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ECONOPHYSICS AND THE COMPLEXITY OF FINANCIAL MARKETS

ABSTRACT:

In this chapter we consider economic systems, and in particular *financial* systems, from the perspective of the *physics* of complex systems (i.e. statistical physics, the theory of critical phenomena, and their cognates). This field of research is known as *econophysics*—alternative names are 'financial physics' and 'statistical phynance.' This title was coined in 1995 by Eugene Stanley, and since then its researchers have attempted to forge it as an independent and important field, one that stands in opposition to standard ('Neo-Classical') economic theory. Econophysicists argue that the empirical data is best explained in terms flowing out of statistical physics, according to which the (stylized) facts of economics are best understood as emergent properties of a complex system. However, some economists argue that the methods used by econophysics are not sufficient to prove the existence of underlying complexity in economic systems. The complexity claim can nonetheless be defended as a good example of an inference to the best explanation rather than a definitive deduction.

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1 INTRODUCTION

Within the 'complexity science' camp there are two broadly distinct ways of modeling the properties and behavior of socioeconomic systems[1]:

- 'Econobiology' ('evolutionary economics') perspective: uses the lessons of evolutionary biology to explain economic phenomena—economic complexity is viewed as analogous to, or grounded in, *biological* complexity.

- 'Econophysics' perspective: applies to economic phenomena various models and concepts associated with the *physics* of complex systems—e.g. statistical mechanics, condensed matter theory, self-organized criticality, microsimulation, etc.

Both of these approaches are 'population-level' ones (*cf.* [Mayr, 1970; Sober, 1980]): they seek to account for 'global' or 'collective' phenomena. They both do so in a 'bottom-up', 'generative' manner: collective ('macroscopic') properties are viewed as the result of interactions at the level of the ('microscopic') constituents. However, the aggregate and its elements are deemed to be of different *kinds* with

[1]Some seem to think that the subsumption of complex socioeconomic behaviour under 'Self-Organized Criticality' counts a distinct third way. However, self-organized criticality can be easily accommodated within both of the economic complexity frameworks I mention and, hence, should not really be considered a separate enterprise. It is, strictly speaking, a part of (non-equilibrium) statistical physics.

causal lives of their own, the former (minimally) being supervenient on the latter. In interesting cases (i.e. where there is complexity) the aggregate system's properties (and dynamics, laws, etc.) are said to be 'emergent' in the sense that they are not reducible to some particular configuration of the constituents (and their properties) despite the fact that some such configurations will be *sufficient* for the generation of said properties—hence, the particular configuration will be sufficient but not necessary for the production of the emergent property. In other words, the properties of the complex system are 'multiply-realizable' by distinct configurations (physicists refer to this latter property as 'universality').[2]

Here I restrict my attention solely to the physics-based approach[3] (i.e. econophysics), an approach generally couched in the language of *statistical* physics.[4] Statistical physics is a framework that allows systems consisting of many (possibly heterogeneous) particles to be rigorously analyzed. In econophysics these techniques are applied to 'economic particles', namely investors, traders, consumers, and so on. Markets are then viewed as (macroscopic) complex systems with an internal (microscopic) structure consisting of many of these 'particles' interacting so as to generate the systemic properties (the microstructural components being 'reactive' in this case, as mentioned already, thus resulting in an *adaptive* complex system).

I further restrict my attention to financial markets since that is where most work in econophysics has been conducted, on account of the availability of copious

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[2]The concepts of 'supervenience,' 'multiple-realization,' and, more so, 'emergence' are still very slippery and I shall avoid them for the most part in this chapter (with the exception of §2). However, economic systems do raise interesting and potentially novel issues *vis-á-vis* these concepts: for example, the 'subvenience basis' of elements responsible for the (supervenient) economic properties and behaviour—that is, the economic agents and their properties—have strategy and foresight, and therefore *respond* to the unitary properties and behaviour they create together. This highlights quite starkly one of the reasons why a simple ('macro-to-micro' or 'micro-to-macro') causal story cannot be told about events involving economic systems and economic agents (and complex systems and their parts more generally): the two form a *co-evolving* pair, updating their behaviour in the light of changes in the others' properties. In this way the 'micro-macro' disconnect of traditional economic theory is overcome.

[3]The econophysics approach cannot be the whole story in and of itself; it cannot (and should not) be considered as completely distinct from other approaches. The underlying *behaviour* that generates the economic data that econophysicists deal with is, after all, generated by socio-biological systems (of a rather special sort, as mentioned in the previous footnote). No doubt there will, at some level, have to be a union of the two perspectives ('physical' and 'sociobiological')—some early progress in this regard has been made in behavioural finance, including 'herding models' [Cont and Bouchaud, 2000] and 'minority game' models [Challet *et al.*, 2005]. My thanks to Clifford Hooker for raising my awareness of the difficult 'integrative' issue (private communication).

[4]There are other physics-inspired approaches to economics that do not utilize this analogy to statistical physics, using an analogy to some other branch of physics—interesting examples are gauge theory [Ilinski, 2001] and quantum field theory [Baaquie, 2004]. However, these approaches, though often referred to as examples of econophysics, do not match what most econophysicists have in mind (nor what I have in mind); namely, an approach that seeks to build *physically realistic* models and theories of economic phenomena from the actual empirically observed features of economic systems. Statistical physics is a many-body theory as, in general, is economics. One doesn't get the same intuitive connection with models based on quantum field theory and gauge theory—though, it has to be said, they do surprisingly well at *reproducing* economic data.

amounts of high-frequency data. Indeed, at the root of most of the work carried out in in econophysics is a family of 'stylized facts' (empirically observable universal generalizations) that are to be found in this economic data—see §6. Econophysicists seek to find new instances of such facts and to explain these and previously known stylized facts using physics-inspired techniques and models, with the ultimate aim of providing them with a theoretical basis. In fact, economists (primarily econometrists and those working in empirical finance) have been well aware of most of the phenomena that econophysicists have 'discovered' for quite some time. This has led to some impatience with econophysicists amongst economists—see, for example, [Gallegati *et al.*, 2006; Lux and Ausloos, 2002]. However, the econophysicists differ from the economists in that they aim to *explain* the various phenomena catalogued in the stylized facts by providing physically realistic ('microscopic') models and underlying theories. Also, the econophysicists tend to view the stylized facts more robustly, as genuine laws (on a par with those of fundamental physics) rather than lesser cousins as economists seem to. Hence, a claim often made by econophysicists is that their models are more 'realistic' than those offered up by economists and econometricians (see, for example, [Stanley *et al.*, 2006] p. 330). This realism is supposed to be a consequence of the physics-based methodology which is more empirical: 'data first, then model'—I consider this claim in §5.2.

My aim in this chapter is simply to present the central ideas of econophysics: to show where they come from (their motivations), and to show how it all fits in with complex systems science. Since the notion of complexity is, to a large extent, still 'up in the air', I shall begin in §2 by getting straight on what I mean by this term within the confines of this chapter and as applied to (financial) economics. In §3 I introduce some elementary facts from statistics that will be used in subsequent sections. The main features of the NeoClassical economics and finance are presented in §4. I then present some of the background to econophysics, and introduce the basic idea behind it in §5. This is followed in §6 by a look at the the statistical puzzles in economic data (known as the stylized facts). Econophysics is a reaction to the standard model and a response to the problems faced by the standard model: we see how this is so in §7. I then consider some more overtly conceptual issues: in §8 I consider the issue of laws and invariance in econophysics and, finally, in §9 I present, and rebut, some recent objections to the econophysicists' inferences to complexity underlying the stylized facts.

2
COMPLEXITY AND COMPLEX SYSTEMS

'Complexity' is a notoriously slippery concept. Usually, precise definitions—in the sense of necessary and sufficient conditions—are avoided. However, whatever complexity may be, complex *systems* are supposed to possess it, so we can reframe the discussion so as to refer to these rather than complexity *per se*. Herbert Simon gives a rough characterization of a complex system as follows:

> by a complex system I mean one made up of a large number of parts that interact in a nonsimple way. In such systems, the whole is more than the sum of its parts, not in an ultimate, metaphysical sense, but in the important pragmatic sense that, given the properties of the parts and the laws of their interaction, it is not a trivial matter to infer the properties of the whole ([Simon, 1981] p.4).

Economic systems are an obvious candidate for the 'complexity treatment': they contain multiple agents, of different types (producers and consumers; risk averse and risk takers; firms and individuals, etc.), all competing for finite resources of some kind or another, and interacting in such a way as to generate the properties and dynamics of economic systems and subsystems. Econophysicists (and a small but growing number of economists) agree that these properties and the dynamics fit the 'complex system' bill: one finds, for example, scaling and universality, criticality, fractal patterns, and (candidates for) emergent properties. All attributes that a good complex system should possess.

2.1 Characteristics of Complex Systems

It is unfortunate that a more precise definition of a 'complex system' is still not agreed upon: there are almost as many definitions as there are discussions—indeed, the difficulty of the problem of definition points, I think, to the fact that we should avoid 'unificatory' approaches to complexity. However, it is reasonably safe to assume a *kernel* that these diverse accounts share. This kernel involves a triplet of characteristics (I hesitate to call them *necessary* conditions):

- A (unit) complex system must contain *many* subunits (the exact number being left vague).
- These subunits must be *interdependent* (at least *some* of the time).
- The interactions between the subunits must be nonlinear (at least *some* of the time).

The properties of the (unit) complex system are understood to be *generated by* or *supervenient on* the properties and interactions of the subunits that constitute it: there is no difference in the unit system without a difference in the subunits (though it is possible that a difference in the subunits does not manifest itself at the unit level). These properties are said to be 'emergent' when they amount to new complex ('systemic') structure that, as Kim puts it, "in some sense transcend[s] the simpler properties of [its] constituent parts" ([2003] p. 556). The subunits need not be identical, and the introduction of heterogeneity can also result in the emergence of higher-order properties of the unit system.[5]

If we are talking about an *adaptive* complex system then we should add the following condition:

[5] The canonical example is Thomas Schelling's study of segregation ([1971]; [1978] p. 147-155). Here, slight differences in the (microscopic) preferences of individuals lead to massive, unexpected (i.e. emergent) macroscopic differences: very slight individual preferences to have neighbours 'like

- The individual subunits modify their properties and behaviour with respect to a changing environment resulting in the generation of new systemic properties that 'reflect' the change that the environment has undergone.

If we are talking about a *self-organizing adaptive* complex system then we should also add:

- The individual subunits modify their own properties and behaviour with respect to the properties and behaviour of the unit system they jointly determine—in other words, there is 'downward causation' operating from the systemic properties to the subunits' properties.[6]

These characteristics certainly seem to be in tune with most contemporary discussions of complex systems.

However, as Latora and Marchiori ([2004] p. 377) point out, these characteristics (and, indeed, most such characterizations) miss out on what they take to be an essential aspect of complex systems: the *network* structure of the subunits. Much recent work, especially on the *modeling* of complex systems and the *reproduction* of 'real-world' economic phenomena such as price dynamics, has focused on the structural features of such networks, rather than on the specific form of the nonlinear interactions between individual subunits—see [Amaral and Ottino, 2004] for further details on the relevance of networks to complex systems science. It is highly likely that future econophysics research will include complex networks as a major component, and this may function as the ontological glue that sticks together econophysics' models and the underlying sociobiological mechanisms responsible for the economic reality these models are intended to represent.

2.2 *Extreme Events as an Indication of Complexity*

There are additional features of complex systems that are involved in economics—in large part these can be derived from the aforementioned features. For example,

themselves' (i.e. in terms of colour, wealth, or in fact any property one cares to choose) can, despite a preference for integration, lead an initially well-integrated population into total segregation with respect to the chosen property. In other words, heterogeneity (with respect to the chosen property) coupled to a preference to be near others like oneself in that property (however weak that preference might be) provides a (self-organizing) clustering mechanism serving to partition the population.

[6] There are problems with the notion of downward causation, notably that any causal chain from an emergent property E to some property P of the subunits (which subunits, you will recall, form the 'base' B 'generating' E) is underdetermined by the base itself. I.e. whenever there is E there is B (or something resembling B in terms of its causal powers and its ability to generate E—E being multiply realizable), so whenever we say that E 'downwardly causes' P we might just as well say B causes P and dispense with the notion of downward causation altogether. This is a general problem with 'population thinking' according to which aggregate-level phenomena have a causal life of their own. However, I agree with O'Connor and Wong [2005] that this problem evaporates once we realize that emergence is not a synchronic relationship between the subunits and the unit but a dynamical process (and, hence, a diachronic relationship). This is indeed borne out by many branches of complexity science where we see that it is *local iterations of processes* that lead to emergent (global) phenomena. For a similar argument see [Hooker, 2004].

complex systems often exhibit large and surprising changes that appear not to have an outside cause, instead arising endogenously—in other words, on a *post hoc* examination there is no sign that the arrival of news has caused the crash nor is there any link up to the dynamics of the financial fundamentals. The corresponding economic phenomenon here is, of course, the stock market crash (see [Sornette, 2003])—speculative bubbles then correspond to a self-organization process (see [Lux, 1996]). Econophysicists argue that stock market crashes, and other economic phenomena (that are often *puzzling* from the perspective of standard economic theory, 'outliers' in fact) are an entirely natural consequence of the view that economic systems, such as financial markets, are complex. Extreme events, involving collective phenomena (resulting from the iteration of nonlinear interactions), such as herding or alignment (as seems to occur in bubbles and crashes), are an integral part of scaling theory (itself a part of statistical physics). They correspond to critical phenomena in which there is long-range dependence between the elements (i.e. diverging correlation length) so that small changes in certain parameter values can result in massive systemic changes. More generally, criticality involves fluctuations of the 'order parameter' (say the returns[7] on some asset) and power law behaviour. Hence, extreme behaviour in a system is a strong indication that complexity is involved. Before we turn to these puzzles and issues, let us first briefly present some basic facts from probability theory and financial economics.

3
PROBABILITY DISTRIBUTIONS

Probability distributions are of vital importance in complex systems research, especially in the investigation of the properties of financial markets. They are what allow us to ascertain the inner workings of complex systems, to uncover their regularities and aspects of their structure.

Given an experiment (or process) with outcome sample space S, a random variable X is a map from outcomes to real numbers—we assume that the map is exhaustive in that every point of S is assigned some (not necessarily distinct) value. Given such a random variable X, a probability density function $\mathcal{P}(x)$ provides information concerning the way the variable is distributed. To work out the probability that the value of X is in between the values a and b one simply computes the integral $\int_b^a \mathcal{P}(x)dx$. Of course, the most well-known example of such a distribution is the Gaussian ('normal') case, with distribution function:

$$\mathcal{P}_{\text{Gauss}}(x) = \frac{1}{\sqrt{2\pi\sigma^2}} \cdot \exp\left[-\frac{(x-\overline{x})^2}{2\sigma^2}\right] \tag{1}$$

[7]Returns are defined as follows: Let $p(t)$ be the price of some financial asset at time t. The return $R_\tau(t)$ from the asset, at time t for scale factor τ (giving the frequency of returns), is the relative variation of it's price from t to $t+\tau$, or: $R_\tau(t) = \frac{p(t+\tau)-p(t)}{p(t)}$. According to the standard model of finance these returns are uncorrelated IID (independent and identically-distributed random) variables; a feature flatly contradicted by the empirical data from real markets.

Here, $\overline{x}$ is the mean (= $\Sigma_{i=1}^{n} x_i/nq$) and σ^2 is the variance. This distribution is ubiquitous in the natural (and social) world because it is linked to the central limit theorem which tells us, roughly, that any stochastic process (understood as the aggregated result of a complex mixture of (independent) random factors) will be characterized by a Gaussian distribution.

A more appropriate distribution for finance is the lognormal distribution which simply involves the (natural) logarithm of x being normally distributed:

$$(2) \quad \mathcal{P}_{\mathrm{LogNorm}}(x) = \frac{1}{\sqrt{2\pi}} \cdot \exp\left[-\frac{(\mathrm{log}x - \overline{x})^2/(2\sigma^2)}{x\sigma}\right]$$

Of course, $\ln(x)$ for $x < 0$ is undefined, which matches the non-negativity of most assets. However, variables that are normally distributed tend to exhibit rather mild fluctuations. They are clearly not capable of dealing with the kinds of large-scale extreme fluctuations that correspond to stock market crashes for example—fine for human weight, not for human wealth.[8] Despite this the (log-) normal distribution is a central component of the standard model of finance, as we see in the next section—recall that the central limit theorem plays a role here too, only with a multiplication of factors replacing the additivity of factors in the normal distribution.

Hence, much of the action in econophysics research tends to focus on probability distributions for variables representing financial observables where it is argued, on the basis of empirical evidence, that certain financial and economic observables do not fit a Gaussian curve, but fit instead some other distribution. The evidence depicts a system with observables that frequently take on 'extreme' values, values that would be counted as incredibly rare (impossible *for all practical purposes*) according to a Gaussian distributed variable.[9]

Examples of alternatives to the normal distribution are the exponential, stretched exponential, and the Lévy distributions (with 'fat' power law tails)—there are *very* many more. The latter class of distribution is of the most importance since it is believed to point to the underlying complexity of the generating process (they are viewed as 'signatures' of complexity). For cases in which x is large (in which case the Lévy distribution possesses 'Pareto tails'), the power law tail distribution function is:

$$(3) \quad \mathcal{P}_{\mathrm{Power}}(x) \propto \frac{\alpha A_{\pm}^{\alpha}}{\|x\|^{1+\alpha}}\ , \quad \|x\| \to \infty$$

[8]I've heard of extraordinary cases of people weighing around seven times my own body weight, and that is truly exceptional: it must mark some natural boundary on possible weight sustainable by the human form. However, there are very many people who earn many orders of magnitude more money than I do: Bill Gates (as of 2007) earns around 750000 times more money per year than me!

[9]The lognormal distribution fares slightly better than the normal distribution by having more probability mass in the tails. However, as I mentioned above, it still radically underestimates the probabilities of extreme events.

Here the (constant) exponent α is the 'tail amplitude' (or 'tail index') which provides information about the tail (it sets the slope of the graph, for example[10])—$\alpha = 2$ corresponds to a Gaussian; as α approaches zero the center becomes more peaked and the tails fatter. The constant $A_\pm$ determines the size of the fluctuations of the relevant observable, x.

Power law distributions are characterized by the slow decay of probability mass in the tails of the distribution—it is for this reason that they are known as 'fat tailed distributions'. We see that the thickness of the tails is one of the stylized facts that the standard model of finance has trouble explaining, since that model is based on a Gaussian (or log-normal) distribution which decays much more rapidly in the tails. I should point out that the log-normal distribution provides a very good fit of the data from a great many situations in economics and finance—indeed, it is often difficult to distinguish log-normal from power law distributions. The problems arise when one considers the extremes of the distribution (e.g. for big earners or for very large fluctuations in stocks prices—bubbles and crashes, that is). Note also that the size of the slope (determined by the power law exponent) varies according to the financial instrument involved: commodities and stocks, for example, appear to demand significantly different values (thus altering the kind of distribution). It is possible, then, that there are different mechanisms generating the behaviour of financial observables for a range of financial systems.

Notably, from the point of view of complexity research, these power law distributions are *scale invariant* (like fractals only in function-space): events (or phenomena) of all magnitudes can occur, with no characteristic scale. What this means is that the (relative) probability of observing an event of magnitude $\|x\| = 1000$ and observing one of $\|x'\| = 100$ does not depend on the standard of measurement (i.e. on the reference units). The ratio between these probabilities will be the same as that for $\|x\| = 1000$ and $\|x''\| = 10000$. Hence, there is no fundamental difference between extreme events and events of small magnitude: they are described by the same law (that is, the distribution *scales*).

Scale invariance of this sort is a feature of the so-called *critical* phenomena (at phase transitions) studied in statistical physics where one has the simultaneous involvement of many (widely) different length scales.[11] Given that this problem

[10] When we plot the distribution of a power law, $\mathrm{Prob}(x) \sim x^\alpha$, on a log($\mathrm{Prob}(x)$) versus log($\|x\|$) plot (where $\|x\|$ is the size of some event or magnitude of some phenomenon (a stock price fluctuation, say) and $\mathrm{Prob}(x)$ is its occurrence probability) we find a straight line of slope α, suggesting (if not quite implying) that the distribution is scale-invariant (the ratio of $\|x\|$ to its occurrence probability—that is, the number of fluctuations of a given magnitude—is invariant under rescaling). That a power law distribution would show up linear on log-log paper follows from the fact that given a power law $f(x) = x^\alpha$, $\log f(x) = \alpha \log x$. Note that one often sees the *cumulative* distribution according to which one considers not the probability of an $\|x\|$-event, but of events greater than or equal to $\|x\|$.

[11] Specifically: near a critical point, fluctuations of the (macroscopic) order parameter will appear at all possible scales. In the case of the liquid-gas phase transition one will have liquid drops and gas bubbles ranging from the molecular level to the volume of the entire system. At the critical point these fluctuations become infinite. The analogous situation in the financial context would be, for example, fluctuations in asset returns at all possible scales. An excellent introduction to the theory of critical phenomena is [Binney *et al.*, 1992].

(of physics at many scales) has been solved, one might expect that the methods used therein can be usefully transferred to the economic case. This explains the interest of statistical physicists, and the *raison d'être* of econophysics.

4
NEOCLASSICAL ECONOMICS

NeoClassical Economics depicts markets as efficient machines, automatically seeking out the configuration that is best for all economic agents. This configuration is an equilibrium state, the one that maximizes utility. The theoretical framework involves several extreme idealizations, not least of which are the assumptions of perfect rationality and omniscience (including unlimited foresight) on the part of the economic agents! Indeed, having rationality is the same thing as maximizing expected utility. Specific economic theories are constructed from this framework by applying the basic postulates to various economic situations. Financial economics is no exception. Neoclasscal finance is based on a random walk model which states that sequences of measurements made to determine the value of some financial observable (returns for example) are such that the present (discounted) value is the best estimate (prediction) one can give for future values.

4.1 The Standard Model of Finance

Johannes Voit [2005] calls "the standard model of finance" the view that stock prices exhibit *geometric Brownian motion*—i.e. the *logarithm* of a stock's price performs a random walk.[12] Assuming the random walk property, we can roughly set up the standard model using three simple ideas: (1) the best estimation of an asset's future price is its current price[13], (2) the distribution of price changes forms a bell-curve ('mesokurtic' or Gaussian condition), and (3) buys balance sales. In the context of finance, these principles of the standard model are encoded in the central tool for pricing options[14]: the 'Black-Scholes-Merton model' [BSM] [Black and Scholes, 1973; Merton, 1973]. This is often viewed as a piece of early econophysics, though not along the lines of todays econophysics which is concerned

[12]This central idea of the standard model, though with an *arithmetic* Brownian motion, can be traced back to the doctoral thesis of Louis Bachelier, a student of Poincaré—this thesis, from 1900, is in print again in an English translation: [Davis and Etheridge, 2006]. This model was, more or less, later rediscovered (independently) by the physicist M. F. M. Osbourne [1959]. For those with no knowledge whatsoever of mathematical finance who wish to learn more, I recommend Ross [2002].

[13]This is known as the *Martingale condition* defined by the conditional probability $E[X_{n+1} \mid x_1, ..., x_n] = x_n$ (where E is the *average* or *expected* value of what is enclosed in square brackets and X_i is a random variable conditioned on outcomes x_j).

[14]In brief, options are contracts that give the owner the right but not the obligation to buy (= 'call option') or sell (= 'put option') some asset (= 'the underlying') for a pre-specified price (= the 'strike price') at some pre-specified time in the future. Hence, the 'payoff' of an option is a function of the future price of the asset (or a group of such)—for this reason they are part of the family of financial instruments known as 'derivatives'.

with 'out of equilibrium' aspects. Moreover, the model is derived from the postulates of the neoclassical theory, rather than the data (independently of an *a priori* theory of how markets ought to behave).

The central idea that underpins BSM is that one views markets as many-body dynamical systems. This insight is then used to draw an analogy with concepts from thermodynamics. In particular, the BSM equation brings over the concept of *thermodynamic equilibrium* into finance. This is defined in the financial context as a steady state reached when the underlying stock and the stock option are balanced in terms of the payoff they yield compared to the risk they entail. The BSM equation describes this relationship[15]:

$$\frac{\partial V}{\partial t} + \frac{1}{2}\sigma^2 S^2 \frac{\partial^V}{\partial S^2} + rS\frac{\partial V}{\partial S} - rV = 0 \tag{4}$$

The solution, $C(S, t)$, of this equation then gives us the cost of constructing an option from the specified stock (or the 'rational value' of the option). Assuming constant r and σ, this is:

$$C(S, t) = SN(d_1) - Le^{-r(T-t)}N(d_2) \tag{5}$$

Here, L is the option's 'strike price' and T is its time to maturity. $N(\ )$ is the cumulative probability distribution function for a (standard) normal random variable. The arguments of the function are:

$$d_1 = \frac{\log(S/L) + (r + \frac{1}{2}\sigma^2)(T-t)}{\sigma\sqrt{(T-t)}} \tag{6}$$

$$d_2 = \frac{\log(S/L) + (r - \frac{1}{2}\sigma^2)(T-t)}{\sigma\sqrt{(T-t)}} = d_1 - \sigma\sqrt{(T-t)} \tag{7}$$

The problem of finding the best price for options is reformulated as a diffusion equation from which one gets the prices of various option-types by imposing various appropriate boundary conditions on the possible solutions.

In terms of the probability distributions from the previous section, then, the relevant function is clearly the log-normal: this is required as a postulate.[16] Hence, modern finance is very much a NeoClassical theory. However, as mentioned, normal distributions cover only fairly mild fluctuations around some central value. Used as a model for ascertaining the riskiness of certain options, the BSM equation will assign vanishingly small probabilities to extreme fluctuations that are, in reality, not all that rare.

[15]The various terms in this partial differential equation are interpreted as follows: V is the value of some specified option (the details change depending on the type of option involved: in this case we consider European call options), σ is the stock's implied volatility (standard deviation of stock returns), S is the current price of the underlying stock, and r is the (risk-free) interest rate.

[16]Further postulates required by the model are: the efficient market hypothesis (see below), constant interest rate, zero commission charges, and no dividend payouts.

4.2 Market Efficiency

This formal framework of BSM is given conceptual foundation *via* the efficient market hypothesis which states that prices always reflect all available information *in actual markets* [Fama, 1970]—the prices themselves emerge (aggregatively) through the consensus amongst a group of perfectly rational agents (the prices themselves are therefore rational). Price changes occur as a result of the exogenous intervention on the market by a piece of news, itself an unpredictable event. It follows that price changes are themselves unpredictable. Or, as Joseph McCauley expresses it: "there are no patterns/correlations in the market that can be exploited for profit" ([2004] p. 88). Let us spell this out in more detail.

The EMH is an inference from (NeoClassical) rational expectations principles: traders will wish to maximize their utility. This implies that they will look to exploit the market. The way to do this would be to spot patterns in price movements and then buy when they expect the price to give higher (than average) returns and sell when they expect lower (than average) returns. The trouble is, in doing this they will change the very patterns they are attempting to exploit: buying increases the price and selling drives the price down. This equalizes the market so that all financial instruments give the same return (modulo risk). In other words, the information that this arbitageur trader had about the market (the patterns) becomes reflected in the market prices. An endgenous process balances the market out. So, while there can be patterns that can be exploited, this is short lived (for the specific pattern), since acting on the information affects the prices and patterns get erased (by a process analogous to Walras' *tâtonnement*). This means that the best estimate for the future is the present price because that price reflects all known information (modulo short term discrepancies). One is left with the problem of what causes the price changes: the answer has to be external factors, and given the vast number and unpredictability of these, they are best modeled as random processes. Therefore, prices changes follow a random walk, and this gives us the foundation of modern (academic) finance and financial risk evaluation.

From the efficient market hypothesis we can quite clearly derive a testable prediction about the behaviour of financial observables (such as prices, returns, etc.): they should follow random walks in time—experience leads one to suggest a *biased* random walk to account for the steady growth over long time scales. However, as we see in the next section, returns don't appear to behave in this way in real markets (*cf.* [LeBaron, 2006] p. 222-4).

5
THE ROUGH GUIDE TO ECONOPHYSICS

The term 'econophysics' was chosen with some care to follow the path of such mergers as 'astrophysics', 'biophysics', and 'geophysics'. The reason for this was to keep the kind of work carried out by econophysicists within physics departments ([Stanley *et al.*, 2006] p. 337—note that it was H. E. Stanley who thus christened the field (in print) in [Stanley *et al.*, 1996a]. Minimally, econophysics is based on the observation of similarities between economic systems and concepts and those from physics. For example, Bertrand Roehner defines it simply as "the investigation of economic problems by physicists" ([2005] p. 3). A slightly less general definition comes from Mantegna and Stanley ([2000]: "The word econophysics describes the present attempts of a number of physicists to model financial and economic systems using paradigms and tools borrowed from theoretical and statistical physics" (p. 355). However, as they go on to say, a "characteristic difference [from traditional approaches to economics and mathematical finance—DR] is the emphasis that physicists put on the empirical analysis of economic data" (*ibid.*). This latter factor is supposed to constitute the 'added value' of econophysics.

5.1 Some Econophysics Pre-History

The existence of a close relationship between physics and economics is nothing new, of course: many of the great economists did their original training in physics, and the influence of physics is clearly evident in many of economic theory's models—see [Mirowski, 1989; Ingrao and Israel, 1990; Cohen, 1994; Schabas, 2006]. I already mentioned too how the centerpiece of modern finance, the Black-Scholes-Merton model, is directly derived from physics. There are, moreover, many instances of physicists who have applied 'the physicist's method' to social phenomena. For example, Daniel Bernoulli found that there were statistical regularities in what are *prima facie* unpredictable events—e.g. the number of letters in the paris dead-letter office (see [Farmer *et al.* , 2005] p. 37). The enigmatic theoretical physicist Ettore Majorana outlined and defended the application of statistical physics to social phenomena [Mantegna, 2005]. One cannot forget, either, Mandelbrot's discovery of scaling behaviour of cotton prices: [Mandelbrot, 1963]. The 'statistical physics connection' also underpins much of this earlier work: the idea is that one can ignore microscopic detail (the individual social entities) in favour of the coarse-grained macro-description (the groups of individuals bound together by social forces), with the aim of deriving macro-level laws. Econophysics is, at bottom, this same thought played out again, only now with the benefit of a rigorously solved theory of multi-scale systems (i.e. renormalization group theory) giving principled reasons to ignore microscopic details in favour of a few choice parameters.

5.2 The Methodology of Econophysics

Econophysics gets itself off the ground as a separate enterprise from economics because, unlike the former, the latter supposedly has an unscientific 'non-empirical' (or 'axiomatic') methodology.[17] As Zhang [1998] puts it, "as a physicist, one may

[17]This brings us to a more general point concerning what Thomas Gieryn calls "Boundary Work" (see, for example, [Gieryn, 1999]). Gieryn argues that when a new discipline comes along, it must strive to separate itself off from other ongoing endeavours and, in particular, aim to demonstrate that it,

get the strange feeling that the theory [the standard model of economics—DR] is detached from the experiment" (p. 51). Likewise, Challett *et al.* write that

> physicists generally feel uneasy about several pillars of mainstream economic theory, such as rational expectations, the efficient market hypothesis and the notion of equilibria to name a few. This approach looks too axiomatic and formal to deal with complex systems as, for example, financial markets. ... [E]conophysicists deny the very rules of the game on which mainstream academic research in economics is based. ([Challet *et al.*, 2005] p. 14)

As this quotation makes plain, econophysics is viewed (by most of its practitioners) as a *revolutionary* reaction to standard economic theory that threatens to enforce a paradigm shift in thinking about economic systems and phenomena.

We have here something akin to the 'principle-theory' versus 'constructive theory' distinction that Einstein made in regard to his 1905 formulation of special relativity ([Einstein, 2002])—the distinction was based on thermodynamics (on the 'principle' side) and statistical mechanics (on the 'constructive' side). The approach of Black, Scholes, and Merton involved a principle-theory-type approach in that the various principles going into their model were given the status of postulates, and no underlying mechanisms for the phenomena were elucidated (*à la* thermodynamics). By contrast, econophysicists, making use of the statistical physics (rather than thermodynamical) analogy, adopt a constructive-theory-type approach: their models are derived from the data and are physically well-founded by providing basic mechanisms for the phenomena. As Johnson *et al.* [2003] state: "[a]s physicists, our tendency is to put our trust in models which are microscopically realistic, and where the model parameters hold some physical meaning" (p. 251).

Stanley *et al.* [1999] claim that econophysics thus approaches economic systems "in the spirit of experimental physics" (p. 157): in contrast to standard methods in economics, econophysicists "begin empirically, with real data that one can analyze in some detail, but without prior models" (ibid.). While almost any philosopher of science would disagree with the details of this statement, the point is well-taken: data first, then model (whether the 'raw data' is itself encoded in a data model or not we can ignore here). As Bouchaud and Potters [2003] put it: "no theoretical model can ever supersede empirical data [in physics]" ... [p]hysicists insist on a detailed comparison between 'theory' and 'experiments' (i.e. empirical results, whenever available)" (p. xvi). However, it is absurd to think that economists would disagree with this in principle: there is an entire field (empirical finance) that adopts this same methodology (often employing 'model free' nonparametric statistics to analyze financial data).

It seems that the specific target for econophysicists' animosity is some form of the (rationalist) 'Austrian-type' economic theory, with its rejection of an empirical

unlike its competitors, is truly scientific. It seems that econophysicists are doing just this in opposing the axiomatic style of NeoClassical economics. However, there's more to economic theory than the axiomatic approach, and in focusing too heavily on this aspect (to draw the boundaries) econophysicists are ignoring many important details. I think in this case we can agree with Gieryn that these objections are rhetorical devices employed to create the illusion of importance and originality with the new field of econophysics.

approach and in favour of the logical derivation of economics from axioms of (individual) human action. This view is, however, not at all mainstream (indeed, it is sometimes labeled 'heterodox'!). The problem is, there are plenty of economists who are equally (if not *more*) uneasy about rational expectations models, utility maximization, the efficient market hypothesis, and general equilibrium—Austrian economics is a case in point. There are plenty of examples of *empirical* economics too: experimental economics being one obvious example in which NeoClassical ideas are put to the test and found to be empirically inadequate—see [Guala, 2005]. Moreover, there are plenty of physicists who appear to be unperturbed about working in a manner detached from experiment: quantum gravity, for example. Here, the characteristic scales are utterly inaccessible, there is no experimental basis, and yet the problem occupies the finest minds in physics.

I don't think we can agree, then, that econophysics adopts a different 'physics based' methodology and this is what distinguishes it from economics. Let us put this aside, though, and consider why complex systems might be difficult for axiomatic approaches (as the above quote from Challett *et al.* suggests)? One can well imagine axioms governing the behaviour of complex systems, with emergent laws and so on. Surely it is the *particular* axioms that NeoClassical economic is based on that are problematic from the perspective of complex systems, not the axiomatic approach *per se*? If this is what is meant, then it is a fair point: the axioms make the wrong predictions about real economic systems. This is hardly an original point, but it points to a *genuine* problem with NeoClassical economics *vis-à-vis* complex systems science.

If we are talking not of econophysics *per se* but the complexity approach in general then we do witness a significant difference between this approach and mainstream economic theory. Moreover, in this case it does turn on a methodological issue: methodological individualism to be exact. NeoClassical economics is based on the idea that the way to understand complex socioeconomic phenomena is to examine the individuals. By synthesizing one's knowledge of the individual level, one can deduce the various phenomena. This is purely a mechanical picture, along the analytical lines of classical mechanics. To understand a complicated entity, decompose it into its parts: the whole is nothing but the individual parts that compose it. In other words, NeoClassical economic theory does not treat economic systems as complex systems where network, structure, interaction and emergence play a central explanatory role and individual details are largely irrelevant. In its avoidance of the importance of non-individualistic matters and interactions, however, NeoClassical theory fails to be empirically adequate. We highlight these flaws in the next section.

STATISTICAL PUZZLES (AKA 'THE STYLIZED FACTS')

Financial time series display some *prima facie* puzzling empirical (statistical) regularities that make their modeling a tricky business. These are called "stylized facts". As Cont [2001] explains, a "stylized fact" is a "set of properties, common across many instruments, markets and time periods" (p. 223). In other words, stylized facts are *universal* regularities, independent of time, place, and many specific compositional details. Coolen [2004] refers to these regularities as "benchmarks, to be met by any theory claiming to explain aspects of financial time series" (p. 234).

This is a puzzle: why should stocks in, say, pork bellies look the same (statistically) as stocks in technology companies? The curious statistical properties of the data are fairly well-known amongst economists, but they *remain* a puzzle for economic theory.[18] This is where physicists come in: whereas some economists had attempted to recover the stylized facts in their models, the models had no empirical grounding; their sole purpose was to *replicate* the statistical properties by any means (admittedly, no small feat in itself given the constraints these stylized facts impose). Econophysicists, by contrast, use the statistical properties as their starting point; the basis from which to construct realistic models: the universality of the statistical properties—i.e. the fact that they reappear across many and diverse financial markets—suggests (to physicists at least) a common origin at work behind the scenes and points towards the theory of critical phenomena (with its notion of *universality*). Many econophysicists view their task as searching for and elucidating this common mechanism. This is also where the connection to contemporary research on complexity comes in: the stylized facts are understood to be emergent properties of complex economic systems. Here, then, we have a genuinely novel and potentially important feature of econophysics: the search for mechanisms underlying the economic phenomena utilizing the direct intuitive link between these phenomena and aspects of statistical physics.[19] Let us finally present these stylized facts on which so much hangs.

[18]The school known as 'behavioural economics' has made some progress along a different ('internal') route to both standard economic theory and econophysics (see [Shefrin, 2002; Shleifer, 2000])—Herbert Simon [1955] did some excellent early work on behavioural models of rational choice, into which he tried to inject some realism concerning actual decision making behaviour (*vis-á-vis* the actual computational powers of humans and their limitations in terms of access to information). There are some overlaps between the behavioural models and the statistical physics models used by econophysicists: in particular, there are analogies between the cooperative or collective phenomena of the physics of critical phenomena and the imitative models used by behavioural economists. Sornette [2003] offers a nice integration of behavioural models and statistical physics.

[19]Again, however, I don't see how this can be sufficient as it stands. Statistical physics denatures the economic agents as severely as any NeoClassical theory, and yet surely the nature of the agents has to play a role in generating the behaviour. True, there might well be emergent effects that enable us to ignore these details when it comes to modeling, but if it is *understanding* we seek, then we cannot ignore the behaviour of the agents. Certainly, if we are to *do* anything of practical importance with econophysics then we need some way of translating the statistical physics talk into talk about real individuals and economic reality. (I am grateful to Allan Walstad for impressing this problem on me—email communication).

6.1 The Facts of the Matter

Here I mention only those stylized facts those most relevant to complexity issues[20]:

Fat Tails: the returns of various assets (evaluated at high frequencies: e.g. a month and less) exhibit fourth moments (kurtosis levels) that are anomalously high when superimposed over a Gaussian distribution. The distributions are roughly bell-shaped but assign greater (than normal) probability to events in the center (i.e. they are more peaked) and at the extremes (i.e. they exhibit heavy tails). In other words, the time series for returns display a significantly larger number of extreme events than a Gaussian process would generate.

- The standard model of finance involves the idea that price changes obey a lognormal probability distribution. This implies that massive fluctuations (crashes or 'financial earthquakes') are assigned a vanishingly small probability: if the world really were like this, then we should not be seeing the kinds of crashes we *do* see.[21]

Volatility Clustering: periods of intense fluctuations and mild fluctuations tend to cluster together: big price changes, of either sign, follow big price changes and little ones, of either sign, follow little ones.

- If the process generating a time series were Gaussian, then we would expect to see a very uniform time distribution of large and small fluctuations. Instead what we see are sequences of periods of large fluctuations and periods of small fluctuations (high and low volatility).[22]

Volatility Persistence ('Long Memory'): there is a dependency between stock market returns at different times. Technically, the volatility has slowly decaying autocorrelations.[23]

[20] There are very many more than I present here, but they will suffice to see how econophysics is supposed to score over the standard model and why the data is believed to point towards a complex systems approach. See [Cont, 2001] for more examples.

[21] In statisticians' terms, if the price changes—or, strictly speaking, their logarithms since we a dealing with a log-normal distribution in the standard model—behaved according to the standard model, the probability distribution would have a kurtosis of around 0 (0 is the value for data that fit the bell curve exactly): such distributions are called "mesokurtic". Distributions of data from *real* markets, with their characteristic 'fat tails,' exhibit positive kurtosis (giving "leptokurtic" probability distributions).

[22] Again, in statisticians' lingo we find the *conditional heteroskedasticity* of returns. These sudden switches bring to mind phase transitions, and indeed this analogy is pressed by many econophysicists—attempts are made to connect this to reality by considering the phenomena to be the result of cooperative ('herding') and competitive effects amongst agents. Again, [Sornette, 2003] is the best place to learn about this.

[23] One characterizes the dependence between points of a time series *via* the Hurst exponent $H = 1 - \alpha/2$ (where α is the tail exponent for a power law distribution): $H = 0.5$ indicates a Brownian motion; $0.5 < H < 1$ indicates positive long range correlations (and an underlying long memory process)—the corresponding data set is known as a fractional Brownian motion. See [Clegg, 2006] for an elementary guide to the Hurst exponent.

- The autocorrelation of returns decays very quickly to zero, providing support for the efficient market hypothesis and for the Brownian motion model of the standard model of finance.

  (8) $\text{Linear Returns} \equiv C(\tau) = \mathbf{Cor}(r(t,\Delta t), r(t+\tau,\Delta t))$

  However, the autocorrelation for squared returns decays much more slowly, and can remain positive for as long as a month. Hence, there exists nonlinear dependence.

  (9) $\text{Squared Returns} \equiv C_2(\tau) = \mathbf{Cor}(|r(t+\tau,\Delta t)|^2, |r(t,\Delta t)|^2)$

Relations: This persistence is obviously related to the above volatility clustering, and is essentially just what one computes to gain a numerical purchase on the clustering phenomenon. The clustering itself generates excess volatility (fat tails). Hence, explaining the clustering and long memory will most likely constitute an explanation of the fat tails. One would like and expect an integrated account of the stylized facts, according to which the same mechanism is responsible for generating multiple stylized facts in a unified manner. This is what econophysicists aim to provide.

In short: price changes change by too much, too often, and with too much 'order' to fit the geometric Brownian motion model that the standard model is based on. There would not be the quantity of large crashes that have been witnessed if that model were true.[24] If we plot the size of price changes against time, we see that there are far more large changes than the standard model suggests. There is too much predictability to the time series for it to be a random walk process generating the data, on account of the clustering and dependence. Mainstream financial economics does not fit the empirical facts.

The probability distribution is best matched by power-law tails rather than a Gaussian distribution. The existence of a power-law distribution often points to some underlying complexity in the system that generates it. It is on this basis that many tools from the theory of critical phenomena and condensed matter physics have been brought over. Often, these are used to provide the *physical* underpinnings of various economic phenomena, in addition to providing the mathematical concepts. It is this feature that makes econophysics so interesting from a philosophical point of view.[25]

[24] Recall that many crashes have no apparent cause, which signals that we might have a complex system. The idea of the EMH, that exogenous factors must be responsible for any price changes (including the massive ones in crashes), does not seem to fit the fact that often no such cause (e.g. in the fundamentals) can be found in 'postmortems' of real cases of crashes. Further, changes, contributing to the volatility, cannot always be due to the impact of some relevant piece of news since the changes are far more frequent than the arrival of such news—see Bouchaud *et al.* ([2004], p. 176).

[25] However, one should be careful in extrapolating too much from the existence of power law behaviour. While complexity may be at work 'behind the scenes', power laws can spring from rather more innocuous sources. Take Zipf's Law for example. This says that the frequency of the nth most common word in some text is inversely proportional to n. Surely we cannot expect this phenomena

6.2 *Significance of the Stylized Facts*

What these features demonstrate is that the underlying mechanism responsible for generating time series data is not one that produces a normal distribution (nor log-normal). The latter simply does not fit the observed statistical properties. It follows that a model based on a normal distribution would assign lower probabilities to extreme events than it ought to to be empirically successful. Given that the standard model of finance involves returns that are log-normally distributed, there is a clear conflict here between theory and evidence. There is every reason to attempt alternative approaches: econophysics is one such, but there are others that do well. It is notable that the other approaches that tend to do well with the stylized facts are complex systems oriented—agent-based computational economics, for example (see [Tesfatsion and Judd, 2006] for a good overview). The idea is to view the stylized facts as emergent properties of a complex system.

There are diverse attempts to reproduce these stylized facts with various methods and models: broadly one can replicate (instrumentalism) or one can explain (realism). Econophysics uses models based on statistical physics and is an attempt along the latter lines. However, this is not the only option: as mentioned, econophysics does not have a monopoly on the stylized facts. For starters, these stylized facts were isolated well before the emergence of econophysics. In fact they go back at least as far as Wesley Mitchell [1913], who identified such properties in the context of his work on business cycles. Large fluctuations in the economy were found to occur roughly every ten years. These fluctuations were thought to be due to external factors. Stanley Jevons [Jevons, 1884] too drew attention to similar regularities in his work on commercial fluctuations. Jevons famously argued (on the basis of a statistical correlation) that the fluctuations were the result of similar fluctuations in the weather (the cycles of sunspots).

Microsimulation models (such as the 'agent-based' models mentioned above) seem to fall somewhere between instrumentalism and realism. For example, the model of Lux (an economist) and Marchesi (an engineer) [1999] involves heterogeneous trading strategies: there are 'noise traders' or 'chartists' on the one hand (whose decisions are based on the price histories) and 'fundamentalists' on the other (whose decisions are based on the notion that there is a *fundamentally* correct price, namely the discounted sum of future earnings). Switches between strategies are also possible. From this setup they are able to recover statistical aspects of real markets. (For a methodologically similar approach see [Bak *et al.*, 1997].) Hence, micro-simulations constitute an attempt to provide an explanation of what were "hitherto mysterious statistical findings like the fat tails and clustered volatility of financial markets" (Lux and Heitger [2001] p. 123). Are these econophysics mod-

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and the stylized facts of financial markets to have a common origin—though Zipf [1949] came close to saying something like this! See Newman [2005] for a very clear-headed review of power laws and their interpretation, including the various mechanisms by which they can be generated. In practice it is also often difficult to distinguish power-laws from other distributions: see [Laherrère and Sornette, 1998; Pisarenko and Sornette, 2006].

els? It seems not, though they are clearly *inspired* by statistical mechanics. Given the availability of other effective models we ought, then, to view econophysics as one of several ways of making sense of the complexity of financial systems. That is not to say that these ways are disparate: microsimulations are obviously used in statistical physics and the conceptual connections are readily apparent—i.e. both involve the idea of *generating* macrophenomena from microbehaviour.

The stylized facts hold further significance on a more conceptual level: the stylized facts encode non-trivial social regularities. It is the *universality* of these regularities is what most interests econophysicists—after all, physicists are in the business of laws and invariances. The stylized facts appear to be invariances that reappear over apparently unrelated systems suggesting that some common underlying mechanism is responsible. The case for comparison here is with the theory of critical phenomena and phase transitions. During a phase transition a system will shift from a relatively disordered global state to a more ordered global state. Or, in other words, parts go from not imitating one another to imitating one another, so that everything depends on everything else (infinite correlation length): a shift in one part propagates (thanks to massive connectivity) to every other part. In such 'critical' conditions the system is said to be 'scale free' so that events of any size can occur, corresponding, of course, to the fat tails which can exhibit themselves as (not infrequent) stock market bubbles and crashes.[26] We turn to the statistical physics explanation in the next section. We consider the issue of the lawhood of the stylized facts in the subsequent section.

7
STYLIZED FACTS ACCORDING TO ECONOPHYSICS

As anyone who reads the finance pages[27] knows, prices of assets fluctuate, sometimes wildly. According to the econophysicist's conception of financial markets the prices (of assets) are viewed as fluctuating macroscopic variables that are determined by the interactions of vast numbers of agents. They are but one of the observables of a complex financial system. Financial time series demonstrate that extreme events are relatively common. A normal distribution would, as we have seen, appear to be inadequate: one needs a distribution with more probability mass in the tails. Power laws appear to fit the bill very well: they possess a scaling property according to which events of all sizes can occur. So far there is nothing to distinguish this analysis as especially econophysical. However, in contrast to the standard model, which views the stylized facts as the result of exogenous factors, the econophysics approach views them as emergent properties resulting from the internal dynamics (the interactions that occur between individual traders). In a

[26]The phase transition suggestion has been taken further with specific applications of spin-system models. The idea here is that stock prices respond to demand in the same way that the magnetization of an interacting spin-system responds to changes in the magnetic field (see [Plerou *et al.*, 2002]).

[27]Or has a computer program that displays the time series as I do!

nutshell: the traders' local interactions (the market's microstructure) determines a global pattern (the market's macrophenomena), which feeds back to the traders' future behaviour. This perspective is developed from parts of statistical physics[28], where one deals with many interdependent parts that 'imitate' each others' behaviour (collective phenomena). In such cases, systems of *prima facie* extremely diverse nature and constitution are found to behave in the same way (the feature physicists call *universality*). According to econophysics, financial markets are part of this class of systems too.

Thus, the ground is set to apply a host of techniques from statistical physics: simply view financial markets as complex systems whose 'particles' are the traders and apply the techniques as usual. As a further step one then constructs 'microsimulations' (or 'agent based models') to test the models and to study the mechanism whereby the macrophenomena emerge from the microstructure ([Levy *et al.*, 2000] offers a comprehensive treatment). These simulations attempt to generate properties of financial markets, such as the stylized facts, from the interactions and properties of traders, thus mimicking statistical physics' models in which particles' behaviour generates properties of a unit complex system—see, again, Lux and Marchesi [1999] for a good example and [Tesfatsion and Judd, 2006] for a more comprehensive treatment.

7.1 The Statistical Physics Analogy

Very often in systems with interacting parts, and whose interacting parts generate properties of the unit system, one finds that the thus generated properties obey scaling laws. Scaling laws tell us about statistical relationships in a system that are invariant with respect to transformations of scale (once certain transformations have been carried out on various parameter values). In statistical physics these scaling laws are viewed as emergent properties generated by the interactions of the microscopic subunits. Scaling laws are explained, then, *via* collective behaviour amongst a large number of mutually interacting components. The components in this financial case would simply be the market's 'agents' (traders, speculators, hedgers, etc...). These laws are 'universal laws', independent of microscopic details, and dependent on just a few macroscopic parameters (e.g. symmetries and spatial dimensions). Econophysicists surmise that since economic systems consist of large numbers of interacting parts too, perhaps scaling theory can be applied to financial markets; perhaps the stylized facts can be represented by the universal laws arising in scaling theory. This analogy is the motivation behind a considerable chunk of work in econophysics; it is through this analogy, then, that the stylized facts receive their explanation—though, as I have said, presumably not their *ul-*

[28]There have been attempts to apply statistical mechanics to economics as far back as 1959 when M. F. M. Osborne developed his Brownian motion model of a stock market [Osbourne, 1959]. Duncan Foley has done extensive work in this area [Foley, 1994]. Farjoun and Machover also develop this analogy [Machover and Farjoun, 1983]. In each case, however, the model is equilibrium statistical mechanics, and it is precisely the equilibrium condition that is thought to be at fault by econophysics.

timate explanation which will involve such things as the agents' psychology, the institutions in which the agents operate, and so on (*cf.* [Lux, 2001] p. 562).[29]

The power-law behaviour of financial instruments can be explained in terms of the scaling laws that arise in the theory of critical phenomena: complex phenomena involving the collective behaviour of a family of subunits produce such power-laws. Power-laws have a particularly simple form, as we have already seen (but we repeat in a modified form for simplicity). For an event (e.g. an earthquake, stock market crash or price fluctuation, etc...) of magnitude (energy, size, etc...) $\|x\|$ (or greater) the probability $\mathrm{Prob}(x)$ that x will occur is given by:

(10) $\mathrm{Prob}(x) \sim \|x\|^{-\alpha}$

Here, the exponent $-\alpha$—now called the *critical* exponent in the context of the theory of critical phenomena—is set by the empirically observed behaviour of the system in question. Systems with identical critical exponents belong to the same 'universality class' and will exhibit similar statistical properties (near their critical points). Such systems are interesting because they do not have an 'average' $\overline{x}$ or a 'width' σ. What this means is that there is a greater chance for complex systems having massive events than there is for systems that fit a normal distribution. Hence, financial markets are complex systems because they exhibit such massive events more often than normally distributed systems in a manner consistent with power law behaviour (often viewed as a 'signature' of complexity).

7.2 Scaling, Universality and Criticality

Most of the research conducted in econophyics is based on an analogy between financial markets and scaling theory and theory critical phenomena. We present the argument explicitly in §7.3, first we spell out some of the details.

Given some observable $\mathcal{O}$ and driving parameter x, a scale-invariant function can be defined by the functional equation (where $x \rightarrow \lambda x$ is an arbitrary rescaling):

(11) $\mathcal{O}(x) = \mu \mathcal{O}(\lambda x)$

The solution of this equation is the functional relation (a power law, in fact):

(12) $\mathcal{O}(x) = x^{\alpha}$

[29] Note that there have been 'ultimate' explanations for these facts that do not involve the sociobiological of the economic agents. The general approach to power-law distributions and complex behaviour in nature (including the biological and social realm) given in the theory of 'self-organized criticality' [Bak, 1996] is supposed to accomplish such a general explanation. The idea is that complex systems spontaneously tune themselves (their parameters) to the critical values required for power law (scale-free) behaviour to emerge. However, this approach is controversial: it is a moot point, to say the least, whether the mechanism could be the same across social, biological, and more fundamental systems. See [Frigg, 2003] for a critical discussion.

That is, equations of this functional form are scale-invariant. They are of interest in statistical physics (and econophysics) because many-body systems that are close to critical (or bifurcation) points obey such power laws. Renormalization group theory analysis shows that there are universal properties in such systems (systems in the same universality class), meaning that diverse systems share the same critical exponents (and scaling behaviour) and so display qualitatively identical macroscopic properties (when approaching criticality). Of course, stock markets are not going to always be poised at a critical point, so one expects to see different regimes separated by phase transitions. Kiyno *et al.* [2006] show that such behaviour can be found in S&P500 market data. When financial markets occupy a regime near to a critical point the behaviour corresponds to the volatility clustering, akin to the spins in a magnet aligning versus pointing in the same direction—otherwise there is disordered behaviour well approximated by (geometric) Brownian motion. The large fluctuations correspond to the scale invariance of systems near to critical points. One sees in this way how a unified account of the three stylized facts emerges from a statistical physics based account.

7.3 Unpacking the Econophysics Argument

It is an uncontested fact that financial market time series display statistical regularities (whether they constitute *laws* or not *is* contested). These regularities have similar characteristics to those obeyed by other complex systems in the physics of critical phenomena. In particular, one can interpret the stylized facts as scaling laws. I think we can discern at the root of a great deal of econophysics research the following argument from analogy:

(P1) Financial markets are made up of a large number of interacting agents
(P2) According to statistical physics, physical (natural) systems that are composed of large numbers of interacting individuals follow scaling laws that are universal
(P3) Financial markets do exhibit universal regularities that show up as stylized facts in their time series

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(C) The stylized facts are scaling laws of the kind found in statistical physics

In other words, given that financial markets have a physical composition like that of systems dealt with in statistical physics (large numbers of interacting individuals) and given, furthermore, that the time series exhibit statistical regularities similar to that of systems dealt with in statistical physics, it follows that a good modeling strategy is to apply statistical physics to financial markets. This argument is presented as a 'plausibility argument':

> Simply put, statistical physicists have determined that physical systems which consist of a large number of interacting particles obey universal laws that are independent of the microscopic details. This progress was mainly due to the development of scaling theory. Since economic systems also consist of a large number of interacting units, it is plausible that scaling theory can be applied to economics. ([Stanley *et al.*, 1996c], p. 415)

A further step is to take statistical physics as providing 'physically realistic' models capable of *explaining* financial phenomena—something Stanley and his group seem to endorse. But clearly the argument is not deductively valid; nor is it intended to be. It is intended to increase our confidence in the application of statistical physics to financial markets and our confidence in the complexity of financial markets. Given that scaling theory and the theory critical phenomena are associated with complex systems, it would further follow that financial markets are complex systems, in the sense that they undergo phase transitions and, at least some of the time, exist near criticality (between order and chaos, if you like that way of speaking). Since this behaviour is seen too, then the position is further bolstered.

8
STYLIZED FACTS AS LAWS OF NATURE

Johnson *et al.* [2003] claim that "[o]ne of the significant contributions of Econophysics has been to establish that the price statistics across a wide range of supposedly different financial markets worldwide, exhibit certain universal properties" (p. 57). The existence of these stylized facts is then taken to point to a shared structure or process generating the statistics in each case (chance, self-organized criticality, etc.). As I mentioned, this is an overstatement: econophysicists have indeed solidified these results, but many were known prior to the advent of econophysics. What distinguishes econophysics from other approaches in economics is the interpretation of the stylized facts. In addition to viewing them as emergent properties of a complex system, it also includes a greater commitment to the stylized facts, treating them as laws rather 'mere' regularities. Part and parcel of this view of the stylized facts as genuine laws is, as Coolen [2004] puts it, that "[i]t is irrelevant...whether the microscopic variables at hand represent coordinates of colliding particles, microscopic magnets, replicating polymers, or (as in the present case) the actions of decision-making agents" (pp. 1-2). That is, the laws themselves are emergent in the sense that they don't depend on the microscopic details of the system.

However, this stance is at odds with (econophysicist) Joseph McCauley [2004], who views the addition of decision-making agents (with free will) as very relevant indeed, even in econophysics: such agents are incompatible with the possibility of invariances, and without invariances there are no symmetries, and without symmetries there are no laws. McCauley argues, therefore, that, like economics, econophysics can at best be a *descriptive* historical science (see also [Rosenberg, 1992]) analyzing what happened in some economic situation. Other econophysicists (most, in fact) believe that they can find some laws for market dynamics, albeit statistical ones of course; but, says McCauley, "[t]here are no known socioeconomic invariances to support that hope" ([2004] p. 4). Whereas laws of nature are independent of initial conditions, socioeconomic behaviour is not: "socioeconomic behaviour is not necessarily universal but may vary from country to country" (ibid.).

It is of course perfectly true that there will be many differences in socioeconomic behaviour between countries, but that does not mean that there are no invariants. As empirical work by econophysicists and econometrists (and sociologists) has revealed, there *are* some surprising commonalities. It is these very commonalities that kick-started econophysics (at least, most of it). There do seem to be examples of statistical laws that *do not* vary from country to country: Pareto's Law, for example. Pareto investigated the statistical properties concerning the distribution of wealth over the individuals in a population. The model involved is the scale-invariant (cumulative) probability distribution function $p(w \geq x) \sim x^{-\alpha}$. This says that the number of people with income w greater than or equal to x is given by x raised to some power $-\alpha$ (which, for a variety of data sets, Pareto found to be around 1.5). Pareto found that the relationship applies to very different nations, of different sizes and composition: "as different as those of England, of Ireland, of germany, and even of Peru" ([Pareto, 1897], §958; quoted in [Stanley, 2003]). In other words, it is *universal*: multiple distinct nations satisfy the same power-law—i.e. the universality class of Pareto's law includes England, Ireland, Germany, and Peru. Closer, and more recent scrutiny finds universal power-law behaviour too, but only in the tails, for the most extreme (i.e. the richest) 5% or so of the distribution: the first 95% or so is characterized by a more traditional 'log-normal' or exponential curve—see the papers in [Chatterjee *et al.*, 2005] for more details.[30]

But there is something right about McCauley's objection; and he is well aware of the example just given of course. His point is that "*markets merely reflect what we are doing economically*, and the apparent rules of behaviour of markets, whatever they may appear to be temporarily, can change rapidly with time" (*ibid.*, p. 200 - emphasis in original). This non-stationarity is, of course, a general feature of adaptive complex systems: "the empirical distribution is not fixed once and for all by any law of nature [but] is also subject to change with agents' collective behaviour" (ibid., p. 185). *Prima facie* the problem appears to be more serious in human systems: as Zhou and Sornette put it, "human beings are not spins, they can learn, that is, adapt the nature and strength of their interactions with others, based on past experience" ([2006] p. 1).[31]

It is, of course, perfectly true that the macroscopic (emergent) distribution would be altered if agents altered their (microscopic) patterns *enough*. However, the issue here is whether there is a lawlike relation between certain patterns of (microlevel) behaviour and the distribution: do we find the same distributions appearing in cases where the patterns are a certain way, and in particular when we have systems

[30]It is rather interesting to note that Schumpeter, writing of Pareto and his work on 'social invariants', suggested the approach (or one like it) that econophysics now seems to be pursuing: "nobody seems to have realized that the hunt for, and the interpretation of, invariants of this type [Pareto's Law—DR] might lay the foundations for an entirely novel type of theory" ([1951] p. 121).

[31]An interesting connection can perhaps be made here with MacKenzie's study of financial markets and their relationship to financial models [2006]: as theories and models about the workings of markets change, the way trading is done changes in response and so, as a consequence, do the financial observables and their distributions. (See also [Johnson *et al.*, 2003] p. 223-4 for a similar point.)

behaving according to the characteristics of complex systems given in §2? Here the econophysicists who believe they are discovering laws would appear to have some evidence in the form of the scaling laws that Pareto first investigated (but that have been found in a much wider variety of economic observables). In truth, the evidence is not conclusive.

An alternative response to McCauley might be to point to the fact that it is the norms and institutions that circumscribe economic behaviour (and these too can arise *à la* complex systems theory). Since there are common norms and institutions in many and varied countries we should expect to find the statistical regularities we do find. These norms and institutions *could* be altered, resulting in the regularities vanishing but while there is some common socioeconomic system in place there will be common emergent properties (these are the stylized facts).[32]

9
ARE FINANCIAL MARKETS *REALLY* COMPLEX?

Do financial markets possess the characteristics of complex systems? Econophysicists (and, indeed, many economists) certainly view financial markets as particularly fine examples of complex systems, as cases of 'complexity in action.' For example, Lisa Borland [2005] writes that "[p]erhaps one of the most vivid and richest examples of the dynamics of a complex system at work is the behaviour of financial markets" (p. 228). Stanley *et al.* [2001] write that "[t]he economy is perhaps one of the most complex of all complex systems" (p. 3). Johnson *et al.* [2003] claim that "it would be hard to find anyone who disagreed with the statement that a financial market is indeed a 'complex' system" (p. 2)—however, there *are* dissenting voices, or at least those who think that the evidence isn't as good as the previous quotations suggest.

Durlauf [2005] argues that the evidence from the econophysicists research and empirical work is not conclusive evidence in favour of economic complexity since the evidence is underdetermined by alternative approaches. Recent work by Pisarenko and Sornette [2006] also pours water on the flames for similar reasons: they show that the power law model at best provides an approximation of the behaviour of market returns. The real story is much more complex. The lesson they

[32]Of course, this ultimately concedes the point to McCauley. It seems that at best only weakened account of laws can be sustained: Laws relative to the socioeconomic system that is in place. This can perhaps be rendered universal in the physicist's sense since we are at liberty to say that whenever the system is set up in *this* way (with large numbers of parts interacting so as to generate a complex system) it will exhibit *these* properties (i.e. the stylized facts). Indeed, this doesn't seem to be at odds with laws in physics since even the most fundamental are hedged in some way: Einstein's law describing the relationship between gravity and mass-energy, for example, is not scale-invariant; it gets modified at the Planck scale (of the order $10^{-33}cm$. Likewise, the field equations of the standard model of particle physics can only be to be taken to apply in a universe without gravity. What I am suggesting here is similar: in a socioeconomic system where there is free-trade and the trading is done in such and such a way, then we get the following distributions of events for the observables. (My thanks to Cliff Hooker for reminding me that physics' laws are hedged too.)

draw is that we shouldn't assume that power law behaviour will extend into the unobserved regions of the distribution's tail. In particular, there are plenty more fat-tailed distributions[33] that offer as good an approximation to many data sets in finance (and social science in general).

Likewise, Brock [1999] is concerned with the kinds of process that are capable of generating the data that enter financial time series. He points to an underdetermination of stochastic processes by scaling laws.[34] In other words, one and the same scaling law is compatible with multiple distributions. This is an *identification problem*: "uncovering and estimating the underlying causal data generating mechanism" ([Brock, 1999], p. 411). The mere isolation of scaling laws in the data is not sufficient to enable the making of inferences about the nature of the data generation process.

He argues that scaling laws are useful in a limited way: they function as constraints on the underlying causal data generating process. They do not serve as a decisive guide to the kind of process responsible for the data. Likewise, the scaling laws can restrict the class of distributions—Gaussian distributions are rendered impotent, for example. So the scaling laws can falsify but not confirm.[35] The scaling law research is, as Durlauf puts it, "consistent with complex systems models" but "[the] evidence is far from decisive and is amenable to alternative interpretations" ([Durlauf, 2005], p. F226). Hence, both Brock and Durlauf are not convinced that econophysics has demonstrated economic complexity.

However, this does not negate the claim that financial markets are complex systems—it just means that the claim that they are should be treated with more care and attention than it has received so far within the econophysics community. It has to be said that for most econophysics researchers the complexity of economic systems is an assumption taken for granted rather than something they seek to prove. It simply seems obvious to them that economic systems are complex systems. However, I think that Brock and Durlauf put too much weight on underdetermination: I think we can defend the complexity view from these objections.

The scaling laws are but one of the stylized facts to be met by an approach. One has to explain the clustering and long-range dependence too (indeed, it seems evident that these will be responsible for the heavy tails). There are a great many

[33] They give the example of the 'stretched exponential' family of distributions defined by: $\mathcal{P}_{SE}(x)_{\geq u} = 1 - \exp\left[-(x/d)^c + (u/d)^c\right]$ (where $x \geq u$). One can 'fatten' the tails of the distribution by playing with the various constants in this expression.

[34] Hence, we have two levels of underdetermination here: (1) concerning whether the correct distribution really is a power law. (2) concerning the mechanism that generated the distribution (assuming it really is a power law).

[35] Philosophers of science are well acquainted with this kind of problem. It is simply the curve-fitting problem (itself a variant of the problem of induction) in a different guise. We can argue that, for a given data set, *any* general theory can only ever be consistent with it (rather than proven from it). However, just because multiple theories are equally well-cnfirmed by the data does not imply that they are equal *simpliciter*. For example, one theory may have better unifying power (it might cover a larger range of phenomena): econophysics seems to have this virtue; it can accommodate multiple stylized facts using the same concepts. Other possible distinguishing marks are simplicity, elegance, cohesion with background knowledge, and so on.

more statistical facts that have to be accommodated by an approach. In principal I think we agree with Brock that the stylized facts function as constraints, but, taken together, this constrains the possible stochastic processes a great deal. It is very difficult to get *any* models that reproduce all of the features. As mentioned in fn. 35, the statistical physics (econophysics) approach involving scaling laws does an exceptional job at covering a great many stylized facts in a unified way. We can understand this as an 'inference to the best explanation' kind of argument: this seems consistent with the way econophysics talk about complexity.

Still, it seems that the ability to definitely demonstrate the existence of a unique mechanism responsible for the fit to the statistical model involving a power law distribution evades us. Brock and Durlauf are quite right to insist on the application of better statistical testing to see what kind of distribution is implied by a data set.[36]

10
CONCLUSION

Intuitively, financial markets appear to be complex and they pass many tests for what we expect a complex system to be like. There is still some doubt as to whether they are *truly* complex: when it comes to more rigorous statistical tests of complexity the jury is still out. Be that as it may, the profusion of data collected about financial markets makes them an ideal specimen from the point of view of complexity research: nowhere else do we possess as much data recorded so accurately and at so many time scales. Econophysics has proven itself to be an excellent way of probing this conjectured complexity. Moreover, the (controversial) revolutionary claims made by econophysicists, *vis-à-vis* traditional economic theory and social laws, make this subject eminently fit for philosophical consumption. It ought, therefore, to play a central role in future discussions of the philosophy of complex systems science.

RESOURCES AND FURTHER READING

Articles on econophysics appear in a variety of journals. The primary ones are: *Physica A*[37], *Quantitative Finance*, *Physical Review E*, and *Europhysics Letters*. One can also find pertinent articles in *The European Physical Journal B*, *Fractals*,

[36] Aaron Clauset, Cosma Shalizi, and M. E. J. Newman have developed some impressive (open source) R and Matlab code that is capable of discriminating between a large number of closely related relevant distributions, and this should put an end to the debate on first level of underdetermination (namely, whether a data set does indeed obey a power law distribution). This is available at `http://www.santafe.edu/~aaronc/powerlaws/`. A paper [Clauset *et al.*, 2007] corresponding to the code, dealing with the identification of power laws, can also be found on this web page.

[37] Volume 324, Issue 1-2, 2003, contains the proceedings of the international econophysics conference held in Bali in August 2002.

Advances in Complex Systems, *International Journal of Theoretical and Applied Finance*, *Physical Review Letters*, and *Nature*.

Internet Resources:

- The Econophysics Forum: `http://www.unifr.ch/econophysics/`.
- Econophysics.org: `http://www.ge.infm.it/~ecph/library/index.php`.
- The Econophysics Blog: `http://econophysics.blogspot.com/`.
- The Electronic Journal of Evolutionary Modeling and Economic Dynamics: `http://beagle.u-bordeaux4.fr/jemed/`.
- The economist/philosopher J. Barkley Rosser has some excellent articles available on his website: `http://cob.jmu.edu/rosserjb/`.
- A great many articles from Stanley's group can be found at the website for the Center for Polymer Studies at Boston University: `http://polymer.bu.edu/~hes/econophysics/`.
- A fairly comprehensive bibliography of relevant material can be found at: `http://www.ge.infm.it/~ecph/bibliography/bibliography.html`.

A more general website, which occasionally features econophysics-related news, is *The Complexity Digest*: `http://www.comdig.com/`.

Textbooks:

There are now many econophysics textbooks on the market. I mention just six of the best here:

- The first textbook on econophysics is Rosario Mantegna and Eugene Stanley's *Introduction to Econophysics: Correlations and Complexity in Finance* (Cambridge University Press, 1999). This is still an excellent way to gain a feel for the nature of econophysics.
- An in-depth guide, primarily focused on risk management, is Bouchard and Potters' *Theory of Financial Risk and Derivative Pricing* (Cambridge University Press, 2003).
- An excellent general guide to the entire field is Voit's *The Statistical Mechanics of Financial Markets* (Springer, 2005).
- A more controversial text covering plenty of philosophical issues is McCauley's *Dynamics of Markets* (Cambridge University Press, 2004).
- In terms of the 'complexity connection' the best books are *Financial Market Complexity* by Johnson *et al.* (Oxford University Press, 2003) and *Minority Games* by Challett *et al.* (Oxford University Press, 2004).

Review Articles:

A philosophically oriented review of econophysics is given in [Rickles, 2007]. More general reviews are: [Gligor and Ignat, 2001] and [Feigenbaum, 2003].

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