The ontological burden of mathematics and scientific realism[*]

Rafael Andres Alemañ Berenguer[†]
rafael.aleman@ua.es

Abstract: Mathematical modelling of nature, due to its accuracy and universality, plays a key role in the scientific inquiry of the world. So important its function is that some authors have defended the existence of an ontological burden in the mathematical formalism used by scientists. According to this opinion, the appeal to certain formalism would entail an implicit commitment to the type of entities that populate the material world. In this paper, the aforementioned thesis will be analysed, as well as other versions of mathematical Platonism, with the conclusion that there are no reasons to support it and that, therefore, it does not pose a threat to a realistic metaphysics in any of its modalities, such as structural realism.

Keywords: ontology, realism, mathematical structure, theoretical representation

1. Introduction: ontologies in mathematics and physics

Galileo's well-known observation that nature is written in mathematical characters is often cited as epitomizing the intellectual substratum that fuelled the Scientific Revolution of the 16th and 17th centuries. It said verbatim as follows (Galileo 1981 [1623], p. 62-63):

> «[…].Philosophy is written in that huge book that we have open before our eyes, I mean, the universe, but it cannot be understood if one does not first learn to understand the language, to know the characters with which it is written. It is written in mathematical language and its characters are triangles, circles and other geometric figures, without which it is impossible to understand a single word; without them it is like spinning vainly in a dark maze.»

However, such a point of view already had a long history when taken on by Galileo. Around the 6th century BC, Pythagoras of Samos had proclaimed that numbers are the ultimate foundation of natural reality, setting a tradition that continues up to our present times in the eccentricities of some few idealists. Plato developed certain aspects of the Pythagorean doctrines to build his own philosophy, presided over by a radical split between the material and the ideal world, where the first realm was regarded as a sad shadow cast by the second one. Platonic dualism left open the question of how ideal

---

[*] To appear in *Principia: an international journal of epistemology.*

[†] Department of Physics, Systems Engineering and Signal Theory. University of Alicante (Spain)

entities were related to material objects, a question that has remained unresolved in all subsequent versions of idealism.

Mathematical Platonism seemed to receive a strong boost during the 20th century in the wake of Willard Van Ornam Quine's (1960, 1969) argument for the real existence of mathematical objects, according to which the quantification of mathematical objects is indispensable for science, either in its formal or factual facet. Now, if we have to admit this quantification as true, an ontological commitment to the veracity of such mathematical entities is derived from it, whose reality we must accept. Quine never came to specifically formulate this thesis, which was later reproduced by Hilary Putnam (1971), even though this author never fully endorsed it (Colyvan 2001; Deacock 2002; Liggins 2008; Sereni 2015; Molinini, Pataut, and Sereni 2016; Bueno 2018).

Quine's indispensability, despite his popularity in certain mathematical and philosophical circles, soon received quite remarkable counterattacks. This is how the work of the Franco-American Paul Benacerraf (1965) was interpreted, in which he insisted, more than on the numbers themselves, on the importance of the structures that arithmetic involved. From this change of perspective arose the so-called identification problem concerning the existence of various candidates –the ordinals of Zermelo and those of Von Neumann– for the treatment of natural numbers in terms of set theory. Later this very author exposed the epistemological problem (Benacerraf 1973), by virtue of which a justification of mathematical truth could be coherent with either a general semantic theory or a general epistemological theory, but not both at the same time.

What was possibly the strongest attack against mathematical Platonism came from the ranks of the fictionists, who refused to grant mathematical entities the same reality as physical objects (Field 1980, 1989; Burgess and Rosen 1997; Balaguer 1998; Yablo 2002; Bueno 2009; Leng 2009; Kasa 2010; Liggins 2010; Nutting 2020; Clarke-Doane 2022). Fictionism holds that mathematical concepts are invented, not discovered, because the discovery itself only occurs when we operate with material systems, which are the only ones existing in this sense. The ideal entities of mathematics, therefore, do not inhabit an otherworldly limbo intellectually apprehensible, but exist only formally as part of the mental (cerebral) activity of specific individuals, who are generally able to communicate and share their ideas. These individuals –on the fictionalist view– possess sufficient cognitive faculties to detect the mathematizable aspects of the objective regularities that govern nature, which would explain the outstanding practical success of modern science (Bunge 1974, 1977; Marquis 2006, 2011, 2012, 2018; Thomasson 1999; Thomas 2000, 2002). The reason that the world turns out to be mathematizable, at least in part, remains outside the walls of fictionism.

The triumph of the empirical-mathematical method that characterizes modern science also invites us to ask ourselves about the ontology of material systems that happen to be so liable to abstract formalization in the hands of theorists. From the Aristotelian duo *substance-accident* to the Kantian pair *noumenon-phenomenon*, the Western tradition has shown a remarkable tendency to divide reality into, say, a window display (appearances) and a mysterious back room (being itself). That this is not the only possibility affordable to us is proved by the fact that, at least since Hume, the metaphysical theory of objects as bundles of properties has been available. From this point of view, material objects are identified with the network of their properties, understood as the "modes of being" of things. There would not be, then, a substratum to which we could reach at the limit –even theoretically– by depriving things of all their properties (Lafrance 2015; Barker and Jago 2018; Robert 2019).

The combination and interconnection between those modes of being that we call properties will provide something more than a simple mereological sum, giving rise to the complex of properties and relationships that we call "object" in its broadest sense. The usual objection consists in pointing out the problem of change: when one of the properties that make it up is altered, the beam changes and, therefore, we would no longer be talking about the same object. This critique is greatly weakened when the physical object is identified with the chronological series of bundles of properties that constitute it at every instant, so that the change is incorporated into its very entity. Consequently, it is the structure of relationships between its properties over time that gives each thing its particular identity.

Having established the foundations of the discussion, we now come to its main core, which is the possibility that the mathematical scaffolding carries an ontological load that influences, perhaps unsuspectedly, our scientific inquiry into the natural world. And if so, what would be the ontological content that the use of one or another mathematical repertoire inevitably provides us with in our theories about the material world?

In the following sections we will address three possible interpretations of the presumed ontological load carried by the mathematical tools with which scientists operate. We will begin in the second epigraph with the most radical version of Pythagoreanism, in whose opinion all mathematical structure must be carried out in some way in the material world, and after rejecting it, in the third section we will move on to a moderate version in which it is argued that, although not every mathematical idea has to manifest itself in nature, everything we can scientifically know about reality does obey some mathematical prescription. The assumption that the choice of a specific mathematical tool anchors our theories on certain reference classes will be examined in the fourth section. The fifth section will consider whether the previous discussions can undermine our confidence in realism as the founding philosophy for scientific work, with special attention to its influence on structural realism. Finally, the sixth section will present some brief conclusions that will close this article.

2. Modern Pythagoreanism and its objections

A first meaning that could be associated with the ontological burden of mathematics gives us back to the Pythagorean spirit, whereby the ultimate foundation of reality is mathematics (Tegmark 2008; MacDonnell 2016; Baron 2023). And since everything arises from it, it would be in some way more real than matter itself. The Swedish-American Max Tegmark, defender of cosmic plurality under the name of “multiverse”, seems to have something like this in mind, a confusing concept that is usually dealed with an ease unbecoming of its intrinsic darkness. In Tegmark's opinion, one can speak of multiple universes in various senses, one of which includes the totality of conceivable mathematical structures, although we do not observe them as physical realities in our universe. It constitutes the most abstract class of possible multiple universes considered by this author and, therefore, the least intuitive of all.

Even if we do not venture into the depths of parallel universes, it would be enough for us to remain in ours and subscribe to a more refined Pythagoreanism by claiming that all self-consistent mathematical ideas will be realized in one way or another in the physical world, although we have not yet discovered them. If so, there would be no ontological burden of mathematics greater than this, as suggested by the discovery of unsuspected physical interpretations in the Riemannian function ζ (Bender; Brody; Müller 2017; Bourgade & Keating 2012) as well as an algorithm for calculating π in the configuration of quantum levels of the hydrogen atom (Friedman & Hagen 2015). However, a closer examination of the situation points in just the opposite direction.

A well-known example is provided by the Majorana fermion, a type of elementary particles theoretically studied at the beginning of the 20th century by the Italian physicist

Ettore Majorana, whose existence has not been yet verified with absolute certainty. This sort of particle is characterized by being itself its own antiparticle, as explained by the detailed mathematical characterization due to its author (Pal 2011). And nonetheless they seem not to exist in nature; almost all known elementary particles violate Majorana's theory, although no one knows why. The only exception is the neutrino, whose assignment in this sense continues to be the subject of controversy.

An even more obvious case of a non-existent physical object, although mathematically well defined, is that of the magnetic monopole. Unlike the two types of electric charge that exist separately in their corresponding elementary particles, there does not seem to be a counterpart to this situation in the physical reality for the magnetic case. The magnetic poles conventionally called north and south always appear in pairs, which is the reason why we speak of magnetic dipoles. From a theoretical point of view, the discovery of even a single monopole would help explain why electric charges appear quantized –that is, in discrete quantities as multiples of a fundamental unit– in the universe. However, even when the mathematical treatment of magnetic monopoles is solid and solvent (Dirac 1931; Malkus 1951;'t Hooft 1974; Drukier and Nussinov 1982; Cho and Maison 1997; Milton 2006), no indication has been shown so far that leads us to think about the existence of a real correlate of such a concept.

In order to apply the entire mathematical apparatus of the infinitesimal calculus, it is essential not only to guarantee the smooth and differentiable character of our base manifold –specifically, physical space-time– but also that the differentiable structure defined on it be unique. Otherwise, we could not be sure that the integration or differentiation operations provide a unique result, and therefore we would not know whether we are comparing the appropriate numerical quantities with the experiments.

However, four-dimensional manifolds such as our space-time, far from having a unique differential structure, admit an uncountable infinity of them (Donaldson 1990; Donaldson & Kronheimer 1990). Manifolds with a lower dimension always have a single differentiable structure, while those with a higher dimension can be classified according to whether they are differentiable or not. The simplest 4-dimensional manifold, R^4, other than accepting atypical differentiable structures, can also possess an uncountably infinite number of non-equivalent differentiable structures (Sładkowski 1996; Taubes 2000).

In quantum field theory (especially when dealing with typically non-abelian Yang-Mills extensions of the Standard Model) computational artifacts, called "ghosts", are routinely introduced to ensure the theory's self-consistency. These quantum ghosts are non-physical states incorporated into the system despite their lack of physical meaning for purely instrumental reasons (DeWitt 2003). It is a kind of formal scaffolding, a useful fiction, which allows us to preserve convenient symmetries and restrictions for the internal coherence of theories as widely used as Feynman path integrals. These "ghosts" constitute further proof of the prolixity of tools offered by mathematics without direct connection to the physical world.

Nor can we forget the problem of supersymmetry, a special type of symmetry that aims to link the two basic types of elementary particles considered in the standard model, fermions and bosons. These two classes are distinguished by the values of the internal quantum number that we call spin, with integer spins for bosons and half-integer spins for fermions. This schism would presumably become closed due to supersymmetric operations capable of allowing exchanges between both families of particles, at the price of introducing a whole new taxonomy of superparticles, hidden companions of the ones already known. And since the bosons substantiate the fundamental interactions, while the fermions constitute matter at its deepest level, supersymmetry comes to offer us a certain equalization between matter and forces. In the last quarter of the 20th century, supersymmetry raised hopes of some grand unified theory (GUT) encompassing all the fundamental forces except gravity. If verified, the supersymmetric features of this family of theories would make it possible to compensate for certain infinite divergences that usually hinder models without supersymmetry.

No matter how beautiful supersymmetric theories may look to their supporters, the unquestionable fact is that nothing tells us that they really occur in nature (or, we would rather say that it is not a property of the theories with which we represent the course of natural phenomena). His defenders do not give up easily and assume that supersymmetry has suffered some kind of breakdown. This means that the superparticles must have a much greater mass than those discovered to date, which is the reason why they have not yet been observed despite the intense experimental efforts dedicated to it. However, after decades of unsuccessful search, it was decided to incorporate supersymmetry as the main ingredient of string theories –thereafter superstrings– although the superstring paradigm itself languishes after more than half a century without conclusive results (Smolin 2016).

A much less noticed mathematical superfluity is called "functional freedom", in relation to the profuse diversity of mathematical expressions with which we can describe the same natural phenomena (Penrose 2017). Without too much rigor it could be said that, when the field of a physical quantity takes its values in a certain manifold V, then what we call a configuration of the field would be identified with a section of the fiber bundle of V over space-time; in other words, we would have a function that associates values of V with points in space-time. The key here is to realize that the space of functions in V –field settings– is a manifold of infinite dimension (assuming that the local domain of these functions is compact in order to avoid major complications). Thus, if the degrees of freedom of a field at a certain point are parameterized by an infinite-dimensional manifold, V, then the spatiotemporal degrees of freedom would also have to be parameterized (mapped) by a space of functions also of infinite dimension.

The dimensionality of V does not always have to be infinite, so V_N could be written to indicate any number N of dimensions. The reluctance of critics such as Penrose in this regard is due in large part to attempts to subsume theories dependent on V_N in others characterized by $V_{N'}$ when usually $N << N'$. Therefore, Penrose's critique stresses that in such a case the vastly greater –sometimes infinitely greater– number of field configurations that N' would allow should manifest themselves in some much more visible way (to the point that, under certain conditions, they could make the models deduced from extended theories physically incoherent). If this is not the case, it should be explained why, as it would be –in Penrose's opinion– the typical situation for superstrings nowadays.

The examples detailed in the previous paragraphs, taken from whatever area of physics, point in the same direction: the mathematical language used to study nature has redundancies and annexes without a straight relation to the real world. This fact suffices to refute the possibility that the ontological burden of mathematics –if such a thing exists– relies on its ability to create reality.

3. Nature as a mathematical book

If we discard the idealist solution that identifies mathematics with the ontological root of the material world, perhaps a softened version of this extreme view could be acceptable. It may not be true that every conceivable mathematical structure –already known or still to be elucidated– has to be carried out in nature, although it could well be that all the phenomena of the universe obey some mathematical guideline, presumably within the reach of human reasoning. Expressed in other words, this is the message of Galileo's oft-repeated observation about the book of nature, mentioned in the introduction.

Perhaps not all the mathematical concepts that inhabit the Platonic limbo have their correlate in the material world, but all existing natural processes would have a mathematical counterpart in that ideal realm. This is a parallel approach to that of natural philosophy understood as the search for a general comprehension of nature through mathematics as a basic conceptual tool (Truesdell 1966, p. 86 – 88):

> «The first objective of natural philosophy is to describe and study natural phenomena by means of the most accurate mathematical concepts. The most accurate ones need not be the most fashionable, although they can be; […]. The second difference in method is deeper. Most physical scientists regard mathematical treatment as belonging only to a later stage in the development of a theory. […]. In modern natural philosophy, the physical concepts themselves are made mathematical from the start, and mathematics is used to formulate theories.»

Whether admitted or not, this seems to have been the implicit creed that has guided modern scientists and many of the ancient philosophers in their inquiries into the cosmos. The very word "cosmos" comes from the Greek word "order" (κόσμος), as opposed to "disorder" or "chaos" (Χάος); and one might wonder where we can find a greater perfection than in the mathematical order. Beyond assessing meanings, repelling inconsistencies and quantifying with precision, the use of mathematics in scientific research tends to exert an irresistible fascination in its most devoted practitioners who seem to be nearly always –a genuine perplexity– on the right track. This is not invariably the case, but with an intriguing frequency the course of nature seems to run along paths previously paved with mathematical ideas whose connection with physics had not been intuited before.

Ptolemy and his followers claimed to describe the celestial trajectories by means of an abstruse superposition of circles, the epicycles and deferents. This model, as long-lived as it was, was surpassed by modern astronomy, although two thousand years later a great French mathematician partially vindicated the Alexandrian astronomer. Thanks to the Fourier series, we now know that an adequate combination of circular functions –sines and cosines– manages to reproduce any closed curve with the desired degree of accuracy, which explains the relative effectiveness of the Ptolemaic method (Bochener 1991). Kepler failed in his attempt to match the orbits of the planets visible at the time to the beauty of the Platonic solids (Koyré 1966), although he did take the first step in discovering that the physically real orbits consist of the conic curves studied millennia earlier by the Greek Apollonius.

Newton and Leibniz created the infinitesimal calculus along with their mathematical studies of motion, studies that culminated in the work of Euler and the eighteenth-century geometers, true architects of rational mechanics in its classical form. The fusion of mathematical analysis with geometry gave rise to differential geometry, today an essential tool in the theories of fields and continuous media. When the German Woldemar Voigt introduced the tensor concept in the 19th century to deal with stresses in elastic solids, he could not imagine that several decades later, in the hands of Einstein, his creation would be used to illuminate a new concept of gravitation and a novel cosmology.

The examples could be multiplied, although if we look for a claim in favour of the indispensability of mathematics in natural science we will find few as powerful as the one offered by the method of gauge symmetries. After an unsuccessful attempt by Herman Weyl in 1918 to incorporate gravity and electromagnetism as properties of space-time, it was soon found that its true utility laid in the then nascent quantum physics (Alemañ 2015). When we take the characteristic phase angle of the wave functions and subject it to a global modification by changing its value by the same amount at all points in space-time, the dynamical equations of the theory remain invariant. This symmetry is lost by relaxing the requirements for the phase change when allowing it to occur locally, that is, differently at every spacetime point. Such a procedure is equivalent to now considering the phase as a field variable, due to which the invariance of the equations will only be preserved by introducing a new field that compensates in every place the arbitrary variation of the phase. This new field, susceptible to quantization, turns out to be the one corresponding to the electromagnetic interaction.

And it is not the only one; by using the currently most successful model, fundamental particles are conceived as localized excitations of quantum fields subject to two types of continuous symmetries. One of them is the space-time symmetry of the Poincaré group $SO^{+}(1,3)$, which requires associating a quantity called spin with every quantum field, with a semi-integer value for fermions and an integer value for bosons. On the other hand, there are gauge symmetries associated with the interactions between particles, as the product of the Lie groups $SU(3)\times SU(2)\times U(1)$. Demanding invariance of the fields under local transformations of these gauge symmetries allows us to deduce the existence of strong, weak and electromagnetic interactions.

It would seem that we are dealing here with an obvious case of ontological determination of a physical entity (for example, the electromagnetic field) by means of a formal requirement (the gauge symmetry). A closer look will reveal that things are not exactly like that. The electromagnetic field –it is true– follows from gauge symmetry when it is imposed to preserve the invariance of the dynamic equations of the theory. But it is no less true that such invariance comes to give a mathematical appearance to characteristics of the physical world, such as the homogeneity and isotropy of space-time or the indistinguishability of some reference frames. It is formally expressing a specific hypothesis about the universe –that can be verified or not– summarized in the claim that, if natural processes can be grouped into certain equivalence classes, the equations that rule them must present certain symmetries that guarantee the equality of their mathematical framework in such equivalent cases.

Analysing the question with the care it deserves, we find a fundamental difficulty in the language used. Is it really correct to say that nature is mathematical in the sense that mathematical formalisms are properties of the material world itself? Perhaps it would be much more prudent to limit ourselves to accepting that the mathematical character belongs to our statements about the universe, and not so much to the universe itself. The question then acquires another dimension, since we would immediately have to question ourselves about the reasons that make the use of a mathematical language so effective in intelligibly representing the events of the cosmos. Kant would not have hesitated for a moment in answering that the concepts of mathematics arise from the a priori forms of our sensibility and for this reason they are inescapably accurate. It is not that mathematical tools have been used in the creation of the cosmos, but rather that human understanding is shaped in such a way that it cannot grasp it in any other way. From this perspective, if there are parcels of reality outside the power of mathematics, we will never be able to know it since our knowledge will never cross the borders of what can be mathematized (Kant 1989, p. 30 – 31).

Those who consider the Kantian proposal largely superseded by the passage of time may be tempted to replace the a priori forms of intuition with Darwinian natural selection, transferring the unknown to a new stage. The possession of great mathematical gifts far exceeds the qualities necessary for our most distant ancestors to survive in the wild, so that it is very doubtful to attribute their origin to the patient labour of natural selection. Arguing that it would be a formidable evolutionary concomitance does not solve the problem either. If the human mind has been evolutionarily forged from inert matter, it is of little use to say that our mathematical aptitudes reflect the character of the regularities to which the matter from which we come is subjected.

With all of this, we are very far from being able to ensure that reality is entirely within the reach of our ability to know it, as Barrow (1997, p. 98-99) acutely points out:

> «We have distinguished between operations that are computable and those that are not. But in real life, being computable might not be very useful if the program that does the required computation takes a million years to do. The world could be mathematical, and even full of computable functions, and yet it could be of such depth and complexity that we would be unable to find them on our fastest computers even if they were running for thousands of years.»

At this point we find ourselves at a crossroads: either we simply admit as a brute fact that the material universe is like this –a fact that a religious temperament would attribute to the divine will– or else we resort to a dubious proliferation of universes among which ours has been selected by a no less debatable postulate, such as the famous “anthropic principle”. Be that as it may, mathematics does not seem to stipulate any ontological constraints on physical reality here, except perhaps that it is comprehensible to the human mind, which would rather be said to be a prerequisite for any rational investigation of the world.

4. Does mathematics require an ontological commitment?

The third possibility to give a clear meaning to the notion of "ontological burden" applied to mathematics comes from the line of thought according to which there is no clear border between the mathematical structure of a scientific theory and its ontology, because the latter is irremediably incorporated into the former. That is to say, by choosing a certain mathematical formalism we would be assuming at the same time, whether we like it or not, an ontological commitment regarding the referents of the theory (Psillos 1995, 1999; Madrid 2010).

In order to guarantee the clarity of the arguments, it would be convenient to specify first what we understand here by ontology as a term opposed to the mathematical structure. Following Bergliaffa, Vucetich and Romero (1993, p. 12):

> «[…]The ontology of the theory is the factual restriction of the set formed by the union of the domains of all the variables related to the logical quantifiers that appear in the axiomatic basis of the theory (by factual restriction we understand a restriction of the domain of the subsets formed by all non-conceptual elements). In the axioms, we quantify on the elements of the generative base or on the conceptual objects generated by it. […]. In our restricted sense, the ontology coincides with the reference class of the theory.»

The reference class of a theory, in its broadest sense, can be made up of conceptual or material objects. In the first case we would have a pure mathematical theory and a physical theory in the second one. Those who advocate the ontological burden of mathematics do not usually reflect on the consequences of applying this notion to mathematics itself; perhaps because it is immediately clear that there is no an inextricable link between a theoretical formalism and its class of reference. An emblematic example is offered by Heisenberg's matrix mechanics, which typically replaces the algebra of functions on the phasic space of classical mechanics by a non-commutative algebra. But non-commutative algebra, considered as a general theory, can be applied in many other situations: the space of irreducible representations of a discrete group, the non-simply connected spaces of a non-abelian fundamental group, the space of sheets in the foliation of a manifold, quantum groups (quantum counterpart of the Lie groups of classical differential geometry), etc. All these cases constitute models of a theory –noncommutative algebra– with a higher degree of abstraction, in a logical sense, not an epistemological one (where "model" would mean the abstract representation of a concrete object).

The traditional geometric connections, from which the concept of curvature derives, expanded its scope due to the notion of bundled spaces (Darling 1994; Taubes 2011). With these tools it was possible to establish links between physical theories based on differential geometry and others apparently far from them, such as the gauge theory of electromagnetism. In the latter, the electromagnetic potential would be equivalent to the connection and the field itself derived from it would play the role of curvature in the pertinent fiber space. Another type of geometry, the so-called symplectic, has turned out to be a basic structure of both classical mechanics and quantum physics, thereby revealing its lack of commitment to any possible external referent.

By granting this freedom to mathematics with respect to itself, is any ontological commitment imposed on physical theories that adopt a certain mathematical language? The answer to this question will critically depend on whether or not a sharp separation between the structure and the ontology of a physical theory is admitted, or rather, between its formal basis and its basis of primitive concepts, which are semantically interpreted by means of the axioms of the theory. In other words, does the choice of a particular formal basis limit the conceptual basis in any way, perhaps restricting the available semantic hypotheses? If so, the same mathematical structure would not be compatible with different ontologies –reference classes– since the ontological neutrality of formalism would be lost.

The history of physics, and even our daily routine, roundly discredits this view. As early as the 19th century, the work of Hamilton and Jacobi demonstrated that the Hamiltonian equations constituted such a non-specific format as to account equally for classical particle mechanics and wave optics (Goldstein 1990). The Heaviside step function and the Dirac delta evolved into a respectable formalism through the theory of Schwartz distributions. Today these generalized functions are used fruitfully in many areas of physics and mathematics far away from their original source.

Especially unfortunate is the recourse to De Broglie's wave-particle duality to maintain that the change in the interpretation of a theory is due to the alteration of its mathematical structure. In the early days of quantum physics, it was not possible to discern that the classical elementary particles should be replaced by time-dependent quantized fields characterized by complex numbers. It is not possible to equate them to elastic or electromagnetic waves (since the Schroedinger equation is not of second order in the derivatives with respect to space and time), so it is extremely wrong to use the phrase "wave mechanics", as is still customary nowadays (Alonso 1994).

Those who reject the possibility of divorcing the mathematical structure of physical ontology –denying that they are essentially independent domains– usually situate the axis of their criticism in the continuous-discontinuous dichotomy, or in terms more typical of natural science, the field-particle antagonism. The thesis that quantum indeterminism broke the marriage between physics and differential equations by introducing discontinuities is belied by the fact that Schroedinger's equation is a differential equation, set aside that eigenvalue equations arose in numerous physical situations –where discontinuities were assiduously handled– long before quantum theory.

Differential equations require a mathematical, not a physical, continuum to be applied to. This means that in order to use them profitably, it is enough to assume that the set of elements under study behaves in some respect as if it constituted a continuous medium, and excellent results are usually obtained with this approximation. For example, the Lotka-Volterra equations for predator-prey dynamics belong to the family of nonlinear first-order differential equations, even though we all know that interacting populations of animals do not form a mathematical continuum at all. Truesdell and Noll (1965, p. 5) underlined this very aptly in the following excerpt, with italics from the original:

> «It is widely misunderstood that continuum theorists believe that matter "really is" continuous, denying the existence of molecules. This is not like this. Continuum physics presumes nothing concerning the structure of matter. It confines itself to relationships between raw phenomena, leaving aside the structure of matter on smaller scales. Whether the continuum approximation is justified, in any particular case, is a matter not for philosophy or the methodology of science, but for experimental testing. […]»

Moreover, since the middle of the 20th century, theorems have been proved according to which the statistical approach –which presupposes a discrete structure– and the continuum one, when applied to material systems, lead to the same macroscopic behaviour. For any set of particles (equal or different from each other, few or numerous, free or affected by any external or internal forces of interaction), subject to any admissible probability distribution, it is possible to define dynamic magnitudes, by means of suitable averages of the variables in the phase space, which exactly satisfy the equations of continuum (Noll 1955; Dahler and Scriven 1963). It does not seem, therefore, that mathematical formalism carries any ontological burden, since it does not incline towards any preference regarding the continuity or discontinuity of its referents and its macroscopic results do not force us to decide between such alternatives.

5. Scientific realism vindicated

Realism, far from being just another metaphysical option, is presented as a precondition of human knowledge in general and of scientific knowledge in particular. No scientist seriously believes that the world whose regularities are explored is a ghostly product of our own consciousness, or that it pops in and out of existence depending on whether someone deigns to observe it. Science deals with enquiring and discovering existing phenomena prior to our knowledge of them, that persist when we do not devote our attention to them, which constitutes the nerve and root of philosophical realism. There are no true "scientific unrealists" and whoever claim to be so only casts doubt on their own intellectual honesty.

The numerous variants of this doctrine lead to as many surnames for it: structural, epistemic, ontic, perspectivist, selective realism, etc. All these nuances agree on the existence of an extramental world independent of our perceptions of it, but they differ on what and how it can be known. The most successful line of thought so far is traced by structural realism, which holds the persistence of the underlying mathematical structures over the theoretical replacements that we call scientific progress. Accordingly, when a scientific theory accepted up to that moment is replaced by another one more closely adapted to the facts, the mathematical structures that formed the fuselage of the relegated theory are maintained in the new proposals. In a famous example, Worrall (1989, p.117) stated: «Fresnel completely misidentified the nature of light, but it is no miracle that his theory enjoyed the predictive empirical success it did; […], Because he […] attributed to light the correct structure».

This is the case due to the evident logical independence between the formalism of the theory, which delimits the conjectured structures for the analysed process, and the interpretation that fixes the reference class –or "ontology", if you prefer to put it that way. There is no place here for the semi-realist position of Chakravartty (1998), who combines realism regarding structures with anti-realism regarding referents. Indeed, it is not possible to accept the reality of certain relationships without also admitting the reality of those individuals to whom they apply. That is why the defense of ontic structuralism in the attempt to substitute the notion of object for that of structure is so counterproductive (French and Ladyman 2003). A world of structures only, without objects, would resemble the Cheshire cat –created by Lewis Carroll for his novel *Alice in Wonderland*– whose smile remains as her face fades away. As Van Fraassen (2007, p. 55) insightfully notes: «Does it make sense to conceive of a structure that is not a structure of something? A structure of nothing is nothing».

The most reasonable position in this field recognizes that the mathematical structures of our most powerful theories reflect in some way the patterns and objective regularities that constitute the way of being of the material world. The structures are not real in the same sense that the tree we stumble upon while absentmindedly wandering through the woods is real. We are the ones who, with more or less sagacity, project onto the universe the structures that our intellect elaborates, while hoping that they will grasp and reveal some of their intimacies to us. The referents of those structures –the ultimate constituents of the cosmos, in the fundamental theories– are only partially known to us, so that the possibility of modifications and amendments to the concepts that we weave around them always remains open.

This lesson is clearly captured in the Indian fable of the three blind wise men who, after inspecting different parts of an elephant by touch, each deduce separately that they are different animals. It would seem absurd that these scholars, faced with the contradiction between their opinions, conclude that the animal whose identity they intend to find out simply does not exist. Our information about the deep nature of reality will always be incomplete and precarious, which does not imply in any way that we should adopt unrealistic beliefs. Each new discovery may shed light on new properties, or new relationships between those already registered, of an object previously known in other aspects. This does not mean that we hold the existence of different objects every time our understanding increases, because what is successively altered are the intellectual constructs that we devise to represent it, not the object itself

The phenomena explained by geometric optics, being the same, acquire a new consistency by virtue of wave optics, which in turn is surpassed in breadth and depth by quantum field theory (although Feynman path integrals or renormalization techniques have yet to acquire a logically closed form). The case of Heisenberg's matrix theory and Schroedinger's wave theory, often used as an example against it, appears as one of the best arguments in favour of the ontological independence between physics and mathematics. It was Heisenberg, and not his mathematical method, that perhaps presupposed the existence of corpuscular particles, although he gave up any mental imagery to focus on the transition frequencies between atomic orbitals. And it was Schroedinger –not the equation that bears his name– who wanted to redirect the nascent quantum theory towards the familiar wave physics, but his failure demonstrated that his equation, free of ontological commitments, could accommodate functions very different from those representing traditional waves. Ultimately, both researchers understood that the concepts of wave and particle only constituted classical limits of a physical object as radically new as the quantum field.

It is a serious mistake to assume that two mathematically equivalent theories –that is, with isomorphic mathematical structures– that equally satisfy the same empirical evidence, must share the same ontological substratum; let us put forwards the case of fluid equations and the electromagnetic field which, despite their mathematical equivalence, point to very different physical referents. Rather, it should be said that the reference classes of both theories are made up of elements, some of whose properties –and relationships between such properties– correspond to each other, thus tolerating the same mathematical structures in certain aspects of their formal description. The key lies in the fact that this correspondence is not isomorphic but rather a homomorphism (Bueno, French and Ladyman 2002), which expresses the always partial and revisable nature of the link between our theories and the real world. And we need nothing more, since it is another serious misunderstanding to require isomorphisms in order to regard as legitimate the realistic inferences about nature from theoretical structures. It is enough for us the deductive possibilities that connect the theory with the material universe, and not the other way around, because the opposite path contains an impossible demand: If we are the ones who project the structures we invent onto nature, we can only hope that the bridge between both domains will be traveled in a single direction.

6. Conclusions

The possible influence of mathematical formalisms on the type of world that we can describe with scientific theories continues to feed a passionate debate that will probably take a long time to be exhausted. In the present work we have examined three possible interpretations of the statement according to which mathematics carries an ontological burden that necessarily orients in a certain direction the choice of the ontological base with which we equip each physical theory. The accumulated historical experience and a cursory analysis of the acquired knowledge leads us to reject the extreme Platonism of those who maintain that all self-consistent mathematical ideas must be realized in nature. Nor does the Galilean metaphor that equates the material world with a book written in mathematical language seem to be justified. Nature is not a text, whatever the language used to describe it,

The third possibility suggests that the mathematical tools used in physical theories entail a certain ontological commitment about the class of entities whose existence such theories presuppose. However, examination of this issue inevitably leads to an unfavourable verdict. Mathematics does not involve any ontological commitment when it is used as the basic framework of our knowledge of the world. It is the semantic hypotheses included in the axiomatic basis of a theory that determines its connection with the material universe, by specifying the physical referents of that theory. There is no inescapable link between the semantic dimension of a physical theory and the mathematical formalism in which it is expressed.

Contrary to the opinion of some authors, this freedom provided by the lack of ontological commitment of mathematical methods does not cause the slightest problem for realism as a metaphysical assumption of scientific knowledge. Physical objects exist independently of our limitations in knowing them, and we are not able to demand from our theoretical representations more than what is logically possible to expect. Our best mathematical theories about the physical world will never be complete, perfect and definitive, and it does not mean that we must refrain from inferring what those very theories allow us to deduce, despite their inevitable deficiencies. The theoretical underdetermination of physical reality, by which it will always be logically possible to develop more than one theory to explain the same set of facts, will always accompany scientific work without an illusory ontological burden of mathematical instruments alleviating what ultimately constitutes an essential ingredient of human knowledge.

References

Alemañ, R. 2015. The imagined nature: Is the world mathematical? Moscow: USSR.

Alonso, M. 1994. The wave-particle duality: Mystery or myth? Spanish Journal of Physics 8(1): 38 – 41.

Balaguer, M. 1998. Platonism and Anti-Platonism in Mathematics. New York: Oxford University Press.

Barker, S.; Jago, M. 2018. Material objects and essential bundle theory. Philosophical Studies 175: 2969–2986.

Baron, S. 2023. Mathematical Explanation: A Pythagorean Proposal. The British Journal for the Philosophy of Science 10.1086/716181.

Barrow, J. 1997. Why is the world mathematical? Barcelona: Grijalbo.

Benacerraf, P. 1965. What Numbers Could Not Be. The Philosophical Review 74(1): 47–73.

Benacerraf, P. 1973. Mathematical Truth. The Journal of Philosophy 70: 661–679.

Bender, C.; Brody, D.; Müller, M. 2017. Hamiltonian for the Zeros of the Riemann Zeta Function. Phys. Rev. Lett. 118, 130201.

Bergliaffa, S.; Vucetich, H.; Rosemary. G. 1993. Axiomatic Foundations of Nonrelativistic Quantum Mechanics: A Realistic Approach. Int. J. Theor. Phys. 32(9): 1507

Bochener, S. 1991. The role of mathematics in the development of science. Madrid: Alliance.

Burgade, P.; Keating, J. 2012. Quantum Chaos, Random Matrix Theory, and the Riemann ζ-function. Progress in Mathematical Physics 66. 10.1007/978-3-0348-0697-8_4.

Bunge, M. 1974. Treatise on Basic Philosophy. Vol. 1. Semantics I: Sense and Reference. Dordrecht: Reidel.

Bunge, M. 1977. Treatise on Basic Philosophy. Vol. 3. Ontology I: The Furniture of the World. Dordrecht: Reidel.

Burgess, J.; Rosen, G. 1997. A Subject with No Object. Oxford (UK): Oxford University Press.

Well, O. 2009. Mathematical Fictionalism. In O. Good; EITHER. Linnebo (eds.), New Waves in Philosophy of Mathematics, p. 59-79. London: Palgrave Macmillan

Bueno, O. 2018. Putnam's indispensability argument revisited, reassessed, revived. Theoria 33(2): 201–218.

Well, O.; French, S.; Ladyman, J. (2002). On representing the relationship between the mathematical and the empirical. Philosophy of Science 69(3):497-518.

Cho, Y.; Maison, D. 1997. Monopole configuration in Weinberg–Salam model. Phys. Lett. B391:360–365.

Clarke-Doane, J. 2022. Mathematics and Metaphilosophy. Cambridge (UK): Cambridge University Press.

Colyvan, M. 2001. The Indispensability of Mathematics. New York: Oxford University Press.
Dahler, H.S.; Scriven, LE 1963. Theory of Structured Continua. Proc. Roy. Soc. London. A 275: 505-527.
Darling, R. 1994. Differential Forms and Connections. Cambridge (UK): Cambridge University Press
Decock, L. 2002. Quine's Weak and Strong Indispensability Argument. Journal for General Philosophy of Science 33(2): 231–250.
DeWitt, B. 2003. The Global Approach to Quantum Field Theory. vols. I–II. New York: Oxford University Press.
Dirac, PAM 1931. Quantized singularities in the electromagnetic field. Proc. R. Soc. London A 133: 60–72.
Donaldson, S. 1990. Polynomial invariants for smooth four-manifolds. Topology 29: 257-315.
Donaldson,S.; Kronheimer, P. 1990. The geometry of four-manifolds. Oxford (UK): Clarendon Press.
Drukier, AK; Nussinov, S. 1982. Monopole pair creation in energetic collisions: is it possible? Phys. Rev. Lett. 49:102–105.
Field, H. 1980. Science without Numbers. Princeton: Princeton University Press.
Field, H. 1989. Realism, Mathematics, and Modality. Philosophical Topics 16(1): 57-107.
French, S.; Ladyman, J. 2003. Remodeling Structural Realism: Quantum Physics and the Metaphysics of Structure. Synthese 136: 31-56
Friedmann, T.; Hagen, C. 2015. Quantum mechanical derivation of the Wallis formula for π. J. Math. Phys. 56: 112101.
Galileo 1981 [1623]. The Assayer. Buenos Aires: Aguilar.
Goldstein, H. 1990. Classical mechanics. Barcelona: I reversed.
Kasa, I. 2010. On Field's Epistemological Argument against Platonism. Studia Logica 96(2): 141-47.
Kant, I. 1989 [1786]. Metaphysical principles of the science of nature. Madrid: Alliance.
Koyré, A. 1966. Etudes Galiléennes, Paris: Hermann.
Lafrance, JD 2015. A bundle of universals theory of material objects. The Philosophical Quarterly 65(259): 202-219.
Leng, M. 2009. Mathematics and Reality. Oxford (UK): Oxford University Press.
Liggins, D. 2008. Quine, Putnam, and the 'Quine–Putnam' Indispensability Argument. Erkenntnis 68(1): 113–127.
Liggins, D. 2010. Epistemological Objections to Platonism. Philosophy Compass 5(1):67-77.
MacDonnell, J. 2016. The Pythagorean World. Why Mathematics is Unreasonably Effective in Physics. New York: Palgrave Macmillan.
Madrid, C. 2010. Mr. structural realist, we have a problem: The ontological load of mathematics. Principia 14(2): 201–209
Malkus, WVR 1951. The interaction of the Dirac magnetic monopole with matter. Phys. Rev. 83:899–905.
Marquis, J.-P. 1997. Abstract Mathematical Tools and Machines for Mathematics. Philosophia Mathematica 5(3):250-272.
Marquis, J.-P. 2006. A Path to the Epistemology of Mathematics: Homotopy Theory. In: J. Gray; J. Ferreiros (eds.), The Architecture of Modern Mathematics, p. 239-260. Oxford (UK): Oxford University Press.

Marquis, J.-P. 2011. Mario Bunge's Philosophy of Mathematics: An Appraisal. Science & Education 21(10):1567−1594.
Marquis, J.-P. 2012. Categorical Foundations of Mathematics: or how to provide foundations for abstract mathematics. The Review of Symbolic Logic 6 (1):51-75.
Marquis, J.-P. 2018. Unfolding FOLDS: A Foundational Framework for Abstract Mathematical Concepts. In: E. Landry (ed.), Categories for the Working Philosopher, p. 136-162. New York: Oxford University Press.
Milton, KA 2006. Theoretical and experimental status of magnetic monopoles. Rep. Prog. Phys. 69:1637–1711.
Molinini, D.; Pataut, F.; Sereni, A. 2016. Indispensability and explanation: an overview and introduction. Synthese 193(2): 317–332.
Noll, W. 1955. Die Herleitung der Grundgleichungen der Thermomechanik der Kontinua aus der statistischen Mechanik. Journal of Rational Mechanics and Analysis 4:627–646.
Nutting, ES 2020. Benacerraf, Field, and the Agreement of Mathematicians. Synthese 197(5):2095-110.
Pal, PB (2011). Dirac, Majorana, and Weyl fermions. American Journal of Physics 79(5):485–498.
Penrose, R. 2017. Fashion, Faith, and Fantasy in the New Physics of the Universe. Madrid: Discussion.
Psillos, S. 1995. Is structural realism the best of both worlds? Dialectic 49: 15-46.
Psillos, S. 1999. Scientific Realism: How Science Tracks Truth. London: Routledge.
Putnam, H. 1971. Philosophy of Logic. New York: Harper & Row.
Quine, WVO 1960. Word and Object. Cambridge (Mass.): MIT Press
Quine, WVO 1969. Existence and Quantification. In: Ontological Relativity and Other Essays, p. 91–113. New York: Columbia University Press.
Robert, G. 2019. Can the Realist Bundle Theory Account for the Numerical Difference Between Qualitatively Non-Discernible Concrete Particulars? Theorem: International Journal of Philosophy 38 (1):25-40.
Sereni, A. 2015. Frege, Indispensability, and the Compatibilist Heresy. Philosophia Mathematica 23(1):11–30.
Sładkowski, J. 1996. Exotic smoothness, noncommutative geometry, and particle physics. International Journal of Theoretical Physics 35(10): 2075-2083.
Smolin, L. 2016. The doubts of physics in the XXI century. Barcelona: Criticism.
Taubes, C. 2000. Seiberg-Witten and Gromov invariants for symplectic fourmanifolds. Somerville (USA): International Press.
Taubes, C. 2011. Differential geometry: Bundles, connections, metrics and curvature. Oxford (UK): Oxford University Press.
Tegmark, M. 2008. The Mathematical Universe. Foundations of Physics 38(2): 101–150
Thomas, R. 2000. Mathematics and Fictions I: Identification. Logique et Analyze 43(171-172): 301-340.
Thomas, R. 2002. Mathematics and Fictions II: Analogy. Logique et Analyze 45(177-178):185-228.
Thomasson, A. 1999. Fiction and Metaphysics. Cambridge (UK): Cambridge University Press.
't Hooft, G. 1974. Magnetic monopoles in unified gauge theories. Nucl. Phys. B 79:276–284.
Truesdell, C. 1966. Six Lectures on Modern Natural Philosophy. New York: Springer-Verlag.

Truesdell, C.; Noll, W. 1965. The Non-Linear Field Theories of Mechanics. New York: Springer-Verlag.

Van Fraassen, B. 2007. Structuralism(s) about Science: Some Common Problems. Proc. Arist. Soc. 81: 45-61.

Worrall, J. 1989. Structural Realism: The Best of Both Worlds? Dialectic 43: 99-124.

Yablo, S. 2002. Go figure: A path through Fictionalism. Midwest Studies in Philosophy 25: 72-102