General Covariance and the Objectivity of Space-Time Point-Events.

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Abstract

"The last remnant of physical objectivity of space-time" is disclosed, beyond the Leibniz equivalence, in the case of a continuous family of spatially non-compact models of general relativity. The *physical individuation* of point-events is furnished by the intrinsic degrees of freedom of the gravitational field, (viz, the *Dirac observables*) that represent - as it were - the *ontic* part of the metric field. The physical role of the *epistemic* part (viz. the *gauge* variables) is likewise clarified. At the end, a peculiar four-dimensional *holistic and structuralist* view of space-time emerges which includes elements common to the tradition of both *substantivalism* and *relationism.* The observables of our models undergo real *temporal change* and thereby provide a counter-example to the thesis of the *frozen-time* picture of evolution.

I. INTRODUCTION

The fact that the requirement of general covariance might involve a threat to the very objectivity of the points of space-time as represented by the theory of gravitation was becoming clear to Einstein even before the theory he was trying to construct was completed. It was during the years 1913-1915 that the threat took form with the famous Hole Argument (*Lochbetrachtung*) [1] [1]. In classical field theories space-time points play the role of *individuals*, but it is often implicit that they can be physically distinguished only by the fields they carry. Yet, the Hole Argument apparently forbids precisely this kind of individuation, and since the Argument is a direct consequence of the *general covariance* of general relativity (GR), this conflict eventually led Einstein to state [3] (our *emphasis*):

> [That] *the requirement of general covariance*, [which] *takes away from space and time the last remnant of physical objectivity*, [is a natural one, will be seen from the following reflexion].

Although Einstein quickly bypassed on purely pragmatic grounds the seeming cogency of the Hole Argument against the implementation of general covariance, the issue remained in the background of the theory until the Hole Argument received new life in recent years with a seminal paper by John Stachel [4]. This paper, followed seven years later by Earman and Norton's philosophical argument against the so-called space-time *manifold substantivalism* [2] [5], opened a rich philosophical debate that is still alive today. The Hole Argument was immediately regarded by virtually all participants in the debate [6] as being intimately tied to the deep nature of space and time, at least as they are represented by the mathematical models of GR. It must be acknowledged that until now the debate had a purely philosophical relevance. From the physicists' point of view, GR has indeed been immunized against the Hole Argument - leaving aside any underlying philosophical issue - by simply embodying the Argument in the statement that *mathematically different* solutions of the Einstein equations related by *passive* - as well as *active* (see later) - diffeomorphisms are *physically equivalent*. Showing that this statement cannot be regarded as the last word on this matter even from the physical point of view, is the main scope of this paper. In the meantime, it must be clear from the start that, given the enormous mathematical variety of possible solutions of Einstein's equations, one should not expect that a clarification of the possible meaning of *objectivity* of space-time points could be obtained *in general*. Specifically, as we shall see, it is essential to consider the family of spatially compact space-times without boundary, separately, from those which are spatially non-compact, like Minkowski space-time. We shall indeed conclude that the main questions we discuss can be clarified for a definite continuous class of generic solutions corresponding to spatially non-compact space-times[3], but *not* for the spatially compact ones.

More generally we aim to show that some capabilities peculiar to the Hamiltonian approach to GR can be exploited for the purpose of better understanding important interpretive issues surrounding the theory. The Hamiltonian approach guarantees first of all that the

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[1] For a beautiful historical critique see Norton 1987 [2].

[2] This is the view that not only the best candidate to interpret the role of space and time in GR is the *bare manifold* M^4 of mathematical points but that, moreover, each point is endowed with the essential properties of a *substance*, the metric being a dynamical field like any other.

[3] The Christodoulou-Klainermann space-times [8]

initial value problem of Einstein's equations is mathematically well-posed, a circumstance that does not occur in a natural way within the configurational Lagrangian framework [9]; furthermore, on the basis of the Shanmugadhasan canonical transformations [10], this framework provides a net distinction between *physical observables*, connected to the (two) intrinsic degrees of freedom of the gravitational field (the so-called *Dirac observables*) on one hand, and *gauge variables*, on the other. The latter, which express the typical arbitrariness of the theory and must be fixed (*gauge-fixing*) before solving the Einstein equations for the intrinsic degrees of freedom, turn out to play a fundamental role, no less than the *Dirac observables*, in clarifying the real import of the Hole Argument. It will be seen that the resulting *gauge* character of GR is a crucial factor in understanding the issue of the objectivity of space-time points, leading to a dis-solution of the Hole Argument, or better, to a philosophical downgrading of it.

Let us report here the very general definition of gauge theories given by Henneaux and Teitelboim [11] (our *emphasis*):

> These are theories in which the physical system being dealt with is described by more variables than there are physically independent degrees of freedom. The physically meaningful degrees of freedom then re-emerge as being those invariant under a transformation connecting the variables (gauge transformation). Thus, one introduces *extra variables* to make the description *more transparent*, and brings in at the same time a gauge symmetry to extract the *physically relevant content*.

The relevant fact in our case is that while, from the mathematical point of view of the constrained Hamiltonian formalism, GR is a gauge theory like any other (e.g., electromagnetism and Yang-Mills theory), from the *physical* point of view it is *radically different*, just because of its invariance under a group of diffeomorphisms acting on space-time itself, instead of being invariant under the action of a local *inner* Lie group. Furthermore, in GR (and in Yang-Mills theory as well) we cannot rely from the beginning on empirically validated, gauge-invariant dynamical equations for the *local* fields, as it happens with electro-magnetism, where Maxwell equations can be written in terms of the gauge invariant electric and magnetic fields. On the contrary, Einstein's general covariance (viz. the gauge freedom of GR) is such that the introduction of extra (*gauge*) variables does indeed make the *mathematical* description of general relativity more *transparent* (through manifest general covariance instead of manifest Lorentz covariance) but, by ruling out any background structure at the outset, it also makes its physical interpretation more intriguing, at least *prima facie*, and conceals at the same time the intrinsic properties of point-events. Indeed in GR the distinction between what is observable and what is not, is unavoidably entangled with the constitution of the very *stage*, space–time, where the *play* of physics is enacted: a stage, however, which also takes an active part in the play. In other words, the gauge-fixing mechanism plays the *dual role* of making the dynamics *unique* (as in all gauge theories), and of fixing the *appearance* of the spatio-temporal dynamical background. At the same time, this mechanism highlights a characteristic functional split of the metric tensor that can be briefly described as follows. On one hand, the Dirac observables specify - as it were - the *ontic* structure of space-time, connected to the intrinsic degrees of freedom of the gravitational field (and - physically - to *tidal-like* effects). On the other, the gauge variables specify the built-in *epistemic* component of the metric tensor (physically related to *generalized inertial effects*). More precisely, any gauge-fixing is equivalent to the constitution of an *extended,* *non-inertial, space-time laboratory with its coordinates* [4], as well as to a *(dynamical !) determination* of the *conventions* about *distant simultaneity*: in particular, different *conventions* within the same space-time (the same *universe*), turn out to be simply *gauge-related options.*

Let us point out that the explicit expression of the gauge variables and the Dirac observables in terms of the metric tensor field and its derivatives is not known. We know nevertheless that such variables are *highly non-local functionals* involving the whole Σ_τ hypersurface. On the other hand, by exploiting the structure of the gauge transformations, we get the inverse canonical transformation explicitly, i.e. the re-expression of the metric tensor in terms of gauge variables and Dirac observables.

Summarizing, the gauge variables play a multiple role in completing the structural properties of the general-relativistic space-time: their fixing is necessary to solve Einstein's equations, to reconstruct the four-dimensional chrono-geometry emerging from the *four Dirac observables* and to allow empirical access to the theory through the definition of a spatiotemporal *laboratory.*

The main result of our analysis is given in Section V where we show how the *ontic* part of the metric (the intrinsic degrees of freedom of the gravitational field) may confer a *physical individuation* onto space-time points[5]. Since - as mentioned before - such degrees of freedom depend in a highly *non-local* way upon the values of the metric and its derivatives over a whole space-like surface of distant simultaneity, point-events receive a peculiar sort of *intrinsic properties* that, nevertheless, are conferred on them *holistically.* Admittedly, the distinction between *ontic* and *epistemic* parts, as well as the *form* of the space-like surfaces of distant simultaneity, are gauge-dependent (non-invariant). Yet, according to a *main conjecture* we have advanced in Ref. [14], a canonical basis of *scalars* should exist, making the above distinction and, therefore, physical individuation of point-events fully invariant and *objective.*

Finally, an additional important feature of the solutions of GR dealt with in our discussion is the following. The ADM formalism [7] on compact space-times implies that the canonical Hamiltonian generates purely harmless gauge transformations connecting admissible 3+1 foliations of space-time, so that it *cannot engender any real temporal change* (and we have the so-called *frozen evolution* description; in this connection see Refs.[15, 16]). However, in the case of the Christodoulou- Klainermann continuous family of spatially non-compact space-times, *internal mathematical consistency* (requiring the addition of the De-Witt surface term to the Hamiltonian [17]) entails that the generator of temporal evolution be instead the so-called *weak ADM energy.* Unlike the canonical Hamiltonian, this quantity *does generate real temporal modifications* of the canonical variables. In conclusion, we offer here a counter-example to the *frozen-evolution* picture, typical of other solutions of Einstein'e equations. This also means, however, that the *frozen-evolution* picture cannot be regarded

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[4] Let us note [12] that such an extended laboratory is a non-rigid, non-inertial frame (the only existing in GR) centered on the (in general) accelerated observer whose world-line is the origin of the 3-coordinates. The gauge-fixing procedure determines the appearance of phenomena because in each point of the non-inertial frame the form of the inertial forces (Coriolis, Jacobi, centrifugal,...) is uniquely fixed.

[5] There is an unfortunate ambiguity in the usage of the term *space-time points* in the literature: sometimes it refers to elements of the mathematical structure that is the first layer of the space-time model, and sometimes to the points interpreted as *physical* events. We will adopt the term *point–event* in the latter sense and simply *point* in the former.

as a philosophically compelling, *typical* and necessary feature of GR.

The technical developments underlying this work have already been introduced in Refs.[12, 14, 18] where additional properties of the Christodoulou-Klainermann family of space-times are also discussed. For a more general philosophical presentation, see Ref.[29].

II. DYNAMICAL SYMMETRIES

Standard general covariance, which essentially amounts to the statement that the Einstein equations for the metric field ${}^4g(x)$ have a tensor character, implies first of all that the basic equations are form invariant under general coordinate transformations, so that the Lagrangian density in the Einstein-Hilbert Action is singular. This entails in turn that four of the Einstein equations be in fact *Lagrangian constraints*, namely restrictions on the Cauchy data, while four combinations of Einstein's equations and their gradients vanish identically (*contracted Bianchi identities*). Thus, the ten components of the solution ${}^4g_{\mu\nu}(x)$ are in fact functionals of only two "*deterministic*" dynamical degrees of freedom and eight further degrees of freedom which are left *completely undetermined* by Einstein's equations *even once the Lagrangian constraints are satisfied*. This state of affairs makes the treatment of both the Cauchy problem of the non-hyperbolic system of Einstein's equations and the definition of observables within the Lagrangian context [9] extremely complicated.

For the above reasons, standard general covariance is then interpreted, in modern terminology, as the statement that *a physical solution of Einstein's equations* properly corresponds to a *4-geometry*, namely the equivalence class of all the 4-metric tensors, solutions of the equations, written in all possible 4-coordinate systems. This equivalence class is usually represented by the quotient ${}^4Geom = {}^4Riem/{}_PDiff\, M^4$, where ${}^4Riem$ denotes the space of metric tensor solutions of Einstein's equations and ${}_PDiff$ is the infinite group of *passive* diffeomorphisms (general coordinate transformations). On the other hand, any two *inequivalent* Einstein space-times are different 4-geometries or "universes".

Consider now the abstract differential-geometric concept of *active* diffeomorphism D_A and its consequent action on the tensor fields defined on the differentiable manifold M^4 (see, for example, Ref. [19]). An *active* diffeomorphism D_A maps points of M^4 to points of M^4: $D_A : p \rightarrow\ p' = D_A \cdot p$. Its tangent map D^*_A maps tensor fields $T \rightarrow\ D_A * \cdot T$ in such a way that $[T](p) \rightarrow\ [D^*_A \cdot T](p) \equiv [T^{'}](p)$. Then $[D^*_A \cdot T](p) = [T](D_A^{-1} \cdot p)$. It is seen that the transformed tensor field $D^*_A \cdot T$ is a *new* tensor field whose components in general will have at p values that are *different* from those of the components of T. On the other hand, the components of $D^*_A \cdot T$ have at p' - by construction - the same values that the components of the original tensor field T have at p: $T^{'}(D_A \cdot p) = T(p)$ or $T'(p) = T(D_A^{-1} \cdot p)$. The new tensor field $D^*_A \cdot T$ is called the *drag-along* (or *push-forward*) of T. There is another, non-geometrical - so-called *dual* - way of looking at the active diffeomorphisms. This *duality* is based on the circumstance that in each region of M^4 covered by two or more charts there is a one-to-one correspondence between an *active* diffeomorphism and a specific coordinate transformation. The coordinate transformation $\mathcal{T}_{D_A} : x(p) \rightarrow\ x'(p) = [\mathcal{T}_{D_A} x](p)$ which is *dual* to the active diffeomorphism D_A is defined so that $[\mathcal{T}_{D_A} x](D_A \cdot p) = x(p)$. Essentially, this *duality* transfers the functional dependence of the new tensor field in the new coordinate system to the old system of coordinates. By analogy, the coordinates of the new system $[x']$ are said to have been *dragged-along* with the *active* diffeomorphism D_A. It is important to note here, however, that the above *dual view* of active diffeomorphisms, as particular *coordinate*-transformations, is defined for the moment only *implicitly*.

In abstract coordinate-independent language, Einstein's equations for the vacuum

$$ {}^4G_{\mu\nu}(x) \stackrel{def}{=} {}^4R_{\mu\nu}(x) - \frac{1}{2}\, {}^4R(x)\, {}^4g_{\mu\nu}(x) = 0. \tag{2.1} $$

can be written as $G = 0$, where G is the Einstein 2-tensor ($G = G_{\mu\nu}(x)\, dx^\mu \bigotimes dx^\nu$ in the coordinate chart x^μ). Under an *active diffeomorphism* $D_A : M^4 \mapsto M^4$, $D_A \in {}_ADiff\, M^4$, we have $G = 0 \mapsto D_A^* G = 0$, which shows that active diffeomorphisms are *dynamical symmetries* of the Einstein's tensor equations, i.e., they map solutions into solutions.

In Ref.[12] we have clarified the explicit relationships[6] existing between passive and active diffeomorphisms on the basis of a nearly forgotten paper by Bergmann and Komar [20] in which it is shown that *the biggest group of passive dynamical symmetries of Einstein's equations* is not ${}_PDiff\, M^4$ $[x^{'\,\mu} = f^\mu(x^\nu)]$ but instead a larger group of transformations of the form[7]

$$ \begin{aligned} Q: \qquad & x^{'\,\mu} = f^\mu(x^\nu, {}^4g_{\alpha\beta}(x)), \\ {}^4g^{'}_{\mu\nu}(x^{'}(x)) \; &= \; \frac{\partial h^\alpha(x^{'}, {}^4g^{'}(x^{'}))}{\partial x^{'\,\mu}} \, \frac{\partial h^\beta(x^{'}, {}^4g^{'}(x^{'}))}{\partial x^{'\,\nu}} \, {}^4g_{\alpha\beta}(x). \end{aligned} \tag{2.2} $$

It is remarkable that, at least for the subset $Q' \subset Q$ that corresponds to *mappings among gauge-equivalent Cauchy data*, the transformed metrics do indeed belong to the *same 4-geometry*, i.e. the same equivalence class generated by applying all *passive diffeomorphisms* to the original 4-metrics: ${}^4Geom = {}^4Riem/{}_PDiff\, M^4 = {}^4Riem/Q'$[8]. The 4-metrics built by using passive diffeomorphisms are, as it were, only a dense sub-set of the metrics obtainable by means of the group Q. On the other hand, the restricted set of active diffeomorphisms passively reinterpreted with Eq.(2.2) belongs to the set of local Noether symmetries of the Einstein-Hilbert action.

In conclusion, what is known as a *4-geometry*, is also an equivalence class of solutions of Einstein's equations *modulo* the dynamical symmetry transformations of ${}_ADiff\, M^4$. Therefore, we can state

$$ {}^4Geom = {}^4Riem/{}_PDiff\, M^4 = {}^4Riem/Q' = {}^4Riem/{}_ADiff\, M^4. \tag{2.3} $$

However, in the case of completely Liouville-integrable systems, dynamical symmetries can be re-interpreted as maps of the space of Cauchy data onto itself. Although we don't have a general proof of the integrability of Einstein's equations, we know that if the initial value problem is well-posed, as it is in the ADM Hamiltonian description, *the space of Cauchy data is partitioned in gauge-equivalent classes of data*: all of the Cauchy data in a

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[6] At least for the infinitesimal active transformations.

[7] Note that an *explicit* passive representation of the infinite group of ${}_ADiff\, M^4$ is necessary anyway for our Hamiltonian treatment of the Hole Argument as well as for any comparison of the various viewpoints existing in the literature concerning the *solutions* of Einstein's equations.

[8] Note, incidentally, that this circumstance is mathematically possible only because ${}_PDiff\, M^4$ is a *non-normal* sub-group of Q.

given class identify a single 4-geometry or *universe*. Therefore, under the given hypothesis, the dynamical symmetries of Einstein's equations fall in two classes only: a) those mapping different *universes* among themselves, and b) those acting within a single Einstein *universe*, mapping gauge-equivalent Cauchy data among themselves (actually, they will be *on shell* gauge transformations[9]). This entails that - at least for the class of solutions of Einstein equations that are dealt with in our ADM Hamiltonian formalism - the same alternative must be predicated for the elements of ${}_A Diff\, M^4$.

III. THE HOLE ARGUMENT

Although the issue could not be completely clear to Einstein in 1916, as shown by Norton (1987) [2], it is precisely the nature of dynamical symmetry of the *active diffeomorphisms* that has been considered as expressing the *physically relevant* content of *general covariance*[10], as we shall presently see.

Remember, first of all that a *mathematical model* of GR is specified by a four-dimensional mathematical manifold M^4 and by a metrical tensor field g, where the latter dually represents *both* the chrono-geometrical structure of space-time *and* the potential for the inertial-gravitational field. Non-gravitational physical fields, when they are present, are also described by dynamical tensor fields, which appear to be sources of the Einstein equations. Assume now that M^4 contains a *hole* $\mathcal{H}$: that is, an open region where all the non-gravitational fields vanish. On M^4 we can define an *active* diffeomorphism D_A^* that re-maps the points inside $\mathcal{H}$, but blends smoothly into the identity map outside $\mathcal{H}$ and on the boundary. By construction, for any point $x \in \mathcal{H}$ we have (in the abstract tensor notation) $g'(D_A x) = g(x)$, but of course $g'(x) \neq g(x)$ (in the same notation). The crucial fact is that from the general covariance of Einstein's equations it follows that if g is one of their solutions, so is the *drag-along* field $g' \equiv D_A^* g$.

What is the correct interpretation of the new field g'? Clearly, the transformation involves an *active redistribution of the metric over the points of the manifold in* $\mathcal{H}$, so the critical question is whether and how the points of the manifold are primarily *individuated*. Now, *if* we think of the points of $\mathcal{H}$ as *intrinsically individuated physical events*, where *intrinsic* means that their identity is independent of the metric - a claim that is associated with any kind of *manifold substantivalism* - then g and g' must be regarded as *physically distinct* solutions of the Einstein equations (after all, $g'(x) \neq g(x)$ at the *same* point x). This is a devastating conclusion for the causality, or better, the *determinateness*[11] of the theory, because it implies that, even after we specify a physical solution for the gravitational and

[9] We distinguish *off shell* considerations, made within the variational framework before restricting to the dynamical solutions, from *on shell* considerations, made after such a restriction.

[10] This is a point of view relying on an abstract coordinate-free use of differential geometry. The dual point of view, making explicit calculations possible, relies on the nature of local Noether symmetries of passive diffeomorphisms, both in the study of the variational principles of the action and in the formulation of the Hamiltonian formalism with Dirac constraints (note that as yet the abstract way has not succeeded in controlling the Lagrangian aspects of gauge theories).

[11] We prefer to avoid the term *determinism*, because we believe that its metaphysical flavor tends to overstate the issue at stake. This is especially true if *determinism* is taken in opposition to *indeterminism*, which is not mere absence of *determinism*.

non-gravitational fields outside the hole - in particular, *on a Cauchy surface for the initial value problem* - we are still unable to predict a unique physical solution within the hole. For one thing, therefore, it is clear that the Hole Argument is unavoidably entangled with the initial value problem[12]. Furthermore, if general relativity has to make any sense as a *physical* theory, there must be a way out of this foundational quandary, *independently* of any philosophical consideration.

According to Earman and Norton [5], the way out of the hole argument lies in abandoning *manifold substantivalism*: they claim that if diffeomorphically-related metric fields were to represent different physically possible universes, then GR would turn into an *indeterministic* theory. And since the issue of whether determinism holds or not at the *physical* level cannot be decided by opting for a *metaphysical* doctrine like *manifold substantivalism*, they conclude that one should go for space-time *relationism*. Diffeomorphically related metric fields must be interpreted as describing the same *universe* (*Leibniz equivalence*). The fact that the Leibniz equivalence seems here no more than a sophisticated re-phrasing of what physicists consider a foregone conclusion, should not be taken at face value, for the real question for the opposing "sensible substantivalist" is whether or not space-time should be simply identified with the bare manifold *deprived of the metric field* instead of with a set of points each endowed with its own *metrical fingerprint*[13]; actually, this substantivalist is willing to sustain the conviction that the metric field, because of its basic *causal structure*, has ontological priority [22] over all other fields and, therefore, it is *not* like any other field, as Earman and Norton would have it.

In agreement with Stachel [23], we believe, however, that asserting that g and $D^*_A g$ represent *one and the same gravitational field* implies that *the mathematical individuation* of the points of the differentiable manifold by their coordinates *has no physical content until a metric tensor is specified* [14]. Stachel stresses that if g and $D^*_A g$ must represent the same gravitational field, they cannot be physically distinguished in any way. Consequently, when we act on g with D^*_A to create the *drag-along* field $D^*_A g$, no element of physical significance can be left behind: in particular, *nothing* that could identify a point x of the manifold itself as the *same point* of space-time for both g and $D^*_A g$. Instead, when x is mapped onto $x' = D^*_A x$, it *carries over its identity*, as specified by $g'(x') = g(x)$. This means, for one thing, that "the last remnant of physical objectivity" of space-time points, if any, should be sought for in the physical content of the metric field itself.

These remarks led Stachel to the important conclusion that *vis á vis* the physical point-events, the metric actually plays the role of *individuating field*. More than that, Stachel stresses that even the topology of the underlying manifold cannot be introduced independently of the specific form of the metric tensor, a circumstance that makes Earman and Norton's choice of interpreting the mere topological and differentiable manifold as *space-time* deprived of the metric even more implausible. Precisely, Stachel suggested that this individuating role should be implemented by four invariant functionals of the metric, which Komar [25, 26] had already considered. Stachel, however, did not follow up on this proposal,

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[12] It is interesting to find that David Hilbert stressed this point already in 1917 [21].

[13] See, for example, Bartels and Maudlin in Ref. [6].

[14] *Coordinatization* is the only way to individuate mathematical points, as stressed by Hermann Weyl [24]: "There is no distinguishing objective property by which one could tell apart one point from all others in a homogeneous space: at this level, fixation of a point is possible only by a *demonstrative act* as indicated by terms like *this* and *there*."

something that we instead will presently do, indicating at the same time the reasons why Stachel's suggestion cannot work as it stands.

Finally, let us stress that the force of the *indeterminacy argument* essentially rests on the fact that the *active* diffeomorphism D^*_A, which is purportedly chosen to be the *identity* outside the *hole* $\mathcal{H}$, is such that it does not *really* alter the *initial data on the Cauchy surface* in any *physically significant* way. But since the non-hyperbolicity of Einstein's equations makes the Cauchy problem nearly intractable in the configuration space M^4, the abstract and purely geometrical nature of the original formulation of the Hole Argument needs further scrutiny, in particular with reference to the various kinds of equivalences of the solutions. As mentioned above, however, whenever the Cauchy problem is well stated (viz. within the Hamiltonian framework, which is a passive viewpoint by definition) - either a D^*_A acts within a single Einstein *universe*, mapping gauge-equivalent Cauchy data among themselves, and must be, therefore, only a harmless *gauge transformation*, or it maps the given *universe* into a different one. It is therefore already clear - to the extent that the Cauchy problem is well-posed - that exploiting the original Hole Argument to the effect of asking ontological questions about space-time points is an enterprise devoid of real philosophical impact, at least concerning the menace of indeterminism. Still, the Hole Argument maintains an interesting open question regarding the issue of the physical (viz. dynamical) *intrinsic* individuation of the point-events of M^4 [15].

IV. THE CHRISTODOULOU-KLAINERMANN CONTINUOUS FAMILY OF SPACE-TIMES, ADM SLICING, AND CANONICAL REDUCTION

The Christodoulou-Klainermann space-times are a continuous family of space-times that are non-compact, globally hyperbolic, asymptotically flat at spatial infinity (asymptotic Minkowski metric, with asymptotic Poincaré symmetry group) and topologically trivial ($M^4 \equiv R^3 \times R$), supporting global 4-coordinate systems.

The ADM Hamiltonian approach starts with a slicing of the 4-dimensional manifold M^4 into constant-time hyper-surfaces $\Sigma_\tau \equiv R^3$, indexed by the *parameter time* τ, each equipped with coordinates σ^a (a = 1,2,3) and a three-metric ${}^3g$ (in components ${}^3g_{ab}$). The *parameter time* τ and the coordinates σ^a (a = 1,2,3) [16] are in fact *Lorentz-scalar, radar* coordinates [13]. The surfaces Σ_τ are described by the embedding functions $x^\mu = z^\mu(\tau, \vec{\sigma}) = X^\mu(\tau) + F^\mu(\tau, \vec{\sigma})$, $F^\mu(\tau, \vec{0}) = 0$[17]. We start at a point on Σ_τ, and displace it infinitesimally in a direction that is normal to Σ_τ. The resulting change in τ can be written as $\triangle \tau = N d\tau$, where N is the so-called *lapse function*. In a generic coordinate system, such a displacement will also shift the spatial coordinates: $\sigma^a(\tau + d\tau) = \sigma^a(\tau) + N^a d\tau$, where N^a is the *shift vector*. Then the interval between (τ, σ^a) and $(\tau + d\tau, \sigma^a + d\sigma^a)$ is: $ds^2 = N^2 d\tau^2 - {}^3g_{ab}(d\sigma^a + N^a d\tau)(d\sigma^b + N^b d\tau)$. The *configurational* variables N, N^a, ${}^3g_{ab}$ together with their 10 conjugate momenta, index

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[15] As Michael Friedman remarked (see Ref.[27], p.663) - if we stick to the simple Leibniz equivalence, "how do we describe this physical situation intrinsically ?".

[16] They are defined with respect to an arbitrary observer, a centroid $X^\mu(\tau)$, chosen as origin, whose proper time may be used as the parameter τ labelling the hyper-surfaces.

[17] An important point to be kept in mind is that the explicit functional form of embedding functions and - consequently - of the geometry of the 3 + 1 slicing of M^4, thought to be implicitly given at the outset, remains arbitrary until the solution of Einstein's equations is worked out in a fixed gauge: see later.

a 20-dimensional phase space[18]. Expressed (*modulo* surface terms) in terms of the ADM variables, the Einstein-Hilbert action is a function of N, N^a, ${}^3g_{ab}$ and their first time-derivatives, or equivalently of N, N^a, ${}^3g_{ab}$ and the extrinsic curvature ${}^3K_{ab}$ of the hypersurface Σ_τ, considered as an embedded manifold.

Since Einstein's original equations are not hyperbolic, it turns out that the canonical momenta are not all functionally independent, but satisfy four conditions known as *primary* constraints (they are given by the vanishing of the *lapse* and *shift* canonical momenta). Another four, *secondary* constraints, arise when we require that the primary constraints be preserved through evolution (the secondary constraints are called the *super-hamiltonian* $\mathcal{H}_0 \approx 0$, and the *super-momentum* $\mathcal{H}_a \approx 0$, $(a = 1, 2, 3)$ constraints, respectively). The eight constraints are given as functions of the canonical variables that vanish on the *constraint surface*. The existence of such constraints implies that not all the points of the 20-dimensional phase space represent physically meaningful states: rather, we are restricted to the *constraint surface* where all the constraints are satisfied, i.e., to a 12-dimensional (20 - 8) *surface* which, however, does not possess the geometrical structure of a true phase space. When used as generators of canonical transformations, the eight constraints map points on the constraint surface to points on the same surface; these transformations are known as *gauge transformations*.

To obtain the correct dynamics for the constrained system, we must consider the Dirac Hamiltonian, which is the sum of the De Witt surface term [17] (present only in spatially non-compact space-times and becoming the ADM energy after suitable manipulations [28]) and of the *primary* constraints multiplied by arbitrary functions (the so-called *Dirac multipliers*). If, following Dirac, we make the reasonable demand that the evolution of all *physical variables* be unique - otherwise we would have real physical variables that are indeterminate and therefore neither *observable* nor *measurable* - then the points of the constraint surface lying on the same *gauge orbit*, i.e. linked by gauge transformations, must describe the *same physical state*. Conversely, only the functions in phase space that are invariant with respect to gauge transformations can describe physical quantities.

To eliminate this ambiguity and create a one-to-one mapping between points in the phase space and physical states, we must impose further constraints, known as *gauge conditions* or *gauge-fixings*. The gauge-fixings can be implemented by arbitrary functions of the canonical variables, except that they must define a *reduced phase space* that intersects each gauge orbit exactly once (*orbit conditions*). The number of independent gauge-fixing must be equal to the number of independent constraints (i.e. 8 in our case). The canonical reduction follows a cascade procedure: the gauge-fixings to the *super-hamiltonian* and *super-momentum* come first (call it Γ_4); then the requirement of their time constancy fixes the gauges with respect to the primary constraints. Finally the requirement of time constancy for these latter gauge-fixings determines the Dirac multipliers. Therefore, the first level of gauge-fixing gives rise to a *complete* gauge-fixing, say Γ_8, and is sufficient to remove all the gauge arbitrariness.

The Γ_8 procedure reduces the original 20-dimensional phase space to a *reduced phase-space* Ω_4 having 4 degrees of freedom per point (12 - 8 gauge-fixings). Abstractly, the reduced phase-space is the quotient of the constraint surface by the 8-dimensional group of gauge transformations and represents the space of variation of the true degrees of freedom of the theory. Ω_4 inherits a symplectic structure (*Dirac brackets*) from the original Poisson brackets and is a *true phase-space* coordinatized by four *Dirac observables* (two configurational and

[18] Of course, all these *variables* are in fact *fields*.

two momentum variables): call such field observables q^r, p_s (r,s = 1,2). These observables carry the physical content of the theory in that they represent the *intrinsic degrees of freedom of the gravitational field* (remember that at this stage we are dealing with a pure gravitational field without matter). Concretely, for any *complete gauge fixing* Γ_8, we get a Γ_8-dependent *copy* of the abstract Ω_4 as a coordinatized realization of it in terms of Dirac observables. Though the Dirac observables are gauge-invariant, their functional form in terms of the original canonical variables depends upon the gauge, so that such observables - a priori - are neither tensors nor invariant under ${}_PDiff$. Yet, *off shell*, barring sophisticated mathematical complications, *any two copies of* Ω_4 *are diffeomorphic images of one-another*. After the canonical reduction is performed, the theory is completely determined: each physical state corresponds to one and only one set of canonical variables that satisfies the constraints and the gauge conditions.

It is important to understand qualitatively the geometric meaning of the eight infinitesimal *off-shell* Hamiltonian gauge transformations and thereby the geometric significance of the related gauge-fixings. i) The transformations generated by the four *primary* constraints modify the *lapse* and *shift* functions which, in turn, determine both how densely the space-like hyper-surfaces Σ_τ are distributed in space-time and the *gravito-magnetism* conventions; ii) the transformations generated by the three *super-momentum* constraints induce a transition on Σ_τ from one given 3-coordinate system to another; iii) the transformation generated by the *super-hamiltonian* constraint induces a transition from one given *a-priori* "form" of the 3+1 splitting of M^4 to another, by operating deformations of the space-like hyper-surfaces in the normal direction.

It should be stressed that the *manifest effect* of the gauge-fixings related to the above transformations emerges only *at the end* of the canonical reduction and *after* the solution of the Einstein-Hamilton equations has been worked out (i.e., *on shell*). This happens because the role of the gauge-fixings is essentially that of choosing the *functional form* in which all the gauge variables depend upon the *Dirac observables*, i.e. - physically - of fixing the form of the *inertial potentials* of the associated *non-inertial frame*. It is only after the initial conditions for the *Dirac observables* have been arbitrarily selected on a Cauchy surface that the whole four-dimensional chrono-geometry of the resulting *Einstein universe* is *dynamically* determined, including the embedding functions $x^\mu = z^\mu(\tau, \vec{\sigma})$. In particular, since the transformations generated by the super-hamiltonian modify the rules for the synchronization of distant clocks, *all* the relativistic *conventions*, associated to the 3 + 1 slicing of M^4 in a given *Einstein universe*, turn out to be *dynamically-determined, gauge-related options*[19].

Two important points must be emphasized.

First, in order to carry out the canonical reduction *explicitly*, before implementing the gauge-fixings we have to perform a basic canonical transformation, the so-called Shanmugadhasan transformation [10], moving from the original canonical variables to a new basis including the Dirac observables as a canonical subset. In practice, this transformation is adapted to seven of the eight constraints [28]: they are replaced by seven of the new momenta whose conjugate configuration variables are the gauge variables describing the *lapse* and *shift* functions and the choice of the spatial coordinates on the simultaneity surfaces. The new basis, then, contains the conformal factor (or the determinant) of the 3-metric,

[19] Unlike the special relativistic case where the various possible conventions are non-dynamical options.

which is determined by the super-hamiltonian constraint (though as yet no solution of this equation, also called the Lichnerowicz equation, has been found) and by the conjugate momentum (the last gauge variable whose variation describes the normal deformations of the simultaneity surfaces).
The Shanmugadhasan transformation is highly *non-local* in the metric and curvature variables: although, at the end, for any τ, the Dirac observables are *fields* indexed by the coordinate point σ^a, they are in fact *highly non-local functionals of the metric and the extrinsic curvature over the whole off shell surface* Σ_τ. We can write, *symbolically*:

$$\begin{aligned} q^r(\tau,\vec{\sigma}) &= \mathcal{F}_{[\Sigma_\tau]}{}^r\big[(\tau,\vec{\sigma})|\ {}^3g_{ab}(\tau,\vec{\sigma}), {}^3\pi^{cd}(\tau,\vec{\sigma})\big] \\ p_s(\tau,\vec{\sigma}) &= \mathcal{G}_{[\Sigma_\tau]_s}\big[(\tau,\vec{\sigma})|\ {}^3g_{ab}(\tau,\vec{\sigma}), {}^3\pi^{cd}(\tau,\vec{\sigma})\big], \quad r,s=1,2. \end{aligned} \tag{4.1}$$

Second: since, as mentioned, in spatially compact space-times the original canonical Hamiltonian in terms of the ADM variables is zero, the Dirac Hamiltonian happens to be written solely in terms of the eight constraints and Lagrangian multipliers. This means, however, that this Hamiltonian generates purely harmless gauge transformations connecting different admissible space-time 3+1 splittings, so that *it cannot engender any real temporal change*. Therefore, in spatially-compact space-times, in a completely fixed Hamiltonian gauge we have a vanishing Hamiltonian, and the canonical Dirac observables are constant of the motion, i.e. τ-independent.

The critical point, however, is that, in the case of spatially non-compact space-times such as those we are dealing with in this work, the generator of temporal evolution is the *weak ADM energy*, which is obtained by adding the so-called De-Witt boundary surface term to the canonical Hamiltonian [20]. Indeed, this quantity *does generate real temporal modifications of the canonical variables*. Thus, the final Einstein-Dirac-Hamilton equations for the Dirac observables are

$$\dot{q}^r = \{q^r, H_{\mathrm{ADM}}\}^*, \quad \dot{p}_s = \{p_s, H_{\mathrm{ADM}}\}^*, \quad r,s=1,2, \tag{4.2}$$

where H_{ADM} is intended as the restriction of the *weak ADM energy* to Ω_4 and where the $\{\cdot,\cdot\}^*$ are the Dirac brackets.

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[20] The ADM energy is a Noether constant of motion representing the *total mass* of the instantaneous 3-universe, just one among the ten asymptotic ADM Poincare' *charges*, the only asymptotic symmetries existing in Christodoulou-Klainermann space-times (due to the absence of super-translations). Consequently, the Cauchy surfaces Σ_τ must tend to space-like hyper-planes, normal to the ADM momentum, at spatial infinity. This means that such Σ_τ's are the *rest frame* of the instantaneous 3-universe, that asymptotic inertial observers exist to be identified with the *fixed stars*, and that an asymptotic Minkowski metric is naturally defined. This *asymptotic background* allows us to avoid a split of the metric into a background metric plus a perturbation, in the weak field approximation (note that our space-times provide a model of either the solar system or our galaxy, but not a model for cosmology). Finally, if gravity is switched off, the Christodoulou-Klainermann space-times collapse to Minkowski space-time and the ADM Poincare' charges become the Poincare' special relativistic generators. These space-times provide, therefore, the natural model of GR for incorporating particle physics. The mathematical background of these results can be found in Refs. [28] and references therein.

In conclusion, within the Hamiltonian formulation it is possible to find a class of solutions in which - contrary to what has been argued by Earman [15, 16] - there is *real temporal change*. But this of course means that the *frozen-time* picture is not a *typical* feature of GR.

V. FINDING THE LAST REMNANT OF PHYSICAL OBJECTIVITY: THE INTRINSIC GAUGE AND THE DYNAMICAL INDIVIDUATION OF POINT-EVENTS

We know that only some of the ten components of the metric are physically essential: it seems plausible then to suppose that only this subset can act as an individuating field, and that the remaining components play a different role.

Consider the following *four scalars invariant functionals* (the eigenvalues of the Weyl tensor), written here in Petrov's compressed notation:

$$\begin{aligned} w_1 &= \mathrm{Tr}\,(gWgW), \\ w_2 &= \mathrm{Tr}\,(gW\epsilon W), \\ w_3 &= \mathrm{Tr}\,(gWgWgW), \\ w_4 &= \mathrm{Tr}\,(gWgW\epsilon W), \end{aligned} \tag{5.1}$$

where g is the 4-metric, W is the Weyl tensor, and ϵ is the Levi–Civita totally antisymmetric tensor.

Bergmann and Komar [25, 26] proposed a set of invariant *intrinsic pseudo-coordinates* as four suitable functions of the w_T,

$$\hat{I}^{[A]} = \hat{I}^{[A]}\big[w_T[g(x), \partial g(x)]\big], \quad A = 0, 1, 2, 3. \tag{5.2}$$

Since they are *scalars*, the $\hat{I}^{[A]}$ are invariant under passive diffeomorphisms. It turns out that the four Weyl scalar invariants, *once re-expressed in terms of the ADM variables*, are independent of the *lapse function* N and the *shift vector* N^a, so that the *intrinsic pseudo-coordinates* are in fact functionals of the only variables ${}^3g_{ab}$ and the conjugated canonical momentum (the extrinsic curvature ${}^3K_{ab}$).

Under the *non-restrictive* hypothesis that *no* space-time symmetries are present - in an analysis of the physical individuation of points, we must consider *generic* solutions of the Einstein equations rather than the null-measure set of solutions with symmetries - the $\hat{I}^{[A]}$ can be used to *label* the point-events of space-time.

This implies that $\hat{I}^{[A]}$ are natural quantities to be used to implement four gauge-fixings constraints *depending only on a single hyper-surface* Σ_τ. On the other hand, in a completely fixed gauge Γ_8, the $\hat{I}^{[A]}$ become *gauge dependent* functions of the Dirac observables of that gauge.

Writing

$$\hat{I}^{[A]}[w_T(g, \partial g)] \equiv \hat{Z}^{[A]}[w_T({}^3g, {}^3\pi)], \quad A = 0, 1, 2, 3; \tag{5.3}$$

and selecting a *completely arbitrary, radar* coordinate system $\sigma^A \equiv [\tau, \sigma^a]$ adapted to the Σ_τ surfaces, we apply the *intrinsic gauge-fixing* defined by

$$\chi^A \equiv \sigma^A - \hat{Z}^{[A]}\big[w_T[(^3g(\sigma^B), {}^3\pi(\sigma^D)]\big] \approx 0, \quad A, B, D = 0, 1, 2, 3; \tag{5.4}$$

to the *super-hamiltonian* (A = 0) and the *super-momentum* (A = 1,2,3) constraints. This is a good gauge-fixing provided that the functions $\hat{Z}^{[A]}$ are chosen to satisfy the fundamental *orbit conditions* $\{\hat{Z}^{[A]}, \mathcal{H}_B\} \neq 0, \quad (A, B = 0, 1, 2, 3)$, which ensure the independence of the χ^A and *carry information about the Lorentz signature.* Then the complete Γ_8 *intrinsic gauge-fixing* leads to

$$\sigma^A \equiv \hat{Z}^{[A]}[w_T(q^a(\sigma^B), p_b(\sigma^D)|\Gamma)], \quad A, B, D = 0, 1, 2, 3; \quad a, b = 1, 2; \tag{5.5}$$

where the notation $w_T(q, p|\Gamma)$ represents the functional form that the Weyl scalars w_T assume in the chosen gauge Γ_8.

The last equation becomes an *identity* with respect to the σ^A, and amounts to a *definition* of the *radar* coordinates σ^A as four *scalars* providing a *physical individuation* of any point–event, in the gauge-fixed coordinate system, in terms of the gravitational degrees of freedom q^a and p_b. In this way each of the point–events of space-time is endowed with its own *metrical fingerprint* extracted from the tensor field, i.e., the value of the four scalar functionals of the *Dirac observables* (exactly four!)[21]. The price that we have paid for this achievement is, of course, that we have broken general covariance!

Note that this construction does *not* depend on the selection of a set of physically preferred coordinates, because by modifying the functions $I^{[A]}$ we have the possibility of implementing *any* (adapted) coordinate transformation. Passive diffeomorphism-invariance reappears in a different suit: we find exactly the same functional freedom of ${}_PDiff\, M^4$ in the functional freedom of the choice of the *pseudo-coordinates* $Z^{[A]}$ (i.e., of the gauge-fixing). *Any* adapted coordinatization of the manifold can be seen as embodying the physical individuation of points, because it can be implemented as the Komar–Bergmann *intrinsic pseudo-coordinates* after we choose the correct $Z^{[A]}$ and select the proper gauge[22].

All this is tantamount to claiming that the *physical role of the gravitational field in the absence of matter* is exactly that of individuating the points of M^4 *physically* as point-events, by means of its four independent phase space degrees of freedom.

As pointed out above, the mathematical structure of the canonical transformation that separates the Dirac observables from the gauge variables is such that the Dirac observables

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[21] The fact that there are just *four* independent invariants for the vacuum gravitational field should not be regarded as a coincidence. On the contrary, it is crucial for the purpose of point individuation and for the gauge-fixing procedure we are proposing.

[22] Note that the individuating relation (5.5) is a numerical identity that has a *built-in non-commutative structure*, deriving from the Dirac–Poisson structure hidden in its right-hand side. The individuation procedure transfers, as it were, the non-commutative Poisson-Dirac structure of the Dirac observables onto the individuated point-events, even though the coordinates on the l.h.s. of the identity are c-number quantities. One could guess that such a feature might deserve some attention in view of quantization, for instance by maintaining that the identity, interpreted as a relation connecting mean values, could still play some role at the quantum level.

are *highly non-local functionals of the metric and the extrinsic curvature over the whole (off-shell) hyper-surface* Σ_τ. The same is clearly true for the *intrinsic pseudo-coordinates* (see Eq.(5.3). Since the extrinsic curvature has to do with the embedding of the hyper-surface in M^4, the Dirac observables do *involve geometrical elements external to the Cauchy hyper-surface* itself. Furthermore, since the temporal gauge (fixed by the scalar $Z^{[0]}$), refers to a continuous interval of hyper-surfaces, the gauge-fixing identity itself is *intrinsically four-dimensional*.

In conclusion, as soon as the Einstein-Dirac-Hamilton equations are solved in the chosen gauge Γ_8, starting from given initial values of the *Dirac observables* on a Cauchy hyper-surface Σ_{τ_0}, the evolution in τ throughout M^4 of the Dirac observables themselves, whose dependence on space (and on parameter time) is indexed by the chosen coordinates σ^A, yields the following *dynamically-determined* effects: i) *reproduces* the σ^A as the Bergmann–Komar *intrinsic pseudo-coordinates*; ii) reconstructs space-time as an (on-shell) foliation of M^4; iii) defines the associated *global (non-inertial) laboratory*; iv) determines a simultaneity convention.

Now what happens if matter is present ? Matter changes the Weyl tensor through Einstein's equations and, in the new basis constructed by the Shanmugadhasan transformation, contributes to the separation of gauge variables from Dirac observables through the presence of its own *Dirac observables*. In this case we have *Dirac observables* for both the gravitational field and the matter fields, which satisfy *coupled Einstein-Dirac-Hamilton equations*. Since the gravitational Dirac observables will still provide the individuating fields for point-events according to our procedure, *matter will come to influence the evolution of the gravitational Dirac observables and thereby the physical individuation of point-events*.

What emerges here is an instantiation of *four-dimensional holism* of space-time (local in the temporal dimension). The underlying dynamically generated stratification depends upon the gauge. In correspondence to every *intrinsic gauge* there is a *distinct gauge-related individuation of point-events* and a *different stratification* in simultaneity 3-spaces and extended laboratories. Yet, according to a *main conjecture* advanced in Ref.[14], a canonical basis should exist that has a *scalar* character as well. If this is true - as an evaluation of the degrees of freedom in connection with the Newman-Penrose formalism for tetrad gravity [29] tends to corroborate - then *the dynamical individuation of point-events* will turn out to be *objective*[23].

At this point we could even say that the existence of physical *point-events* in our models of general relativity appears to be synonymous with the existence of the Dirac observables for the gravitational field. We advance accordingly the *ontological* claim that - physically - *Einstein's vacuum space-time is literally identifiable with the autonomous degrees of freedom of such a structural field*, while the specific (gauge-dependent) functional form of the *intrinsic pseudo-coordinates* maps such coordinates into the points of M^4. The intrinsic gravitational degrees of freedom are - as it were - *fully absorbed in the individuation of point-events*. Thus, in this way - point-events of space-time also keep a *special kind of structuralistic (non-point-like) intrinsic properties*, even more so if our *main conjecture* is true.

Finally, let us emphasize that, even in the case with matter, time evolution is still ruled by the *weak ADM energy* rather than by the simple canonical Hamiltonian. Therefore, the

[23] Objective in the sense of coordinate (or *gauge*) independence. One should not forget, however, that there is anyway a built-in *frame-dependence* in the concept of *radar coordinates* themselves[13].

temporal variation corresponds to a *real change* and not merely to a harmless gauge transformation as in other models of GR. The latter include, for instance, the spatially compact space-time without boundary (or simply closed models) which are exploited by Earman in Ref.[15]. Since in these spatially compact models the Dirac observables of every completely fixed gauge are τ-independent, the first of the gauge fixings (5.5) is inconsistent: it is impossible to realize the *time*-direction in terms of Dirac observables, and the individuation of point-events breaks down. This is compatible with the Wheeler-DeWitt interpretation according to which we can speak only of a local time evolution (in the direction normal to Σ_τ) generated by the super-hamiltonian constraint: in other words the local evolution would coincide with a continuous local change of the convention about distant clock synchronization!

We acknowledge that the validity of our results is restricted to the class of models of GR we worked with. Yet, we are interested in exemplifying a question of principle, and we claim that there is a basic class of models of GR embodying both a *real notion of temporal change* and a *new structuralistic and holistic view of space-time* [30].

Concerning the Hole Argument, a deeper analysis of the correspondence between symmetries of the Lagrangian configurational approach and those of the Hamiltonian formulation shows the following. Solutions of Einstein's equations that within the Hole, in the configurational approach, differ by elements of the subset of *active* diffeomorphisms that can be properly connected to the initial value problem, once seen at the Hamiltonian level are simply solutions differing by a *harmless Hamiltonian gauge transformation on shell.* Therefore, since outside and *inside the Hole* the gauge must be completely fixed *before* solving the initial-value problem and thereby finding the solution of the field equation throughout M^4, it *makes little sense to apply active diffeomorphisms to an already generated solution to obtain an allegedly "physically different" Einstein universe*. Conversely, it should be possible to generate these *different* solutions, corresponding to the same *universe*, by appropriate choices of the initial gauge fixing (the functions $\hat{Z}^{[A]}$). If, on the other hand, the *active* diffeomorphism is not a mere gauge transformation, it must modify the Cauchy data intrinsically, thus leading to a really different *Einstein universe*, yet violating the assumptions of the Hole Argument.

VI. CONCLUDING REMARKS

First of all, we point out that the isolation of the superfluous structure hidden behind the Leibniz equivalence, which surfaces in the physical individuation of point-events, renders even more glaring the ontological diversity of the gravitational field with respect to all other fields, even beyond its prominent causal role. It seems substantially difficult to reconcile the nature of the gravitational field with the standard approach of theories based on a *background space-time* (to wit, string theory and perturbative quantum gravity in general). Any attempt at linearizing such theories unavoidably leads to looking at gravity from the perspective of a spin-2 theory in which the graviton stands at the same ontological level as other quanta. In the standard approach of background-dependent theories of gravity, photons, gluons and gravitons all live on the stage on an equal footing. From the point of view set forth in this paper, however, *non-linear gravitons* are at the same time both the stage and the actors within the causal play of photons, gluons, and other *material characters* such as electrons and quarks.

Second, we believe that our results cast some light over the *intrinsic structure* of the general relativistic space-time that had disappeared behind Leibniz equivalence. Since space was uniform for Leibniz, he could exploit the principle of sufficient reason, while in GR the upshot is that space (space-time) is not *uniform* at all and shows a *rich structure*. In a way, in the context of GR, Leibniz equivalence ends up hiding the very nature of space-time, instead of disclosing it.

Third, remember what Bergmann and Komar wrote in Ref.[20]:

> [...] in general relativity the identity of a world point is not preserved under the theory's widest invariance group. This assertion forms the basis for the conjecture that some physical theory of the future may teach us how to dispense with world points as the ultimate constituents of space-time altogether.

Indeed, would it be possible to build a fundamental theory that is grounded in the reduced phase space parametrized by the Dirac observables? This would be an *abstract and highly non-local theory* of classical gravitation but, transparency aside, it would lack all the *epistemic machinery* (the gauge freedom) which is indispensable for empirical access to the theory. Indeed, once Einstein's equations have been solved, the metric tensor and all of its derived quantities, in particular the light-cone structure, can be re-expressed in terms of *Dirac observables* in a gauge-fixed functional form. Yet, if we look at the reduction procedure the other way around, we could imagine starting with a given choice of initial values for the *Dirac observables* (i.e., the germ of a *universe*), and adding all the required *gauge* variables as suitable independent variables, so as to obtain at the end a space-time expression for the *local field* $g_{\mu,\nu}(x)$. Since the relation between all tensor expressions and Dirac observables depends on the gauge, the gauge freedom would represent also the *flexibility* of the final local description of the *deep non-local structure* of the theory, a local description that supports the empirical access to the theory. In other words the *gauge* structure could be seen as playing a crucial role in the *re-construction* of the spatiotemporal continuum representation from a non-local structure. We see, therefore, that even in the context of classical gravitational theory, the spatiotemporal *continuum* plays the role of an *epistemic precondition* of our sensible experience of macroscopic objects, playing a role which is not too dissimilar from that enacted by Minkowski *micro-space-time* in the local relativistic quantum field theory. From the philosophical point of view, we find much more substance here than a simple instantiation of the relationship between canonical structure and locality that pervades contemporary theoretical physics.

Can this basic freedom in the choice of the *local realizations* be equated with a "taking away from space and time the last remnant of physical objectivity," as Einstein suggested? We believe that, discounting Einstein's "spatial obsession" with *realism as locality (and separability)*, a significant kind of spatio-temporal objectivity survives. It is true that - if our *main conjecture* is not verified - the *functional form* of the Dirac observables depends upon the particular choice of the latter (or, equivalently, of the gauge); yet, there is anyway no a-priori *physical* individuation of the manifold points independently of the metric field, so we cannot say that the individuation procedures corresponding to different gauges individuate *different* point-events. Given the conventional nature of the primary mathematical individuation of manifold points through 4-tuples of real numbers, we could say instead that the *real point-events* are *constituted* by the non-local values of gravitational degrees of freedom, while the underlying point structure of the mathematical manifold may be changed at will. A *really different* physical individuation should only be attributed to different initial conditions for the *Dirac observables*, (i.e., to a different *universe*). We can, therefore, say that general covariance represents the horizon of *a priori* possibilities for the physical constitution of space-time, possibilities that must be actualized within any given solution of the dynamical equations.

We conclude spending a few words on the implications of our results for the traditional debate on the absolutist/relationist dichotomy.

First of all, let us recall that, in remarkable diversity with respect to the traditional historical presentation of Newton's absolutism *vis á vis* Leibniz's relationism, Newton had a much deeper understanding of the nature of space and time. In a well-known passage of *De Gravitatione* (see Ref. [31] Hall and Hall (1962)), he expounds what could be defined as an original *structuralist view* of space and time. He writes (our *emphasis*):

> Perhaps now it is maybe expected that I should define extension as substance or accident or else nothing at all. But by no means, for it has its own manner of existence which fits neither substance nor accidents [...] the parts of space derive their character from their positions, so that if any two could change their positions, they would change their character at the same time and each would be converted numerically into the other *qua* individuals. The parts of duration and space are only understood to be the same as they really are because of their mutual order and positions (*propter solum ordinem et positiones inter se*); nor do they have any other *principle of individuation* besides this order and position which consequently cannot be altered.

We could surmise that a new kind of *holistic and structuralist* conception of space-time (see Ref. [29]) emerges from our analysis, including elements common to the tradition of both *substantivalism* (space-time has an autonomous existence independently of other bodies or matter fields) *and relationism* (the physical meaning of space-time depends upon the relations between bodies or, in modern language, the specific reality of space-time depends (also) upon the (matter) fields it contains). The points of general-relativistic space-times, quite unlike the points of the homogeneous Newtonian space, are endowed with a remarkably rich *non-point-like* and *holistic* structure furnished by the metric field and its derivatives. Therefore, the general-relativistic metric field itself or, better, its independent degrees of freedom, are able to characterize the "mutual order and positions" of points *dynamically*, since - as it were - each point-event "is" the "values" of the intrinsic degrees of freedom of the gravitational field. This capacity is even stronger, since *such mutual order is altered by the presence of matter*. On the other hand, even though the metric field does *not* embody the traditional notion of *substance*, it *exists* and plays a role for the individuation of point-events by means of its *structure*.

Finally, we agree of course with the thesis according to which the Hole Argument is a blow against strict *manifold substantivalism*. Yet, this result appears now to be rather trivial, and certainly it does not hold under the threat of *indeterminism*. For, to the extent in which the Cauchy problem of GR is well-posed, the *active diffeomorphisms* cannot generate *really different solutions* corresponding to the *same* initial conditions of Einstein's equations, the difference being only a *gauge equivalence* that must be *fixed* before finding the solution itself. If, on the other hand, they happen to modify the initial conditions, the Argument obviously does not apply. In the same sense, of course, the *intrinsic gauge* shows that *active diffeomorphisms* do not map point-events into point-events that are *physically* distinct.

On the other hand, the isolation of the intrinsic structure hidden behind the Leibniz equivalence - leading to our *point-structuralism* - does not support even the standard relationist view. As a matter of fact, by referring to Earman's third criterion (R_3) for relationism (see

Ref. [32]): "No irreducible, monadic, spatiotemporal properties, like 'is located at space-time point p' appears in a correct analysis of the spatiotemporal idiom", we can observe that: if 'spacetime points' mean our *physically individuated point-events* instead of the naked manifold's points, then - because of the autonomous existence of the intrinsic degrees of freedom of the gravitational field (a basic ingredient of GR) - the above-mentioned spatiotemporal property should be admitted to our spatiotemporal idiom.

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