Is Simultaneity Conventional Despite Malament's Result?[1]

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Rather than answering this question directly, I'll let you make up your own minds on the basis of the considerations adduced in what follows. Before getting to them, I need briefly to lay out some background, particularly for those of you that don't have a clue as to what the question means.

In his groundbreaking 1905 paper "Zur Elektrodynamik bewegter Körper,"[2] which set out for the first time the special theory of relativity, Einstein begins the body of the paper by examining the operational meaning, at least in principle, of the standard use of coordinate assignments in kinematics. A rectilinear system of spatial coordinates can be established by the employment of rigid rods and the rules of Euclidean geometry. Differences in the time coorinate at any fixed triple of spatial coordinates can be determined by means of a standard clock positioned at that point. In order to describe motion, however, one must have a criterion for judging when standard clocks at *different* spatial locations are synchronous. Such a criterion, according to Einstein, is not available without the introduction of some further stipulation. Thus, he introduces, as a *definition*, the stipulation that the time required for light to travel in vacuo from a point A to a point B is the same as that required for light to travel in the reverse direction from B to A. Since the distance to and fro is the same, this is equivalent to stipulating that the speed of light is the same in both directions. Given this stipulation, it follows that a clock at A synchronizes with an identically constituted clock at B just in case the following holds. If a light ray emitted from point A when the clock there reads t_A is reflected back to A from point B when the clock there reads t_B, then it arrives back at A when the clock there reads

$$t_A{}' = t_B + (t_B - t_A).$$

In the form made famous by Reichenbach in the 1920's,[3] this is to say that the

[1] Read for the author by Steve Gimbel at a meeting of the Canadian Society for the History and Philosophy of Science at Université Laval, Quebec City, 24 May 2001.

[2] *Annalen der Physik* **17** (1905): 891-921.

[3] First in his *Axiomatik der relativistischen Raum-Zeit-Lehre* (Braunschweig: Vieweg u. Sohn, 1924), p. 26 and p. 34, where it is used on the way toward establishing a global time function even before the notion of an inertial frame is defined, and later in *Philosophie der Raum-Zeit-Lehre* (Berlin u. Leipzig: Walter de Gruyter, 1928) p. 151 where it is not clear from the discussion whether Reichenbach tacitly assumes that an inertial frame has already been selected. It may be overstatement to say that the fame derives from these works rather than from the translation of the latter into English by Maria Reichenbach and John Freund, *The Philosophy of Space and Time* (New York: Dover, 1958) or perhaps from the writings of Grünbaum and Salmon, who were already acquainted with the formulation.

two clocks synchronize when $\epsilon = 1/2$ in the equation

$$t_B = t_A + \epsilon({t_A}' - t_A).$$

Reichenbach, one of the earlier philosophical defenders of the conventionality of distant simultaneity, pointed out that different choices of values for ϵ (restricted to the open interval between 0 and 1) correspond to alternative stipulations concerning the one-way speed of light.

In 1977, David Malament published a result[4] that seemed to many to establish that no such stipulative definition concerning one-way light speeds is needed in order to secure a criterion of distant simultaneity. Malament's result builds on the earlier work of the English mathematician Alfred A. Robb, who in 1914 developed a synthetic axiomatization of Minkowski spacetime using, as his only non-logical primitive, the relation of causal connectibility between spacetime points.[5] One of Robb's accomplishments was to show that it is possible in his system to define a generalization of the ordinary notion of spatial orthogonality, one which applies to time-like and light-like vectors as well as to space-like vectors. In his honor, let us refer to this generalized relation as *Robb othogonality*. It has the following distinguished property. Given any inertial world-line W and any point P on W, the set of all points Q such that the line PQ is Robb-orthogonal to W is just the set of all points simultaneous with P using the standard Einstein ($\epsilon = 1/2$) synchronization procedure in the inertial frame of W.

Now Malament's result is that Robb-orthogonality is the *only* relation definable from causal connectibility and the world line W that carves up Minkowski space-time into 3-dimension spatial hypersurfaces. That is, the standard $\epsilon = 1/2$ Einstein synchronization criterion is the one and only simultaneity relation definable for a given inertial frame using the primitive relation of causal connectibility. Thus, if one accepts the principle that a relation in Minkowski spacetime is conventional only if it is not uniquely definable from causal connectibility (a principle which Malament does not commit himself to, but which is widely accepted by others), then it follows that simultaneity relative to an inertial frame is not a matter of convention.

The way I have stated this argument is a bit sloppy in at least one respect. Strictly speaking there is a suppressed premise to the effect that (although I hate putting it this way) if a relation is definable in extension, then it is definable in intension. Since talk about intensions is pretty murky stuff, as is talk about what

[4]'Causal Theories of Time and the Conventionality of Simultaneity', *Noûs* **11** (1977): 293–300.

[5]*A Theory of Time and Space* (Cambridge: at the University Press, 1914). Robb later published a less technical survey of his work under the title *The Absolute Relations of Time and Space* (Cambridge: at the Univerity Press, 1921). Although Robb introduced a strictly asymmetric relation of causal connectibility, and hence a temporal orientation on Minkowski space-time, what follows remains correct for the symmetric closure of the relation and hereafter we will mean by 'causal connectibility' the symmetric closure of Robb's relation.

it means for something to be conventional, let's try to work our way through this without using such terminology.

What's pretty clear so far is this. If I'm just given the causal structure of Minkowski spacetime, a world line W, and a pair of space-like separated points Q_1 and Q_2, then it's clear that it's straightforwardly true or false whether or not Q_1 and Q_2 are $\epsilon = 1/2$ simultaneous relative to W's inertial frame. Thus I know whether or not Q_1 and Q_2 are simultaneous *simpliciter* (relative to that frame) *if for any pair of stationary world lines in that frame, the one-way speed of light from the one to the other is the same as in the reverse direction*. So, in order to know whether Q_1 and Q_2 are simultaneous *simpliciter* (in that frame, anyway), I need to know something about the one way speed of light in that frame. And how will I figure this out? Or is it somehow *a priori* true in virtue of the fact that Robb-orthogonality is definable from causal structure alone?

It will be easier to make sense out of this if we try to develop a contrasting case that uses a different (putative) stipulation concerning the one way speed of light but is the same as special relativity in all it's other assumptions. Such an example is available from what I call the Houdini class of stationary aether theories, according to which every inertial frame has the same fixed velocity with respect to the aether and thus if there are observable aether-drift effects, they are quantitatively the same for everyone. Of interest for us is one of these theories having no observable aether-drift effects. This results from adopting the following one-way speed of light assumptions in an inertial frame:

$$+\hat{x} : (c - v)/(1 - v^2/c^2)$$

$$-\hat{x} : (c + v)/(1 - v^2/c^2)$$

$$\pm y : c$$

$$\pm z : c,$$

where c is the average round-trip speed of light and $v < c$ is some fixed speed that one can picturesquely think of, more or less, as the rate of travel through Houdini's magic aether. If you like (in fact it's better for what follows) simply set $v = c/2$.

The simultaneity sheets that arise for this inertial frame from these one-way speed of light assumptions are those of the imaginary aether frame, and hence not Robb-orthogonal to the stationary world lines of the frame. But since all inertial frames use precisely the same parameters for the one-way speed of light (in accord with the Principle of Relativity), different inertial frames disagree on what frame is the rest frame of Houdini's aether. Thus, just as with the standard $\epsilon = 1/2$ stipulation, each frame has a different standard of simultaneity. Furthmore, one can derive the set of coordinate transformations that carry you from one inertial frame to another by requiring that the equation for the propagation of light has the same form in each of them. What these look like is not important here. Suffice it to say that under composition they form a group isomorphic to the Lorentz group. In fact, under the naturally induced spacetime metric, the resulting spacetime is isomorphic to Minkowski spacetime and thus, at least up to isomorphism, has the same symmetry group. The appearance of spatial anisotropy is just that — an appearance, since the associated simultaneity sheets are not intrinsic to the induced spacetime structure. However, in order to have a convenient tag to discuss them, I'll use the label 'H-simultaneity'.

Having done all this, let's now go back and ask ourselves the same question about points Q_1 and Q_2 and world line W, only now with regard to H-simultaneity rather than standard $\epsilon = 1/2$ simultaneity. Are Q_1 and Q_2 H-simultaneous relative to W? Obviously, the question has no answer until we are told how the coordinate axes have been grafted onto W's inertial frame. More precisely, we don't know what to say until we at least know how the x axis has been oriented.

Nonetheless, what we *can* say is whether or not Q_1 and Q_2 are H-simultaneous relative not only to W and but also to a given spatial direction. Thus, what we have is a simultaneity relation that is relative to the choice of both an inertial frame and a spatial direction. Now, without saying precisely how, I'll simply point out that it is possible to introduce non-standard one-way speed of light assumptions that yield simultaneity relations that are not relative (in part) to a choice of direction, but instead, say, to a choice of coordinate origin and temporal orientation.

This said, I think we can safely generalize on what we have seen. In order to say what the one-way speed of light is between a given pair of spatial points (i.e., co-moving inertial world lines), we obviously need a family of simultaneity sheets onto which we can parallel project light-like vectors. If we require that simultaneity be relative to a choice of inertial frame alone and nothing else, then only one such family of simultaneity sheets will do, namely, that consisting of Robb-orthogonal hypersurfaces. It then follows that the one-way speed of light is the same in all directions. No further stipulation is required.

BUT, I think it deserves to be asked, where does the requirement that simultaneity be relative *only* to the choice of inertial frame come from? Is this a postulate of the theory of relativity? Can we put it to test? Is it an *a priori* truth? Or is it rather an artifact of having gotten so used to working with the standard one-way speed of light assumption that we take it entirely for granted and consequently overlook the possibility that the requirement is itself just one of serveral available conventions, one in fact equivalent to the standard Einstein convention?