What Branching Spacetime Might Do for Physics

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ABSTRACT

In recent years, the branching spacetime (BST) interpretation of quantum mechanics has come under study by a number of philosophers, physicists and mathematicians. This paper points out some implications of the BST interpretation for two areas of quantum physics: (1) quantum gravity, and (2) stochastic interpretations of quantum mechanics.

1. Introduction

Today, most physicists accept the Copenhagen interpretation as the correct way of understanding quantum mechanics. Nevertheless, other interpretations of quantum mechanics exist and remain subjects of active research. One of these alternative interpretations, known as the *many-worlds interpretation* [DeWitt & Graham, 1973], is widely discussed by physicists and philosophers, and has proven useful in quantum cosmology and in the theory of quantum computing (see [Deutsch & Lockwood, 1994]). Another interpretation, generally known as the *stochastic interpretation*, has a very long history that extends back to the time of Einstein (see [Jammer, 1974]) and continues in our time in recent research (see, for example, [Pavon, 2001]; [de Angelis, et al., 1986]; [Nagasawa, 1993]; [Garbaczewski, 1990]; [Vigier, 1989]; [De Angelis & Jona-Lasinio, 1982]; [Lehr & Park, 1977]; [de la Peña and Cetto, 1975]; [Nelson, 1985]; [de la Peña-Auerbach, 1971]; [de la Peña-Auerbach, 1969]).[1] Several years ago, Smolin [1986] applied the stochastic interpretation to black hole physics, to understand quantum phenomena for which the Copenhagen doctrine does not appear to provide a sensible interpretation.

Related to the many-worlds and stochastic interpretations is a newer interpretation of quantum mechanics, sometimes called the *branching spacetime interpretation*. During the last dozen years or so, this interpretation has come under intensive study by physicists, mathematicians, and philosophers of science (see, for example, [Belnap, 2003]; [Kowalski & Placek, 2000]; [Placek, 2000]; [Kowalski & Placek, 1999]; [Belnap and Szabó, 1996]; [Douglas, 1995]; [McCall, 1995]). In the branching spacetime (or BST) interpretation [Belnap, 2003][2], the observed indeterminism of quantum mechanics is associated with a nonclassical structure for spacetime, which contains multiple branches that allow for different possible outcomes of observations. The BST interpretation resembles the many-worlds interpretation in that it postulates a branching of history into multiple alternative futures. However, the BST interpretation is not identical to the many-worlds

interpretation; the main difference is that the BST interpretation takes the branching of history to be a feature of the topology of the set of events with their causal relationships [Belnap, 2003], rather than a consequence of the separate evolution of different components of a state vector.

In spite of its relative newness, the BST interpretation already has proven to be physically useful for understanding certain features of quantum mechanics. In particular, this interpretation has yielded a new and revealing analysis of Bell-like theorems, including the GHZ theorem ([Kowalski and Placek, 2000]; [Placek, 2000]; [Kowalski and Placek, 1999]; [Belnap and Szabó, 1996]). Anyone who objects to the scientific use of the notion of branching spacetime should keep these applications in mind, along with the successes of the many-worlds interpretation, which also uses the idea of branching history.

In this paper, I wish to suggest some other ways in which the BST interpretation might eventually be useful to quantum physics. The potential uses that I will mention here are intended only as possibilities for future research; they have not been worked out in detail, and hence remain speculative. Nevertheless, these applications may be worthy of further study, since in each of them the BST interpretation appears to be able to simplify a known problem or to resolve a known conceptual difficulty in physics.

2. BST and Quantum Gravity

A. *BST and spacetime geometry*

The chief obstacle which any theory of quantum gravity must overcome is the apparent incompatibility between quantum mechanics and general relativity. At the heart of this problem is the difficulty of reconciling quantum mechanics with the conception of spacetime geometry used in general relativity. Familiar principles of quantum theory, when applied to the spacetime manifold, suggest that spacetime geometry fluctuates so

severely at small scales that the manifold structure of spacetime is lost. This is expected to happen over regions of the scale of the Planck length, $L_P \sim 10^{-35}$ m (see, for example, [Misner et al., 1973]).

Some models of BST appear to be able to sidestep this problem to some extent. The BST models which may be able to do this are the ones in which there is a fixed time constant governing the rate of branching of the spacetime -- in other words, models in which the branching of spacetime occurs on a discrete set of non-intersecting hypersurfaces, with a fixed time scale T for the time (extremal proper time) between consecutive branchings. In such a BST, any region bounded by hypersurfaces of branching, and having no hypersurfaces of branching in its interior, is a Hausdorff manifold -- unlike the total spacetime, which is non-Hausdorff. (Following my usual practice, I will call such a Hausdorff region a *branch*.) In a branching spacetime of this kind, the motion of a particle, as seen by an observer whose time resolution is much coarser than T, must in the general case be described by a probability distribution (see [Sharlow, 2003]). However, an observer who probes the particle's motion at time resolution < T will find that the particle moves on a branching sharp trajectory. This latter kind of motion is not what we normally call quantum mechanical motion -- at least if we were reared on the Copenhagen interpretation. However, this kind of motion cannot fairly be called "classical," since classical physics is based on the concept of a unique actual history.

In BST models of this kind (as in other BST models), the quantum behavior of physical systems results from the branching of spacetime. Hence within any single branch, there are no quantum fluctuations of any sort. It follows that T acts as a cutoff scale for quantum fluctuations of spacetime geometry. If we assume that T is at least as large as than the Planck time T_P, then the quantum fluctuations of geometry are much less severe than a naive application of quantum ideas to general relativity would suggest. In particular, if the time (extremal proper time) between any two consecutive hypersurfaces of branching is > T_P, then there are no quantum fluctuations at the Planck scale, and the breakdown of the manifold structure appears to be forestalled. Geometry still may

fluctuate, but the fluctuations will be relatively tame. This recipe for taming fluctuations in geometry probably can be implemented in a wide range of models of BST.

Needless to say, this non-rigorous qualitative argument is just what it seems to be: a non-rigorous qualitative argument. Nevertheless, this argument is highly suggestive. One always must remember that existing formulations of quantum gravity, such as superstring theory and loop quantum gravity, have their own ways of taming the metric fluctuations. However, the fact that BST can tame the fluctuations *by itself* suggests that it might be fruitful to try to develop a version of quantum gravity based on BST, along the lines of existing ideas about BST quantum mechanics of particles. A natural candidate for such a theory would use a BST with a Lorentzian (and perhaps Einsteinian?) geometry on each branch, combined with random changes in the geometry at the hypersurfaces of branching. Since we know so little about quantum gravity, this approach cannot be excluded *a priori*. A theory of this sort might be unpalatable to some (especially to those who have worked long and hard on one of the existing approaches to quantum gravity), but the existence of another alternative formulation of quantum gravity might be healthy for research in the field. The idea of a stochastically fluctuating metric is not new; it appears in the work of Namsrai [1986] -- along with the suggestion that these fluctuations make particle motion stochastic -- and in some other spacetime theories cited in [Namsrai, 1986] (see p. 9).

The question of the correspondence between classical spacetime and BST inevitably arises at this point. I will not analyze this question in depth here, but will simply point out the following. In a BST, each maximal set of consecutive branches forms what amounts to a single alternative history of the universe. Adapting the terminology of Belnap [2003], I will call the union of such a set a *history* in the spacetime. One can think of a history as an ordinary non-branching spacetime, though of course it differs from a classical spacetime because of what happens at the hypersurfaces of branching. If we look at a single history from a coarse-grained standpoint (that is, with time resolution $\gg T$), we will see what appears to be a classical spacetime. We may approximate this with a smooth, classical manifold, and may assign events in the history to places in this manifold. Thus, a "coarse"

observer (with time resolution >> T) will be free to interpret a single history in a branching spacetime as a classical spacetime manifold -- though some unexpected things may happen in this manifold, due to the (unobserved) presence of the hypersurfaces of branching.

B. BST and topology change

Another problem facing quantum theories of spacetime geometry is the thorny issue of topology change ([Visser, 1995]; [Misner et al., 1973]; [Wheeler, 1968]). This issue arises because it is mathematically possible for space to have many different topologies, including multiply connected topologies containing wormholes. If quantum mechanics is applicable to spacetime, then one would expect space configurations with wormholes to contribute to the time evolution of spacetime geometry. In particular, wormholes might be created or destroyed, resulting in changes in the connectivity of space. This process is known to be difficult to understand within the framework of a classical spacetime manifold.

In BST, it is much easier to understand (at least in an intuitive way) how topology change might occur. In a sense, any BST already contains topology change of a certain sort. A particle in a BST repeatedly goes into places (the hypersurfaces of branching) that differ topologically from surrounding regions of spacetime. These hypersurfaces are not just topologically different from their surroundings; they also are discontinuous in a certain sense -- at very least, they represent sudden temporary changes in the structure of space. Further, the structure of these hypersurfaces makes them natural candidates for the places where topology change might occur. The following argument shows what I mean by this.

Mathematically speaking, one can think of the region near a hypersurface of branching as constructed from three different sheets of ordinary, non-branching spacetime, glued together by identification of points.[3] (A very similar construction, not involving identification of points but giving the same results, is carried out in a different context in [Visser, 1995].) There are many different ways of doing this gluing. One way is to take a

pair of isometries f:B_1→B_2 and f:B_1→B_3, where B_1 is the future boundary of the initial

sheet and B_2, B_3 are the past boundaries of the final sheets, and then glue together points

related by the isometry (that is, identify x∈ B_1 with f(x) and g(x)). This will result in a

boundary that is geometrically smooth. However, this way of joining the boundaries is

rather arbitrary. Instead of using isometries, we can use non-isometric mappings between

B_1 and B_2 and between B_1 and B_3. If we use arbitrary homeomorphisms, then we can

create discontinuous changes in geometry across the junction [Sharlow, 2003]. If we use

mappings that are not even homeomorphisms, then we can create junctions at which

topology change occurs. This is done by gluing together sheets with different numbers of

wormholes. Depending upon the topology of the sheets that are glued together, the

resulting spacetime may have topology change at its hypersurfaces of branching .

From a mathematical standpoint, it is possible to construct a BST having random,

discontinuous changes in geometry across hypersurfaces of branching [Sharlow, 2003].

Once one has entertained this possibility, there is no obvious reason why topology also

could not change abruptly and randomly. Certainly, topology change is much more

plausible in a spacetime that already has a kind of discontinuous topology change than in a

spacetime that is everywhere smooth.

One problem facing the idea of topology change is the potentially nonlocal character of the

process. If a wormhole comes into existence, what (if anything) governs the relative

positions of the wormhole's two mouths? Can the mouths spring into existence far from

each other? What causality problems would this cause? To avoid all these questions, one

might want to assume arbitrarily that topology change follows only the pathways shown in

Figure 1.

Figure 1. "Local" pathways for topology change.
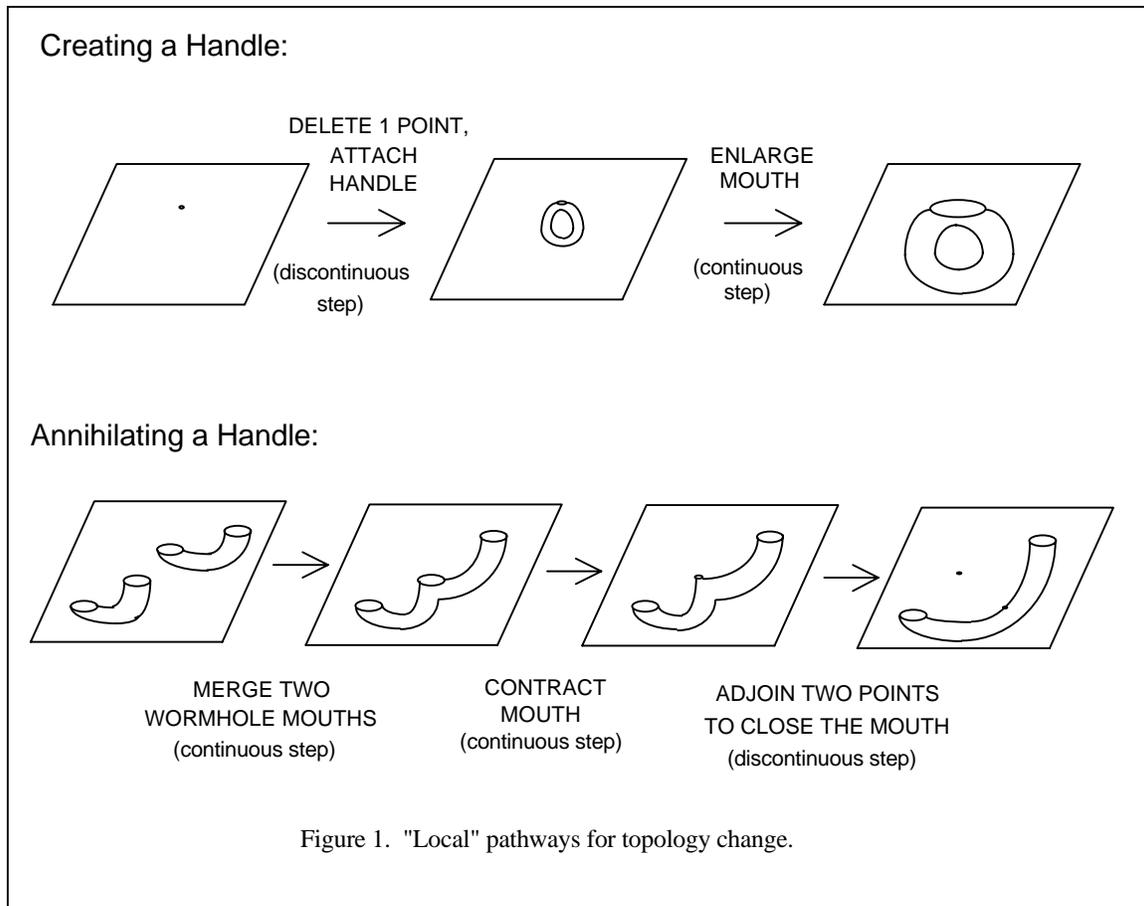
In the first of these pathways, a wormhole is created "all in one place," with its two mouths initially merged into a single mouth. Wormhole creation of this kind could begin at a single event in spacetime, as shown in Figure 1. In the second pathway shown in Figure 1, the total number of wormholes in space is reduced by one when two mouths belonging to two different wormholes merge, and the resulting mouth closes, leaving behind a single combined wormhole. (The same thing could happen with two mouths of the same wormhole, leaving no wormhole at all -- the reverse of the first pathway in Figure 1.) Both of these processes, wormhole creation and wormhole annihilation, are local in character. Each of them is discontinuous only at a single event in spacetime. Of course, I am not suggesting that topology change really occurs via these pathways; we do not know enough about topology change to dare to venture such a suggestion. I am only suggesting that if topology change did occur only in these ways, then topology change might be less messy, both mathematically and physically, than we think.

3. BST and Stochastic Mechanics

Another area of physics in which BST might be useful is stochastic mechanics, and particularly the application of stochastic mechanics to the stochastic interpretation of quantum mechanics. Since the stochastic interpretation of quantum mechanics may be unfamiliar to some readers, I will explain briefly what this interpretation is.

A. *The stochastic interpretation: an introduction*

At least since the mid-twentieth century, some physicists have suggested that quantum mechanics, with its probabilistic description of matter, might be interpreted as a description of some kind of stochastic motion of particles (see [Jammer, 1974]). According to this idea, the reason that we must describe the motion of particles probabilistically is that the particle's motion really is random. Particle motion, according to this view, is a stochastic process, similar (in spirit if not in detail) to Brownian motion. The wave function then is taken to be a description of the probabilities connected with this motion. The stochastic interpretation of quantum mechanics is the view that this is the correct way to understand physically the equations of quantum mechanics. Several different versions of this interpretation are known (see [Jammer, 1974]).

This mental picture of quantum mechanics may seem utterly unacceptable to those of us who grew up on the Copenhagen interpretation. However, as I stated in Section 1, the Copenhagen interpretation is not the only possible interpretation of quantum mechanics, and the stochastic interpretation still is an active field of research. Despite its historical importance and its current live status, the stochastic interpretation appears to be unfamiliar to most physicists today. The chief reasons for its unfamiliarity are, I think, the widespread acceptance of the Copenhagen interpretation and the demise of local hidden variable theories (see [Shimony, 1988]) -- although the stochastic interpretation in itself

does not imply a local hidden variable theory, and appears to be able to bypass the usual objections to such theories (see [Nelson, 1985]). The stochastic interpretation has been a subject of intensive recent research, and appears to have a significant, and possibly growing, constituency among physicists (recall the list of references in Section 1). It has been shown that most of the wave equations of quantum theory can be interpreted as descriptions of stochastic motion of various sorts. Such results have been obtained for the Schrödinger equation ([Nagasawa, 1993]; [Nelson, 1985]; [de la Peña and Cetto, 1975]; [de la Peña-Auerbach, 1971]; [de la Peña-Auerbach, 1969]; see also [Ord, 1997b]), the Klein-Gordon equation ([Pavon, 2001]; [Lehr & Park, 1977]), the Maxwell-Proca equation ([Garuccio & Vigier, 1981]; see also [Ord, 1997a] for a special case), and the Dirac equation (see [de la Peña-Auerbach, 1971]; see also [de Angelis, et al., 1986], [Ord, 2002] and [Ord, 1997b] for the (1+1)-dimensional case). In addition, progress has been made toward an understanding of quantum measurement in terms of stochastic mechanics [Nelson, 1985]. The stochastic interpretation even has found use in black hole physics; Smolin [1986] has suggested that the stochastic interpretation may be useful in understanding certain aspects of the quantum theory of black holes.

The main conceptual difference between the stochastic interpretation and the Copenhagen interpretation of quantum mechanics lies in the understanding of the nature of quantum probability. According to the Copenhagen interpretation, a particle's position and other dynamical variables can be genuinely indefinite, in the sense that there is no fact of the matter as to which value of the variable is the correct one at a given time. According to the stochastic interpretation, there *is* a fact of the matter -- but it is a fact which we, because of the limitations of our observational capacities, cannot know. Neither the stochastic interpretation nor the Copenhagen interpretation requires the kind of hidden determinism that one often associates with hidden variable theories. It should be mentioned that the stochastic interpretation is not a purely philosophical interpretation of quantum mechanics, but actually represents an attempt to reduce quantum mechanics to an underlying theory of stochastic mechanics.

Today, the Copenhagen interpretation is so widely accepted that its ideas even pervade introductory courses in quantum mechanics. Popularized discussions of quantum mechanics also tend to rely upon the Copenhagen interpretation, often as the basis for mystical conclusions. In an intellectual atmosphere of this sort, it is all too easy to forget that the Copenhagen interpretation is only an interpretation of quantum mechanics, and is not the same as quantum mechanics itself. The fact that other interpretations are possible is evidenced by the growth of interest in the many-worlds interpretation (see, for example, [Deutsch & Lockwood, 1994]).

The idea of interpreting quantum mechanics as a description of a stochastic motion of particles may be upsetting to persons used to the Copenhagen interpretation. Such persons need to remind themselves that the question of the correct interpretation of quantum mechanics is not yet closed. (If Smolin's suggestion contains even a grain of truth, then we may not have a choice in the matter.)

In this paper I will not try to build a case for or against the stochastic interpretation of quantum mechanics. Instead, I will point out some ways in which BST might contribute to our understanding of stochastic mechanics and of the stochastic interpretation. My discussion of the stochastic interpretation will be disproportionately long, simply because there are several points of contact that need to be discussed.

B. Modeling stochastic motion in BST

The main way in which BST could affect our understanding of the stochastic interpretation is by providing models of spacetime geometry in which particles *spontaneously* undergo stochastic motion. Some models of BST already have stochastic particle motion "built in" in a natural way; for example, the model in [Sharlow, 2003] even allows explicitly for stochastic motion of the sort that a stochastic interpretation of quantum mechanics would require. Normally, the stochastic interpretation of quantum mechanics is formulated in non-branching spacetime. Even those versions of the

interpretation which incorporate stochastic features into spacetime (as in [Namsrai, 1986] and [Prugovečki, 1984]) do not use branching spacetime. However, branching spacetime may have a conceptual advantage over non-branching spacetime models as a setting for the stochastic interpretation of quantum mechanics. BST does not just provide a way to incorporate stochastic motion into spacetime; it also has an independent rationale as a model for the understanding of quantum mechanics ([Belnap, 2003]; [Kowalski and Placek, 2000]; [Placek, 2000]; [Kowalski and Placek, 1999]; [Belnap and Szabó, 1996]; [Douglas, 1995]; [McCall, 1995]) and of indeterminism in general ([Belnap, 2003]; [Belnap, 1998]; [Douglas, 1995]; [McCall, 1995]).

One of the main conceptual difficulties with the stochastic interpretation is the problem of the physical origin of the random changes in particle motion. In some versions of stochastic mechanics [Nelson, 1985], an electromagnetic background field may be responsible for these changes. In other versions, new kinds of fields or particles are invoked (see [Jammer, 1974]). If we adopt a branching picture of spacetime, then another hypothesis is possible. In BST, every particle in spacetime is constantly encountering hypersurfaces of branching. The geometry on these hypersurfaces can carry degrees of freedom, distinct from gravity, that do not exist elsewhere in spacetime [Sharlow, 2003]. Hence, this geometry might well be the source of disturbances in particle motion. Once one admits that spacetime might be a BST, there is no further need to search for a background field to supply the random kicks that stochastic quantum mechanics requires. Random variations in the geometry of the hypersurfaces of branching can perform that task. (Of course, to formulate a real theory of stochastic mechanics in BST, we would have to figure out what sort of geometry on the hypersurfaces of branching could produce the kind of jumps necessary for the stochastic theory. Because there is a lot of freedom in the choice of this geometry, it seems very likely that we could replicate the jumps needed for any sensible theory of stochastic mechanics [Sharlow, 2003].)
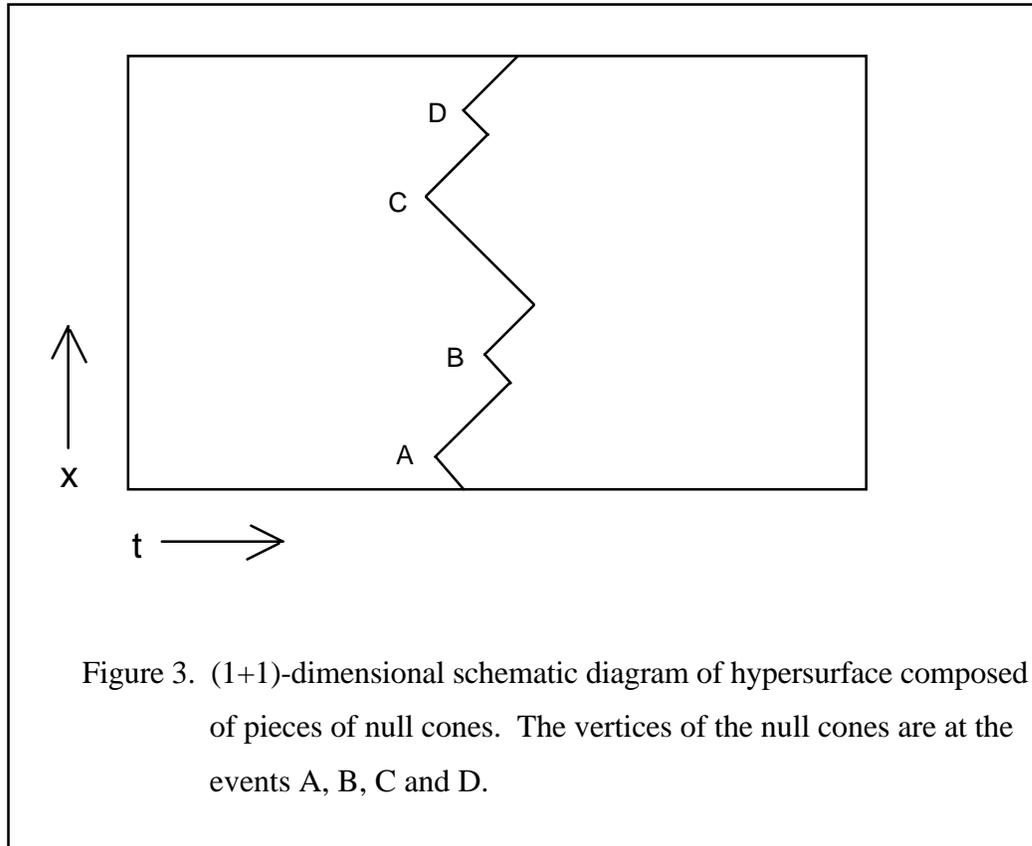
C. Modeling timelike jumps in BST

Another way in which BST might help stochastic mechanics is by providing a ready mechanism for a certain kind of jump that some stochastic theories require, but that seems implausible in ordinary spacetime. Some formulations of relativistic stochastic mechanics make use of jumps in the time coordinates of particles, as well as in the space coordinates (see [Ord, 2002]; [Ord, 1997b]). At first glance, these time coordinate jumps may appear to pose the threat of time travel, with its usual consequences of causal loops and paradoxes. It is possible to reinterpret the backward jumps Feynman-style, in terms of pair creation and annihilation; this is done in [Ord 1997b]. However, if the jumps occur in BST, then this reinterpretation is not necessary. In some versions of BST, a particle can undergo a backward jump in its time coordinate without any threat of real time travel.

To see how this can happen, refer to Figure 2. Note that a hypersurface of branching can in principle change the position, as well as the state of motion, of a particle that crosses it. Mathematically speaking, we can construct a hypersurface that does this by gluing together the edges of sheets of spacetime in certain ways. For example, we can glue points that are close to each other on the edge of a past sheet A to points that are far from each other on the edge of a future sheet B. (Note that we do not have to deform the geometry of either sheet to do this; both sheets might have flat metrics.) Now suppose that each of the two glued edges is a future null cone. Then the hypersurface of branching also has the shape of a future null cone. A particle that hits the hypersurface might undergo a jump along that cone. If the particle jumped toward the vertex of the cone, then the particle would undergo a negative jump in its time coordinate as measured from either of the two glued-together branches. Note that such a jump could *not* lead to any of the usual time travel paradoxes, because once the particle crossed the junction it could not go back. All particles would begin on the past side of the junction and end up on the future side; hence no particle could meet itself in the past.

Figure 2. A spacetime that allows time-reversed jumps without causal loops.
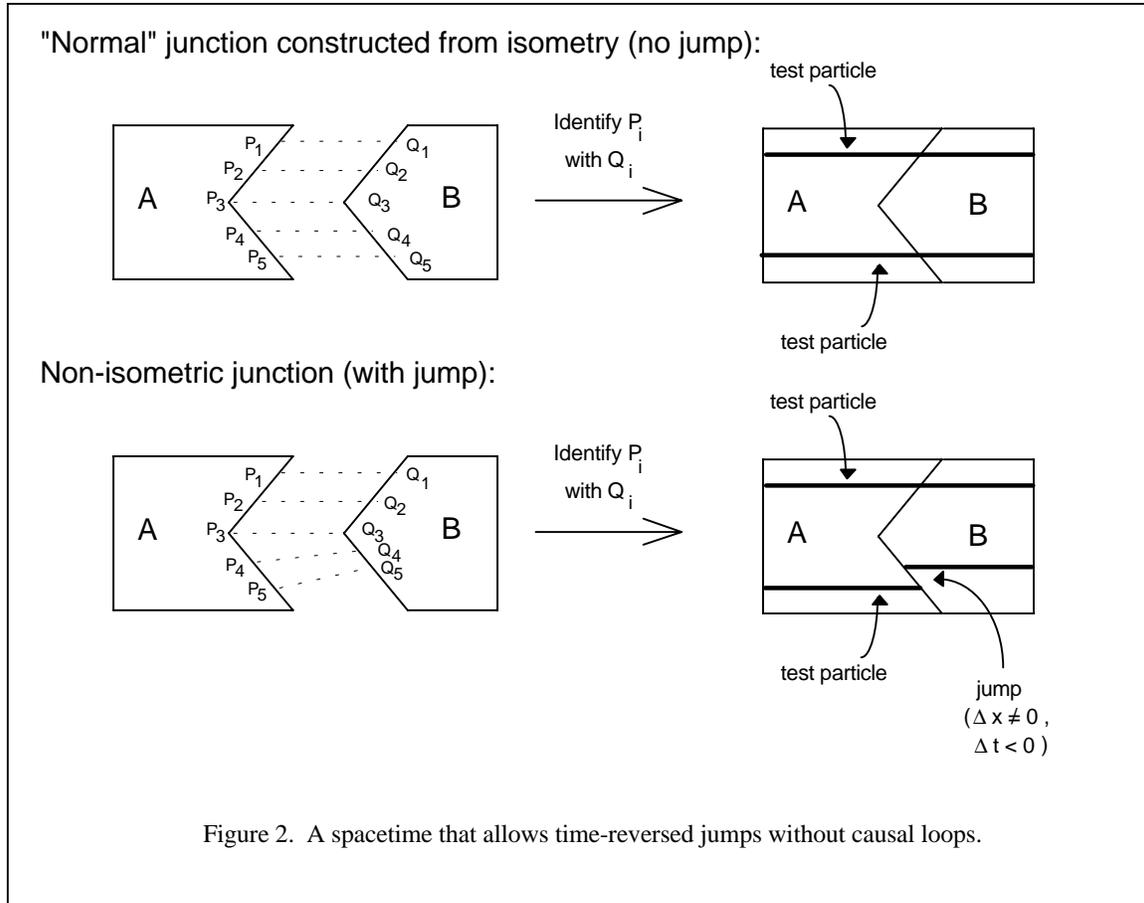
This mechanism for jumping also could provide the jumps at light speed that some stochastic models of relativistic quantum mechanics require ([Ord, 2002]; [Ord, 1997a]; [de Angelis et al., 1986]; [Lehr & Park, 1977]). In the BST just described, particles could jump forward or backward along the cone; these jumps would have a formal velocity of c.

One can get the same results if the hypersurfaces of branching are unions of pieces of null cones instead of single future null cones (see Figure 3).

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Figure 3. (1+1)-dimensional schematic diagram of hypersurface composed of pieces of null cones. The vertices of the null cones are at the events A, B, C and D.

D. Tunneling

Another way in which BST might help the stochastic interpretation of quantum mechanics is by resolving some of the known objections against that interpretation. One such objection, raised by Baublitz [1997], is that some versions of the stochastic interpretation cannot adequately explain tunneling. Baublitz pointed out that "classical stochastic theories" explain tunneling as an effect of fluctuations in the energy of the tunneling particle; the particle happens to gain energy before crossing the barrier and thereby becomes able to cross the barrier. This picture, Baublitz claimed, leads to predictions incompatible with what we know about tunneling. Baublitz argued against one possible way to avoid such predictions, namely, to assume that the particle loses its excess energy after crossing the barrier.

In my opinion, BST provides two possible ways around this objection.

First, note that from the standard point of view of quantum field theory, the potential through which a particle tunnels is not really a classical potential, but is a result of the exchange of field quanta. Thus, if a particle enters a classically forbidden region and is reflected, it is not really a smooth classical potential that throws the particle back, but discrete events of emission and/or absorption of virtual quanta. If we think of the field quanta as real entities governed by stochastic mechanics (as the stochastic derivations of the Klein-Gordon, Maxwell and Dirac equations suggest we might), then it is a matter of chance how far the particle gets into the barrier region before it undergoes an interaction. Thus, the tunneling particle can undergo some real tunneling (entry into the classically forbidden region), at least over a very short distance, even if it does not have the energy to cross the barrier classically. Hence it is not obvious that a particle in the stochastic interpretation must have an energy that is classically sufficient to enter the barrier.

The preceding argument does not depend on BST, but upon a consistent application of the stochastic interpretation to the barrier as well as to the particle. In BST, however, tunneling would be even easier, as the following argument suggests.

We have seen that in some versions of stochastic mechanics, particles undergo jumps at the speed of light. There are two possible physical interpretations for such jumps, depending upon what kind of spacetime model one adopts.

(1) In classical spacetime, one must think of these jumps as real motions through space. According to special relativity, this implies that the particle executing the jump actually is massless -- so the observed mass of the particle is not equal to the particle's "true" mass. It is not at all clear that a relativistic jump of this peculiar sort would be blocked by the classical, nonrelativistic potential in the way that Newtonian mechanics, or even nonrelativistic quantum mechanics, would demand.

(2) In BST, we could think of these jumps in the same way as in (1), but there is another alternative: we can think of the jumps as displacements of the particle along a hypersurface of branching, due to the non-isometric "gluing" of the hypersurface (recall Figure 2). If the jumps are like this, then the particle effectively jumps from one spatial position to another without crossing the region in between. If a particle can jump in this way, then there is nothing to keep the particle out of a classically forbidden region -- though the particle's energy after the jump could, of course, affect the particle's behavior after it gets there.

Regardless of whether we adopt alternative (1) or (2), it is clear that the possibility of real tunneling (instead of just high-energy fluctuations) exists. It would be interesting to work out the details of this mechanism of tunneling and find out whether it really circumvents Baublitz's argument.

E. Rethinking the Madelung equations

Another objection to stochastic theories was proposed by Wallstrom [1994], who found an argument against a wide class of stochastic derivations of the Schrödinger equation. (Baublitz [1997] also discusses Wallstrom's objection.) This objection focuses on the derivation of the Schrödinger equation from the Madelung equations, which are differential equations governing the functions $R(\mathbf{x}, t)$ and $S(\mathbf{x}, t)$ defined by $\psi = e^{(R + iS)}$, where ψ is of course the wave function for a particle. That derivation plays a central role in many versions of the stochastic interpretation of the Schrödinger equation. According to Wallstrom's objection, the presence of the azimuthal factor $e^{im\varphi}$ (m = integer) in wave functions with nonzero angular momentum introduces a term $m\varphi$ into S, thereby making S multiple-valued. This, in turn, makes it impossible to derive the Schrödinger equation from classical stochastic mechanics unless we assume a certain seemingly arbitrary quantization condition for the particle velocity field. Wallstrom has noted that this objection depends crucially upon the assumption that S is continuous. He avoids discarding this assumption, noting that if S is discontinuous, then the equality $\nabla \psi = (\nabla R +$

$i\nabla S)\psi$, which plays a role in the derivations of stochastic theories, makes $\nabla\psi$ singular.

The BST picture of spacetime suggests a possible way to circumvent this objection. I will explain this way in the next few paragraphs.

First, we note that even if S is discontinuous, ψ does not need to be discontinuous if S only undergoes jumps that are multiples of 2π. By subtracting step functions of height 2π from S (for example, by setting $S_{new} \equiv S \mod 2\pi$), we can make S single-valued without changing the values of ψ at all. Because such a change in S does not change ψ, it also cannot make $\nabla\psi$ singular; the equality $\nabla\psi = (\nabla R + i\nabla S)\psi$ cannot be used to infer that $\nabla\psi$ is singular, because this equality does not generally hold when the argument of the exponential is not differentiable. Thus, for any given S, we can construct a new S that is single-valued, gives exactly the same ψ, and has discontinuities (in S, not in ψ) only on a set of space points of measure zero.

Given any single-valued ψ, if we adjust S in this way to allow for discontinuities, then we find that the velocity field v, which is proportional to ∇S, develops singularities. In ordinary spacetime this would be unphysical. However, in a BST model (or at least in a BST of the kind we have been discussing in this section), one can argue that a discontinuity of this sort lacks its usual physical meaning. Consider the physical significance of a one-particle wave function $\psi = R(r,z)e^{im\varphi}$ according to the stochastic interpretation in BST. The coordinates r, z, φ are defined on the spacetime as seen by a coarse-grained observer; ψ itself is a statistical quantity, reflecting the behavior of the particle as it crosses very many hypersurfaces of branching. The piece of spacetime on which ψ is defined actually is not a placid, unchanging classical spacetime; instead, it is a history in a BST, in which any object or observer repeatedly hits hypersurfaces of branching and undergoes discontinuous changes there. If the hypersurfaces of branching are built up from null cones (as in Figure 3), then those hypersurfaces would appear, to a three-dimensional observer who could see them, as multiple spherical "shock fronts"

expanding at v = c and ultimately merging with each other. Of course, real coarse-grained observers such as ourselves will not see these hypersurfaces; we would be unable to resolve the "shock fronts" at all, and would see a smooth effective spacetime geometry. Given that a coarse-grained observer cannot see these discontinuities in space, one wonders whether such an observer could notice a singularity in v which occupies a set of measure zero in space, just as the other (unobservable) discontinuities do! Physically, the singularity in v amounts to a sudden very large jump in particle velocity, followed instantaneously by the loss of the same velocity. This is something that might easily happen at a hypersurface of branching, if we choose the geometry there judiciously. We do not know if it makes any physical sense to assume such a geometry at some hypersurfaces of branching -- but at least we cannot rule it out. Thus, it is not obviously impossible that our BST model would allow for a jump in particle velocity corresponding to an unobserved singularity in v, and hence would allow S to be discontinuous in the way required to get around Wallstrom's argument.

BST also raises doubts about Wallstrom's objection from another angle. We have not specified the exact geometry of the hypersurfaces of branching. Hence, one might ask whether the quantization condition on v, to which Wallstrom's argument leads, could be derived from some constraint on the geometry of the hypersurfaces of branching. This geometry controls the velocities of the jumps; hence a judicious choice of this geometry might (for all we know) impose a quantization condition on particle velocities.

Wallstrom [1994] also gives another argument for the view that the Madelung equations cannot rule out multiple-valued S. This other argument is based on the fact that S is not defined at the nodal surfaces of the wave function; if we consider only the part of space on which S is defined, we get a non-simply connected space which allows for a multiple-valued S. The possibility of converting a multiple-valued S to a single-valued one, as we did for the first objection, casts doubt upon this objection too.

Of course, the above remarks about Wallstrom's objection do not add up to a rebuttal of

that objection. We have not worked out these ideas in enough detail to know whether they really undermine the objection. However, these remarks do show that the cogency of Wallstrom's objection is much less obvious in a branching spacetime than in a classical, non-branching one.

F. Nonlocality

Some versions of the stochastic interpretation of quantum mechanics require nonlocal interactions (see, for example, the remarks in [Wallstrom, 1994] and [Nelson, 1985]). I am not going to argue for or against the physical plausibility of such interactions. Instead, I will merely point out that if one wants nonlocal interactions in one's picture of physics, BST can accommodate these interactions in a natural way -- without postulation of any new fields, and apparently without causality violations.

Earlier I pointed out a way in which a particle in a BST can jump forward or backward in time without any threat of a causal loop. The trick is that this jumping takes place only on a hypersurface of branching which is a union of subsets of null cones; the particle arrives at that hypersurface from the past, makes a jump, and then departs into the hypersurface's future. One can envision particles jumping in this way along a hypersurface of branching, resulting in an apparently instantaneous interaction between events on that hypersurface -- but still without any possibility of an observable causality violation.

Even without signals of this kind, there might conceivably be correlations among distant events on a hypersurface of branching. As yet, we know nothing about the details of the geometry of hypersurfaces of branching in a BST. Thus, we cannot rule out the possibility that this geometry contains long-range regularities which would cause correlations in the motion of particles crossing the hypersurface at distant points. If these correlations are restricted to hypersurfaces of branching, then the threat of causal loops is greatly reduced.

At present, we do not know whether either of these mechanisms can lead to physically

interesting kinds of nonlocality, or can explain the nonlocality known to exist in quantum mechanics. My only point in describing these extremely speculative mechanisms is to point out that BST can accommodate nonlocality in a relatively natural way -- as correlations that occur only on hypersurfaces of branching, where they pose no great threat to causality.

4. Concluding Remarks

In this paper, I have suggested a number of ways in which the BST interpretation of quantum mechanics might eventually prove to be useful in various areas of physics. At present, all of the suggestions given here are highly speculative; none of them has been worked out in any detail, and in many cases we do not know enough about the theories involved to work out the suggestions in detail. The only thing that we can say with near certainty is that many of the suggestions made here will turn out to be wrong. Nevertheless, I believe that these potential uses of BST are worthy of further investigation despite their conjectural nature. Judging by the current intensity of research in quantum gravity, the ongoing effort to understand stochastic mechanics, and the growth of interest in BST, it would be interesting to see what happens when these three fields are brought closer together.

Notes

1. This list is not meant to be exhaustive; it is intended only to show the scope of past and present work in this field. Works not included here are not being slighted.

2. I adopt the abbreviation "BST" from Belnap [2003]. In the present paper I have used this abbreviation to cover all branching spacetime models.

3. A way of joining branches different from the one described here is given in [Sharlow, 1998]. This alternative way of joining branches may soften some of the philosophical problems about the identities of observers and objects with branching histories (see [Sharlow, 1998]).

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