U_N(t)$ can be defined as follows:

$$U_N(t)|0\rangle_{Alice} = \alpha(t)|0\rangle_{Alice} + \beta(t)|1\rangle_{Alice}, \tag{3}$$

$$U_N(t)|1\rangle_{Alice} = \alpha'(t)|0\rangle_{Alice} + \beta'(t)|1\rangle_{Alice}, \tag{4}$$

where $\alpha(0) = 1$, $\alpha(T) = 0$, $\alpha'(0) = 0$, $\alpha'(T) = 1$, $\beta(0) = 0$, $\beta(T) = 1$, $\beta'(0) = 1$, and $\beta'(T) = 0$. Note that the unitarity of $U_N(t)$ will keep the orthogonality of the two states of Alice during the evolution.

Now the state of Alice and Bob at each instant t during the evolution is

$$[\alpha(t)|0\rangle_{Alice} + \beta(t)|1\rangle_{Alice}]|0\rangle_{Bob} + [\alpha'(t)|0\rangle_{Alice} + \beta'(t)|1\rangle_{Alice}]|1\rangle_{Bob} \tag{5}$$

This state can also be written as follows:

$$[\alpha(t)|0\rangle_{Bob} + \alpha'(t)|1\rangle_{Bob}]|0\rangle_{Alice} + [\beta(t)|0\rangle_{Bob} + \beta'(t)|1\rangle_{Bob}]|1\rangle_{Alice} \tag{6}$$

Then, $U_N(t)$ will equivalently evolve the states of Bob as follows:

$$U_N(t)|0\rangle_{Bob} = \alpha(t)|0\rangle_{Bob} + \alpha'(t)|1\rangle_{Bob} \tag{7}$$

$$U_N(t)|1\rangle_{Bob} = \beta(t)|0\rangle_{Bob} + \beta'(t)|1\rangle_{Bob} \tag{8}$$

This means that the unitarity of $U_N(t)$ will ensure that the two states of Bob are also orthogonal during the evolution.[2]

Now an interesting question arises: what worlds does the state (5) or (6) correspond to in MWI? By (5), since the two states of Alice, $\alpha(t)|0\rangle_{Alice} +$

[1]For a Hilbert space with dimension greater than two, the swap operator U_N can be accomplished in many ways, such as with a 180 degree rotation about the ray halfway between the two state vectors. Admittedly U_N involves anti-thermodynamic manipulation of macrocopically many degrees of freedom. But for a unitary theory like MWI, U_N can be accomplished in principle, although the accomplishment is extremely difficult. Note that a similar thought experiment involving the swap operator U_N was first proposed and discussed by Gao (2019).

[2]In a two-dimensional Hilbert sub-space, $U_N(t)$ can be represented as $\begin{pmatrix} \alpha(t) & \alpha'(t) \\ \beta(t) & \beta'(t) \end{pmatrix}$. Then its unitarity implies the relation $\alpha(t)\beta^*(t)+\alpha'(t)\beta'^*(t) = 0$, which means that the two states of Bob, $\alpha(t)|0\rangle_{Bob} + \alpha'(t)|1\rangle_{Bob}$ and $\beta(t)|0\rangle_{Bob} + \beta'(t)|1\rangle_{Bob}$, are also orthogonal.

$\beta(t)\,|1\rangle_{Alice}$ and $\alpha'(t)\,|0\rangle_{Alice} + \beta'(t)\,|1\rangle_{Alice}$, are orthogonal and thus Bob's two result states are decohered,[3] there are two (or two sets of) worlds, in one of which Bob obtains a definite result 0, and in the other Bob obtains a definite result 1. This is the answer from the point of view of Bob. On the other hand, by (6), due to the similar reason, there are also two (or two sets of) worlds, in one of which Alice obtains a definite result 0, and in the other Alice obtains a definite result 1. This is the answer from the point of view of Alice. Since the two answers are incompatible with each other, there is a contradiction here.

It is worth emphasizing that the state (5) or (6) only corresponds to two worlds and it does not correspond to four worlds according to the decoherence requirement. For example, the first two terms in (5), namely $[\alpha(t)\,|0\rangle_{Alice} + \beta(t)\,|1\rangle_{Alice}]\,|0\rangle_{Bob}$ does not correspond to two worlds for Alice, since the decoherence condition is not satisfied for her.

There is another formulation of the contradiction concerning the changes of worlds. According to MWI, the initial state of Alice and Bob corresponds to two worlds. In one world, Alice obtains a definite result 0 and Bob obtains a definite result 0, and in the other world, Alice obtains a definite result 1 and Bob obtains a definite result 1. Similarly, the final state of Alice and Bob also corresponds to two worlds. In one world, Alice obtains a definite result 0 and Bob obtains a definite result 1, and in the other world, Alice obtains a definite result 1 and Bob obtains a definite result 0. Then, how do the states of Alice and Bob change in each world during the above time evolution?

Since the states of Alice and Bob in the branches of both the initial state and the final state are symmetrical, the answers to this question for Alice and Bob must be the same. But, as we will see, this is impossible. By Eqs. (3) and (4), since the states of Alice, $\alpha(t)\,|0\rangle_{Alice} + \beta(t)\,|1\rangle_{Alice}$ and $\alpha'(t)\,|0\rangle_{Alice} + \beta'(t)\,|1\rangle_{Alice}$, are always orthogonal during the time evolution and thus the decoherence condition is satisfied for Bob, Bob's state should not change in each world during the evolution. On the other hand, by Eqs. (7) and (8), since the states of Bob, $\alpha(t)\,|0\rangle_{Bob} + \alpha'(t)\,|1\rangle_{Bob}$ and $\beta(t)\,|0\rangle_{Bob} + \beta'(t)\,|1\rangle_{Bob}$, are always orthogonal during the time evolution and thus the decoherence condition is also satisfied for Alice, Alice's state should not change in each world during the evolution either. However, the time evolution (2) shows that Alice's and Bob's states cannot both keep unchanged in each world; when Alice's state keep unchanged, such as from $|0\rangle_{Alice}$ to $|0\rangle_{Alice}$, Bob's state must change, such as from $|0\rangle_{Bob}$ to $|1\rangle_{Bob}$, and vice versa. This is a contradiction.[4]

It can be seen that the above contradiction results from the decoherence

[3] We can regard Alice as Bob's environment, and vice versa.

[4] The contradiction can also be formulated in a higher-dimensional Hilbert space. For example, in a 3D Hilbert space, the time evolution will be $|0\rangle_{Alice}\,|1\rangle_{Bob} + |1\rangle_{Alice}\,|2\rangle_{Bob} + |2\rangle_{Alice}\,|0\rangle_{Bob} \rightarrow |1\rangle_{Alice}\,|1\rangle_{Bob} + |2\rangle_{Alice}\,|2\rangle_{Bob} + |0\rangle_{Alice}\,|0\rangle_{Bob}$.

condition required for the emergence of many worlds. If the requirement is dropped, then we can avoid the contradiction by assuming that the state (5) or (6) corresponds to four worlds, in each of which Alice and Bob both obtain a definite result, either 0 or 1. In other words, for example, the first two terms in (5), namely $[\alpha(t)\,|0\rangle_{Alice} + \beta(t)\,|1\rangle_{Alice}]\,|0\rangle_{Bob}$ correspond to two worlds for Alice, even if the decoherence condition is not satisfied for her.

To sum up, I proposed a thought experiment and argued that the decoherence condition is not necessary for the emergence of many worlds; otherwise there will be a contradiction.

Acknowledgments

I wish to thank David Deutsch, Meir Hemmo, Don Page, Simon Saunders, Paul Tappenden, and Lev Vaidman for helpful discussion. This work was supported by the National Social Science Foundation of China (Grant No. 16BZX021).

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