

You Can't Mean That: Yablo's Figuralist Account of Mathematics

Sarah Hoffman

**Department of Philosophy
University of Saskatchewan
sarah.hoffman@usask.ca**

Burgess and Rosen argue that Yablo's figuralist account of mathematics fails because it says mathematical claims are really only metaphorical. They suggest Yablo's view is implausible as an account of what mathematicians say and confused about literal language. I show their argument isn't decisive, briefly exploring some questions in the philosophy of language it raises, and argue Yablo's view may be amended to a kind of revolutionary fictionalism not refuted by Burgess and Rosen.

First, a quick run through of some of the terminology in this area, to refresh our memories: Fictionalism in the philosophy of mathematics is the view that mathematical objects are mere fictions, talk of them mere *façon de parler*. Most fictionalists believe that mathematical propositions are false. Fictionalists are often, but not always, motivated by nominalist scruples. These are not Yablo's reasons; his view—figuralism—is a kind of fictionalism, one that seeks to explain how mathematics and its language operate without positing abstract mathematical objects. He does this by interpreting mathematical language as figurative rather than literal. This supplies the central argument advanced against his account: it fails because it claims mathematicians only pretend to assert mathematical claims, but really advance them only as metaphors or metaphorically true. Burgess and Rosen (1997, 2005) and Burgess (2004) in particular make this complaint, suggesting that Yablo's view is both implausible as an account of mathematics, and confused about the nature of literal language. I show their attempt to undermine Yablo is not decisive, briefly exploring some more general questions in the philosophy of language raised by this dispute. In particular I take up the claim

about literality. Further, I argue that Yablo's account can be amended to a form of revolutionary fictionalism, a kind not refuted by Burgess and Rosen.

Yablo's (2001, 2002, 2005) philosophy of mathematics takes up some central elements of Kendall Walton's (1990, 1993) pretense account of fiction. Walton's theory explains fictional discourse using the concepts of games of make-believe, props, pretense, and imagining. According to Walton, works of fiction are props in games of make-believe; their function is to operate in those games to make certain things fictional. In virtue of the properties it has, for example, *The Golden Compass* makes it fictional that there are daemons (small creatures that take the form of different animals and are the embodiment of the soul of their humans.). What it means for this to be fictional is that in all the authorized games of make-believe associated with *The Golden Compass*, it is *to be imagined* that there are daemons. When engaging in such games of make-believe it is appropriate to pretend that this is the case, that is, *to be as if you believed that there are daemons*. Walton can thus provide an account of the various sorts of things people say about fictions without requiring the existence of fictional objects.

Walton also provides the conceptual resources to explain the different stances and aims that we take up in relation to fictions. Sometimes we fully engage in the pretense of a fiction, our interest is in its content, sometimes we don't, or our interest is in the props and how they generate content. Walton (1993) takes up this distinction between prop and content-oriented make-believe and argues that it can help us understand (at least some, central) cases of metaphor. A metaphor, according to Walton, invokes or reminds us of a game of make-

believe, one oriented towards props.¹ (Juliet is the sun). It “is an utterance that represents its objects as being *like so*: the way that they *need* to be to make the utterance ‘correct’ in a game that it itself suggests.” (Yablo, 2005, p. 98) This leads Yablo to develop his fictionalism about mathematics as a kind of figuralism. So, an example:

“Talking to philosophers wakes up the butterflies in my stomach.”

Stomach butterflies are the creatures of existential metaphor. That is, *we invoke them (metaphorically) merely as representational aids*. This allows us to express some real content non-literally. How does this work for math? Like stomach butterflies, reference to mathematical objects occurs because they serve as representational aids. There are strong parallels between our mathematical talk and such “(certifiably figurative) talk” (2001, 91). The real content of “The number of Canadian provinces is 10” is “There are 10 Canadian provinces,” the truth of which doesn’t require number objects, any more than stomach butterflies have to exist for mine to be battering my insides.

We can get more of mathematics by seeing that quantification over numbers arises because we want to “express... infinitely many facts in a finite compass” (2005, 94). Mathematical objects are also introduced, however, as representational aids for already posited mathematical objects, when “descriptive needs arise w.r.t., not the natural world, but our system of representational aids as so far developed” (2005, 96). So we get numbers operating both as the things represented and as representational aids (e.g. “The number naturals is the same as the number of rationals.”) So we have an explanation of how number talk functions, and how we could be saying something true when we utter existential

¹ Walton understands the framing effect of metaphors, by which they point us towards understanding one thing in terms of another, “as consisting in the metaphor’s implication or introduction or reminder of a game of make-believe.” (1993, p. 75 in Kalderon).

mathematical sentences, even if there are no numbers. The same holds of other mathematical objects, including those introduced in parasitic games to help represent facts about mathematical objects we treat as objects represented.

Yablo's claim that mathematical existence claims are examples of figurative language has drawn sharp criticism, notably in Burgess (2004) and Burgess and Rosen (2005).² Burgess identifies Yablo's figuralism as a kind of hermeneutic fictionalism, the view that "*the mathematicians' own understanding* of their talk of mathematical entities is a form of fiction, or akin to fiction: mathematics is like novels, fables, and so on in being a body of falsehoods *not intended to be taken for true*" (2004, 23). This differs from revolutionary forms of fictionalism, which maintain that mathematicians are systematically mistaken in understand their own mathematical talk literally.

Both versions of mathematical fictionalism fail, according to Burgess and Rosen. Revolutionary fictionalism can't work because it doesn't pass naturalist muster: What business or competence do philosophers have to attempt reform of mathematics? None, since, "[t]here is no philosophical argument powerful enough to override or overrule mathematical and scientific standards of acceptability" (2005, 517). Hermeneutic fictionalists, on the other hand, advance an implausible claim about what mathematicians mean. It is "in actual fact very doubtful... whether mathematicians who assert that there are prime numbers greater than $10^{10^{10}}$ intend their assertion only as something 'non-literal'" (Burgess 2004, 26). In fact, by maintaining that mathematicians do not literally assert what they say, the hermeneutic fictionalist also makes a mistake about the nature of literalness. There is a defeasible

² The central objection to mathematical fictionalism I take up in what follows is also found in Cole (2005a, 2005b). A very useful critical discussion of general hermeneutic fictionalism is found in Stanley (2001).

“presumption that people mean and believe what they say” and, moreover, “[o]ne doesn’t have to think anything extra in order to speak literally: One has to think something extra in order to speak non-literally” (Burgess 2004, 26). Unless mathematicians are doing this extra thing, we must understand them as literally asserting when they make mathematical existence claims. Moreover, there simply is no evidence that mathematicians are doing anything beyond meaning and believing just what they say: “in all clear cases of figurative language ... the nonliteral character of the linguistic performance will be perfectly obvious as soon as the speaker is forced to turn attention to the question of whether the remark was meant literally” (2005, 533). Call this the “perfect obviousness” test for the figurative use of language. Mathematical language fails this test, so hermeneutic fictionalism collapses. Hermeneutic fictionalism entails the possibility of *misunderstanding* mathematicians’ existence claims as literal. We would expect mathematicians to correct such misunderstandings, saying something like “No, no, I don’t mean that numbers literally exist. I am only using ‘exist’ figuratively.” Burgess and Rosen contend that no actual mathematicians would respond this way, in contrast with, say, a teacher faced with a student asserting that Shakespeare is ridiculous, Juliet isn’t the sun, she’s too small and cold.

So, to summarize: Yablo’s figuralism should be rejected—understood as revolutionary, it fails to be naturalist, understood hermeneutically it is falsified by the way mathematicians really talk.

A number of avenues of response are open. First, Yablo’s figuralism could be reinterpreted as revolutionary fictionalism, and the naturalist argument against revolution met. I take this up below. An alternative line takes Yablo at face value when he says his view “is

put forward in more of a hermeneutic spirit” (2001, 85), but notice that he qualifies this claim in a way that illuminates the fundamental dispute here: It is not that mathematicians operate as fictionalists and just don’t know it, but that they are “something closely related.” They (and we) are “people apt to speak figuratively” (2001, 85). This is a denial of the default presumption of literalness. That Yablo intends this as a general claim about how we use language, not one restricted to mathematical talk, is clear from his suggestion that figurative speech is the default or normal way in which we (all of us) talk and “if it is non-figurative speech that is special... then figurative fictionalism might be just what the doctor ordered” (2001, 85). So the dispute is really a general one about language and meaning, rather than specifically about mathematics.

Consider a bit more of what Burgess says about literal meaning: “To mean what one says literally is simply to mean what one says, just as to be a genuine antique is simply to be an antique” (2004, 26) Is *what one says* quite so easily identifiable as this claim blithely presupposes? But what kind of *saying* is Burgess invoking? Is uttering “Can you pass me the salt?” literally inquiring about your abilities, or literally requesting that you pass me the salt? Burgess would pick, I presume, the former, in line with the standard conception of the literal/non-literal distinction as a matter of language *use*, rather than *meaning*. But even if he is right that the default case is a speaker meaning the semantic content of the sentence they utter, this itself may not be so easily fixed, as the debate between contextualism and minimalism shows us. I do not mean to raise here the old referential ambiguity problem many have used in attempts to refute realist accounts of mathematics (i.e., Which omega sequence are *you* talking about?). Though clearly related, the problem here is more general. To use

Searle's (1978) example: What is the literal meaning of a sentence like "The cat is on the mat"? Does it express a minimal proposition which is true iff the cat is on the mat? Or is it more context sensitive, such that it has different, more specific truth (or assertability or what have you) conditions on different occasions of its utterance? I don't have a horse in this particular race, but to the extent that semantic contextualism is persuasive, it seems that Burgess is offering an only *apparently* much simpler and more straightforward alternative to Yablo's view. Semantic contextualism requires pragmatic processes operating not just post-semantically to determine implicatures and so on, but also in the determination of the truth (or assertability or whatever) conditions of an utterance. This means that even if Burgess is right that mathematicians mean just what they literally say, we are still left with the problem of pinning down just what they do literally assert. So, semantic minimalism may offer an escape route, but it has its costs as well.

Nevertheless, Yablo does talk about figurative language as an example of indirect speech (1998, 291) If so, then for language to be figurative is for it to be *used* figuratively, thus its user must intend that what they are saying not be taken literally. And if so, then the objection goes through, since such intentions are unlikely to be present in the minds of (many most?) of those who make mathematical existence claims. So the problem here is that figurative speech is normally taken as a central example of indirect speech. However, this idea doesn't seem to sit very well with Yablo's apparent reversal of the normally assumed relationship between literal and figurative language. (Recall his "if it is non-figurative speech" rather than figurative "that is special.") But if figurative language is the default and literal the special case, then one would suspect that the more likely case of indirectness would be the

literal. But that can't be right. The pragmatic mechanisms that indirect speech exploits all take as input the literal meaning directly expressed by an utterance. Yablo, however, points out that this model of our comprehension of indirect speech appears to have been falsified by empirical evidence.³

In any case, the issue of indirectness is something of a red herring. Instead, what Yablo may have in mind is that figurative language is regularly our way of operating, because when we have novel meanings to express, as we frequently do, we often don't have the lexical resources to do so literally. Consider, for example:

“Just north of Memphis, Highway 61 has soft shoulders.”

Is this an example of literal or figurative speech? It certainly doesn't seem very figurative.

Compare the (arguably) more figurative:

“First philosophy is in some quarters rudely again shouldering its way past naturalism.”

What are we to make of these examples? The first documented use of shoulder to refer to the strip along the side of a road is 1933.⁴ By now it can be used quite literally in this way. Was its first such use metaphorical? I would maintain it must have been—even if not then consciously intended as such—and even if it almost immediately become literal. What is clear is that there are ways in which figurative uses become (at least *more*) literal. Semantically

³ Yablo (2001) cites Gibbs (1994) on this point.

⁴ One of the earliest example the OED cites is Shakespearean: *Hamlet*. I. iii. 56 “Aboord, aboard for shame, The winde sits in the shoulder of your saile.” As applied to a road, the earliest is: **1933** *Sun* (Baltimore) 27 Dec. 8/7, “I... stayed well over on the shoulder. But... only one of the numerous cars... bothered to move nearer the middle of the road. Repeatedly, I stepped back into the bushes and mud.”

As a matter of fact ‘shoulder’ is among the few hundred earliest surviving English words (dating from at least 700 CE). Its earliest meaning is indeed the human body part/area. Its application to things without arms and necks does not appear until hundreds of years later in the 16th century

deviant uses start out as metaphors, some are dropped, some continue as metaphors and some become new literal meanings. This is just the way of languages.

Burgess and Rosen might well respond that the most this shows is that there are unclear cases about which we can have differing intuitions. Their perfect obviousness test is formulated for *clear* cases of figurative language. The perfect obviousness test is thus an objection to Yablo only if he is committed to mathematics being a clear case of figurative language. Is he? Clearly not:

These mathematical metaphors prove so useful that they are employed on a regular basis. As generation follows on generation, the knowledge of how the mathematical enterprise had been launched begins to die out and is eventually lost altogether. People begin thinking of mathematical objects as genuinely there. (2005, 108)

What this suggests is that the mathematical metaphor might be dead. A new literal meaning has arisen. Mathematical objects are not so much the creatures, but the *orphans* of existential metaphor. (So it isn't surprising that in talking about them we do not normally register their metaphorical origins.) And the shoulder example is instructive in another way. It highlights the dynamic nature of the semantics of real language. New meanings get introduced through metaphorical extension—this year's metaphor becomes next year's literal truth. This might seem to concede Burgess and Rosen's point, since mathematical language turns out to be literal after all. But I'm not so sure this follows.⁵ To show why, I'll now turn to the question of

⁵ There are important general questions here. A lot depends, for example, on the account of metaphor to which one is committed. The Burgess and Rosen argument seems to presuppose a commitment to some kind of substitution view, in which the semantic content of a metaphor is capturable by some (probably quite cumbersome) literal expression. But Yablo denies this view. Literally understood mathematical sentences are false. Their real content is however, true, he seems to think, but not always capturable by a true sentence that is free of reference to mathematical objects. (This is what the representational power of abstract objects gets us.)

whether Yablo's figuralism should really be considered hermeneutic rather than revolutionary.

Yablo suggests no revision of the practices of mathematicians, and neither do I. By that criterion we are no revolutionaries. Notwithstanding Burgess and Rosen, however, one *can* consistently maintain that, literally understood, the language of mathematics does have commitments to the existence of mathematical objects, that mathematicians (in the main) intend to be literal and to be understood literally, that they are systematically mistaken in this, but that *mathematicians should nevertheless continue talking and practicing as they have been*.

Leng (2005) points this out, also arguing that "even in cases where mathematicians are clear about what they mean by their assertions, many such mathematicians still don't know whether to believe what they say" (280). This calls into question the out-of-hand dismissal of revolutionary fictionalism as an example of ridiculous philosophical arrogance. Revolutionary fictionalism of this form does not seek *mathematical* change, but philosophical. It attacks the claim that we *must* believe that there are mathematical objects in order to understand the way that (pure and applied) mathematics functions. It is not so clear that naturalism rules out such a philosophically motivated revolution in the interpretation of mathematics. Passeau (2005) distinguishes the two forms of naturalism in the philosophy of mathematics relevant to this point, identifying them as reconstruction naturalism—the view that "philosophy cannot legitimately change standard mathematics—and reinterpretation naturalism—the view that "philosophy cannot legitimately sanction a reinterpretation of mathematics" (377). It is the latter that matters here, and Passeau argues persuasively against it, debunking the kind of

inductive ‘failure argument’ stemming from Lewis (and found in Burgess and Rosen’s rejection of revolutionary nominalism). His central point is that to the extent that the examples of philosophical failure put forward are persuasive, it is because they are taken to be reconstructive rather than reinterpetive. If Passeau is on the right track here, and I think he is, a philosophical *interpretive* revolution may not be so unpalatable, even to naturalists.

Yablo’s view is therefore most persuasive understood as revolutionary. This philosophical revolution will require not only explaining how mathematics can be so successfully applied in science without being true, but also explaining why the aims of pure and applied mathematics are all the same better served by (most or many) practicing mathematicians and scientists continuing to operate as realists. This task goes beyond the scope of my paper. It is important to recognize how much has already been achieved, especially on the application front. Field (1980) shows how we can understand the inferential role of mathematics in science without believing mathematics is true. Most importantly for my case here, Yablo has shown how we can understand applied number talk without commitment to the existence of numbers.⁶ His account also gives us a good model for understanding the way mathematics develops through using its own concepts reflexively to articulate and create the mathematical properties of mathematical objects (again, without requiring that mathematical objects exist).

⁶ Balaguer (1996, 1998) also presents an important fictionalist account of applied mathematics.

References

- Balaguer, M. [1996]: "A Fictionalist Account of the Indispensable Applications of Mathematics", *Philosophical Studies*, vol. 83, no. 3, pp. 291-314.
- Balaguer, M. [1998]: *Platonism and Anti-Platonism in Mathematics*, New York: Oxford University Press.
- Burgess J. and Rosen G. [1997]: *A Subject with No Object*, New York: Oxford University Press.
- Burgess, J. [2004]: "Mathematics and Bleak House," *Philosophia Mathematica* (III), Vol 12, 18-36.
- Burgess and Rosen [2005]: "Nominalism Reconsidered," in S. Shapiro (ed.), *The Oxford Handbook of Philosophy of Mathematics and Logic*, Oxford University Press, pp. 515-535.
- Cole, J. [2005a]: "Social Construction: The Neglected Option", *Notre Dame Journal of Formal Logic* Vol. 46 No. 2, pp. 235-247.
- Cole [2005b]: *Practice Dependent Realism and Mathematics*, PhD Dissertation, The Ohio State University.
- Field, H. [1980]: *Science Without Numbers*, Princeton: Princeton University Press.
- Gibbs, R. [1994]: *The Poetics of Mind: Figurative Thought, Language and Understanding*, New York: Cambridge University Press.
- Leng, M. [2005]: "Revolutionary Fictionalism: A Call to Arms," *Philosophia Mathematica* (III) Vol. 13, pp. 277-293.
- Passeau, A. [2005]: "Naturalism in Mathematics and the Authority of Philosophy," *British Journal of the Philosophy of Science* 56, pp. 377-96.
- Searle, J. [1978]: "Literal Meaning," *Erkenntnis* 13, pp. 207-224.
- Stanley, J. [2001]: "Hermeneutic Fictionalism," *Midwest Studies in Philosophy* XXV, pp. 36-71.
- Walton, K. [1990]: *Mimesis as Make-Believe: On the Foundations of the Representational Arts*. Cambridge: Harvard University Press.

- Walton, K. [1993]: "Metaphor and Prop Oriented Make-Believe," *European Journal of Philosophy* I.I, pp. 39-57. Reprinted in M. Kalderon (ed.) [2005]: *Fictionalism in Metaphysics*, New York: Oxford University Press, pp 63-87.
- Yablo, S. [2001]: "Go Figure: A Path Through Fictionalism," *Midwest Studies in Philosophy* XXV, pp72-102.
- Yablo, S. [2002] "Abstract Objects: A Case Study," *Nous Supplement: Philosophical Issues*, vol. 12, pp. 220-240.
- Yablo, S. [2005]: "The Myth of Seven," in M. Kalderson (ed.) *Fictionalism in Metaphysics*, New York: Oxford University Press, pp 88-115.