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# Utilitarianism, Degressive Proportionality and the Constitution of a Federal Assembly

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## 1 Introduction.

A federal assembly consists of a number of representatives for each of the nations (states, *Länder*, cantons,...) that make up the federation. How many representatives should each nation receive? What makes this issue worth quibbling about is that the model of representation that is instituted will have an impact on the welfare distribution over the nations in the federation that will ensue over due course. We will investigate what models of representation yield welfare distributions that score higher on a utilitarian measure. First, we construct a continuum of models of representation ranging from equal to proportional representation. In between these extremes are models of *degressive proportionality*. We run a Monte-Carlo simulation in which a large number of motions are voted up or down within different contexts of evaluation and investigate how well the resulting welfare distributions score on the utilitarian measure. Subsequently, we will provide matching analytical results for a slightly idealized case. We conclude with a discussion of the significance of our results and of the role of simulations and analytical results and point to further work.

## 2 The Federation, its Constituent Nations and Models of Representation.

Let there be a federation that has a total population of $S$ people. It is divided into $N$ nations, some of them larger, some of them smaller. Each nation $i$ has a population size of $s_i$. The federal assembly is the decision-making organ for the federation. Our model can be readily generalized, but just to have some definite numbers, we will run our simulations with the actual population sizes of the European Union (see Table 1) and with the actual number of representatives in the Council of Ministers of the European Union before the 2004 enlargement. By using the pre-enlargement data we can avoid computational complexity and nothing is lost since the European Union is just a token federation in our investigation.

To represent the continuum between equal representation and proportional representation in the assembly, we construct the following measure, which determines the proportion of representatives of nation $i$ in the assembly:

$$(1) \qquad r_i(\alpha) = \frac{x_i^{\alpha}}{\sum_{i=1}^{N} x_i^{\alpha}} \qquad \text{for } x_i = s_i / S \text{ and } \alpha \in [0, 1]$$

$\alpha$ is a measure of the degree of proportionality. $r_i(0) = 1/N$ and there is equal representation in the assembly. $r_i(1) = x_i$ and there is proportional representation in the assembly. Intermediate values of $\alpha$ represent models of representation of degressive proportionality that are located on the continuum between both extremes. Obviously,

$\sum_{i=1}^{N} x_i = 1$ and $\sum_{i=1}^{N} r_i(\alpha) = 1$ for any value of α. We follow a simple procedure to turn these ratios $r_i(\alpha)$ of representatives into whole numbers of representatives $R_i(\alpha)$.[1]

## 3 Voting on Motions

A motion affects the people of the respective nations in different ways. A motion to improve the defense of the federation may benefit each nation to the same extent. But a motion to improve the highway system in some nation on the periphery of the federation does little more than benefit the nation in question, while it constitutes a cost to the other nations. A motion can be thought off as a utility vector $\langle v_1, ..., v_i, ..., v_N \rangle$ in which each $v_i$ represents the expected utility that the motion will bring to an arbitrary person of nation $i$ if the motion were adopted.

There is a certain threshold value of utility so that all the representatives of a nation will vote in favor of the motion if the utility that this motion will bring to the nation in question exceeds the threshold value. They vote against the motion if the utility drops below the threshold value. They will abstain if the utility equals the threshold value. Let us say that the threshold value is the point at which the costs balance out

[1] This is a complex question in voting theory, but for our purposes the following simple system suffices. We multiply the number of representatives $T$ in the Council with the ratio $r_i(\alpha)$. We assign, in a first step, $[r_i(\alpha)T]$—i.e. the whole number smaller than or equal to $r_i(\alpha)T$—representatives to each nation $i$. The number of remaining seats is $k(\alpha) = T - \sum_{i=1}^{N}[r_i(\alpha)T]$. Clearly $k(\alpha) < N$. These $k(\alpha)$ seats are distributed as follows. We order the nations according to the relative sizes of the decimal parts $r_i(\alpha)T - [r_i(\alpha)T]$, going from larger to smaller. We now assign to each of the first $k(\alpha)$ nations in this ordering precisely one additional seat. Let $R_i(\alpha)$ be the number of seats that each nation $i$ receives in the assembly on the proportionality measure α.

against the benefits for the nation of question. Costs and benefits should be understood broadly. They may also reflect feelings of altruism between the nations in question.

## 4 Contexts of Evaluation

David Hume (1888 [1739]: Book III, Part II, Section II) notoriously believed that questions of justice only arise if we can expect moderate selfishness (and not benevolence or extreme selfishness) and in times of relative scarcity (and not in times of extreme scarcity or abundance). We will not follow Hume's contention that questions of justice only arise under these conditions, but his taxonomy comes in handy in distinguishing between alternative *contexts of evaluation*:

(i) *Benevolence and Abundance*. In times of economic prosperity, or amongst nations that genuinely care about the well being of the other nations, the benefits that nations receive when a motion is adopted tend to outweigh the costs more often than not. There is money enough to go around so that costs matter minimally and there is a positive disposition towards political initiatives in general so that each nation's utility from a motion receives an added bonus. To model this situation, we let the utility values in the vector that represents a motion be random numbers generated under a uniform distribution over the range [-.5, 1] and we set the threshold value for acceptance at $v_t = 0$. Hence, the chance that an arbitrary nation will vote for a motion is 2/3. Let us name this the context of *generous* voters.

(ii) *Extreme Scarcity and Extreme Selfishness*. In times of economic recession, or amongst nations that are strictly concerned with their own welfare, the costs of a motion tend to outweigh the benefits more often than not. The nations are wary of expenditures and they only benefit from the implementation of motions in support of projects that directly affect their own welfare. We let the utility values be random numbers generated under a uniform distribution over the range [-1, .5] with $v_t = 0$. Hence, the chance that an arbitrary nation will vote for some motion is 1/3. This is the context of *stingy* voters.

(iii) *Moderate Selfishness and Relative Scarcity*. This context is intermediate between the previous two poles of the continuum. We let the utility values be random numbers generated under a uniform distribution over the range [-1,1] with $v_t = 0$. The chance that an arbitrary nation will support a motion is 1/2. This is the context of *balanced* voters.

Clearly there are alternative ways of understanding and modeling Hume's conditions. As we shall see later, the only thing that is required for our purposes is that particular specifications of three parameters of these distributions provide a meaningful characterization of certain types of voters. The Humean labels are no more than mnemonic aids.

## 5 Evaluating Models of Representation by Means of Monte-Carlo Simulations

Models of representation in a federal assembly are social arrangements. Each value of $\alpha$ constitutes an alternative social arrangement. We start with the model of equal representation ($\alpha = 0$) and increase the value of $\alpha$ with increments of $\Delta\alpha$ (which we set in our computer simulation at .01) until we reach the model of proportional representation ($\alpha = 1$). We consider $m$ motions for some sufficiently large $m$, say $m$ = 10,000, in our calculations. That is, we generate $m$ $n$-dimensional vectors $\langle v_1^k, ..., v_i^k, ..., v_N^k \rangle$ for $k$ = 1,..., $m$ with random numbers in the ranges that correspond to the respective contexts of evaluation. For each motion, a vote is taken by an assembly whose constitution is based on a particular value of $\alpha$. The representatives of a nation $i$ will all vote for a motion if $v_i^k$ exceeds the threshold value $v_t$ that we take to be same for all nations; they will all vote against the motion if $v_i^k$ is lower than $v_t$; and they will abstain if $v_i^k$ equals $v_t$. If the motion $k$ is accepted, each nation $i$ is assigned a utility value $v_i^k$. If the motion is not accepted, then it is discarded and each nation remains unaffected by the motion. After the $m$ motions have all been considered, we divide the sum of the utilities that each nation has accrued by $m$: the resulting vector $\boldsymbol{u}(\alpha) = \langle u_1(\alpha), ..., u_i(\alpha), ..., u_N(\alpha)\rangle$ contains the utilities $u_i(\alpha)$ that a person in nation $i$ can expect from a motion, given a particular model of representation represented by a specific value of the parameter $\alpha$. At the end of this process we have a vector of utility distributions associated with the values of $\alpha$, viz. $\langle \boldsymbol{u}(0), \boldsymbol{u}(\Delta\alpha), \boldsymbol{u}(2\Delta\alpha),..., \boldsymbol{u}(1)\rangle$, or, more specifically, in our computer simulation, $\langle \boldsymbol{u}(0), \boldsymbol{u}(.01), \boldsymbol{u}(.02), ..., \boldsymbol{u}(1)\rangle$. The measure that is to be maximized is the sum of the component utility values $u_i(\alpha)$ in the utility vector $\boldsymbol{u}(\alpha)$ weighted by the respective population proportions $x_i$:

$$(4) \qquad \mathrm{M}^{\mathrm{util}}[\boldsymbol{u}(\alpha)] = \sum_{\boldsymbol{i}=1}^{\boldsymbol{N}} x_i u_i(\alpha)$$

The model of representation $\alpha$ that maximizes this measure is the social arrangement that is supported by the utilitarian conception of justice. The results of our simulations are represented in Figures 1, 2 and 3.

## 6 Matching Analytical Results

The measure $M^{util}$ is an expectation, viz. the expected utility $E[U]$ from an arbitrary motion. We will compute this expectation by conditioning on the propositional variables $\boldsymbol{A}$ and $\boldsymbol{C}$. The variable $\boldsymbol{A}$ equals $A$ when the motion is accepted and $\neg A$ when the motion is not accepted. To define the variable $\boldsymbol{C}$, construct all the combinations of $i$ nations voting for the motion and $N - i$ nations voting against the motion. From combinatorial analysis, we know that there are $\sum_{i=0}^{N} \binom{N}{i} = 2^N$ such combinations. The variable $\boldsymbol{C}$ equals $C_1$ when all the nations vote for the motion, $C_2$ when all nations except for nation $N$ vote for the motion,..., and $C_{2^N}$ when all nations vote against the motions. By the probability calculus,

$$(5) \qquad E[U] = \sum_{\boldsymbol{A}=A,\neg A} \; \sum_{\boldsymbol{C}=C_1,\ldots,C_{2^N}} E[U|\boldsymbol{A}, \boldsymbol{C}]P(\boldsymbol{A}, \boldsymbol{C}).$$

Notice that $E[U|\neg\boldsymbol{A}, \boldsymbol{C}]$ equals 0 for any values of $\boldsymbol{C}$, since the expected utility of a rejected motion is 0. Furthermore, by the chain rule, $P(\boldsymbol{A}, \boldsymbol{C}) = P(\boldsymbol{A}|\boldsymbol{C})\mathrm{P}(\boldsymbol{C})$. Hence,

$$(6) \qquad E[U] = \sum_{\boldsymbol{C}=C_1,\ldots,C_{2^N}} E[U|A, \boldsymbol{C}]P(A|\boldsymbol{C})P(\boldsymbol{C}).$$

In Table 2 we illustrate this calculation for a federation of two nations named '1' and '2'. Each row lists the factors within each term of the sum in (6). First, let $u^+$ be the utility that a nation derives from an accepted motion assuming that they voted for the motion. $u^+$ equals 1/2 for *generous* and *balanced* voters and 1/4 for *stingy* voters. Let $u^-$ be the utility that a nation derives from an accepted motion assuming that they voted against the motion. $u^-$ equals –1/2 for *generous* and *balanced* voters, and –1/4 for *stingy* voters. On row 2 of the table, nation 1 voted for and nation 2 voted against. Hence, assuming that the motion is accepted, the expected utility from this motion is the sum of the $u^+$ and $u^-$, weighted by the population proportions of the respective nations. Second, the chance that the motion will be accepted depends on the proportion of the representatives in the assembly. The function $g(y)$ is defined as before, i.e. it equals 1 if $y > 0$ and 0 if $y \leq 0$. The chance that a motion is accepted equals 1 if a majority supports the motion, i.e. if $R_1 - R_2 > 0$, and equals 0 if the majority does not support the motion, if i.e. $R_1 - R_2 \leq 0$. Note that the values of $R_i$ are a function of $x_i$ and $\alpha$ for $i = 1, 2$. Third, let $p$ be the chance that an arbitrary nation will vote for a motion. We have seen before that $p$ equals 1/2 for *balanced* voters, 2/3 for *generous* voters and 1/3 for *stingy* voters. On row 2, the chance that the particular combination of nation 1 voting for and nation 2 voting against the motion equals $p(1 - p)$. In the last column we construct the product of these factors on each row and on the last row we construct the sum of these products.

The computational time in constructing a plot for $\alpha \in [0, 1]$ can be substantially reduced by assuming that the assembly has an infinite number of members, so that we can actually conduct a vote by means of the ratios $r_i(\alpha)$. This may seem like an unrealistic idealization, but the fact of the matter is that this idealization makes very little difference. We calculate $E[U]$ for *stingy*, *generous* and *balanced* voters in the European Union for $\alpha \in [0, 1]$ and have plotted these functions in Figures 4, 5 and 6. Note that the functions in Figures 3, 4 and 5 approximate the simulation results in Figures 1, 2 and 3.

It is worth noting that the function $E[U]$ is fully determined by the parameters $u^+$, $u^-$ and $p$ for a particular federation. In our simulation we specified a uniform distribution for $v_i$ for $i = 1,..., N$. But the only features of this distribution that are relevant are the probability $p$ that an arbitrary nation will accept a motion, the expected utility $u^-$ of an accepted motion for a nation that voted against the motion and the expected utility $u^+$ of an accepted motion for a nation that voted against the motion. As long as we keep these parameters fixed, the particular shape of the distribution is of no consequence for the quantities of interest in this paper.

## 7 Discussion

What is surprising about these results is the following. One might expect that utilitarianism would support proportional representation and that the only reason for instituting degressively proportional models is to protect the interests of smaller nations in the federation. In other words, it is a concern for equality or for the plight of the underdog that makes us move away from strict proportional representation. However, it turns out that also a strict utilitarian should support models of degressive proportionality once we give up the presumption of *balanced* voters and move in the direction of *stingy* or *generous* voters.

We have decided to present both the simulation results and analytical results. The reason for including the simulation is that we can obtain results for a finite number of representatives and that the assumption of independent and identically distributed utility distributions can be readily relaxed. If we want to model dependencies between the utilities for certain nations (e.g. Mediterranean nations) or let different nations have different expectations from motions, then we can generate vectors of utility values under a multivariate distribution in which the i.i.d. assumption does not hold. The reason for including the analytical result is that it shows how the results are invariant under alternative utility distributions under the i.i.d. assumption. The only parameters that matter are the parameters for the probability of acceptance, for the expected utility of a motion for a nation given that it will vote for the motion and for the expected utility of a motion for a nation given that it will vote against a motion.

We have restricted ourselves here to models of degressive proportionality that can be characterized by the $\alpha$ parameter and in which motions are accepted by a simple majority vote. The actual models that have been proposed for the EU Council of Ministers are much more complex. In Beisbart, Bovens and Hartmann (forthcoming) we present a utilitarian evaluation of various models of representation with different quotas of acceptance that have been discussed in academic and political contexts.[2]

## References


Beisbart, C., Bovens, L. and Hartmann, S. forthcoming. A Utilitarian Assessment of Alternative Decision Rules in the Council of Ministers. *European Union Politics*.

Hume, D. 1988 [1739]. *A Treatise of Human Nature.* Oxford: Clarendon.



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# Figures

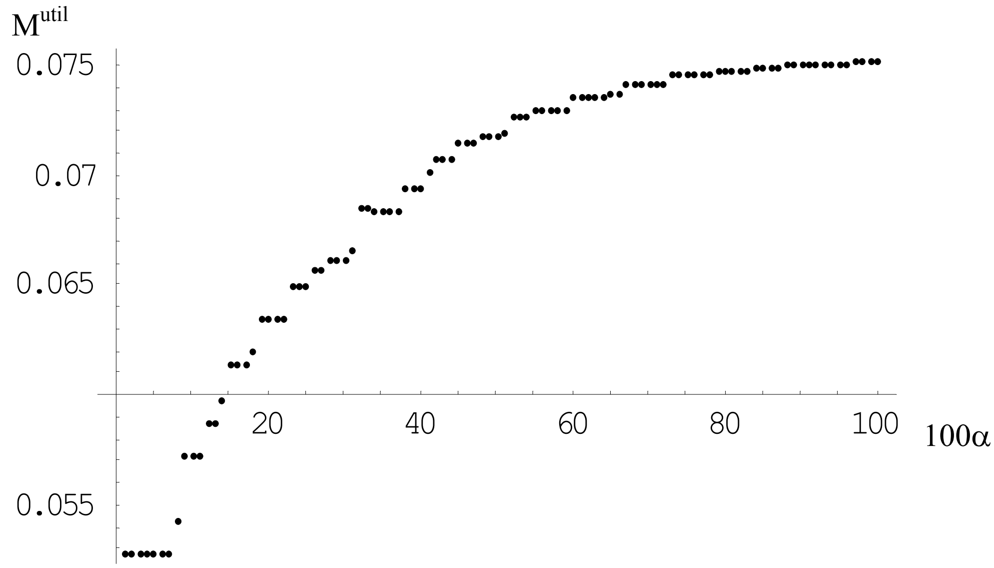


Fig. 1: The Utilitarian Measure for *Balanced* Voters for Europe and 87 Representatives in the Council of Ministers

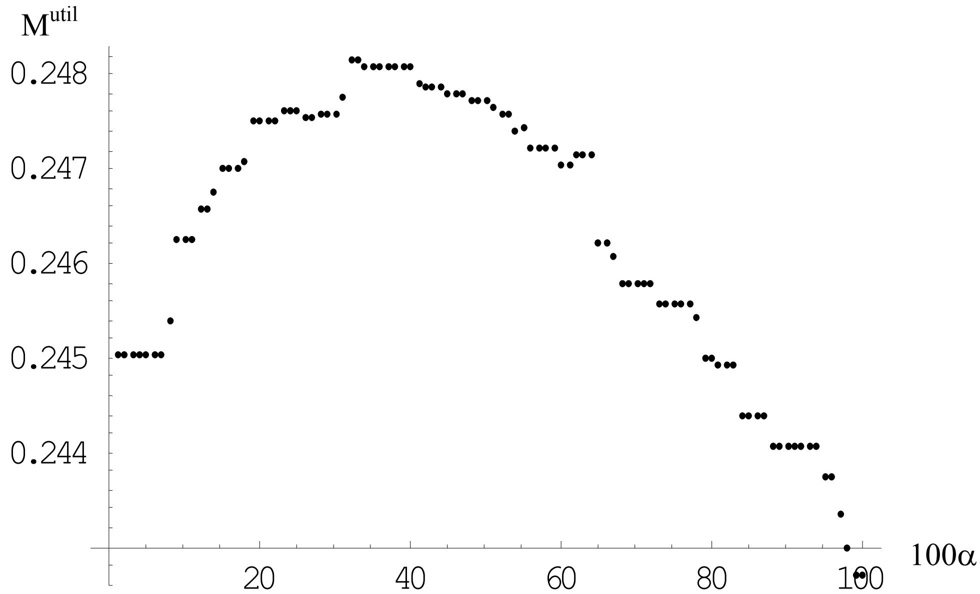


Fig. 2: The Utilitarian Measure for *Generous* Voters for Europe and 87 Representatives in the Council of Ministers

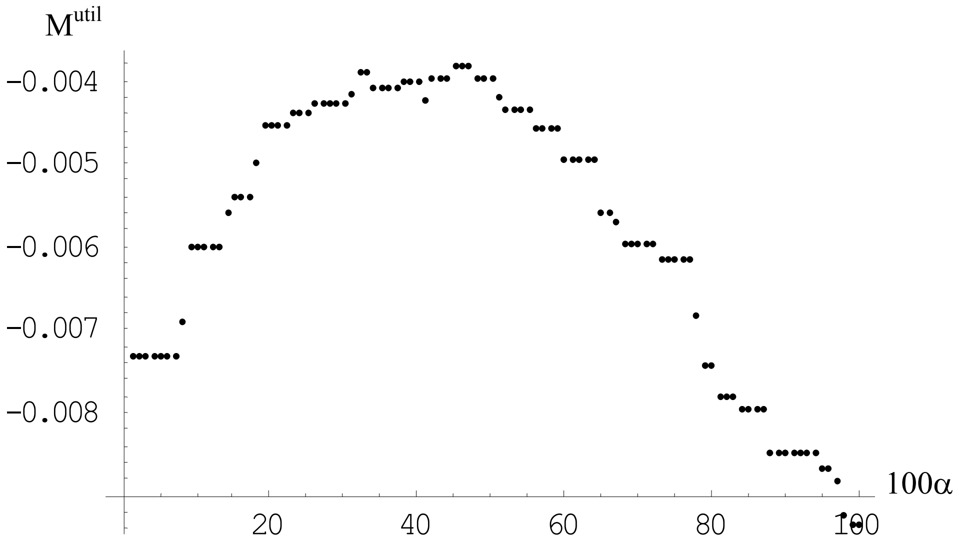


Fig. 3: The Utilitarian Measure for *Stingy* Voters for Europe and 87 Representatives in the Council of Ministers

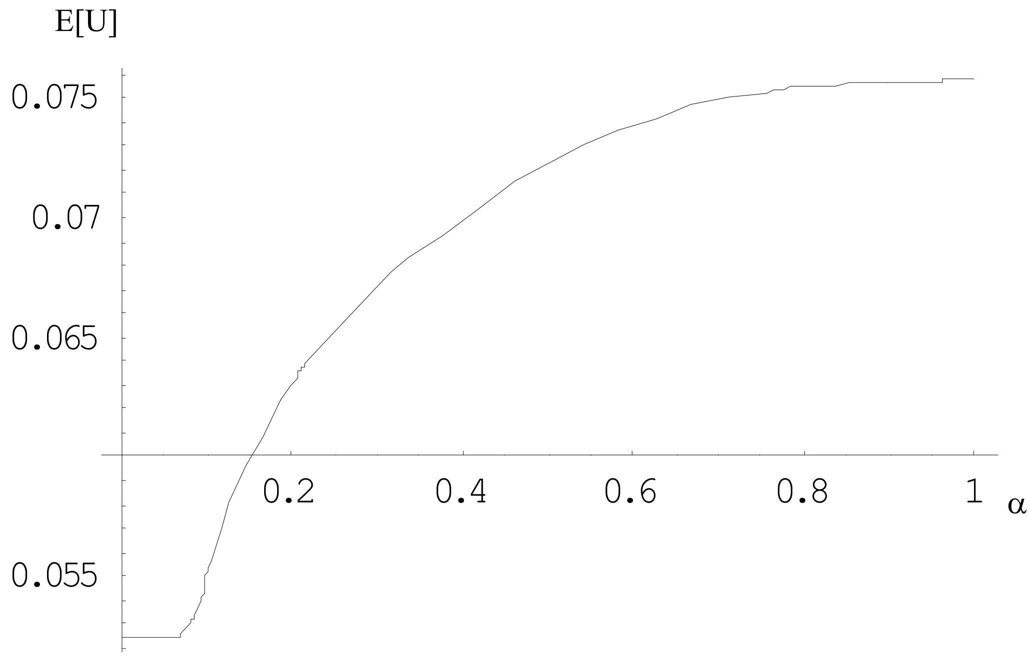


Fig. 4: The function E[U] for *Balanced* Voters for Europe and an Infinite Number of Representatives

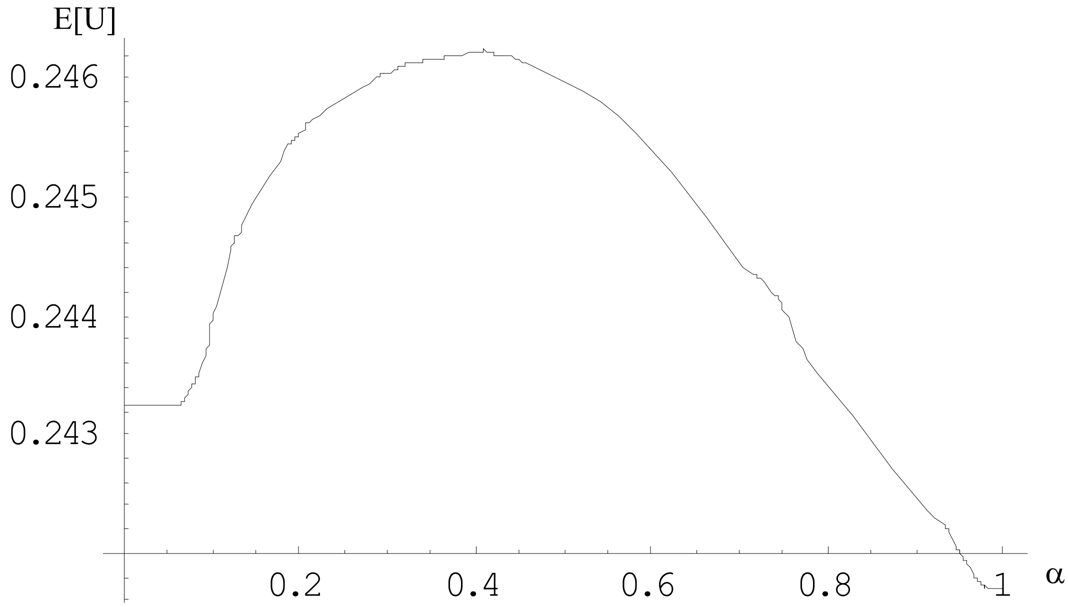


Fig. 5: The function E[U] for *Generous* Voters for Europe and an Infinite Number of Representatives

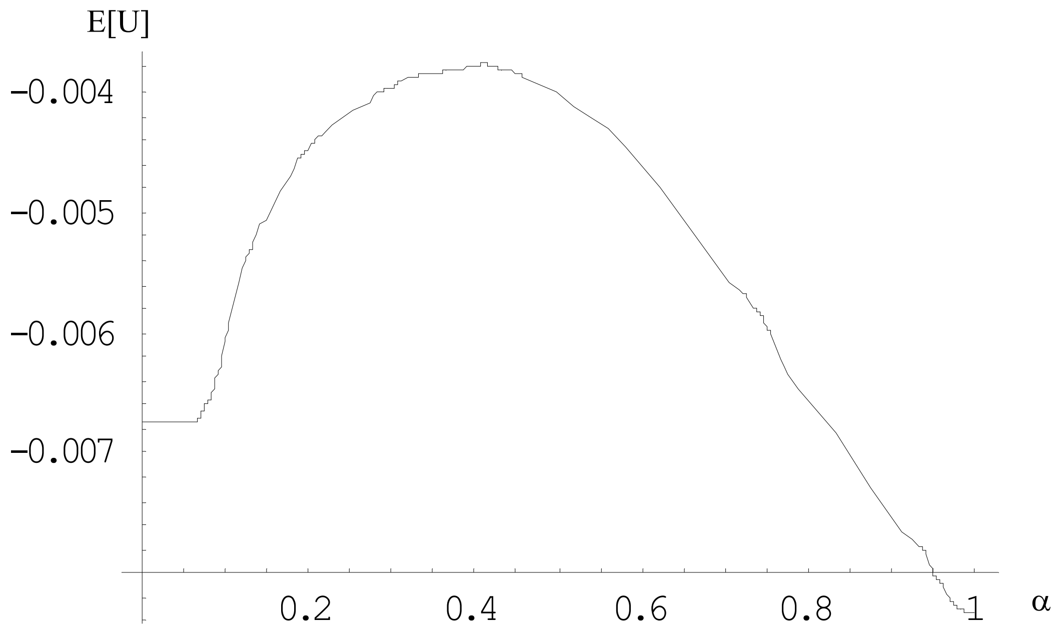


Fig. 6: The function E[U] for *Stingy* Voters for Europe and an Infinite Number of Representatives

# Tables

| | | |
|---|---|---|
| Austria | 7.9 | .0214 |
| Belgium | 10 | .0271 |
| Denmark | 5.2 | .0141 |
| Finland | 5 | .0136 |
| France | 57.2 | .1550 |
| Germany | 81.2 | .2201 |
| Greece | 10.2 | .0276 |
| Ireland | 3.5 | .0095 |
| Italy | 57.8 | .1566 |
| Luxembourg | .3897 | .0011 |
| Netherlands | 15.1 | .0409 |
| Portugal | 9.8 | .0266 |
| Spain | 39.1 | .1060 |
| Sweden | 8.8 | .0238 |
| UK | 57.6 | .1561 |
| Total | 369 | 1 |

Table 1: Population sizes in Millions (Second Column) and Population Proportions (Third Column) of the Constituent Nations of the EU in 2004

| | 1 | 2 | $E[U\|A, C_i]$ | $P(A\|C_i)$ | $P(C_i)$ | $\Pi$ |
|---|---|---|---|---|---|---|
| $C_1$ | + | + | $u^+x_1 + u^+x_2$ | $g(R_1 + R_2)$ | $p^2$ | |
| $C_2$ | + | – | $u^+x_1 + u^-x_2$ | $g(R_1 – R_2)$ | $p(1 – p)$ | |
| $C_3$ | – | + | $u^-x_1 + u^+x_2$ | $g(–R_1 + R_2)$ | $(1 – p)p$ | |
| $C_4$ | – | – | $u^-x_1 + u^-x_2$ | $g(–R_1 – R_2)$ | $(1 – p)^2$ | |
| | | | | | | $\Sigma$ |

Table 2: Construction of the Function $E[U]$ in Equation (6)