I. Introduction

The line of argument pursued in this paper is to proceed from Einstein’s fundamental problem situation to a consideration of scientific representation with respect to the Special theory of relativity (STR). Einstein’s fundamental problem situation, which is Kantian in spirit, is how the conceptual freedom of the scientist is compatible with the need for an objective representation of an independently given material world. To solve this philosophical issue Einstein employs a number of constraints, which are central to the STR. The issue of scientific representation leads to a consideration of the notion of reality and to the realistic commitments implied in the STR. From this point of view, the paper concludes that Einstein was committed to a kind of ‘structural’ realism.

II. Concepts, Facts and Constraints

Einstein’s fundamental philosophical position arises from the age-old puzzle of how concepts are related to facts. More generally, how do scientific theories represent empirical reality? Einstein warned against the tendency to regard concepts as thought necessities. Once certain concepts have been formed, often on the basis of experience, there is a danger that they will quickly take on an independent existence. People are tempted to invest them with some kind of Kantian necessity. Concepts, however, just like theories, are always subject to revisions. Einstein complained that

Philosophers had a harmful effect upon the progress of scientific thinking in removing certain fundamental concepts from the domain of empiricism, where they are under our control, to the intangible heights of the a priori. (Einstein 1922, 2)

What Einstein had in mind were the classical notions of space and time. Newton had regarded it as necessary to introduce the notions of absolute and universal space and time into his mechanics in order to make sense of his laws of motion. These notions had become part and parcel of classical physics. Kant turned them into thought necessities, although in his Critique of Pure Reason he rejected the Newtonian view that space and time had an existence outside of the human mind. The Special theory arrived at a different result. Temporal and spatial measurements became relativitized to particular
reference frames. This was a necessary consequence of embracing the principle of relativity and taking the velocity of light as a fundamental postulate of the theory. Through his own work Einstein had witnessed how such fundamental philosophico-physical notions as space and time required conceptual revision. This made him forever suspicious about the sway that such notions could hold over the minds of physicists and philosophers.

As is well-known Einstein characterizes scientific theories and the fundamental notions of physics - energy, event, mass, space, and time - as free inventions of the human mind. No amount of inductive generalizations can lead from empirical phenomena to the complicated equations of the theory of relativity. Science, however, assumes the existence of an external world. Furthermore, scientific theories are meant to entail objective statements about the external world. Although the fundamental notions of physics are logically speaking free inventions of the human mind, they must be mapped onto the data given by empirical reality through experiments and observation. (Einstein 1920, 141) Thus Einstein faced the fundamental Kantian position of finding a synthesis between reason and experience. This problem situation poses the question of scientific representation. The notion of constraint is of particular importance in an assessment of how the theory of relativity deals with the representational link between concepts and facts, between models and physical systems.

In an article written for the London Times Einstein introduces the now famous distinction between constructive theories and principle theories. (Einstein 1919) Constructive theories employ relatively simple formalisms, which are meant to represent the hypothetical structure of a physical system. The role of a constructive theory is to propose hypothetical (or as-if) models, which assign an underlying structure to the observable phenomena. The hypothetical structure is meant to explain the observable phenomena. The kinetic theory of gases models the behaviour of gas molecules as if they were billiard balls. Early atom models modelled atoms as if they were tiny planetary systems. A constructive theory, in order for its models to represent the observable phenomena, introduces in its formalism a number of idealizations and abstractions. The models represent the phenomena as if they only consisted of the components, which the
model introduces. Nevertheless, for the representation to succeed the models must retain a degree of approximation to the systems modelled.

Einstein was mostly concerned with theories of principles. Principle theories employ very general features of natural systems, from which mathematical criteria follow, which natural events and their models must obey. The role of a principle theory is to propose well-confirmed fundamental physical principles: the laws of thermodynamics, the principles of relativity, of covariance and invariance, and the constancy of light. These principles forbid the occurrence of certain physical events, like the propagation of signals beyond \( c \) or perpetual motion machines. They constitute constraints on the construction of models and theories and the postulation of laws of physics. Constraints can be understood as restrictive conditions, which such symbolic constructs must satisfy in order to qualify as admissible scientific statements about the natural world. Principle theories seek to represent physical systems under the constraint of these principles. If principle theories differ from constructive theories, it is to be expected that they employ more sophisticated models to represent aspects of the external world.

Einstein implicitly talks about various kinds of models, associated with constructive and principle theories respectively. Theories seem to represent via different kinds of models; in this representation, different kinds of constraints seem to be involved. The idea that constructive and principle theories represent different aspects of natural systems raises immediate questions about realism. To which extent can the models of the theory of relativity be regarded as realistic representations of natural systems? Attention should be directed to a number of constraints, which arise from the theory of relativity. The focus on constraints implies a view of scientific representation: that representation is a question of fit. Einstein hints at a notion like ‘fit’.

We have thus assigned to pure reason and experience their places in a theoretical system of physics. The structure of the system is the work of reason; the empirical contents and their mutual relations must find their representation in the conclusions of the theory. In the possibility of such a representation lie the sole value and justification of the whole system, and especially of the concepts and fundamental principles which underlie it. (Einstein 1933, 272)

A scientific theory constructs a coherent and logically rigid account of the available empirical data. Its coherence may always come under threat with new empirical discoveries. There is nothing final about the representation of a scientific theory of the
external world. In his philosophical writings Einstein insists on the logical simplicity of a theory and testability as constraints to be imposed on admissible scientific theories (Einstein 1949a, 22) Logical simplicity is a methodological constraint. Compatibility with available and new evidence is an empirical constraint. Although Einstein claims that ‘the world of phenomena uniquely determines the theoretical system’ (Einstein 1918b, 226), it is clear from a study of the theory of relativity that, apart from ‘external confirmation’ and ‘inner perfection’, further constraints come into play. Einstein sees the importance of principle theories in the introduction of fundamental principles – like the relativity principles – which act as constraints or limiting principles. (Einstein 1920, 99; Einstein 1950, 352) For instance, he speaks of the requirement that the laws of physics must be invariant ‘with respect to the Lorentz transformations’:

This is a restricting principle for natural laws, comparable to the restricting principle of the non-existence of the perpetuum mobile which underlies thermodynamics. (Einstein 1949a, 56)

Einstein holds that the interplay of such constraints – and others like covariance, invariance– creates a fit of the theory or model with the evidence extracted from the external world. (Einstein 1949a, 23; Einstein 1918b, 226, Einstein 1944, 289) The representation is described in terms of fit, which is understood in terms of satisfaction of constraints. A theory ‘represents’ a section of the empirical world, if it satisfies a certain number of constraints.

In order that thinking might not degenerate into ‘metaphysics’, or into empty talk it is only necessary that enough propositions of the conceptual system be firmly enough connected with sensory experiences and that the conceptual system, in view of its task of ordering and surveying sense-experience, should show as much unity and parsimony as possible. (Einstein 1944, 289; Einstein 1949b, 669, 680)

The representation is not an image, nor need it be perfect or absolute. Fit is a matter of degrees. It changes with the changing nature of constraints. In his discussion of principle theories Einstein explicitly states that such theories employ principles ‘that give rise to mathematically formulated criteria which the separate processes or the theoretical representations of them have to satisfy.’ (Einstein 1919, 228) The more constraints are imposed on scientific constructs, the greater the chance that representation will succeed.
The physical world is represented as a four-dimensional continuum. If I assume a Riemannian metric in it and ask what are the simplest laws which such a metric can satisfy, I arrive at the relativistic theory of gravitation in empty space. If in that space I assume a vector-field or an anti-symmetrical tensor-field which can be derived from it, and ask what are the simplest laws which such a field can satisfy, I arrive at Maxwell’s equations for empty space. (Einstein 1933, 274)

Let the empirical facts, the mathematical theorems, methodological rules and the physical postulates constitute a constraint space; and consider that scientific theories and their models must be embedded into this space. Einstein was one of the first physicists to become fully aware of the power of constraints, operating as restrictive conditions on scientific constructs. His emphasis on theories of principles, like the theory of relativity, was particularly helpful in this respect. Although Einstein himself did not always clearly distinguish between them, from the modern point of view he imposes four constraints on physical constructs in the theory of relativity. Any admissible theory must satisfy such constraints.

♦ Empirical constraints. These constraints comprise Einstein’s postulation of the constancy of ‘c’ in vacuum and his famous predictions: the red shift of light as a function of gravitational field strengths and the bending of light rays in the vicinity of strong gravitational fields. His GTR also explains the perihelion advance of Mercury and other planets.

♦ Principles of Relativity. Einstein characterizes reference frames as ‘mechanical scaffolds’ or grids, according to which the spatio-temporal location of bodies can be determined. (Einstein/Infeld 1938, 156) No reference frame must serve as a preferred basis for the description of natural events in the STR. For this reason Einstein abandoned Newton’s absolute space and time and 19th century ether theories. Even his Special theory gave an unjustifiable preference to inertial systems and Euclidean geometry. The General theory extends the principle of relativity to all – inertial and non-inertial – coordinates systems. In its general form the principle states that all coordinate systems, which represent physical systems in motion with respect to each other, must be equivalent from the physical point of view. In other words, the laws which govern the changes that happen to physical systems in motion with respect to
each other are independent of the particular coordinate system, to which these changes are referred. (Einstein 1905)

♦ Invariance and Symmetry. Invariance is related to the symmetry principles of the relativity theory. In the STR symmetries result from the operations of transformation rules between inertial frames. Reference frames serve as idealized physical systems in the theory of relativity. Compared with the many types of symmetries, which are recognized today (global, local, external, internal, continuous and discrete symmetries, see Castellani 2003, Ch. 26.6; Kosso 2000; Brading/Brown 2004), Einstein only deals with space-time symmetries of a global (STR) or local (GTR) kind. The Lorentz transformations deal with space-time transformations of a global kind: they are constant throughout space and time. The Lorentz transformations represent transformations of the inertial frames, say a boost from a system at rest to an inertially moving system. Symmetry transformations form symmetry groups. Symmetry groups (like the Lorentz transformations) require the physical equivalence of various inertial systems: as we subject inertial frames to transformations (rotation and translation in space-time) certain features remain invariant, others change. The symmetry operations show which physical features remain invariant under the operation of transformations and which features change in the transition between reference frames. As we shall see in the next section many physicists regarded what remains invariant under symmetry operations as the ‘real’. According to Einstein only space-time coincidences can claim physical reality. It seems that the ‘invariant’ provides a new criterion of what physicists are to count as physically real. A consideration of symmetries also point to the importance of structures in the theory of relativity.

♦ Covariance. The relativity principles state that all inertial and non-inertial systems are to be treated as equivalent from a physical point of view. The invariance principle states that symmetry transformations performed, say, on inertial frames must return some values of parameters as invariant. Einstein introduces covariance as ‘form invariance’ of the laws of physics. (Einstein 1916; Einstein 1922) The laws must retain their form whether they are considered from different coordinate systems or described in different mathematical languages. This intuition reflects Einstein’s demand that the laws of physics remain ‘covariant’ with respect to different coordinate
systems of the theory of relativity. We can express the laws of nature in different mathematical languages, for instance in the form of Euclidean or Riemannian geometry. Lorentz covariance means that the form of physical laws must remain invariant as reference systems undergo symmetry operations with respect to their spatial and temporal coordinates. But the covariance constraint takes on its true importance in the GTR. The space-time coordinates are abstract notations, $x_1, x_2, x_3, x_4,$ and the space-time laws are required to remain unchanged under the quite general transformations of the space-time coordinates, which the GTR allows. (Norton 1993, 794-5) Einstein often illustrates covariance with respect to the space-time interval $ds^2$. (Einstein 1922, 28) In Minkowski space-time, the space-time interval $ds^2$ is expressed as an invariant expression in what remains essentially a quasi-Euclidean space; for the propagation of light it is:

$$ds^2 = \sum_{\nu=1}^{3} (\Delta x_{\nu})^2 - c^2 \Delta t^2 = 0 \quad (1).$$

If the expression satisfies covariance it must remain form-invariant under the substitution of a primed coordinate system, i.e. $ds^2 = 0 = ds'^2$:

$$ds^2 = \sum_{\nu=1}^{3} (\Delta x'_{\nu})^2 - c^2 \Delta t^2 = 0 = d's^2 = \sum_{\nu=1}^{3} (\Delta x'_{\nu})^2 - c^2 \Delta t'^2 \quad (2).$$

The equation for the space-time interval, $ds^2$, remains form-invariant if $K$ is substituted by another quasi-Euclidean inertial frame, $K'$, as indicated by the coordinates $\Delta x'_{\nu}$.

The space-time interval, $ds^2$, is expressed, in Minkowski space-time, by the invariant line element:

$$ds^2 = c^2 dt^2 - dx^2 - dy^2 - dz^2 \quad (3)$$

Equation (3) captures Einstein’s desire to call his theory ‘theory of invariants’ rather than ‘relativity theory’. Laws must remain covariant under arbitrary transformations of the coordinate systems. It is not easy to say what form invariance actually means. For present purposes it suffices to say that such a change in symbolic form should not affect the objective relations, which the laws encode. Covariance expresses the requirement that
equivalent expressions of the laws of nature must remain objective. (Cf. Weinert 2007) The requirement that the physical laws in the STR and GTR must remain ‘form-invariant’ under the transformation of space-time coordinates is a further hint, to be developed later, that structures play a significant part in the theory of relativity.

The reason for the imposition of the constraints is to increase the fit between the theory and the world of experience. If the number of constraints and their interconnections can be increased, then many scientific theories will fail to satisfy the constraints. (Einstein 1933, 272; Einstein 1936, 18-9; Einstein 1944, 258) This process of elimination will usually leave us with only one plausible survivor. The General theory of relativity was able to explain the perihelion advance of Mercury, where both classical mechanics and the STR had failed. It would be exaggerated to claim that there exists such a tight fit between the theory and the world that there is a one-to-one mapping of the theoretical with the empirical elements. Due to the need for approximations and idealizations in the theoretical constructs, which are ‘free inventions’, there will always be theoretical structure, for which there is no direct empirical evidence. But Einstein holds that one theory always satisfies the constraints better than its rivals. It does not follow from this argument that the survivor – let us say the theory of relativity – will be true. It does follow that the process of elimination will leave us with the most adequate theoretical account presently available. New experimental or observational evidence may force us to abandon this survivor. The desire for unification, logical simplicity and the clash with experience may persuade us to develop alternative theoretical accounts. Einstein’s extension of the principle of relativity from its restriction to inertial reference frames in the Special theory to general coordinate systems in the General theory is a case in point. Although Einstein does claim that there is one correct theory, he cannot mean this in an absolute sense. (Einstein 1918b, 226) His insistence on the eternal revisability of scientific theories speaks against this interpretation. What he must mean is that there is always one theory, at any one point in time, which better fits the available constraints. This one theory settles better into the constraint space, which theory and evidence erect, than its rivals. Clearly, Einstein regarded the theory of relativity as a superior theory at
his time; proponents of this theory also claimed that it committed them to an invariance view of reality.

III. Views of Reality

The laws of physics must express the invariant features, which remain as coordinate systems undergo space-time transformations. Einstein explicitly claims that the laws of physics are statements about space-time coincidences. In fact only such statements can ‘claim physical existence’. (Einstein 1918a, 241; Einstein 1920, 95) As a material point moves through space-time its trajectory is marked by a large number of co-ordinate values $x_1, x_2, x_3, x_4$. The requirement of covariance allows it to be equally well described in terms of the primed coordinates $x'_1, x'_2, x'_3, x'_4$. This is true of any material point in motion. It is only where the space-time coordinates of the systems coincide that they ‘have a particular system of coordinate values $x_1, x_2, x_3, x_4$ in common’. (Einstein 1916, 86; Einstein 1920, 95) In terms of observers, attached to different coordinate systems, it is at such points of intersection that they can agree on the temporal and spatial measurements of the respective systems. This is Einstein’s point-coincidence argument. From this argument, many physicists, including Einstein, concluded as a philosophical consequence of the symmetries of the relativity theory that only the invariant can be regarded as the physically real. (Einstein 1920, Appendix II) This is now a common-place view:

All the appearances are accounted for if the real object is four-dimensional, and the observers are merely measuring different three-dimensional appearances and sections; and it seems impossible to doubt that this is the true explanation. (Eddington 1920, 181)

(... the objective features of the world must be represented by invariant quantities.’ Why? Because frame-dependent quantities 'change from reference to reference frame' and are, in part, artefacts of convention. (Maudlin 2002, 34)

If two frames from which the universe can be accurately described disagree on a certain matter, then that matter cannot be an objective fact. (Lange 2002, 207; cf. Belashov 1999, 2000)

Yet as Nozick (2001, 329 Fn 11) correctly points out, while frame-specific temporal and spatial measurements in the Special theory of relativity are not invariant but perspectival, they are objective. What effect does this concession have on the invariance view of reality?
**Perspectival Reality.** Is it true that only the ‘invariant is real’? What happens, say in the STR, to the clock and meter readings in particular inertial frames? As they differ from frame to frame, should we conclude that these events are ‘unreal’ in the respective inertial frames? Note that the question of the reality or unreality of events in space-time does not depend on observers’ perceptual relativity. Different systems in motion with respect to each other register different values for rod lengths and clock times. These measurements do not depend on what observers perceive; they depend on the behaviour of physical systems in motion. For measuring observers in the respective systems, these measurements have *perspectival* reality. Observers in time-like related frames, moving at a constant velocity with respect to each other, can observe that their respective clocks ticks at different rates and their measuring rods do not measure the same lengths. The ticking rate of the clocks and the behaviour of measuring rods show that perspectivalism is not observer-dependent but frame-dependent. It depends on the behaviour of rods and clocks in particular frames. Only the reading and comparison of clocks depends on the presence of conscious observers. The perspectival realities of physics are the result of a combination of frame-dependent features – the ‘3+1’ view of observers, due to their perspectival lamination of space-time - and frame-independent parameters of inertial frames - the invariant features of four-dimensional Minkowski space-time.

If we adopt perspectival realities, what becomes of the physicist’s criterion that *only* the invariant is to be regarded as real? The adoption of perspectival, frame-dependent realities enhances the invariance criterion of reality. The Minkowski space-time structure has both invariant and perspectival aspects. In Minkowski space-time, the non-tilting light cones, emanating from every space-time event, are invariant for every observer. The space-time interval, $ds^2$, is invariant across inertially moving frames. The particular perspectives then result from attaching clocks and rods to the ‘scaffolds’. That is, they result from the particular ‘slicing’ of space-time by the world lines of inertial systems in relative, constant motions with respect to each other. The space-time symmetries tell us what is invariant across inertial frames, and what is perspectival. Once we know what features remain invariant across different inertial frames, we can derive the perspectival aspects, which attach to different inertial frames, as a function of velocity. Such a modified view of
physical reality can be derived from the Minkowski presentation of the theory of relativity. Max Born compared the perspectival realities to projections, which must be connected by transformation rules to determine what remains invariant. The projections are reflections of frame-dependent properties. But there are also frame-independent properties, which are invariant in a number of ‘equivalent systems of reference’.

In every physical theory there is a rule which connects projections of the same object on different systems of reference, called a law of transformation, and all these transformations have the property of forming a group, i.e. the sequence of two consecutive transformations is a transformation of the same kind. Invariants are quantities having the same value for any system of reference, hence they are independent of the transformations. (Born 1953, 144)

The Lorentz transformations show, Born adds, that perspectival quantities ‘like distances in rigid systems, time intervals shown by clocks in different positions, masses of bodies, are now found to be projections, components of invariant quantities not directly accessible.’ (Born 1953, 144)

The theory of relativity leads to the invariance view of reality, which can be modified by incorporating perspectival realities. But Einstein rejected such perspectival views, as they appear in the Copenhagen interpretation of quantum mechanics (QM). In his opposition to the Copenhagen view he appears to adopt a much more traditional view of reality.

In his opposition to the Copenhagen interpretation of quantum mechanics, Einstein is committed to a complete, direct description of reality. (Einstein 1940, 924) By this he means a direct representation of the actual space-time events, rather than a probability distribution of possible outcomes of measurements. Such a complete description of actual events in space-time will avoid non-local effects, the spooky action-at-a-distance, which Einstein found objectionable in QM. For it will be subject to the ‘strict laws for temporal dependence.’ (Einstein 1940, 923; Einstein 1948, 323; Einstein 1949a, 86) In physics the ‘strict laws for temporal dependence’ are typically expressed in differential equations, which trace the evolution of some parameter as a function of time. A complete description of quantum reality would recover the differential equations, which describe the temporal evolution of real physical systems in space-time. The Schrödinger equation is of course a differential equation, which spells out the temporal evolution of quantum systems. However, this does not satisfy Einstein, because the Schrödinger equation describes temporal evolution in an abstract Hilbert space. His opposition to the
Copenhagen interpretation of QM led him to a more classical *separability view of reality*: spatially separated system, A and B, which obey Einstein locality, possess physical properties, which are not immediately affected by external influences on either of the systems. (Einstein 1948) This view of reality also transpires in the much-quoted definition of reality in the EPR paper. (See Einstein 1949a, 82-6)

IV. The Importance of Structure

If the models of the theory of relativity *represent* both invariant and perspectival aspects of reality, the question of realism imposes itself. It is generally agreed that Einstein’s position shifted from an early sympathy for Mach’s positivism to a later commitment to realism. (Einstein 1949a, 10; Holton 1965; Fine 1986; Scheibe 1992, 119; Scheibe 2006, 167-61; but see Howard 1990; 1993) In his ‘Autobiographical Notes’ he criticizes Mach for having misunderstood the ‘essentially constructive and speculative nature of scientific thought’ (Einstein 1949a, 20). But the question is which kind of realism the theory of relativity supports.

The *invariance view of reality*, which is a consequence of the introduction of symmetries in the STR, is in good agreement with a certain version of realism, which is expressed in many of Einstein’s philosophical announcements. This position simply regards scientific theories as hypothetical constructs, free inventions of the human mind. But science is committed to the existence of an external world, irrespective of human awareness. To be scientific, theories are required to represent reality via models. This version of realism need not claim that the theories, its models and laws are true mirror reflections of the natural world and its regularities. Einstein rejected ‘naïve realism’. (Einstein 1944, 280) There only needs to be the objectivity assumption that the models and laws of physics are good approximations and idealizations of the systems modelled. (See Einstein 1949a, 21-2) The models of the Special theory of relativity are idealized representations of kinematic aspects of physical systems. The models of the theory represent specific aspects of the physical systems modelled in the theory.

To focus on the representational aspects of models it will be convenient to distinguish between the topologic and algebraic structure of models. In the simplest case, a model represents the topologic structure of a system; e.g. a heliocentric scale model of the solar
system represents the spatial arrangement of the planets around the sun. The models used in the theory of relativity are more sophisticated structural models, which combine a topologic with an algebraic structure. The algebraic structure of the model expresses the mathematical relations between the components of the model. (Weinert 1999; Weinert 2006)

An analysis of the theories of relativity clearly shows that physics is concerned with physical systems, which are modelled in the STR by inertial reference frames and more general coordinate systems in the GTR. The reference frames, characterized by Einstein as ‘mechanical scaffolds’, select structural aspects of the systems modelled by way of their coordinates; in the STR these are kinematic relations between reference frames in inertial motion. Einstein emphasized his belief in the structure of the real world (e.g. relata and relations) in a number of places:

‘Without the belief that it is possible to grasp the reality with our theoretical constructions, without the belief in the inner harmony of our world, there could be no science.’ (Einstein/Infeld 1938, 296)

‘Physics is the attempt at the conceptual construction of a model of the real world, as well as its lawful structure.’ (Quoted in Fine 1986, 97; italics in original; Einstein 1948, 321)

The greatest change in the axiomatic basis of physics – in other words, of our conception of the structure of reality – since Newton laid the foundation of theoretical physics was brought about by Faraday’s and Maxwell’s work on electromagnetic phenomena. (Einstein 1931, 266)

There is clearly a concern with structure in Einstein’s physics, which is highlighted by the use of coordinate systems as models of reality. The concern with structure is further emphasized by a consideration of Einstein views on structure laws. According to this view the equations of the theory of relativity and electrodynamics can be characterized as structure laws, which apply to fields. (Einstein/Infeld 1938, 236-45) Structure laws express the changes which happen to electromagnetic and gravitational fields. These structure laws are local in the sense that they exclude action-at-a-distance. ‘They connect events, which happen now and here with events which will happen a little later in the immediate vicinity.’ (Einstein/Infeld 1938, 236) The Maxwell equations determine
mathematical correlations between events in the electromagnetic field; the gravitational equations specify mathematical correlations between points in the gravitational field. The postulates of quantum mechanics, like the Born rule, encode the probability of quantum events. Einstein submits that structure laws have the form ‘required of all physical laws.’ (Einstein/Infeld 1938, 238, 243) According to such a structural view of laws, the laws of physics capture structural aspects of natural systems. That is, they symbolically express the structure of a class of natural systems by showing how their relata are mathematically related to each other. Wigner was similarly aware of the importance of structure ‘in the events around us,

that is correlations between the events of which we take cognizance. It is this structure, these correlations, which science wishes to discover, or at least the precise and sharply defined correlations. (Wigner 1967, 28; cf. Weinert 2007)

By associating correlations with structure, Wigner emphasizes that the correlations between events can be mathematically determined; it is the mathematical determination, which provides the structure of the correlation. Generalizing the Einstein-Infeld-Wigner view we can therefore say that structure laws govern how the components (or relata) of physical systems modelled in the theory are mathematically related to each other.

Einstein clearly believes in the existence of a lawlike, structured reality, a physical world consisting of a network of systems, which can be described and explained by physical theories. The constructs of physical theories (axioms, constraints, coordinate systems, laws, models, theorems) express the structure of natural systems in mathematical form. The laws of physics determine the relations between the relata: for instance whether the relations are linear or non-linear, whether they involve quadratic or polynomial functions. Einstein seems to have believed in the reality of classical objects, fields and the structure of space-time, insofar as it is determined by the matter-energy contents of the universe. Apart from space-time events, the relata may refer to objects like planets (as in Kepler’s laws), electromagnetic or gravitational fields or to quantum systems in the wave function, \( \psi \). Einstein declares that ‘the concepts of physics refer to a real external world, i. e. ideas are posited of things that claim a ‘real existence’ independent of the perceiving subject (bodies, fields etc.)’ (Einstein 1948, 321, transl. Howard 1993, 238; Einstein 1944, 290)
If the natural systems in the physical world display various kinds of structure, then the models of scientific theories must represent this structure through their algebraic and topologic structures. The mathematical representation of three-dimensional Euclidean space, for instance, takes the form \( \mathbb{R}^3, d \), where \( \mathbb{R}^3 \) represents the Euclidean coordinate systems and \( d \) is the Pythagorean distance function. Space-time models can be represented in the general form \( \langle M, O, CS \rangle \), where \( M \) represents the differentiable manifold of space-time points – the topology of space-time points in local neighbourhoods – and the \( O_i \)'s various geometric objects, like spatio-temporal metrics and the \( CS \) represent the coordinate systems of the theory of relativity. The STR is represented by the mathematical structure \( \langle \mathbb{R}^4, h_{ik} \rangle \), where \( h_{ik} = \text{diag}(-1, -1, -1, +1) \), which is the matrix of the line element \( ds^2 = -dx_1^2 - dx_2^2 - dx_3^2 + dx_4^2 \). (See Norton 1992, 283, 289; Scheibe 2006, 110-12; Smolin 2006, 205-7)

Such issues of representation suggest that a consideration of the STR naturally leads to some version of structural realism, which predates current debates in the philosophy of science. Structural realism is a thesis about (knowledge of) structural relations. But such relations must, according to Einstein’s principle theories, obey constraints. Symmetries constitute one type of constraint. A consideration of the symmetries involved in the STR therefore suggests that the mathematical relations between relata must include the geometric symmetries, which are important in the STR. Symmetries constitute important elements of structure.

V. **Symmetries and Structure**

Physical systems can be regarded as manifestations of structures. A physical structure consists of relata and relations. But the modified invariance view of reality tells us that structures can have both frame-specific and frame-invariant features. What holds the relata together and binds them into specific structures are the mathematical relations. A system, like the solar system, consists of relata (the planetary bodies and the sun) and relations (Kepler’s or Newton's laws). The relations prescribe the elliptical orbits of the

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1 This section summarizes the results of a longer unpublished paper, ‘Invariance and Reality’ available on my website at http://www.brad.ac.uk/acad/ihs/Invariant&Real.pdf
relata. The job of science is to employ mathematical structures, also consisting of relations and relata, which represent the physical structures in an approximate and idealized form. Einstein, for instance, wrote that coordinate frames are modelled as ‘representatives’ of rigid bodies in mechanics. (Einstein 1925, 538) But Einstein also recognized that the algebraic relations are subject to constraints. Consider, for instance, the effect of symmetries.

Physics is interested in frame-invariant realities, because the frame-specific realities can be obtained from them by the transformation rules of a particular theory. But it seems that all our experience of reality is perspectival or frame-dependent, because of the ‘3+1’ slicing of space-time by observers. In our efforts to obtain frame-invariant realities, symmetries play an important part. Symmetries result from the application of transformation groups, which will leave all frame-invariant parameters unchanged. Frame-invariant parameters are those, which prove to be immune to the possible changes expressed in symmetry operations, like translation in time and space, rotation and mirror imaging. Given appropriate constraints, structures tend to be frame-invariant. But structure also has frame-variant perspectival manifestations. The relations between objects may not be structure-preserving, as for instance the distance relation between two objects may change; but the Euclidean distance, $r^2$, remains invariant.

A transformation group satisfies 3 logical criteria: reflexivity, symmetry, and transitivity. Symmetries can be distinguished according to different properties. Rotation, reflection, spatial and temporal translations and space-time symmetries are typical geometric symmetries. They take events, things and properties as their objects. A better name may be external (global) symmetries. External symmetries result from the operation of space-time transformation groups. They are external to their reference objects because they govern the invariance of their objects with respect to an 'external' change of space-time reference systems. So according to the Galileo transformations, an event that happens at a location $x$, can equally be transferred to a location $x'$, because the two locations are related by the equation $x' = x - vt$. Whatever event takes place at location $x$, its physical

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structure, expressed in the laws of physics, will not change as a result of its transport to $x'$. 

The characterization of structure as 'relata & relations' in philosophy of science debates cannot be restricted to geometric or physical relations between events and objects alone. The relations themselves are subject to symmetry constraints. The invariance of the relata (fields, objects, properties) are governed by their algebraic relations (laws of nature, symmetries principles, mathematical theorems), which makes the relations structural principles. For the relata to be governed by the relations means for the relations to put constraints on the relata - structural constraints since they determine the type of relata, which are allowed to enter the relations. But the space-time relations themselves are governed by space-time symmetries. Amongst the relations, we find for instance conservation laws, and these follow from symmetry principles, according to Noether’s theorem. With respect to the relata, the symmetries are higher-order principles. If the symmetries preserve invariant parts of structure, then they are constraints on structure. The structure consists of relations and relata, so that symmetries are higher-order constraints on relata and relations. As higher-order principles the symmetries give us the invariant structures, in which physics is interested. Following Leibniz, we can adopt a relational approach to structure. Such an approach emphasises that structure is born of a union of relata and relations; relations and relata are equally important for the formation of a structure. We have a triad of relata, relations and higher order principles, such as symmetries. The algebraic relations and symmetries act as constraints on the admissible relations and relata, with the result that, if the constraints do their job, the relata and relations refer to the components of physical structures, albeit in an approximate and idealized way.

An analysis of the STR shows that its consideration in terms of structural realism has to take into account the role of relata - reference frames and coordinate systems – and relations – laws and symmetry principles. 

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3 According to Noether's theorem symmetries and conservation laws are related. The laws of conservation are consequences of the space-time symmetry operations. Conservation of momentum results from invariance with respect to spatial translations, conservation of energy from invariance with respect to temporal translation, conservation of angular momentum from invariance with respect to spatial rotation and conservation of the centre of mass from invariance with respect to uniform motions. See Mainzer (1996), 350; Feynman (1997), 29-30; Rosen (1995), 150-3, Wigner (1967), 18ff.
of symmetries. Symmetries offer a deeper insight into the nature of reality, because they automatically yield frame-specific (perspectival) and frame-invariant properties of physical reality. A suitably modified invariance view of reality may be regarded as support for some ontic version of structural realism, which, however, assumes (against the standard ontic version) the existence of structured physical systems, e.g. the reality of relata and relations, which models aim to represent. This representation is not a plea for naïve realism or even isomorphism between theoretical structures and empirical substructures. The strength of structural realism resides in an awareness of the approximations and idealizations necessarily built into modelling. The only claim made in structural realism, as derived from a consideration of the theory of relativity, is that a certain ‘fit’ must exists between the theoretical structure whose job is to represent a physical structure. (Weinert 2006)

According to the relational view of structure, relata and relations interrelate in such a way that it can be misleading to claim that structures are prior to relata. Events, objects, properties and systems are needed as inputs to structures. Symmetries help to determine the invariant parts of structures. But invariant structures also need the input of relata.

VI. Conclusion. Looking at Einstein’s constructive work and some of his diverse statements on realism in science, we have argued that the STR commits its proponents to a certain form of structural realism. Such a commitment is implicit in the representational claims associated with Einstein’s focus on principle theories, their models and constraints. The invariance view, suitably modified to include perspectivalism, anticipates the emphasis on structure, which has dominated the recent debate about Structural Realism. In particular the invariance view highlights the role of space-time symmetries, which result in invariant and perspectival aspects of the systems, to which the transformation groups are applied. Symmetries also constitute an important form of constraint. An analysis of the Special theory of relativity tells philosophers much about science, which has not been sufficiently analyzed in the literature, in particular, the central role of constraints in scientific theorizing.
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