PEIRCE IN THE LONG RUN:
REMARKS ON KNOWLEDGE *A ULTERIORI* [1]

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Abstract: Knowledge *a priori* has an important role in rationalistic schools: it pre-establishes truth in order to justify a system of correlated ideas. Empiricism usually concerns knowledge *a posteriori*, for experience itself is what we have actually known. Peirce's probabilistic approach to science was based on necessity in the long run but it has no clear place in the categorization of knowledge either as *a priori* or as *a posteriori.* This deficit should be overcome by introducing a new category — synthetic knowledge *a ulteriori*, defined as what is known about an indefinite number of cases but not about isolated instances.
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1. Introduction

This paper intends to clarify the basis of a pragmatic account of probability and induction. The movement called pragmatism has been associated to several philosophical conceptions, thus it is difficult to isolate and present one of them as an undisputable definition. But there is a general idea that seems to be a point of convergence among pragmatists: namely, that pragmatism is a type of empiricism concerned with the future, so much so that "habit" and "experience" must be evaluated by their consequents rather than by their antecedents. John Dewey emphasized this

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notion in "The Development of American Pragmatism" (Dewey, 1981, p. 50): "Pragmatism, thus, presents itself as an extension of historical empiricism, but with this fundamental difference, that it does not insist upon antecedent phenomena but upon consequent phenomena; not upon the precedents but upon the possibilities of action". This reference to the future creates a link between the pragmatic school and the field of the philosophy of probability. Pragmatists always affirm that their philosophy assumes the probabilistic character of the future — the *only* problem is that they often disagree on what probability actually means.

The pragmatic maxim — that is, to consider the sensible effects of intellectual concepts — was first stated around 1878 in the well-known essay "How to make our ideas clear" (Peirce 1992a). For our purposes, it is important to note that the application the pragmatic maxim to probabilistic matters was one of Peirce's favorite subjects (Peirce, 1992b; 1992c; 1992d; 1992e; 1998a, 1998b). At first sight, probability makes the future uncertain and unpredictable. But Peirce insisted that the former claim is not completely true. In probabilistic matters, we are not able to predict singular cases, but we can have accurate knowledge about frequencies in the long run. In "The Maxim of Pragmatism", Peirce (1998) described his pragmatic approach to probability:

> *What is meant by saying that the probability of an event has a certain value, p?* According to the maxim of pragmatism, then, we must ask what practical difference it can be make whether the value is *p* or something else. Then we must ask how are probabilities applied to practical affairs. The answer is that the great business of insurance depends upon it. Probability is used in insurance to determine how much must be paid on a certain risk to make it safe to pay a certain sum if the event insured against should occur. Then, we must ask how can it be safe to engage to pay a large sum if an uncertain event occurs. The answer is that the insurance company does a very large business and is able to ascertain pretty closely out of a thousand risks of a given description how many in any one year will be losses (…) Now in order that probability may have any bearing on this problem, it is obvious that it must be of the nature of a *real fact* and not a mere *state of mind*. For facts only enter into solution of the problem of insurance. And this fact must evidently be a fact of statistics (…) Probability is a statistical ratio

> (…) of the number of experiential occurrences of a specific kind to the number of experiential occurrences of a generic kind, in the long run." (p. 136-137, italics in the original).

Indeed, Peirce's opinions about probability shifted along his lifetime. Initially he considered the subjective approach of probability as degree of rational belief. According to Levi (2004, p. 262), Peirce "admits that inquirers may assign subjective degrees of probabilistic belief to hypothesis provided those degrees of belief can be ground or justified by knowledge of objective, statistical, or frequency probability". But his method of evaluating the degree of rational belief was not Bayesian.[2] He thought that a strict numerically determination of subjective probability was somewhat fanciful and he "understood the matter to be one of a comparative preference" (Levi, 2004, p. 274). In the end of his career, he adopted the notion of probability as propensity, as real dispositions towards a definite outcome, a conception related to some aspects of Popper's later works (Popper, 1985; 1988). Apropos, Hans Reichenbach (1939, p. 189) also noted that certain points of peircean approach to probability anticipates his own formulation. In fact, some aspects of a pragmatic account of probability link Peirce, Reichenbach and Popper (Tiercelin, 1999). For a comprehensive account of Peirce's ideas about probability and induction, see "Beware of syllogism" of Isaac Levi (2004).

Nevertheless, for the most part of Peirce's writings, it is assumed an approach based on long run frequencies, such as that above quoted. We will confine our analysis to the frequentist period of his work because it raises the problem that we wish to discuss. The notion of long run frequencies gives support to probabilistic determinism and accommodates contemporary descriptions of causal relations in terms of probabilities. But, if we make a "Kantian question" — namely, how is possible to make probabilistic judgments? — difficult problems arise. A classical analytic interpretation of probabilities, as that of Laplace, would answer that, indeed, a probabilistic statement

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[2] Peirce's method of estimating scientific beliefs was based on his conception of *chance*. Peirce defines the chance (C) of an event as the direct division of the favorable cases by the unfavorable cases of an event. This quantity varies between $0 < C \leq 1$ when unfavorable cases are greater than the favorable ones. Conversely, when favorable cases are more numerous than unfavorable cases, this quantity varies between $1 \leq C < \infty$. For cases favorable and unfavorable equally, the chance is 1 (in terms of probability it would be 0.5). If it is applied a logarithm to C, then log (C) is negative in the interval $0 < C \leq 1$ (which expresses a belief against the event, a negative preference), and it is positive in the interval $1 \leq C < \infty$ (which expresses a belief in favor of the event, a positive preference), and it is 0 for C = 1 (it neither adds nor subtract the degree of belief). Moreover, Peirce's estimation of C varies according to Fechner's Law (which also is a logarithmic function) providing a possible link with the neurobasis of reasoning. See Peirce 1992c, p. 156 – 161.

is not amplificative; but then it is hard to see why such statements would be so well-fitted to physical phenomena described, for instance, by quantum mechanics and statistical mechanics. Conversely, if probabilistic statements are considered as having synthetic, amplificative nature, then it becomes difficult to classify them either as *a priori* or as *a posteriori*. Probabilistic determinism makes statements that require collective necessity in the long run, and this type of necessity has a peculiar characteristic: it does not determine single cases, but it constrains large collections of cases in the long run. Hereafter it is intended to remark three points: that most of Peirce's writings about probability were based on the notion of collective n the long run; that collective necessity in the long run does not fit in the Kantian traditional categorization, thus we need to enlarge our categorical vocabulary in order to comprise those additional conceptions involved in probabilistic theories; and that such reformation opens the door to our contemporary scientific theories supported by probabilistic determinism.

2. Peirce's pragmatic approach to probability

Probability has always suggested two distinct meanings historically interlaced (see, for instance, Carnap, 1950; Hacking, 1975). In first place, there is a mathematical sense concerning chance and distributions of characters, properties, etc, which can be expressed by relative numbers, accordingly cases favorable divided by the total of cases. This mathematical probability is brought upon analysis of oddities, and it is considered objective. As Peirce (1992b) once noted in "The doctrine of chance", unique events do not have degrees of probability — either it occurs or it fails to occur. Probability, objectively speaking, is asserted of indefinite series: when it is said that the probability of throwing a "five" with a dice is 1/6, it is understood that in the long run one sixth of throws will turn up five. With each throwing, however, either a "five" turns up or it does not, and we are unable to anticipate what will be. But, at same time, we can be sure that the distribution will converge to a stable ratio when the number of throwings increases. In other words, mathematical probability supports the appearance of some stable frequency in the long run (in accordance with Bernoulli's Weak Law of Large Numbers). Note that we are not defending the gambler's fallacy, that is: "the fallacy of supposing, of a sequence of independent events, that the probabilities of later outcomes must increase or decrease to compensate for earlier outcomes" (Kennedy, 1999, p. 339). We are just highlighting that we can expect some kind of objective convergence of probabilistic designs. It has a collective necessity that constrains the general result of larges collections, a well-known phenomenon in gambling and matters alike.

A brief illustration will show it. During the last FIFA World Cup 2006, a world-wide fast-food corporation promoted a curious instant game that consisted in a scratchcard with nine (3 X 3 columns) covered up small squares, each one with either a ball or a cross hidden by a thin grey ink. You had to scratch only three squares of different columns, and if three balls were found, you got a prize. In fact, no one could anticipate his own upshot, but the fast-food corporation had means of estimating the collective result of its promotional campaign. (Peirce's example of insurance companies follows the same line of reasoning.) The important point is that such collective necessity of large collections actually has empirical expression, therefore statements based on collective necessity presents an amplificative nature and should be considered of synthetic nature.

On the other hand, probability has also an epistemic sense concerning an evaluation of belief making process: for instance, which degree of reliability is provided by a particular experimental design. In this second sense, degrees of probability are attributed to our statements, propositions, predictions, etc., as a way of evaluating the "weight" of beliefs. A remarkable example of evaluation of belief making process is provided by Bayesian approach to probability. If inquirers could obtain more and more empirical data, under adequate statistical standards, it would increase the probability attributed to their statements, propositions and predictions; and, in the limit, certainty could be eventually reached. But inquirers hardly could obtain enough data, in view of the fact that they always deal with their individual and finite amount of experience. Peirce (1992b) concludes that science should not rely on finite individual experience, but on the endless experiences of an unlimited community of inquirers. It could be argued that, in fact, there can be no such community. Nevertheless, in some cases, "extrapolation" from limited available data is mathematically legitimate — it is as if we had unlimited experience.

Although these distinct meanings can be seen as covering different domains, if, in the long run, distributions of characters converged towards a stable ratio, and the reliability of beliefs converged towards its maximum, then those distributions could be known, and with large degree of certainty. That is what supports Peirce's claim that: "the opinion which is fated to be ultimately agreed to by all who investigate, is what we mean by truth, and the object represented in this opinion is the real" (Peirce, 1992b, p.

139). In another paper, called *The Probability of Induction*, Peirce employed the notion of *necessity in the long run* in order to give support to synthetic knowledge by means of inductive reasoning:

> Though a synthetic inference cannot by any means be reduced to deduction, yet that the rule of induction will hold good in the long run may be deduced from the principle that reality is only the object of the final opinion to which sufficient investigation would lead. That belief gradually tends to fix itself under the influence of inquiry is, indeed, one of the facts with which logics sets out (Peirce, 1992c, p. 169).

Here Peirce adopted a long run procedure of inquiry in the realm of reality as well as in the realm of beliefs. Accordingly, there would be one reality to be known by all inquirers. There is no problem that this reality be a frequency, a proportion, a relative number that express the ratio of certain cases divided by the total of cases. The only requirement is that this ratio must reach a stable limit in the long run. Now, in the realm of beliefs, although Peirce considered problematic a precise quantitative measurement of the degree of belief in Bayesian terms, he believed in reaching a good reliability of inductive process in the long run. "As all knowledge comes from synthetic inference, we must equally infer that all human certainty consists merely in our knowing that the [inductive] process by which our knowledge has been derived are such as must generally have led to true conclusion". The objective side of Peirce's conception of probability is related to the collective necessity of certain classes of statistical phenomena; the subjective side concerns the problem of the validity of inductive inferences, and it is related to the *modus operandi* of acquiring empirical data; notwithstanding, the most important point is that both sides demands that inquiry succeed in the long run.

3. There is no category for collective necessity in the long run

Synthetic knowledge *a priori* has an important role in rationalistic schools: it is a way of establishing truth at the outset in order to justify a system of correlated ideas. On the other hand, empiricists usually are occupied with knowledge *a posteriori*, for experience itself concerns what we have known from the past: which sensations impressed our minds, which ideas were associated to those impressions, and which habits we have acquired during our lives. But pragmatism intends to be a philosophical method occupied with the future of experience; thus, although considering that *a posteriori* knowledge deserves close consideration, pragmatists tend to stress that any event has a component which is not completely implied in the past.

Peirce's view, indeed, does not fit in the Kantian traditional classification of synthetic knowledge as *a priori* and *a posteriori*. For sure, mere numerical accounts can be appropriately described as synthetic *a posteriori*. And necessity is not required here: for instance, that one fifth of world population is Chinese does not involve any *a priori* law. Thus probability, as Peirce understood it, surely is not synthetic *a posteriori* because implies collective necessity — so, perhaps they should be seen as *a priori*. But probabilistic statements cannot be properly synthetic *a priori* since their necessity only applies to large series, but not to single cases. Here, our central concern is that there is no clear place for the collective necessity implied by probabilistic statements in the Kantian framework. In fact, Kant usually thought of probability as something secondary. For instance, there is an emphatic passage of Prolegomena that states:

> Nothing can be more absurd, than in metaphysics, a philosophy from pure reason to think of grounding our judgments upon probability and conjecture. Everything that is to be known *a priori,* is thereby announced as apodictically certain, and must therefore be proved in this way. We might as well think of grounding geometry or arithmetic upon conjectures. As to the doctrine of chances in the latter, it does not contain probable, but perfectly certain, judgments concerning the degree of the probability of certain cases, under given uniform conditions, which, in the sum of all possible cases, infallibly happen according to the rule, though it is not sufficiently determined in respect to every single chance.[3]

In others words, Kant himself discerned that probability involves necessity in the long run, "though it is not sufficiently determined in respect to every single chance". Here we do not pretend make a case against Kant. Our goal is only to show that, in probabilistic statements, we always have to stress that they do not constrain single

[3] Kant, 1783, section "Solution Of The General Question Of The Prolegomena: "How Is Metaphysics Possible As A Science?". Accessed in 03/05/2006, at http://webexhibits.org/causesofcolor/ref/Kant.html

events, but constrain the whole class in the long run. We are highlighting that there is a collective necessity implied in probabilistic statements, but it is not exactly *a priori*: so much so that it is always required the qualification “in the long run”. We defend that such a deficit can be overcome by the introduction of a new category — namely, synthetic knowledge *a ulteriori*, defined as “what is known about an indefinite number of cases, but not about isolated instances”. In short, knowledge *a ulteriori* consists in a category that could embrace the notion of collective necessity in the long run.

There is a peculiar semantic detail in the comparison between words “ulterior” and “posterior”. In my native language, these terms denote different relations of ordinal arrangements. For Portuguese speakers, *posterior* means “definitely after” in time and/or in space (the antonyms, in English, are “earlier” and “anterior”, respectively). Now, *ulterior* means “indefinitely after” in time and/or space (the antonyms are “primaeval” and “internal”, respectively, but this “internal” is related to the idea of closeness). For instance, nautical explorers talked about the “Mar Ulterior” as the furthermost sea beyond all known limits; conversely, the Romans called the Mediterranean Sea Mare Internum ("Internal Sea"). In the case of the word “posterior”, its meaning is slightly different. For instance, — imagine a queue — if you are posterior to me then you are behind me, that is, you are definitely placed after me. For time relations, a “posterior research” denotes an actual research made definitely after certain earlier one — for instance, “my research is posterior of yours” — meanwhile an “ulterior research” would mean a “research to be indefinitely done in the future”. For sure, there are several problems in such linguistic analysis since Portuguese differs from English in many aspects. If someone has a better English word to represent this matter, I will adopt it with sincere gratitude.

Despite such problem, two points must be remarked. First, the word “ulteriori” should understood in terms of both space and time — that is, it does not concerns exclusively to temporal series or to spatial arrangements. Second, the difference concerns definiteness versus *in*definiteness: in temporal series, the idea of *a ulteriori* carries the notion of a indefinite temporal sequence taken in the long run; for spatial arrangements, *a ulteriori* suggests an indefinite large collection of points, coordinates, cases or instances taken altogether.

In the last two years I have thought about this apparent categorical blank in the Kantian system, and during this time I have recollected different criticisms. In first place, it is possible to affirm that the formal axiomatization of probability already provided an analytic ground for inductive procedures. For instance, Rudolf Carnap is clear when say that "all principles and theorems of inductive logic are analytic; hence the validity of inductive reasoning is not dependent upon any synthetic presuppositions like the much debated principle of the uniformity of nature" (Carnap, 1950, p. v). This point of view denies the supposed synthetic character of probability in inductive procedures and, therefore, no problem arises at all. But then we have to face the mystery of the practical applications of probabilities, as well as we have to surrender to the general problem of the "metaphysical" relation between mathematics and the world of experience. Denying the synthetic and amplificative character of probability implies taking them as purely verbal formulas. It is true we can deduce probabilities by means of mathematical calculus, but the main problem is whether we could induce probabilities in order to draw inferences beyond our actual analytic powers. It seems that scientific beliefs based on probabilities involve more than pure calculus, because there is expectancy about future events. So, at least provisionally, we do continue supposing the synthetic nature of probability.

The second manner of criticizing the notion of knowledge *a ulteriori* consists in admitting the synthetic character of probability, but, at same time, applying the Occam's razor to the question: since we have a well-grounded distinction of *a priori* and *a posteriori*, why would we entertain a new category? Therefore, knowledge *a ulteriori* is just an illusion that could be reduced to an *a priori* formulation. Alas, it was said that knowledge *a ulteriori* is based on the Law of Large Numbers and the latter clearly has an aprioristic character. In fact, most of the attempts to 'dissolve' problematic character of probability follows this line of reasoning: to apply certain aprioristic principle in order to justify a matter that *seems* to belong to *a posteriori* ground. For instance, the principle of the uniformity of nature is an example of aprioristic principle that would explain why events are what they are.

This criticism is very strong but there is an important detail about the Law of Large Numbers. Since it demands indefinite collections, so it appeals for potential infinity, and such appeal, in my opinion, has a character different from others mathematical procedures. For the Greeks, for instance, potential infinity was not desirable; indeed, it was considered fallacious. Their mathematics was based on postulates and axioms laid on at the outset, and further statements are deduced in accordance with the validity of its original basis. But in the case of the Law of Large Numbers, although it is achieved by deductive means, its content states that potential infinity can provide validity in the long run. Moreover, the Law of Large Numbers does not constrain particular results of single cases for its necessity has a collective character. If we do not realize such difference, there is no need for a new category and therefore knowledge *a ulteriori* is reduced to *a priori*. But if we perceive that there is something strange in appealing to potential infinity in order to analyze indefinite series, then we can see the supposed blank in Kantian scheme.

It is important to say that we are not defending that knowledge *a ulteriori* has the same *power* of knowledge *a priori*: indeed, the collective necessity of *a ulteriori* judgments does not constrain the facts as strongly as *a priori* ones, for the latter has a constitutive nature and it marks the limits of experience. Nevertheless, knowledge *a ulteriori* is not merely contingent like *a posteriori* knowledge — bare facts ordinarily collected and accumulated. Collective necessity seems to present an intermediary character, for it is not completely certain at the outset, but at same time it is not the merely expression of contingency. Again, that "one fifth of world population is Chinese" could actually be true but it would not be necessary. Now, for a fair die, that "one sixth of throws of will turn up certain face in the long run" involves collective necessity *a ulteriori* when the number of events increase indefinitely.

In order to perceive the importance of the new category proposed, ask yourself the following questions: How is possible to make probabilistic judgments? What is the place of probabilistic judgments in the Kantian traditional categorization? These questions concern the foundation of several contemporary probabilistic theories and if someone has a good answer, I would be glad to know it as soon as possible, because I have being baffled by these questions for long time.

4. Probabilistic determinism and contemporary theories

Collective necessity in the long run is distinctively implied when we discuss theories based on probabilistic determinism. For instance, thermodynamics take advantage of probability in order to predict the whole behavior of a collection in the long run, although does not provide any particular information about a single molecule. The procedure of calculation of probability in the long run is practically essential for the analysis of dynamical systems by statistical mechanics. In the microscopic domain, quantum mechanics works in view of the fact that the wave-function prescribes the probability of distribution of results. But these forms of scientific knowledge do not involve *a priori* knowledge because there is no constitutive necessity, from the out-set, but a collective necessity in the long run.

Before the analysis of contemporary theories in terms of knowledge *a ulteriori*, it is useful to draw a general sketch of the matter. The following an analogy was proposed to me, in a personal conversation, by Professor Bento Prado Jr., unfortunately recently deceived. He was a Bergsonian (see, e.g., his book *Presence et champ transcendantal*) with a marked ability of proposing images which capture a synoptic description of philosophical matters. He proposed the following analogy.

Take the atomic bombing of Hiroshima. *A priori*, scientists were perfectly sure, at the outset, that every human being directly exposed to that catastrophic event would necessarily die. After the bombing, a certain number of people actually have died and it is possible to collect data in order to estimate the tragedy. Such number, whatever, has no necessary character. It is *post facto* and is achieved by *a posteriori*. But an atomic bombing has a peculiar side-effect: radiation increases the number of diseases such as leukemia, malformations, and all sort of genetic damages that are distributed among the population. As matter of fact, no one can predict who will be affected, even more because it depends on the specific births that will eventually occur. For instance, Tomiko either may espouse Akira or may not; Mariko may marry the intelligent and sweet Tanaka, but perhaps she would prefer to be taken away by the virile but impetuous Matsuo — by the way, what algorithm governs the heart of women? As said by Cervantes (Chapter XX, p. 110), in the novel Don Quixote: “That is a natural condition of women to disdain those that love them, and to affect those which hate them”. But the fact that love affairs are events governed by capricious laws promotes good basis for probabilistic predictions about the genetic pattern of populations making possible estimation of health assistance: we can estimate how many cases of diseases related to radiation should be expected, how many hospitals are needed and how many doctors should be trained and employed in this service. It concerns the statistical average of a large population of human beings, but it does not specify precisely who will manifest health problems. The usual linkage between health assistance and insurance is not accidental.

Prado’s analogy summarizes the main characteristics of different types of synthetic knowledge, but hereafter we need to focus on the scientific aspects of reasoning *a ulteriori*. For instance, there is voluminous literature about Schrödinger’s cat and the combination of quantum states. In its original formulation, it resembles a problem of probability applied to single case: for the half-life of the isotope, the cat either is alive or dead, as well as a coin either drops tail or head. But instead of facing this paradox by using the usual arguments, let us analyze it in *a ulteriori* fashion. So, I say: I really do not know if *this* cat is alive when the half live of the isotope is fulfilled; however, give me a million of cats and I will affirm with good reliability — based on the notion of half-life and in the concept of collective necessity — that, in that precise moment, half of them will be alive and the other half will be dead; and if you give me more and more cats, more precise will be the prediction. This approach to the paradox does not explain the question of combination of quantum states, but it dissolves most of difficulties involved in the problem. The half-life is a statistical concept that concerns large collections and it causes troubles as applied to single cases. It is important to remark, however, we do not pretend to present a panacea; we are just highlighting how this new category could dissolve some puzzling questions.

Another scientific matter that could be considered *a ulteriori* is thermodynamical systems. Thermodynamics states that in the long run an isolated system tends toward a state of maximum disorder, its maximum entropy, which is its higher state of probability. However, the increasing of entropy is not an unavoidable strictly rule that may not be violated in small scale. It is a probabilistic description that we can rely with certainty in the long run, but occasional and local fluctuations are in fact expected. If one tries to ground it in aprioristic terms, one should be able of predict case by case, molecule by molecule, and thermodynamics is far from being such a minute calculation. However, in the long run, we can be sure that the system will reach its state of equilibrium and such result apparently revives the ancient Aristotelian physics — particularly, the idea of "natural place". In fact, our modern physics only admits efficient causes, but the French physicist Pierre Duhem (1989 [1905]) once remarked thermodynamics has certain Aristotelian traces.

> *What do we find, then, of truly essential in the theory of the natural place of the elements?* We find the assertion that it is conceivable a state in which the order of universe would be perfect; that this state would be, for the world, a state of equilibrium, e even more, it should be a stable equilibrium. [It means] to constrain the universe to this state of ideal balance, in a manner that the final cause is, at same time, its efficient cause (…) When we conceive an assemble of

> material bodies, which are supposed free from any extern influence, each state of this assemble corresponds some value of its entropy. In a certain state, its entropy would be greater than other else. This state of maximum entropy would be a state of equilibrium, and stable equilibrium. Every movement, every event inside this isolated system, produce increasing of entropy (…) How could we not recognize this surprising analogy between Aristotle's cosmology and the achievements of thermodynamics? (p. 153-154)

Related to thermodynamics is the idea of *attractors* in dynamical systems. An attractor is a region of the state of phases to which several different trajectories converge and can not escape from. Roughly speaking, imagine a surface with valleys and mountains; if you drop a ball from a mountain, it will roll down and eventually stop in a valley. Now, if many balls dropped at different places go to certain valley, this valley usually is said to be the attractor of the system. Nevertheless, the concept of attractor is not much proper, since it also involves the idea of "natural place" and teleological causation — events fulfilling given ends. Indeed, attractors and maximum entropy (both considered as "natural places") imply collective necessity in the long run, but here we need to make some clarifications. When we say that attractors and maximum entropy imply collective necessity in the long run, we are not defending the actual existence of some kind of teleological materialism — for instance, one could say, in accordance with classical thermodynamics, that the heat death of the universe is an example of *final* attractor and that it would involve a teleological assumption based only on material bodies. We do not wish to make this strong cosmological assumption, for we are just suggesting, by *modus tollens*, that without the notion of collective necessity in the long run there is no clear explanation of such concepts.

5. Conclusion

Peirce's pragmatic approach to science, including theories about probabilistic matters, holds that we should discover true and justified beliefs if scientific inquiry were pursued indefinitely in the long run. This conception involves the domain of our scientific beliefs as well as the domain of the empirical facts. Concerning empirical facts, he defended that relative frequencies must reach a stable ratio in the long run, independently of what numerical value it will be (for the cases of dice and coins, it will be 1/6 and 1/2 respectively). Now, in the domain of our scientific beliefs, the more we collect data, the more we achieve statistical reliability about them. At any rate, in both domains the role of the long run is crucial.

Such type of certainty due to "long run procedures" is what I mean by knowledge *a ulteriori*. We have defended that it is a third category that does not fit in the traditional Kantian framework. Since it is not necessary at the outset, thus it is not *a priori*; but, at same time, it is not merely *a posteriori*, because it does involve collective necessity in the long run. I do not wish to be dogmatic on semantic details about the word *a ulterior*. Perhaps such word is not a good choice, so I am open to adopt any other term. Moreover, Peirce in fact never proposed that there is such a blank in Kant's category. I have acted on my own responsibility.

Once I heard that Kant's philosophy has a so systematic architecture that sometimes he created false windows. In my opinion, however, there is a real but unforeseen chamber that we, admirers of Kant, still need to explore. Scientific theories based on probabilistic determinism were not entertained in 18th century; but nowadays they are recognized as practically essential in many fields. Thus I insist that we should ask ourselves: how is possible probabilistic judgments? I have made my suggestion, but taking Peirce (1992e) as model, "I earnestly beg that whoever may detect any flaw in my reasoning will point it out to me, either privately or publicly; for if I am wrong, it much concerns me to be set right speedily" (p. 311).

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