Correct exposition of complementarity in Unruh's and Afshar's experiments

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Abstract

We discuss the multiple pass interferometer setup proposed by Unruh, and clarify some of the fundamental issues linked with complementarity. We explicitly state all mathematical instructions for manipulating the quantum amplitudes and assessing the probability distribution functions. In this respect we show that certain purely math logical limitations (requirement for consistency) prevent one to argue that there is one-to-one corespondence between paths 1 and 2 and the exit gates 10 and 9 ("which way" interpretation), and at the same time insist on pure state density matrix, i. e. existent nonmeasured interference in the second building block of Unruh's interferometer. Furthermore one cannot even argue that Unruh's setup is described by mixed density matrix that keeps the one-to-one correspondence between the paths 1 and 2 and the exit gates. This last claim is mathematically consistent, however is experimentally disprovable - because one may potentially distinguish mixed quantum state from pure quantum state. One just lets the two beams captured at the exit gates cross each other. If interference can be observed the two exit gates were coherent and provide beams in pure state (superposition), while if interference cannot be observed, the state of the exit gates was mixed one. Since the captured beams at the exit gates in Unruh's experiment could interfere this implies that the whole setup is characterized with pure state density matrix and does not preserve the one-to-one correspondence between the entry points and exit gates, even in case where the destructive interference in arm 5 of the interferometer is not measured. Therefore the correct (experimentally plausible and mathematically consistent) exposition of complementarity introduced by Georgiev in 2004 is that Unruh's setup is characterized by pure state density matrix and does not keep the suggested by Unruh one-to-one correspondence. As an appendix is shown the equivalence between Unruh's setup and Afshar's setup and correct analysis of Afshar's experiment is also provided.

1 Introduction of basic definitions

Before we introduce the Unruh's setup let us define mathematically two basic terms: *distinguishability* and *indistinguishability*, as well as the physical actions of half-silvered and fully silvered mirrors on the incoming quantum state of photon (following Roger Penrose, 1991; 1994).

1.1 *Distinguishable states*

Two states ψ_1 and ψ_2 are said to be *distinguishable* if they satisfy the logical XOR gate. Here we provide the truth table of the XOR gate:

ψ_1	ψ_2	XOR output
0	0	0
0	1	1
1	0	1
1	1	0

The provided definition is mathematically strict representation of the idea of distinguishability in classical physics!

1.2 *Indistinguishable states*

Two states ψ_1 and ψ_2 are said to be *indistinguishable* if they satisfy the logical XNOR gate. This is just a logical negation of the distinguishable state logical gate, thus if you show that two states ψ_1 and ψ_2 are both *distinguishable* and *indistinguishable* at the same time, the formal system that you have used will necessarily be mathematically *inconsistent*.

ψ_1	ψ_2	XNOR output
0	0	1
0	1	0
1	0	0
1	1	1

It is obvious that two non-existing (or not involved in some experiment) states are *indistinguishable*, yet in our further discussion we will discuss only existing (and involved in experiment) physical states. Hence we arive at more strict formulation of *indistinguishability* as states ψ_1 and ψ_2 satisfying the AND gate, and this definition will be true *if and only if* the discussion is limited only on states involved in experimental setups.

ψ_1	ψ_2	AND output
0	0	0
0	1	0
1	0	0
1	1	1

Thus one sees that by definition two *non-superposed states* are *distinguishable*, while two *quantum superposed states* are *indistinguishable*.

1.3 *Half-silvered mirror* (/)

If we denote the incoming quantum amplitude by ψ, then the forwarded by the mirror straight ahead quantum amplitude is $\frac{1}{\sqrt{2}}\psi$, while the reflected at right angle quantum amplitude is $\imath\frac{1}{\sqrt{2}}\psi$.

1.4 *Fully silvered mirror* (//)

If the incoming quantum amplitude ψ falls at angle of $\frac{\pi}{4}$ onto the mirror, then the reflected perpendicularly quantum amplitude is $\imath\psi$.

1.5 "Which way" claims

"Which way" (*welcher weg*, in German) claim is defined as existent *one-to-one correspondence between elements of two sets.*

2 The building block of multiple pass interferometer

Now we introduce the essential building block of the interferometer, from which more comlicated interferometers can be built up (Miguel Carrion Alvarez, 2000). The setup is symmetric and contains two half-silvered and two fully silvered mirros as shown on the next FIGURE 1.

$$
\begin{array}{cccccccc}
 & & & & & 5 & & \\
 & & & 3 & & \uparrow & & \\
 & // & \cdots & \cdots & \cdots & / & \rightsquigarrow & 6 \\
 & \vdots & & & & \vdots & & \\
1 & \vdots & & & & \vdots & 4 & \\
 & \vdots & & & & \vdots & & \\
\rightsquigarrow & / & \cdots & \cdots & \cdots & // & & \\
 & \uparrow & & 2 & & & &
\end{array}
$$

FIGURE 1. Building block of Unruh's interferometer. Half-silvered mirrors (/), fully silvered mirrors (//), and incoming quantum amplitude Ψ at entry point $\rightsquigarrow$. As one can see the quantum amplitude Ψ quantum mechanically *self-interferes* in order to produce its own full cancelation at exit $\uparrow$ and recover entirely itself at exit $\rightsquigarrow$.

It can be shown that the beam $\rightsquigarrow$ exits always towards GATE 6 $\rightsquigarrow$, while the beam $\uparrow$ exits always towards GATE 5 $\uparrow$. The observation is that the splitted beam Ψ *quantum mechanically destructively self-interferes* at one of the exit gates, while it *quantum mechanically constructively self-interferes* at the other exit gate.

Let the incoming amplitude at the entry point $\rightsquigarrow$ denoted with Ψ is normalized so that $|\Psi|^2 = 1$.

Then the amplitude reflected in the vertical branch 1 of the interferometer building block will be $\psi_1 = \imath\frac{1}{\sqrt{2}}\Psi$, written in *Dirac's ket notation* as $\imath\frac{1}{\sqrt{2}}|\psi_1\rangle$.

The amplitude forwarded in the horizontal branch 2 of the interferometer building block will be $\psi_2 = \frac{1}{\sqrt{2}}\Psi$, written in *Dirac's ket notation* as $\frac{1}{\sqrt{2}}|\psi_2\rangle$.

Let us investigate a flash of light Ψ incoming through entry $\rightsquigarrow$. In every arm $1 - 6$ of the interferometer we can write the quantum state in *Dirac's ket notation*, where $|\psi_1\rangle$ refers to passage along path 1 and $|\psi_2\rangle$ refers to passage along path 2:

$$1: \quad \imath\frac{1}{\sqrt{2}}|\psi_1\rangle$$

$$3: \quad -\frac{1}{\sqrt{2}}|\psi_1\rangle$$

$$6: \quad -\frac{1}{2}|\psi_1\rangle - \frac{1}{2}|\psi_2\rangle$$

$$2: \quad \frac{1}{\sqrt{2}}|\psi_2\rangle$$

$$4: \quad \imath\frac{1}{\sqrt{2}}|\psi_2\rangle$$

$$5: \quad \imath\frac{1}{2}|\psi_2\rangle - \imath\frac{1}{2}|\psi_1\rangle$$

One easily sees that in 6 one gets *constructive quantum interference*, while in 5 one gets *destructive quantum interference*.

Note that when it comes for Ψ it *quantum self-interferes*, while it comes to ψ_1 and ψ_2 they *quantum cross-interfere*.

Thus flash of light through path $\rightsquigarrow$ will always goes to GATE 6 $\rightsquigarrow$ and if we flash beam through path $\upharpoonleft$ it will always go to GATE 5 $\upharpoonleft$. In this sense there is one-to-one correspondence between the entry point and exit gate. However this one-to-one correspondense is result of negative quantum interference of the two wavefunctions ψ_1 and ψ_2 at the exit gates, therefore the entry points are one-to-one mapped to the exit gates, yet the two paths ψ_1 and ψ_2 are *indistinguishable* and quantum interfere. The *indistinguishability* of ψ_1 and ψ_2 allows for *quantum self-interference* of Ψ at the exit gates.

Thus in order to have one-to-one correspondence between entry point and exit gate you need both arms of the interferometer open because the one-to-one correspondence is critical on quantum interference between ψ_1 and ψ_2! If you block one of the splitted beams ψ_1 or ψ_2, or you label ψ_1 and ψ_2 by polarization filters L and R, you will lose the quantum interference at the exit gates and the one-to-one correspondence between entry points and exit gates will be lost.

Thus we have encountered the bizzare phenomenon of *complementarity*:

- (i) we can measure the interferometer paths, hence $\psi_1 \text{XOR} \psi_2$ (*distinguishable* ψ_1 and ψ_2), and destroy the one-to-one correspondence between entry points and exit gates (*indistinguishable* GATES 5 and 6).

- (ii) we don't measure which path of the interferometer has been taken and allow quantum interference of amplitudes at the exit gates coming from both interferometer paths, hence $\psi_1 \text{AND} \psi_2$ (*indistinguishable* ψ_1 and ψ_2), thus keeping the one-to-one correspondence between entry points and exit gates (*distinguishable* GATES 5 and 6).

In this case the two *observables* are said to be *complementary*: you cannot get both which path the photon has taken $\psi_1 \text{XOR} \psi_2$ (*distinguishable* ψ_1 and ψ_2) and keep at the same time the one-to-one correspondence between the entry points and exit gates (*distinguishable* GATES 5 and 6).

3 Complementarity principle in mathematical form

- If quantum states ψ_1 and ψ_2 are *distinguishable* (not quantum coherent), that is you say $\psi_1 \text{XOR} \psi_2$, then the probability distribution is given by $\mathcal{P} = \frac{1}{2}|\psi_1|^2 + \frac{1}{2}|\psi_2|^2$. The *(reduced) density matrix* of the setup is then one of mixed type $\hat{\rho} = \begin{pmatrix} \frac{1}{2} & 0 \\ 0 & \frac{1}{2} \end{pmatrix}$ and you have $\hat{\rho} \neq \hat{\rho}^2$. You say that *the discussed two quantum states do not quantum mechanically interfere*, but just sum up classically.

- If quantum states ψ_1 and ψ_2 are *indistinguishable* (quantum coherent), that is you say $\psi_1 \text{AND} \psi_2$, then the probability distribution is given by $P = |\psi_1 + \psi_2|^2$. The *density matrix* of the setup is then one of pure type $\hat{\rho} = \begin{pmatrix} \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} \end{pmatrix}$ and you have $\hat{\rho} = \hat{\rho}^2$. You say that *the discussed two quantum states do quantum mechanically interfere.*

4 Unruh's experiment

Unruh's thought experiment is a setup that tries to create more understandable version of Afshar's experiment. Indeed Unruh's setup is crystally clear version of Afshar's setup and completely analogous (Afshar's setup will be discussed later in the text). In the following section we will explain Unruh's setup in some detail.

Let us have multiple pass interferometer with two elementary building blocks, such the one described in the previous paragraph. On the following FIGURE 2 every arm of the interferometer is labelled with a number, and the mathematical expressions for the quantum states in the interferometer arms are given in the text following the figure. The complementarity picture however cannot be built up without knowing whether Ψ_1 and Ψ_2 are *distinguishable* or *indistinguishable*! *Distinguishability* can be obtained for example via insertion of different polarization filters in the arms 1 and 2, yet in Unruh's setup such *distinguishability via "labelling"* is not suggested. Therefore one can question whether Ψ_1 and Ψ_2 are *distinguishable* at all as suggested by Unruh. This is a very deep argument raised by Georgiev (Wikipedia, 2004), that was completely neglected by physicists. In the discussion following we will show that neglecting the deep mathematical formulation of complementarity as already stated in the beginning of this article, may lead into interpretational "delusions". This seems to happen in Unruh's reasoning. Therefore we will discuss the Unruh's setup, having in mind the next complete FIGURE 2.

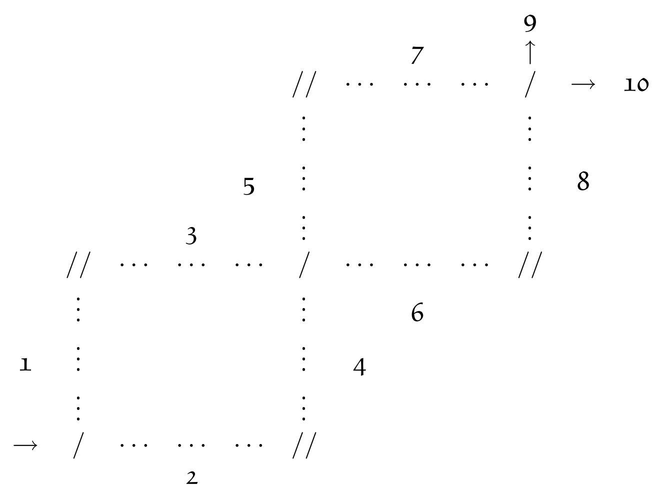

FIGURE 2. Unruh's version of a multiple pass interferometer setup that captures the essence of Afshar's experiment. It is composed of two elementary building blocks described above in the text, and the incoming quantum wave Ψ at entry point $\rightarrow$ goes to both exit gates $\uparrow$ and $\rightarrow$.

Application of *Feynman's sum over histories approach* leads us to the correct quantum mechanical description of the experiment. The *Dirac's ket notation* for the quantum states in the first building block coincide with the above derived states $1-6$. However there is a small (and important) change to be noted, the used wavefunctions Ψ_1 and Ψ_2 have their own quantum branches in the second building block of Unruh's interferometer.

$$1: \quad \imath \frac{1}{\sqrt{2}}|\Psi_1\rangle$$

$$3: \quad -\frac{1}{\sqrt{2}}|\Psi_1\rangle$$

$$6: \quad -\frac{1}{2}|\Psi_1\rangle - \frac{1}{2}|\Psi_2\rangle$$

$$2: \quad \frac{1}{\sqrt{2}}|\Psi_2\rangle$$

$$4: \quad \imath\frac{1}{\sqrt{2}}|\Psi_2\rangle$$

$$5: \quad \imath\frac{1}{2}|\Psi_2\rangle - \imath\frac{1}{2}|\Psi_1\rangle$$

Here we will add the expressions for the quantum states $7-10$:

$$7: \quad \frac{1}{2}|\Psi_1\rangle - \frac{1}{2}|\Psi_2\rangle$$

$$9: \quad -\imath\frac{1}{\sqrt{8}}|\Psi_2\rangle + \imath\frac{1}{\sqrt{8}}|\Psi_1\rangle - \imath\frac{1}{\sqrt{8}}|\Psi_2\rangle - \imath\frac{1}{\sqrt{8}}|\psi\Psi_1\rangle$$

$$8: \quad -\imath\frac{1}{2}|\Psi_1\rangle - \imath\frac{1}{2}|\Psi_2\rangle$$

$$10: \quad +\frac{1}{\sqrt{8}}|\Psi_1\rangle + \frac{1}{\sqrt{8}}|\Psi_2\rangle + \frac{1}{\sqrt{8}}|\Psi_1\rangle - \frac{1}{\sqrt{8}}|\Psi_2\rangle$$

Before we investigate the mathematical structure of the expressions $7-10$ let us introduce the Unruh's reasoning.

4.1 "Which way" interpretation: one-to-one correspondence $\Psi_1 \leftrightarrow$ GATE10 and $\Psi_2 \leftrightarrow$ GATE9 + mixed state density matrix

Unruh puts obstacle on path 1 and corectly argues that the photons coming from the source that pass the first half-silvered mirror and take path 2 (that is they are not reflected to be absorbed by the obstacle located at path 1), will all end at gate $\uparrow$ (GATE 9). These are exactly 50% of photons! So Unruh shows that there is corespondence between path Ψ_2 and gate $\uparrow$ (GATE 9). See the following FIGURE 3.

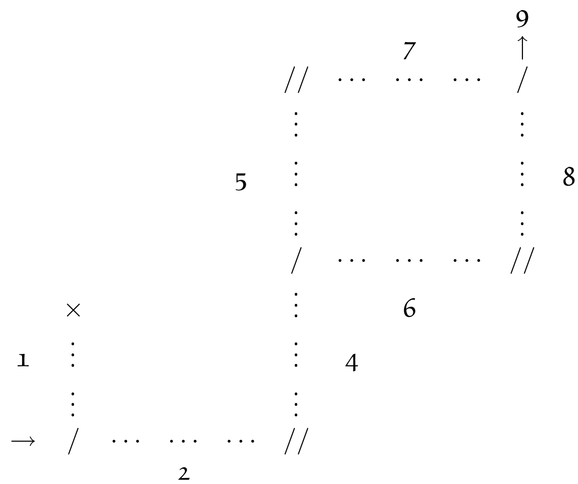

FIGURE 3. Unruh's version of a multiple pass interferometer with path 1 blocked by obstacle.

Similarly he argues that inverted setup with photons coming from the source and take path 1 (that is they are not absorbed at obstacle located on path 2) will end at gate $\rightarrow$ (GATE 10). This suggests one to one correspondence between Ψ_1 and gate $\rightarrow$ (GATE 10). See the following FIGURE 4.

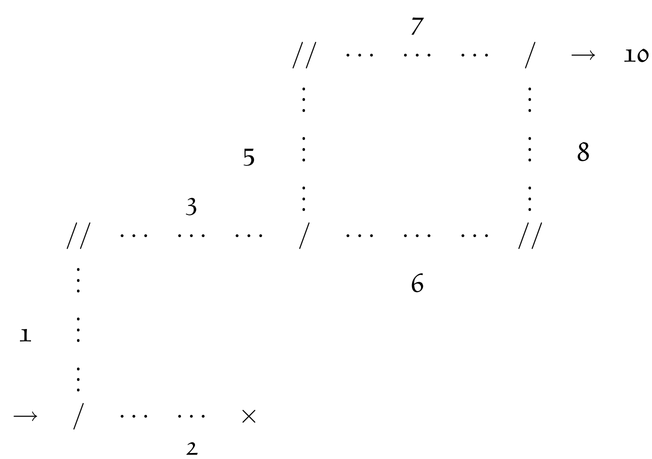

FIGURE 4. Unruh's version of a multiple pass interferometer with path 2 blocked by obstacle.

Note at this stage that Unruh investigates a statistical mixture of two single path experiments. Therefore the case is $\Psi_1 \mathrm{XOR} \Psi_2$, both paths Ψ_1 and Ψ_2 are *distinguishable* because of the *existent obstacle*, and the Ψ_1 and Ψ_2 do not quantum *cross-interfere* with each other in the second block of the interferometer (in the first block they are separated spatially, in the second branch they are separated temporally). Thus in the mixed setup there is one-to-one correspondence between paths and exit gates due to the fact of *distinguishability* of $\Psi_1$and Ψ_2, that is no quantum interference between $\Psi_1$and Ψ_2 in the second building block of Unruh's interferometer.

Then Unruh opens of both paths Ψ_1 and Ψ_2 and considering the mixture of single path experiment argues that photons that end up at gate $\rightarrow$ (GATE 10) have taken path Ψ_1, while those ending at gate $\uparrow$ (GATE 9) come from path Ψ_2. The logic is that the second building block of the interferometer has both its arms open, and the one-to-one correspondence is result of *self-interference* of Ψ_1, respectively *self-interference* of Ψ_2.

The problem is how to secure the conclusion that "which way" information in the form of one-to-one correspondence between paths $\Psi_1$and Ψ_2 and exit gates still "remains" when both paths $\Psi_1$and Ψ_2 are open?

Indeed Unruh's reasoning of such "which way" information can be justified only if taking these two statements as axioms:

Statement 1: Ψ_1 and Ψ_2 do not *quantum cross-interfere* with each other.

Statement 2: $\Psi_1$and Ψ_2 *can only quantum self-interfere.*

Concisely written together both statements reduce to one logical formula Ψ_1XORΨ_2. Thus Unruh's "which way" statement is equivalent with the statement that the density matrix of the photons at the exit gates is *mixed one*!

Conclusion: *Unruh's "which way" thesis is equivalent with the postulation of mixed state density matrix for the whole setup (even without any additional "labelling" of the beams Ψ_1 and Ψ_2).*

Thus stated Unruh's thesis for mixed state density matrix of the setup is vulnerable by experimental testing. Quantum mechanically one may perform experiments to find whether two incoming beams are coherent (pure state) or not (mixed state). Thus Unruh's thesis is experimentally disprovable, and in order to keep that his thesis is correct, Unruh must immunize it against experimental testing.

However in order to be immunized against experimental disproof the Unruh's interpretation must postulate that one cannot experimentally distinguish mixed state from pure state.

Otherwise one may decide to let the two beams from the exit gates 9 and 10 cross each other. If an interference pattern is build up then one will have experimental verification that the density matrix of the setup is not of *mixed type* (Ψ_1XORΨ_2, hence $\hat{\rho} \neq \hat{\rho}^2$), but one of *pure type* (Ψ_1ANDΨ_2, hence $\hat{\rho} = \hat{\rho}^2$).

Thus Unruh's interpretation for "which way" + mixed state density matrix is experimentally disprovable. One may immunize Unruh's thesis against experimental disproof *if and only if* a metaphysical principle is accepted according to which mixed states cannot be experimentally distinguished from pure states. Needless to say that such principle will be furiously stigmatized by most physicists as theology and not recognized as science!

4.2 "No which way" interpretation: lack of correspondence $\Psi_1 \leftrightarrow$ GATE10 and $\Psi_2 \leftrightarrow$ GATE9 + pure state density matrix (Georgiev's thesis)

Look now the expressions 9 and 10 for the quantum amplitudes at the exit gates. We have already shown that if one argues that there is "which way" correspondence, he must accept that Ψ_1 and Ψ_2 are *distinguishable*, hence they will not be able to interfere at arms 5 and 7 of the interferometer.

Now we will show the opposite, that postulating of "unmeasured destructive interference" in the arms 5 and 7 of the interferometer regardless of the fact that the interference is not measured, is enough and sufficient to erase completely the "which way" information. One see that in both expressions 9 and 10 there participate both amplitudes Ψ_1 and Ψ_2. Postulating of quantum interference in the arms 5 and 7 is equivalent to postulate *indistinguishability* of Ψ_1 and Ψ_2, that is equivalent to say that Ψ_1 and Ψ_2 can anihilate each other.

For the expression at gate 9 we have:

$$9: \quad -\imath\frac{1}{\sqrt{8}}|\Psi_2\rangle + \imath\frac{1}{\sqrt{8}}|\Psi_1\rangle - \imath\frac{1}{\sqrt{8}}|\Psi_2\rangle - \imath\frac{1}{\sqrt{8}}|\Psi_1\rangle$$

The first two members in the expression have met each other earlier, so they anihilate each other leaving *zero*. What remains is $-\imath\frac{1}{\sqrt{8}}|\Psi_2\rangle - \imath\frac{1}{\sqrt{8}}|\Psi_1\rangle$ and when squared it provides half of the full intensity of Ψ, where $\frac{1}{4}$ contribution comes from Ψ_1 and $\frac{1}{4}$ contribution comes from Ψ_2. Now is clear why one cannot hold consistently both the existence of "which way" one-to-one correspondence and existentent but undetected interference at paths 5 and 7.

- If one postulates Ψ_1XORΨ_2 then $\imath\frac{1}{\sqrt{8}}|\Psi_1\rangle - \imath\frac{1}{\sqrt{8}}|\Psi_1\rangle$ will interfere at the exit and the obtained half the original intensity of Ψ will come from squaring the doubled $-\imath\frac{1}{\sqrt{8}}|\Psi_2\rangle$ i.e. only from path 2.

- If one postulates Ψ_1ANDΨ_2 then $-\imath\frac{1}{\sqrt{8}}|\Psi_2\rangle + \imath\frac{1}{\sqrt{8}}|\Psi_1\rangle$ will interfere first, and the obtained half the original intensity of Ψ will come from squaring of $-\imath\frac{1}{\sqrt{8}}|\Psi_2\rangle - \imath\frac{1}{\sqrt{8}}|\Psi_1\rangle$ i.e. both paths 1 and 2.

The "mixing of the two channels" at gate 10 is analogous.

$$10: \quad +\frac{1}{\sqrt{8}}|\Psi_1\rangle + \frac{1}{\sqrt{8}}|\Psi_2\rangle + \frac{1}{\sqrt{8}}|\Psi_1\rangle - \frac{1}{\sqrt{8}}|\Psi_2\rangle$$

- If one postulates Ψ_1XORΨ_2 then $\frac{1}{\sqrt{8}}|\Psi_2\rangle - \frac{1}{\sqrt{8}}|\Psi_2\rangle$ will interfere at the exit and the obtained half the original intensity of Ψ will come from squaring the doubled $\frac{1}{\sqrt{8}}|\Psi_1\rangle$ i.e. only from path 1.

- If one postulates Ψ_1ANDΨ_2 then $\frac{1}{\sqrt{8}}|\Psi_1\rangle - \frac{1}{\sqrt{8}}|\Psi_2\rangle$ will interfere first, and the obtained half the original intensity of Ψ will come from squaring of $\frac{1}{\sqrt{8}}|\Psi_1\rangle + \frac{1}{\sqrt{8}}|\Psi_2\rangle$ i.e. both paths 1 and 2.

4.3 Inconsistent interpretation: "which way" + pure state density matrix

William Unruh (2004), as well as, Alfred Ramani (Wikipedia, 2006) suggests that by looking the expressions of 5 and 7 one can conclude that there is *undetected destructive quantum interference* between Ψ_1 and Ψ_2, and at the same time can hold the "which way" interpretation in which Ψ_1 and Ψ_2 are *distinguishable*. Ramani suggests that only measuring of the interference at arms 5 and 7 is disturbing the "which way" interpretation, and if the destructive quantum interference is not measured it can peacefully co-exist with the "which way" interpretation. Mathematically formulated the claim is that there is "which way" one-to-one correspondence between paths 1 and 2, with exit gates 10 and 9 respectively, while at the same time the whole setup is described by pure state density matrix. Afshar (2004) claims equivalent statement for his setup insisting on "which way" + pure state density matrix, which is shared also by Aurelien Drezet, Carl Looper, and others (Wikipedia, 2006).

We will show that assuming "which way" + pure state density matrix leads to mathematical inconsistency! Inconsistent mathematical model cannot subserve the function of a physical theory, by inconsistent mathematical system one can prove everything!

The following text is *mathematical proof of inconsistency (a theorem)*, and is not result of my own misunderstanding of interpretational details introduced by Unruh, Afshar, Ramani, Drezet, Looper, and others. All of the mentioned scientists have repeatedly stated that the whole setup is described by *pure state density matrix*, yet the "which way information is there"! The mathematical proof of inconsistency is derived within the standard formulation of quantum mechanics, so the conclusion must be that the standard mathematics of quantum mechanics is not consistent with the suggested interpretation in terms of "which way" + pure state density matrix, and if one is to accept such bizzare interpretation he must revisit fundamentally the mathematical fundament of quantum mechanics (none of the four authors belives in this last claim).

In order to show *where the inconsistency is born* we should re-write the expressions $9-10$ in a fashion where each of the wavefunctions Ψ_1 and Ψ_2 is written as a superposition of its own branches $|\psi_{15}\rangle, |\psi_{16}\rangle$ and $|\psi_{25}\rangle, |\psi_{26}\rangle$, respectively, where the second index 5 or 6 denotes a branch in the second building block of Unruh's interferometer.

$$9*: \quad -\imath\frac{1}{\sqrt{8}}|\psi_{25}\rangle + \imath\frac{1}{\sqrt{8}}|\psi_{15}\rangle - \imath\frac{1}{\sqrt{8}}|\psi_{26}\rangle - \imath\frac{1}{\sqrt{8}}|\psi_{16}\rangle$$

$$10*: \quad +\frac{1}{\sqrt{8}}|\psi_{16}\rangle + \frac{1}{\sqrt{8}}|\psi_{26}\rangle + \frac{1}{\sqrt{8}}|\psi_{15}\rangle - \frac{1}{\sqrt{8}}|\psi_{25}\rangle$$

From the "which way" claim follows that the contributions to the final intensity (squared amplitude) detected at gates 9 and 10 must come from Ψ_1 or Ψ_2 only! This is possible *if and only if* the individual branches 5 or 6 of each function are *indistinguishable*, so that claim mathematically yields quantum destructive interference (anihilation) between ψ_{15} and ψ_{16}, and between ψ_{25} and ψ_{26}, respectively!

However to postulate at the same time that there is "undetected negative quantum interference" at branch 5 between Ψ_1 and Ψ_2 is equivalent to say that paths 5 and 6 are *distinguishable*. One should remember that *destructive quantum interference* is supposed to work for the *distinguishablity* of Ψ_1 and Ψ_2 (see also how the elementary building block of the interferometer functions where one-to-one correspondences are result from self-interference)! We have arrived at logical inconsistency!

Paths 5 and 6 cannot be both *distinguishable* and *indistinguishable* for the quantum state Ψ - this is what complementarity principle says! (See also section 1.2 defining these two terms in the language of logical gates)

Therefore we have proved that within standard quantum mechanics one cannot claim both "which way" and pure state of the density matrix at the same time. Georgiev (Wikipedia, 2004) was the first to show that whether the quantum interference at branch 5 is measured or not does not matter! Its consistent postulation is sufficient to rule out the "which way information".

It is obvious that the mentioned physicists suffer a very severe disease, in which they believe that "measurement" (*a posteriori* knowledge) is above everything else, including mathematical axioms (*apriori* knowledge). A funny situation occurs - *a mathematical axiom is not valid until experimentally measured*!!!

I will try to be as objective as possible, and conclude this subsection with the theorem *"postulation of which way information in Unruh's setup + postulation of pure state density matrix of the whole setup is apparent violation of mathematical logic"*. For a professional mathematician it is clear that postulation of certain fact as axiom does not further need experimental verification to establish its truthness. Newton's theory of gravity fails to predict some experimentally verified relativistic effects, yet this does not make it *mathematically inconsistent*. That is why Newton's theory still can be used as approximation for nonrelativistic velocities. However *a mathematically inconsistent* theory cannot serve the needs of physics!

5 Retrospective reconstructions and complementarity

Now notice that arguing that photons possess "which way" information implies that the photon density matrix at detectors is that of *mixed type* (Unruh's reasoning). We have denoted the quantum amplitude through path 1 with Ψ_1, and the quantum amplitude through path 2 with Ψ_2. Therefore when we *retrospectively reconstruct* the photon probability distribution function we should use the correct complementarity rule $\mathcal{P} = \frac{1}{2}|\psi_1|^2 + \frac{1}{2}|\psi_2|^2$, and we must logically and consistently argue that there is no negative interference at the path 5 - simply we do not add Ψ_1 to Ψ_2 but first square each of those amplitudes. (In this respect the proposed graphic by Unruh in his figure 2 is flawed!). Basicly if the two paths Ψ_1 and Ψ_2 are *distinguishable* then the interference terms must be *zero*, and the (reduced) density matrix will be of *mixed type* $\hat{\rho} = \begin{pmatrix} \frac{1}{2} & 0 \\ 0 & \frac{1}{2} \end{pmatrix}$. To accept that there is "which way" information is equivalent to accept that the setup with both paths open is a statistical mixture of the two single path setups with obstacles so the complementarity rule for making retrospective predictions is $\mathcal{P} = \frac{1}{2}|\psi_1|^2 + \frac{1}{2}|\psi_2|^2$. Thus alternative formulation of principle of complementarity is in a form of instruction how to make correct retrospective reconstruction of *mixed state* setup - it says that *mixed state setups* should be retrospectively reconstructed with $\mathcal{P} = \frac{1}{2}|\psi_1|^2 + \frac{1}{2}|\psi_2|^2$ distribution.

However, if the beams along paths 1 and 2 interfere so that no photons are expected along paths 5 and 7 the setup is "no which way" *pure state* setup. In this case retrospectively the photon probability distribution should be calculated as $P = |\psi_1 + \psi_2|^2$. Thus the alternative formulation of principle of complementarity in a form of instruction how to make correct retrospective reconstruction of *pure state* setup is - *pure state setups* should be retrospectively reconstructed with $P = |\psi_1 + \psi_2|^2$ distribution.

Taken together the above two instructions provide clear idea of complementarity - you cannot recover retrospectively given setup with both types of probability distributions $\mathcal{P} = \frac{1}{2}|\psi_1|^2 + \frac{1}{2}|\psi_2|^2$ and $P = |\psi_1 + \psi_2|^2$ at the same time, otherwise you will arrive at mathematical inconsistency.

One sees that in some special cases for a given plane x both probability distributions coincide, thus $\mathcal{P}(x) = P(x)$, and one has the choice to retrospectively reconstruct any way he likes. However it is unwise to retrospectively reconstruct a *pure state* setup with $\mathcal{P} = \frac{1}{2}|\psi_1|^2 + \frac{1}{2}|\psi_2|^2$ probability distribution. This is what happens in Unruh's setup and is done by Unruh. One will not arrive at direct experimental contradiction if he looks only within the plane where $\mathcal{P}(x) = P(x)$. Yet, any measurement outside this plane will reveal the improper retrospective reconstruction.

Needless to say that the importance of the fact that in the image plane of Afshar's setup one has $\mathcal{P}(x) = P(x)$ was not recognized by anybody except Georgiev in 2004 when results from Afshar's result was announced (see blogs of Luboš Motl, William Unruh, Shahriar Afshar, etc.). Indeed this was the central argument of the current paper when we discussed the expressions 9, 10 and 9∗, 10∗.

Appendix I: Afshar's setup

Shahriar Afshar claimed (erroneously) that Unruh's setup is not equivalent to Afshar's setup, therefore the *"plane constructed by Unruh has no wings"*. At first glance a striking difference is that in Afshar's setup at *image* $\mathcal{A}$ comes only amplitude from *pinhole* 1, and *zero* amplitude from *pinhole* 2, and at *image* $\mathcal{B}$ comes amplitude from *pinhole* 2 and *zero* from *pinhole* 1.

Fundamental difference between Unruh's setup and Afshar's setup at first glance seems this one:

- *Afshar's setup: image $\mathcal{A}$*: $\frac{1}{\sqrt{2}}\psi_1 + 0 \times \psi_2$ and *image $\mathcal{B}$*: $\frac{1}{\sqrt{2}}\psi_2 + 0 \times \psi_1$. The zero looks "physically unstructured", not result from negative interference of positive and negative amplitudes contributed from the alternative pinhole.
- *Unruh's setup:* GATE 9: $\frac{1}{\sqrt{2}}\psi_2 + \frac{1}{\sqrt{8}}\psi_1 - \frac{1}{\sqrt{8}}\psi_1$ and GATE 10: $\frac{1}{\sqrt{2}}\psi_1 + \frac{1}{\sqrt{8}}\psi_2 - \frac{1}{\sqrt{8}}\psi_2$. In this case the zero seems "with physical structure", and is result from negative interference of positive and negative amplitudes contributed from the alternative path.

If one shows that Georgiev's "no which way" interpretation applied to Unruh's setup is not applicable to Afshar's setup, he will also show that Unruh's plane is without wings!

If in contrast, one can prove that in the Afshar's setup the zero at the opposite image is generated by *negative quantum interference*, he will show that Unruh's setup is completely equivalent to Afshar's setup. Thus Georgiev's criticism towards Afshar will be the same as in Unruh's case - logical fallacy and math error to claim both *pure state* and "which way".

Now we will show that Afshar's setup is equivalent to Unruh's setup. In brief Afshar has a double slit, a lens and detectors at the image plane of the lens where they record photons lead away from the pinhole images (Afshar, 2004). Analogously to Unruh's setup one closes *pinhole* 1 and sees that light goes only to *image $\mathcal{A}$*. Then closes *pinhole* 2 and sees that light goes only to *image $\mathcal{B}$*. One analogously to Unruh's setup may *inconsistently postulate* "which way" + pure state density matrix. However one should note that in the single pinhole experiments at the image plane of the lens the *zero* light intensity outside the central Airy disc of the pinhole image is result of *destructive quantum interference*! There are many faint higher order maxima and minima outside the central Airy disc result from quantum interference. In order for the two pinholes to be *resolvable*[1] the image of the second pinhole must be outside the central Airy disc, and located in the first negative Airy ring of the first pinhole image (or further away). Therefore in the case of both pinholes open the quantum amplitude at *image $\mathcal{A}$* is not $\frac{1}{\sqrt{2}}\psi_1 + 0 \times \psi_2$, but $\frac{1}{\sqrt{2}}\psi_1 + \frac{1}{\sqrt{8}}\psi_2 - \frac{1}{\sqrt{8}}\psi_2$. Now one has to "choose" which amplitudes will anihilate, and which will remain to be squared.

Thus Afshar is wrong to say that *"Unruh's plane is without wings"*. Afshar's setup is equivalent to Unruh's setup. The treatment of complementarity is analogous. In the case with both pinholes open there is no "which way" information in Afshar's experiment (Georgiev, 2004).

[1] One should be extremely cautious to note that *resolvable* does not mean *distinguishable*! *Resolvable* means that two dots can be seens as two separated dots, not fused into a single spot. The meaning of *distinguishable* has already been defined rigorously in the introduction of the current manuscript.

Appendix II: Note on visibility and distinguishability

Afshar claimed he has violated the duality relation $V^2+D^2 \leq 1$, where V stands for *visibility* and D stands for *distinguishability* and are defined as:

$$D = \frac{\left||\psi_1|^2 - |\psi_2|^2\right|}{|\psi_1|^2 + |\psi_2|^2}$$

$$V = \frac{2|\psi_1||\psi_2|}{|\psi_1|^2 + |\psi_2|^2}$$

Since the duality relation is a mathematically true statement (theorem) then it cannot be disproved by experiment and certainly means that Afshar's arguments through which he violates the duality relation are inconsistent. Luboš Motl (2005) qualifies Afshar's claim in the following fashion: *"It's a silliness - the same silliness like saying that "x" commutes with "the derivative with respect to x". Nevertheless, this silliness was described as a "quantum bombshell"."*

Unruh's thesis

In view of the provided explanation earlier in text for Unruh's interpretation with *mixed density matrix* yields simply $D = 1$ and $V = 0$.

Gate 9: $|\psi_1| = 0$, $|\psi_2| = \frac{1}{\sqrt{2}}$

Gate 10: $|\psi_1| = \frac{1}{\sqrt{2}}$, $|\psi_1| = 0$

Thus the two paths 1 and 2 are claimed *distinguishable* yet they *do not quantum mechanically interfere*! This is however experimentally disprovable interpretation since the density matrix of the setup is not a mixed one. One needs to additionally postulate an immunization against experimental disproof.

Georgiev's thesis

The correct Georgiev's analysis of Unruh's setup suggests *pure state density matrix* and amplitudes for each of the exit gates being $|\psi_1| = |\psi_2| = \frac{1}{\sqrt{8}}$. Thus one gets $D = 0$ and $V = 1$.

Gate 9: $|\psi_1| = \frac{1}{\sqrt{8}}$, $|\psi_2| = \frac{1}{\sqrt{8}}$

Gate 10: $|\psi_1| = \frac{1}{\sqrt{8}}$, $|\psi_2| = \frac{1}{\sqrt{8}}$

The two paths 1 and 2 are *indistinguishable*, and they *quantum mechanically interfere*!

Since the Unruh's setup is equivalent to Afshar's setup the above stated mathematical expressions by Georgiev are applicable for Afshar's setup also.

Personal statement

Much more can be said on the meaning of complementairty, however the essential importance of this paper is to show *mathematical inconsistency* of some widely used interpretations of pure states in terms of "which way" one-to-one correspondences. In case of lens in double slit setups, papers have been published by leading scientists such as Anton Zeilinger (1999) that insist that *focal plane of lens* is involved in measurement of "no which way" type while the *image plane of lens* is involved in "which way" measurements. Such claim is generally *false* because the type of experiment is decided by the *density matrix of the photon*, and even in the focal plane polarized photons will cary "which slit" information, while even in the image plane coherent photons will not cary "which slit" information. Maybe it is commonly accepted and written in many textbooks that the lens focal and image planes are with complementary functions, and possibly Bohr, and others have been involved in such discussions. Also Drezet, Afshar and others, may quote these wide spread sources - this however does not make these widespread concepts mathematically consistent! Something should be changed in the teaching of complementarity and the current paper is aimed to be a first correct step in this direction. The presented material is partially based on my own reading of the perfect lectures by prof. Bob Eisenstein available online.

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